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QUARK PARTON MODEL WITH LARGE PARTON k_T

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A B S T R A C T

The quark-parton model is generalized to allow for arbitrarily large parton k_T . Since it is expected that $\langle k_T \rangle$ will rise with Q^2 in (highly virtual) photon mediated processes, this generalization is necessary to restore the applicability of the quark-parton model. By treating k_T as an essential kinematical variable, we have been led to the introduction of a new scaling variable z . Together with Bjorken's x variable, we give a unified kinematical description of the four distribution functions: hadron structure functions and jet decay functions for spacelike and timelike photons. Possibility of a simple interpolating universal function is considered. Phenomenological determination of that function is examined in detail. Predictions on R , parton $\langle k_T \rangle$, hadron $\langle p_T \rangle$ in jets, etc., are made with the dimuon $\langle q_T \rangle$ being used as an input. The usual relation $\langle q_T \rangle = \sqrt{2} \langle k_T \rangle$ is shown to be false in the region where k_T is not small compared to k_L , a situation which prevails in the production of dileptons recently measured. The k_T distributions for timelike and spacelike cases are shown to be not identical. The model is consistent with nearly all relevant data on virtual photon processes.

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1. - INTRODUCTION

In the quark parton model (QPM) ¹⁾ it is customary to assume that the parton transverse momentum k_T is small. Indeed, a k_T cut-off is necessary in field theory ²⁾ to prove the scaling behaviour in deep inelastic scattering (DIS). Phenomenologically, the limited transverse momentum of the pions produced in high energy collisions suggests that the parton k_T is also limited. Moreover, the average value of $R = \sigma_L / \sigma_T$, which was reported by Taylor ³⁾ in 1975 to be 0.14 ± 0.06 , is consistent with $\langle k_T^2 \rangle \lesssim 0.25$ (GeV/c)².

The situation is, however, significantly changed as a result of recent experimental findings. In the first place the nucleon structure functions are found not to scale ⁴⁾⁻⁶⁾, so the theoretical necessity of a k_T cut-off is lost. Secondly, in massive lepton-pair production (LPP) by hadrons in high energy collisions the average transverse momentum $\langle q_T \rangle$ of the dimuons is large ⁷⁾ for large dimuon mass, $M_{\mu\mu} > 4$ GEV/c². Assuming the Drell-Yan mechanism ⁸⁾ for the production of dimuons, one can infer that $\langle k_T \rangle \approx 1$ GeV/c, which is a value much larger than $\langle p_T \rangle$ of the pions produced in the central plateau and is of the same order as the longitudinal k_L for the range of $M_{\mu\mu}$ covered experimentally. Finally, and perhaps most disturbingly, the new value of R was recently reported ⁹⁾ to be 0.35 ± 0.16 for values of Q^2 (momentum transfer squared) reaching as high as 20 (GeV/c)². In the naive QPM one has ¹⁾

$$R = \frac{4}{Q^2} (\mu^2 + \langle k_T^2 \rangle) \quad , \quad (1.1)$$

where μ is the parton mass. Thus, assuming μ to be small, we have at large Q^2

$$\langle k_T^2 \rangle \approx \frac{R}{4} Q^2 \quad . \quad (1.2)$$

This implies that $\langle k_T \rangle$ not only is large but would get even larger at higher Q^2 if R remains relatively constant.

Collectively, the various pieces of experimental information mentioned above, when confirmed by further analysis and better data, would put severe strain on the validity of the naive QPM in which $\langle k_T \rangle$ is assumed negligible.

The theoretical situation is partially improved by the consideration of scale breaking effects ¹⁰⁾⁻¹²⁾ in asymptotically free quark-gluon gauge theory (QCD) ^{13),14)}. In such a theory one has

$$R \approx \frac{16}{25} \frac{(1-x)}{\ln(Q^2/\Lambda^2)} \quad (1.3)$$

which is weakly dependent on Q^2 , as is apparently the case experimentally.

However, it is inadequate on two counts :

- (i) for the usual value of Λ at about 0.5 GeV/c, the predicted value of R is too small compared to the new experimental number ⁹⁾ ;
- (ii) the dependence of $\langle q_T \rangle$ on $M_{\mu\mu}$ in LPP, which was calculated on the basis of a simple-minded parton model and (1.3), does not agree with the data ^{7),10)-12)}. While the predicted $\langle q_T \rangle$ increases indefinitely with $M_{\mu\mu}$ on account of (1.2), the data of Ref. 7) indicate a saturation of $\langle q_T \rangle$ at 1.2 GeV/c as $M_{\mu\mu}$ increases.

There have been more recent investigations on the subject in QCD ¹⁵⁾⁻²¹⁾. Reasonable fits of the $\langle q_T \rangle$ data ⁷⁾ can be obtained ¹⁵⁾⁻¹⁷⁾ if the low order perturbative calculation in QCD is regularized by the assumption of a soft q_T component with appropriately chosen $\langle q_T \rangle_{\text{soft}}$. The gluon correction diagrams that contribute to the "hard" components, when taken into account collectively in the leading log approximation, were shown by Sachrajda ²¹⁾ to reinstate the basic Drell-Yan form. Thus the parton model calculation based on the Drell-Yan mechanism remains valid, provided the QPM can be generalized to incorporate large parton $k_{\perp}$.

Apart from specific QCD calculations, there are general considerations which suggest that $\langle k_T^2 \rangle$ increases with Q^2 as

$$\langle k_T^2 \rangle \sim O(Q^2) \quad (1.4)$$

which, if true, would necessitate a proper treatment of the large k_T problem. This growth of $\langle k_T^2 \rangle$ with Q^2 has been argued on physical grounds by Kogut and Susskind ^{10),22)} stressing that the resolution of the virtual photon probe is of order Q^{-1} . That line of argument can be extended to timelike photons as well, since the size of the constituents

which couple to the photon as point-like objects in an annihilation process should also be proportional to Q^{-1} , assuming the transverse dimension and binding energy are inversely related ¹⁰⁾. Thus, if (1.4) is taken to describe a general property of the partons at high Q^2 independent of the underlying theory, then we have to consider large k_T even if R in (1.2) is small (but finite). The largeness of R reported recently ⁹⁾ has only made more pressing the necessity to reformulate the QPM so that k_T may be arbitrary.

The above motivation is theoretical. Phenomenologically the data of Ref. 7) indicate that $\langle q_T \rangle$ is independent of $M_{\mu\mu}$ (and therefore of Q^2) for fixed initial energy-squared s . The naive interpretation

$$\langle k_T \rangle \sim \frac{1}{\sqrt{2}} \langle q_T \rangle \quad (1.5)$$

which we shall show to be inaccurate, would suggest that $\langle k_T \rangle$ is also independent of Q^2 , in direct contradiction to (1.4). Such a dilemma can be resolved only if the large k_T problem is treated with care.

Numerically, if we take $\langle k_T \rangle$ to be ~ 1 GeV/c at $M_{\mu\mu} \sim 4$ GeV/c², in the experiment of Ref. 7), corresponding to $k_T \sim 2$ GeV/c, the angle between the momentum vectors of the parton and proton is appreciable (almost 30°). Thus the naive QPM is inadequate even though the absolute value of $\langle k_T \rangle$ may not be huge.

In this paper we present both the formulation and phenomenology of the generalized QPM with arbitrary k_T . When k_T is included as an additional variable, the number of degrees of freedom in the problem is increased by one, not counting the trivial azimuthal angle. Therefore, it is reasonable that we should expect to find two scaling variables ²³⁾ in the general case. In that connection it was recently pointed out ²³⁾ that four virtual photon processes are intimately related. Among them, two are DIS and LPP already mentioned. The other two are lepton pair annihilation (LPA) and deep inelastic fragmentation (DIF) in lepton-hadron semi-inclusive reactions. These processes that are mediated by spacelike or timelike photons describe either the parton structure in a hadron or the jet structure in a quark fragmentation. In terms of the two scaling variables the relationship among the kinematical regions of the four processes becomes manifest. This leads us to consider the possibility that their

dynamical description might also be simply related. We conjecture the existence of a universal function which interpolates among the "physical" functions appropriate for the different processes as the two scaling variables range over the respective regions.

In the phenomenological application of the generalized model we shall use essentially three pieces of data to fix the parameters of our universal function : they are νW_2 in DIS, and $d\sigma/dM_{\mu\mu}$ and $\langle q_T \rangle$ vs. $M_{\mu\mu}$ in LPP. We then calculate R , $\langle k_T \rangle$ and the q_T distributions. A satisfactory unification of DIS and LPP can be achieved. The connection with LPA and DIF, on the other hand, is hard to test due to the lack of appropriate data ; however, an effort to link the structure functions with the jet fragmentation functions will be made.

Apart from the question of unification, the phenomenological investigation of this work has yielded a parton distribution function which is consistent with nearly all relevant data on the photon-mediated processes. We have gone beyond the usual analysis in the x distribution. More specifically, we have determined the parton distribution function for the proton in terms of the spherical co-ordinates, i.e., a radial scaling variable and an angle. The results should be useful in its own right in many other phenomenological applications.

The paper is organized as follows. In Section 2 we discuss the QPM with arbitrary k_T and the four virtual photon processes that the model will relate. The two scaling variables are introduced in Section 3 and their physical interpretations in particular Lorentz frames given. Measurable quantities in physical processes are derived in our formalism in Section 4. Up to this point the formulation of the problem is quite general ; no assumption is introduced besides the basic ones of the QPM (minus any statement about k_T limitations). In Section 5 we motivate a universal form for the distribution function in the hope that a unification of the four virtual photon processes might be possible. Phenomenology in the framework of the generalized QPM is carried out in Section 6. Our conclusions are given in the last section.

2. - QUARK PARTON MODEL

Apart from the assumption of negligible k_T , we adopt the basic tenets of the standard parton model ¹⁾. Thus, in a suitably chosen infinite momentum frame we apply the impulse approximation to the interaction between the virtual photon and parton in DIS. The parton is therefore to be put on-mass shell both before and after its interaction with the photon, whatever the transverse momentum of the parton may be. We note that this view differs from the emphasis of the covariant parton model ^{24),25)}. In that model the constituent system of the proton minus the struck parton is required to have a positive invariant mass ($s' > 0$) with the consequence that the struck parton must go off its mass shell in certain kinematical regions. However, since the residual system does not constitute a physical state and is not disconnected from the struck parton before the whole system evolves into the final hadronic state, it is not imperative that s' be positive. We shall therefore in that respect proceed in accordance with the naive parton model ¹⁾ and assume that the contribution from the off-mass shell effects associated with the struck parton is small.

An essential notion in the naive QPM is the longitudinal momentum fraction ξ

$$\xi = \frac{k_L}{P} \quad (2.1)$$

where k_L is the longitudinal momentum of the parton in some "infinite momentum frame" where the hadron's momentum P is large. If k_T is small as is usually assumed, then ξ can be identified ¹⁾ with the Bjorken variable x

$$x = \frac{Q^2}{2M\nu} \quad (2.2)$$

where M is the hadron mass and ν is the energy of the virtual photon in the rest frame of the hadron. That ξ may have this Lorentz invariant identification is possible only if M^2 , μ^2 and k_T^2 are all negligible compared to Q^2 , P^2 and $M\nu$ ^{*)}. If k_T is not small but proportional

*) Strictly speaking, ξ can have the Lorentz invariant meaning and coincide with x only when $k^\mu = \xi P^\mu$, that is in the case of vanishing k_T ($k_T = 0$) and varying quark mass ($\mu = \xi M$). Otherwise, the identification with x is violated by some amount of order μ/Q or k_T/Q .

to Q , then the transverse mass of the parton, $(\mu^2 + k_T^2)^{1/2}$, can be greater than M or even k_L . Hence, ξ is not manifestly Lorentz invariant. The simplicity of the naive QPM is therefore lost. In QCD one nevertheless identifies ξ with x on the ground that $\langle k_T^2 \rangle \sim Q^2 / \ln(Q^2/\Lambda^2)$; consequently $\langle k_T^2 \rangle / P^2$ vanishes as $1/\ln(Q^2/\Lambda^2)$. However, for the Q^2 values currently achieved this $\ln(Q^2/\Lambda^2)$ suppression is insufficient to justify the identification $\xi = x$. We give in this paper a Lorentz invariant formulation of the QPM with arbitrary k_T .

Let P^μ , k^μ and q^μ be the momentum four-vectors of the hadron, the struck parton and the virtual photon, respectively, in a DIS process. The relationship among these momenta is depicted in Fig. 1a. For notational uniformity for all processes to be considered, we adopt the convention that q is directed away from the parton line. We define a function $G(P, k, q)$ so that $G(P, k, q) k_0^{-1} d^3k$ is the number of partons in the volume element d^3k at $\vec{k}$ when the hadron with P^μ is probed by a virtual photon with momentum q^μ . The conditions for the validity of the QPM demand that the consideration be made in a frame in which the magnitude of the momenta discussed are all large. We note that we have included q in the definition of G because the description of a state of a system is not in general independent from the probe in accordance with the principle of quantum mechanics. More specifically, the nature of the constituent carrying momentum k depends on the resolution of the photon. We shall make no particular assumption about that dependence, but for the sake of completeness in the description of the parton's momentum distribution, we include all three relevant momenta in the definition of G . In that connection, Fig. 1a should not be interpreted to suggest factorization of the q dependence from the bubble; instead, one should regard q in G as a supplement to the labels, which collectively specify the type of parton being described, but have been suppressed (along with the hadron label).

The utility of G may be exemplified by how it enters in the expression for the structure functions W_1 and W_2 :

$$W_i = \int \frac{d^3k}{k_0} G(P, k, q) w_i, \quad i = 1, 2 \quad (2.3)$$

where w_1 is the corresponding "structure" function for the lepton-quark scattering. The equation is Lorentz invariant and no restriction on k_T has been imposed. The detailed treatment of (2.3) as well as other equations for some other processes is deferred until Section 4.

So far our discussions are on DIS. As we mentioned in the Introduction, there are three other virtual photon processes that are intimately related. They are : (1) lepton-pair production (LPP) through the Drell-Yan mechanism, (2) hadron production in lepton pair annihilation (LPA), and (3) deep-inelastic fragmentation (DIF) of quarks into hadrons. The parts that involve a hadron, a parton (or quark) and a photon in these processes are shown in Figs. 1b, 1c and 1d, respectively. In these figures we have used a vertical (horizontal) wavy line to denote the spacelike (time-like) photon. All momentum lines are defined with the same senses in all four cases. In DIS and LPP the partons are constituents of the parent hadrons labelled by P , while in LPA and DIF the hadrons labelled by p are fragmentation products of the parent quarks. Kinematically there are no overlaps of the regions in which these processes are defined, a point that we shall return to in more detail in the next section. In each case there is a corresponding $G(P,k,q)$ or $G(p,k,q)$ function. The latter refers to the number of a particular hadron at p in a jet associated with a parent quark with momentum k struck or created by a photon with momentum q . Since the four G functions are defined in four non-overlapping regions and are in general different, we shall use the symbol G only when referring to them collectively. In specific cases we use the notation S_- for DIS, S_+ for LPP, J_+ for LPA, and J_- for DIF [where S stands for structure and J stands for jet, while $- (+)$ designates spacelike (time-like) photons being involved].

3. - TWO SCALING VARIABLES

Since $G(P,k,q)$ is a Lorentz scalar, it can depend only on the invariants formed by the momenta. It is taken as understood that P is replaced by p in the cases of LPA and DIF. With the assumption that the struck parton is on-mass shell before and after the interaction with the virtual photon, i.e., $k^2 = (k-q)^2 = \mu^2$, the interesting scalar variables are q^2 , $P \cdot q$ and $P \cdot k$. Out of these three invariants two independent scaling variables can be defined

$$x = \frac{q^2}{2 P \cdot q} \quad , \quad (3.1)$$

$$z = 1 - 2 \frac{P \cdot k}{P \cdot q} \quad . \quad (3.2)$$

Thus in general we consider $G(x, z, q^2)$.

In DIS x agrees with the Bjorken variable (2.2) since $q^2 = -Q^2$, $P \cdot q = -M\nu$, and q is directed toward the lepton vertex. z is a new variable. In the naive parton model where k_T is ignored so that one may use the approximation $k^\mu = xP^\mu$, the variable z reduces to

$$z = 1 + \frac{2xM}{\nu} \quad . \quad (3.3)$$

It is therefore not an independent variable and approaches one in the Bjorken limit, $\nu \rightarrow \infty$ with x fixed. Thus the deviation of z from 1 measures k_T .

The same definition of x and z as given by (3.1) and (3.2) applies to the other three virtual photon processes also. For a precise interpretation of x and z in all these processes, we must evaluate them in specific Lorentz frames. They are considered separately below.

3.1. - Deep inelastic scattering (DIS)

The frame in which k_T is decoupled from k_L for fixed x and Q^2 is the Breit frame between the virtual photon and parton. This is obvious since the frame is defined only up to a transverse boost. To fully specify the frame, we further require that the photon and hadron be collinear. In that frame the kinematics is independent of the azimuthal angle α of $\vec{k}$ relative to $\vec{P}$, and the two essential degrees of freedom may be expressed either in terms of k_L and k_T , or the invariants x and z .

More precisely, we have

$$\left. \begin{aligned} q^\mu &= (0, \vec{0}, -Q) \\ P^\mu &= (P_0, \vec{0}, -P) \\ k^\mu &= (k_0, \vec{k}_T, -k_L) \end{aligned} \right\} \quad (3.4)$$

and

$$k_L = \frac{Q}{2} \quad , \quad (3.5)$$

from which follows

$$x = \frac{k_L}{P} \quad , \quad z = \frac{P_0 k_0}{P k_L} \quad (3.6)$$

These relations can be simplified when P and Q are large compared to the masses of the hadron (M) and the quark (μ). Let us define r to be the radial scaling variable

$$r = \frac{k}{P} \quad , \quad (3.7)$$

where k and P are the magnitudes of the three-momenta of the parton and hadron, respectively. Moreover, let β be the angle between these two momentum vectors. These quantities are defined in the Breit frame specified by (3.4). In terms of these geometrical (albeit frame-dependent) variables we have from (3.6)

$$x = r \cos \beta \quad , \quad z = \frac{1}{\cos \beta} \quad (3.8)$$

or, conversely

$$r = x z \quad , \quad \cos \beta = \frac{1}{z} \quad (3.9)$$

The kinematical constraints

$$0 \leq r \leq 1, \quad 0 \leq \cos \beta \leq 1 \quad (3.10)$$

restrict the domain in x and z where DIS is defined :

$$0 \leq xz \leq 1, \quad z \geq 1 \quad (3.11)$$

This is shown in Fig. 2.

It should be pointed out that the domain (3.11) follows from the application of the conditions (3.10) in the special Breit frame defined in (3.4), but not in any other frame. This dependence on the Lorentz frame is a consequence of the frame dependence of r and β . It also reflects the imprecision of the parton model ; indeed, in an extreme and inadmissible example, if we were allowed to go to the rest frame of the hadron, r would be infinite and therefore invalidate the condition in (3.10) for DIS. However, the frame of (3.4) is actually the unique choice among the infinite momentum frames if we demand that x and z depend only on r and β but not on the azimuthal angle α of $\vec{k}$ relative to $\vec{P}$.

3.2. - Lepton pair production (LPP)

In this case the virtual photon is timelike, so it is natural to go to the rest frame of the photon. Thus we have

$$\left. \begin{aligned} q^\mu &= (Q, \vec{0}, 0) \\ P^\mu &= (P_0, \vec{0}, P) \\ k^\mu &= (k_0, \vec{k}_T, k_L) \end{aligned} \right\} \quad (3.12)$$

and

$$k_0 = \frac{Q}{2} \quad (3.13)$$

It then follows that

$$x = \frac{k_0}{P_0} , \quad z = \frac{P k_L}{P_0 k_0} , \quad (3.14)$$

which implies, for large momenta

$$x = r , \quad z = \cos \beta . \quad (3.15)$$

The conditions (3.10) therefore restrict the domain of LPP to

$$0 \leq x \leq 1 , \quad 0 \leq z \leq 1 \quad (3.16)$$

as shown in Fig. 2.

3.3. - Lepton pair annihilation (LPA)

Next we consider the fragmentation of a quark created by a timelike photon. Again, the natural frame is in the rest system of the photon. Since the quark is now the parent, we align the z axis along the quark momentum, which specifies the jet axis. Thus we have

$$\left. \begin{aligned} -q^\mu &= (Q, \vec{0}, 0) , \\ -k^\mu &= (k_0, \vec{0}, k) , \\ -p^\mu &= (p_0, \vec{p}_T, p_L) . \end{aligned} \right\} \quad (3.17)$$

and

$$k_0 = \frac{Q}{2} , \quad (3.18)$$

where the negative signs in (3.17) are due to our adherence to the uniform directions of the momentum lines in Fig. 1. We then get

$$x = \frac{k_0}{p_0} , \quad z = \frac{p_L k}{p_0 k_0} . \quad (3.19)$$

Now, in LPA and DIF the hadron momenta are less than that of the parent quark, so the appropriate definition of the radial scaling variable in these cases is

$$r = \frac{p}{k} \quad (3.20)$$

The angle β retains the same definition. Hence, (3.19) implies in the infinite momentum limit,

$$x = \frac{1}{r}, \quad z = \cos \beta \quad (3.21)$$

The conditions (3.10) then imply

$$x \geq 1, \quad 0 \leq z \leq 1 \quad (3.22)$$

for the domain of LPA as shown in Fig. 2.

3.4. - Deep inelastic fragmentation (DIF)

Finally we consider the jet structure of a quark knocked out of a hadron by a high Q^2 spacelike photon as in a deep inelastic process. As in DIS we use the photon-parton Breit frame, but unlike DIS we must choose one in which the photon and parton momenta are collinear since this time the parton (quark) is the parent. In that frame x and z are independent of the azimuthal angle α . In order to avoid confusion with the variables of DIS when we later combine DIS with DIF, let us replace k^μ by ℓ^μ here. Thus we have

$$\left. \begin{aligned} -q^\mu &= (0, \vec{0}, Q) \\ -\ell^\mu &= (\ell_0, \vec{0}, \ell) \\ -p^\mu &= (p_0, \vec{p}_T, p_L) \end{aligned} \right\} \quad (3.23)$$

and

$$l = \frac{Q}{2} \quad (3.24)$$

The scaling variables in this case are

$$x = \frac{l}{p_L} \quad , \quad z = \frac{p_0 l_0}{p_L l} \quad (3.25)$$

which in the infinite momentum limit imply

$$x = \frac{1}{r \cos \beta} \quad , \quad z = \frac{1}{\cos \beta} \quad (3.26)$$

or alternatively

$$r = \frac{z}{x} \quad (3.27)$$

We then obtain from (3.10) the restriction

$$1 \leq z \leq x \quad (3.28)$$

for the domain of DIF. This is also shown in Fig. 2.

Because q^μ in (3.23) is invariant under a transverse boost, one could again, as in DIS, raise a question about other possible frames. The same conclusion applies. Independence on the azimuthal angle α uniquely selects out the Lorentz frame specified by (3.23).

3.5. - Summary

For the four virtual photon processes discussed above, we have considered special Lorentz frames in which to interpret the two scaling variables. They are in each case related simply to the radial scaling variable r and the angle β . The relationships are summarized in the Table for ease of reference. To emphasize that those relationships exist in four different Lorentz frames, we shall refer to them as the preferred frames.

4. - PHYSICAL PROCESSES

We have so far discussed a function $G(x, z, q^2)$ which, when evaluated in four non-overlapping regions in the x - z space, are to be identified with the four distribution functions $S_{\pm}(x, z, q^2)$ and $J_{\pm}(x, z, q^2)$. While an underlying theory will most certainly relate these functions, we have made no assumption about that relationship. Proceeding along the same general line, we derive in this section the formulae which express the measurable quantities in terms of the four distribution functions. The formulae will be general expressions in the QPM, and will reduce to the usual ones when k_T is put to zero and the hadron and quark masses are ignored. Any assumptions about the nature of the four distribution functions will be deferred until the next section.

4.1. - Deep inelastic scattering (DIS)

In the standard way ¹⁾ we define the structure functions W_1 and W_2 through the Lorentz covariant tensor $K^{\mu\nu}$

$$\begin{aligned} K^{\mu\nu} &= \sum'_X \langle P | J^\nu(0) | X \rangle \langle X | J^\mu(0) | P \rangle \\ &= \frac{4\pi}{M} \left[W_2 \left(P^\mu - q^\mu \frac{P \cdot q}{q^2} \right) \left(P^\nu - q^\nu \frac{P \cdot q}{q^2} \right) \right. \\ &\quad \left. - M^2 W_1 \left(q^\mu q^\nu - \frac{q^\mu q^\nu}{q^2} \right) \right] \cdot 2\pi \delta(M_X^2 - (P - q)^2) \end{aligned} \quad (4.1)$$

where the prime on the summation sign denotes spin average of the target hadron, which, for definiteness, will be assumed to be a proton. By impulse approximation we relate $K^{\mu\nu}$ to the tensor $L^{\mu\nu}$ describing the scattering of a pointlike parton by the virtual photon, weighted by the Lorentz invariant probability $S_-(P, k, q)$ appropriate for the kinematical configuration. Thus we have

$$P_0^{-1} K^{\mu\nu} = \int \frac{d^3k}{k_0} S_-(P, k, q) k_0^{-1} L^{\mu\nu} \quad (4.2)$$

where the factors P_0^{-1} and k_0^{-1} account for the difference in the normalizations of the proton and parton states. With the parton on-mass shell before and after its interaction with the photon, we have

$$\begin{aligned}
 L^{\mu\nu} &= \sum_l' \langle k | j^\nu(0) | l \rangle \langle l | j^\mu(0) | k \rangle 2\pi \delta(\mu^2 - (k-q)^2) \\
 &= \left[(k^\mu - q^\mu \frac{k \cdot q}{q^2}) (l^\nu - q^\nu \frac{l \cdot q}{q^2}) + (\mu \leftrightarrow \nu) \right. \\
 &\quad \left. - k \cdot q (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) \right] 4\pi \delta(\mu^2 - l^2),
 \end{aligned}
 \tag{4.3}$$

where $l^\mu = k^\mu - q^\mu$ and the prime on the summation sign denotes spin average of the scattering quark. The mass-shell condition implies $\delta(2k \cdot q - q^2)$, which by virtue of (3.1) is proportional to $\delta(x - x_L)$ where

$$x_L = \frac{k \cdot q}{P \cdot q} \tag{4.4}$$

In the preferred frame specified by (3.4), x_L becomes just the longitudinal momentum fraction

$$x_L = \frac{k_L}{P} \tag{4.5}$$

Nevertheless, as defined in (4.4), x_L is a Lorentz invariant quantity.

To extract from (4.2) the expressions for W_1 and W_2 we need to evaluate the tensor for at least two independent components. We do this by going to the preferred frame and noting from (3.4) that

$$P^\mu - q^\mu \frac{P \cdot q}{q^2} = (P_0, \vec{0}, 0),$$

$$g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} = (1, -1, -1, 0)_{diag.},$$

$$k^\mu - q^\mu \frac{k \cdot q}{q^2} = l^\mu - q^\mu \frac{l \cdot q}{q^2} = (k_0, \vec{k}_T, 0).$$

Substituting these into (4.1) and (4.3), we obtain with the help of (4.2)

$$W_1(x, Q^2) = \int \frac{d^3 k}{k_0} S_-(x, z, Q^2) \frac{1}{2M} \delta(x - x_L) \cdot L(x, \beta, Q^2) \left[1 + \frac{1}{2} \tan^2 \beta \right], \quad (4.6)$$

$$\nu W_2(x, Q^2) = \int \frac{d^3 k}{k_0} S_-(x, z, Q^2) x \delta(x - x_L) L(x, \beta, Q^2) \cdot \left[1 + \frac{4\mu^2}{Q^2} + \frac{3}{2} \tan^2 \beta \right] \left(1 + \frac{4x^2 M^2}{Q^2} \right)^{-1}, \quad (4.7)$$

where

$$L(x, \beta, Q^2) = \left[\frac{1 + \frac{4x^2 M^2}{Q^2}}{1 + \frac{4\mu^2}{Q^2} + \tan^2 \beta} \right]^{1/2}. \quad (4.8)$$

In these equations we have used

$$x^2 = \frac{Q^2}{4P^2} \quad (4.9)$$

and

$$\tan^2 \beta = \frac{4k_T^2}{Q^2} \quad (4.10)$$

which follow from (3.5) and (3.6).

If further we make use of the Table and identify

$$\tan^2 \beta = z^2 - 1, \quad (4.11)$$

then (4.6) and (4.7) are manifestly Lorentz invariant expressions for W_1 and W_2 .

Before we discuss the implications of (4.6) and (4.7), it should be pointed out that S_- represents an average parton distribution function which contains a sum over all flavours. More precisely, if S_-^f and $S_-^{\bar{f}}$ denote the distributions of quarks and antiquarks with flavour f in a proton, and e_f is the corresponding charge in unit of e , then we have

$$S_- = \sum_f e_f^2 (S_-^f + S_-^{\bar{f}}) \quad (4.12)$$

where f is summed over u, d and s .

We now remark that the factor $L(x, \beta, Q^2)$ appears in (4.6) and (4.7) because of P_0/k_0 in (4.2) since

$$\left. \frac{P_0}{k_0} \right|_{x_L=x} = \frac{1}{x} L(x, \beta, Q^2). \quad (4.13)$$

Note from (4.8) that L does not approach one, even as $Q^2 \rightarrow \infty$, if β is non-vanishing. It exhibits the difference between the parton flux and proton flux that the virtual photon sees when the two are not collinear.

We further remark that due to the presence of $\tan^2 \beta$ in (4.6) and (4.7) the Callan-Gross relation ²⁶⁾

$$2x M W_1 = \nu W_2 \quad (4.14)$$

is no longer valid even in the Bjorken limit. This will make expressions for semi-inclusive cross-sections somewhat more complicated. It also leads to essential and non-vanishing contributions to the value of R , as we now see.

The ratio of the longitudinal to the transverse cross-sections is

$$R = \frac{\sigma_L}{\sigma_T} = \left(1 + \frac{\nu^2}{Q^2} \right) \frac{W_2}{W_1} - 1 \quad (4.15)$$

Substituting (4.6) and (4.7) into (4.15) yields

$$R(x, Q^2) = \frac{\int \frac{d^3k}{k_0} S_-(x, z, Q^2) \delta(x - x_L) L(x, \beta, Q^2) \left[\frac{4\mu^2}{Q^2} + \tan^2 \beta \right]}{\int \frac{d^3k}{k_0} S_-(x, z, Q^2) \delta(x - x_L) L(x, \beta, Q^2) \left[1 + \frac{1}{2} \tan^2 \beta \right]} \quad (4.16)$$

If one ignores the L factors in the above and uses (4.10), one obtains

$$R(Q^2) \simeq \frac{4(\mu^2 + \langle k_T^2 \rangle)}{Q^2 + 2 \langle k_T^2 \rangle} \quad (4.17)$$

which is the usual formula in the low k_T approximation. From (4.16) we see that at high Q^2 a non-vanishing value of R is possible if and only if β is non-vanishing.

4.2. - Lepton pair production (LPP)

We assume the Drell-Yan mechanism ⁸⁾ for lepton pair production in hadronic collisions. The leptons are produced by quark-antiquark annihilation through the exchange of a massive timelike photon. If S_+^f and $S_+^{\bar{f}}$ are the distribution functions for the quark and the antiquark with flavour f , then the differential cross-section for LPP is

$$d\sigma = \sum_f \frac{1}{3} \int \frac{d^3k}{k_0} \frac{d^3l}{l_0} S_+^f(P_1, k, q) S_+^{\bar{f}}(P_2, l, q) d\hat{\sigma}_f + (f \leftrightarrow \bar{f}) \quad (4.18)$$

where $1/3$ is the colour factor and $d\hat{\sigma}_f$ is the differential cross-section for the subprocess $q_f + \bar{q}_f \rightarrow \gamma \rightarrow \mu^+ + \mu^-$, and has the form

$$d\hat{\sigma}_f = \frac{4\pi\alpha^2 e_f^2}{3q^2} \delta^3(\vec{k} + \vec{l} - \vec{q}) d^3q \theta(q_0) \delta((k+l)^2 - q^2) dq^2, \quad (4.19)$$

where α is the fine structure constant. Substituting (4.19) into (4.18) yields

$$\frac{q_0}{d^3q} \frac{d\sigma}{dq^2} = \sum_f \frac{2\pi\alpha^2 e_f^2}{9q^2} \int S_+^f(p_1, k, q) S_+^{\bar{f}}(p_2, l, q) d^2\Phi + (f \leftrightarrow \bar{f}), \quad (4.20)$$

where $d^2\Phi$ is the differential element of the two-dimensional phase space

$$d^2\Phi = \delta^4(k+l-q) \frac{d^3k}{k_0} \frac{d^3l}{l_0}. \quad (4.21)$$

In analogy with S_- in (4.12) we can decompose S_+ similarly :

$$S_+ = \sum_f e_f^2 (S_+^f + S_+^{\bar{f}}). \quad (4.22)$$

For the sake of simplifying (4.20) so that the sum over flavours does not appear explicitly, we assume that the distributions $S_+^{\bar{f}}$ for antiquarks in a proton are independent of flavour [i.e., the sea is $SU(3)$ symmetric] and define

$$\bar{S}_+ = \sum_f e_f^2 S_+^{\bar{f}} = \frac{2}{3} S_+^{\bar{f}}. \quad (4.23)$$

Then we have

$$\sum_f e_f^2 S_+^f S_+^{\bar{f}} = \frac{3}{2} (S_+ - \bar{S}_+) \bar{S}_+. \quad (4.24)$$

Putting this in (4.20), we obtain finally

$$\frac{q_0}{d^3q} \frac{d\sigma}{dq^2} = \frac{\pi\alpha^2}{3q^2} \int \left[\left\{ S_+(x_1, z_1, q^2) - \bar{S}_+(x_1, z_1, q^2) \right\} \bar{S}_+(x_2, z_2, q^2) + (1 \leftrightarrow 2) \right] d^2\Phi \quad (4.25)$$

The separation of $\bar{S}_+$ from S_+ will be self-evident when we consider phenomenology in Section 6.

Because (4.25) is Lorentz invariant, the integration can be performed in any convenient frame, usually in the centre-of-mass system of the incident hadrons where data are presented. However, that is not the preferred frame for LPP discussed in Section 3.2, which is the centre-of-mass system of the leptons. It is therefore important for us to state here the procedure that we shall follow for calculations in LPP as well as in DIF later. As we shall discuss in the next section, $G(P, k, q)$ is to be specified in the preferred frame for each process. After it is expressed in terms of the invariants x , z and q^2 , we can then use it in any frame. The domains of definition of the functions $S_{\pm}(x, z, q^2)$ and $J_{\pm}(x, z, q^2)$ remain as shown in Fig. 2. But when (4.25), for example, is evaluated in the centre-of-mass system of the incident hadrons, $d^2\Phi$ will have its own phase space which depends on the incident energy, dilepton mass, $\vec{q}$, etc. The integration will, of course, be performed over the intersection of the two regions. The phase space for $d^2\Phi$ turns out to be an ellipsoid as depicted in Fig. 3.

4.3. - Lepton pair annihilation (LPA) *)

We consider here the inclusive hadron cross-section in LPA. In the QPM two jets are produced due to the fragmentations of a quark and an antiquark created via one-photon exchange. Let $J_+^{h/f}$ denote the distribution function for finding a hadron h in a jet initiated by a quark with flavour f , and similarly $J_+^{h/\bar{f}}$ for an antiquark with the same flavour. In the centre-of-mass system of the leptons (the preferred frame of LPA) the momentum sum rule is

*) In this subsection, for the sake of physical transparency, p^μ , k^μ and q^μ are taken to be the physical momentum vectors, defined with a sign convention opposite to the one adopted in Sections 2 and 3.

$$\sum_h \int \frac{d^3 p}{p_0} J_+^{h/f}(p, k, q) \vec{p} = \vec{k} \quad (4.26)$$

On the other hand we also have

$$\int \frac{d^3 p}{p_0} J_+^{h/f}(p, k, q) = \langle n^h \rangle_f \quad (4.27)$$

which is the average multiplicity of hadron h in a jet of f flavoured quark.

Let ℓ^μ be the momentum of the antiquark opposite that of the quark so that if $k^\mu = (k_0, \vec{k})$, then $\ell^\mu = (k_0, -\vec{k})$. The inclusive distribution for detecting a hadron h with momentum p^μ in the centre-of-mass system is

$$\begin{aligned} \frac{p_0}{\sigma_T} \frac{d\sigma^h}{d^3 p} &= \left(\frac{16\pi}{3} \sum_f e_f^2 \right)^{-1} \sum_f e_f^2 \int d^2 \Omega_k (1 + \cos^2 \theta) \\ &\quad \left[J_+^{h/f}(p, k, q) + J_+^{h/\bar{f}}(p, \ell, q) \right], \end{aligned} \quad (4.28)$$

where σ_T is the total cross-section for producing hadrons, θ is the angle between the quark and the incident leptons, and the integration is over all orientations of the quark momentum $\vec{k}$. On account of that integration the momentum ℓ^μ in the integrand of (4.28) may be replaced by k^μ . In analogy to the definitions of $S_\pm$ in (4.12) and (4.22) in which the label for the parent hadron has been suppressed for brevity, we now define

$$J_\pm^h(p, k, q) = \sum_f e_f^2 \left[J_\pm^{h/f}(p, k, q) + J_\pm^{h/\bar{f}}(p, k, q) \right] \quad (4.29)$$

with the label for the detected hadron h appearing explicitly. Equation (4.28) can then be expressed simply as

$$\frac{p_0}{\sigma_T} \frac{d\sigma^h}{d^3p} = \frac{9}{32\pi} \int d^2\Omega_k (1 + \cos^2\theta) J_+^h(p, k, q). \quad (4.30)$$

We note that with the help of (4.26) $J_+^h(p, k, q)$ satisfies the momentum sum rule

$$\sum_h \int \frac{d^3p}{p_0} J_+^h(p, k, q) \vec{p} = \frac{4}{3} \vec{k}, \quad (4.31)$$

but for the inclusive distribution we have

$$\sum_h \int \frac{d^3p}{p_0} \left(\frac{p_0}{\sigma_T} \frac{d\sigma^h}{d^3p} \right) \vec{p} = 0 \quad (4.32)$$

in view of the integration over the jet orientation.

It is evident that more information can be extracted from the inclusive distribution if the jet axis is always realigned along a fixed direction for all events in LPA, as is done experimentally ²⁷⁾. Thus, fixing $\pm \vec{k}$ along the z axis as in the preferred frame and using $d\sigma^h$ to denote the cross-section measured according to that procedure, we have

$$\frac{p_0}{\sigma_T} \frac{d\sigma^h}{d^3p} = \frac{3}{2} J_+^h(x, z, q^2), \quad (4.33)$$

where the kinematical variables on the two sides of this equation are related by the last column in the Table. The average multiplicity of the hadron h in LPA is then

$$\begin{aligned} \langle n^h \rangle &= \int \frac{d^3p}{p_0} \left(\frac{p_0}{\sigma_T} \frac{d\sigma^h}{d^3p} \right) \\ &= \frac{3}{2} \sum_f e_f^2 \left[\langle n^h \rangle_f + \langle n^h \rangle_{\bar{f}} \right]. \end{aligned} \quad (4.34)$$

4.4. - Lepton hadron semi-inclusive reaction *)

We consider finally the semi-inclusive reaction, $\ell + N \rightarrow \ell' + h + X$, which is a combination of the subprocesses DIS and DIF. Recall first the inclusive cross-section in which no hadrons are detected :

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{4\pi\alpha^2}{q^4} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} W_2 + 2 \sin^2 \frac{\theta}{2} W_1 \right], \quad (4.35)$$

where W_1 and W_2 are given by (4.6) and (4.7). For notational ease, let us condense (4.6) and (4.7) to the form

$$W_i(x, Q^2) = \int \frac{d^3k}{k_0} S_-(x, z, Q^2) w_i, \quad i=1, 2, \quad (4.36)$$

where

$$w_1 = \frac{1}{2M} \delta(x - x_L) L(x, \beta, Q^2) \left[1 + \frac{1}{2} \tan^2 \beta \right], \quad (4.37a)$$

$$w_2 = \frac{x}{\nu} \delta(x - x_L) L(x, \beta, Q^2) \left[1 + \frac{4\mu^2}{Q^2} + \frac{3}{2} \tan^2 \beta \right] \cdot \left(1 + \frac{4x^2 M^2}{Q^2} \right)^{-1}. \quad (4.37b)$$

Now, for a semi-inclusive process in which p is the momentum of the detected hadron h , the cross-section is

$$\begin{aligned} \mathcal{F}^h(p, Q^2, \nu, E) &\equiv p_0 \frac{d}{d^3p} \left(\frac{d^2\sigma}{dq^2 d\nu} \right) \\ &= \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} p_0 \frac{d}{d^3p} W_2^h(x, Q^2, p) \right. \\ &\quad \left. + 2 \sin^2 \frac{\theta}{2} p_0 \frac{d}{d^3p} W_1^h(x, Q^2, p) \right], \quad (4.38) \end{aligned}$$

*) The footnote made with reference to subsection 4.3 concerning the momentum convention applies here also.

where

$$p_0 \frac{d}{d^3 p} W_i^h(x, Q^2, p) = \int \frac{d^3 k}{k_0} w_i T_-^h(x, z, x', z', Q^2), \quad i=1, 2, \quad (4.39)$$

$$T_-^h = \sum_f e_f^2 \left[S_-^f(x, z, Q^2) J_-^{h/f}(x', z', Q^2) \right.$$

$$\left. + \bar{S}_-(x, z, Q^2) J_-^{h/\bar{f}}(x', z', Q^2) \right]. \quad (4.40)$$

In the above equation the scaling variables x and z for DIS and x' and z' for DIF are as defined in Sections 3.1 and 3.4. They are related to the spherical scaling variables (r, β, α) and (r', β', α') in their respective preferred frames as indicated in the Table, in terms of which the invariant phase spaces are

$$\text{DIS: } \frac{d^3 k}{k_0} = \frac{Q^2}{4x^2} \frac{r^2}{r_0} dr d\cos\beta d\alpha, \quad (4.41)$$

$$\text{DIF: } \frac{d^3 p}{p_0} = \frac{Q^2}{4} \frac{r'^2}{r'_0} dr' d\cos\beta' d\alpha', \quad (4.42)$$

where r_0 and r'_0 are, respectively, the scaled energy variables

$$r_0 = \sqrt{r^2 + \frac{4x^2 \mu^2}{Q^2}}, \quad r'_0 = \sqrt{r'^2 + \frac{4m^2}{Q^2}}, \quad (4.43)$$

m being the detected hadron mass.

To simplify (4.40) and especially to express it in terms of S_- measured in DIS and J_- of DIF (which is closely related to J_+ measured in LPA), let us consider, in particular, the case of π^+ and π^- production. For economy of notation we temporarily omit the subscripts +

or - on the J function, since the following consideration applies to both. Because of isospin and charge conjugation invariance there are only three independent J functions :

$$J_1 \equiv J^{\pi^+/u} = J^{\pi^+/\bar{d}} = J^{\pi^-/d} = J^{\pi^-/\bar{u}} \quad (4.44a)$$

$$J_2 \equiv J^{\pi^+/d} = J^{\pi^+/\bar{u}} = J^{\pi^-/u} = J^{\pi^-/\bar{d}} \quad (4.44b)$$

$$J_3 \equiv J^{\pi^+/s} = J^{\pi^+/\bar{s}} = J^{\pi^-/s} = J^{\pi^-/\bar{s}} \quad (4.44c)$$

It therefore follows from (4.29) that for either π^+ or π^- we have

$$J^{\pi^\pm} = \frac{5}{9} (J_1 + J_2) + \frac{2}{9} J_3 \quad (4.45)$$

Moreover, (4.40) can be reduced to the following form if one considers the sum of the contributions to π^+ and π^- :

$$T_-^{\pi^+} + T_-^{\pi^-} = S_- (J_1 + J_2) + \frac{1}{9} (S_-^s + S_-^{\bar{s}}) (2J_3 - J_1 - J_2) \quad (4.46)$$

If one ignores the contributions of the strange quarks to (4.45) and (4.46), one has the simple result ²⁸⁾

$$T_-^{\pi^+} + T_-^{\pi^-} = \frac{9}{5} S_- J_-^{\pi^\pm} \quad (4.47)$$

In that approximation one obtains from (4.38) and (4.39)

$$\begin{aligned} \mathcal{J}^{\pi^+} + \mathcal{J}^{\pi^-} = & \frac{36\pi\alpha^2}{Q^4} \frac{E'}{E} \int \frac{d^3k}{k_0} \left[\cos^2 \frac{\theta}{2} w_2 \right. \\ & \left. + 2 \sin^2 \frac{\theta}{2} w_1 \right] S_-(x, z, Q^2) J_-^{\pi^\pm}(x', z', Q^2), \end{aligned} \quad (4.48)$$

where S_- contributes directly to W_1 and νW_2 through (4.6) and (4.7), while $\bar{J}^{\pi^\pm}$ is a spacelike counterpart of the jet function measured in LPA and given in (4.33).

On the other hand, if one assumes an $SU(3)$ symmetric sea (consistent with the approximation procedure adopted in Section 4.2), the contribution of the strange quarks are not negligible ; in fact, one has in that approximation $J_2 = J_3$. The truth lies somewhere between the two extremes. To see the consequences of that approximation, we introduce a two-component description of $J^{h/f}$ analogous to the valence-sea two-component description of $S^{f/h}$ (the hadron label h having been suppressed in the discussions on S^f in Sections 4.1 and 4.2). In (4.23) we defined for the proton the function $\bar{S}$ which, being the antiquark contribution to S , represents half of the sea. The valence contribution is therefore $S - 2\bar{S}$. Now, for the J function we similarly define two parts. Specifically, for fragmentation into $\pi^\pm$ we define in analogy to (4.23)

$$\bar{J}^{\pi^\pm} = \sum_f e_f^2 J_2 = \frac{2}{3} J_2 \quad (4.49)$$

which is, figuratively speaking, reciprocal to $\bar{S}^{\pi^\pm}$. Unlike $J^{\pi^\pm}$ it is not directly measurable. Inverting (4.45) and (4.49) with $J_2 = J_3$ we have

$$J_1 = \frac{9}{5} \left(J^{\pi^\pm} - \frac{7}{6} \bar{J}^{\pi^\pm} \right), \quad (4.50)$$

$$J_2 = \frac{3}{2} \bar{J}^{\pi^\pm} \quad (4.51)$$

Substituting these into (4.46) and using (4.23) we finally obtain

$$\begin{aligned} T_-^{\pi^+} + T_-^{\pi^-} = \frac{3}{5} \left[S_- (3J_-^{\pi^+} - \bar{J}_-^{\pi^+}) \right. \\ \left. - \bar{S}_- (J_-^{\pi^+} - 2\bar{J}_-^{\pi^+}) \right] \end{aligned} \quad (4.52)$$

Hence, in the $SU(3)$ approximation the inclusive cross-section for charged pions is

$$\begin{aligned} \mathcal{J}^{\pi^+} + \mathcal{J}^{\pi^-} = \frac{12\pi\alpha^2}{5Q^4} \frac{E'}{E} \int \frac{d^3k}{k_0} \left(\cos^2\frac{\theta}{2} W_2 + 2 \sin^2\frac{\theta}{2} W_1 \right) \\ \left[S_-(3J_- - \bar{J}_-) - \bar{S}_-(J_- - 2\bar{J}_-) \right], \end{aligned}$$

(4.53)

where J_- and $\bar{J}_-$ refer to either π^+ or π^- . Evidently, (4.48) differs from this only by the absence of the last three terms in the square bracket. Phenomenological implication of (4.53) will be investigated in Section 6.

5. - UNIVERSAL FORM FOR $G(x, z, q^2)$

So far our consideration has been general in the sense that no restrictions have been imposed on the nature of the four distribution functions $S_{\pm}(x, z, q^2)$ and $J_{\pm}(x, z, q^2)$. They are defined in four non-overlapping regions in the x - z plane and pertain to four different processes. Thus, insofar as phenomenology is concerned, they may be regarded as four unrelated functions, which jointly define a global function $G(x, z, q^2)$ in the union of the four regions.

Theoretically, on account of the similarity among the four diagrams in Fig. 1, they cannot be expected to be unrelated. However, there exists no reliable determination of that relationship on the basis of a generally accepted theory. In this section we motivate a simple universal form for all four functions. The degree to which it is realistic is tested in the following section.

In the application of the naive QPM to the virtual photon processes, no distinction is made of the timelike or spacelike aspects of the photon. Since k_T is also ignored, the approximations made in the naive QPM are tantamount to identifying S_+ with S_- , and J_+ with J_- . In

our generalized version of the model, the timelike or spacelike nature of q^μ is incorporated in the definition of $G(P,k,q)$ and in the choice of the preferred frames where the geometrical interpretations of x and z are made. When expressed in terms of the radial variable r , the angle β and the scale Q^2 (defined as $|q^2|$), the function $G(r,\beta,Q^2)$ has no manifest dependence on the sign of q^2 , although in identifying with $S_\pm$ and $J_\pm$ in the respective regions the information on the nature of q^μ is restored. In the approximation of $k_T=0$, we would have $\beta=0$ and

$$G(r, \beta=0, Q^2) = S_+(x, z=1, q^2=Q^2) = S_-(x, z=1, q^2=-Q^2) \quad (5.1)$$

for $r=x \leq 1$. A similar relation exists also for $J_\pm$ if $r^{-1}=x \geq 1$. Thus, with (5.1) serving as a boundary condition we conjecture that even when z is away from 1, i.e., $\beta \neq 0$, $S_\pm(x,z,q^2)$ can be described by one $G(r,\beta,Q^2)$ function and $J_\pm(x,z,q^2)$ by another, using the Table as a dictionary that translates the variables r and β into x and z in the appropriate regions.

It is tempting to go further and conjecture that even $S_\pm$ and $J_\pm$ can be unified and are described by one universal single $G(r,\beta,Q^2)$ function. This step of unification is (in the case of $\beta=0$) equivalent to assuming that the reciprocity relation of Gribov and Lipatov²⁹⁾ is valid for the entire range of r . So far there is no theoretical or experimental evidence for or against the reciprocity relation. The symmetry of the four processes in our analysis makes it very inviting to consider its possibility and we shall scrutinize its phenomenological implications in the following.

Our working hypothesis is then that a single $G(r,\beta,Q^2)$ function describes all four distribution functions $S_\pm$ and $J_\pm$. For each region in Fig. 2 one should use the appropriate column in the Table to translate the variables r and β into x and z . The resultant $S_\pm(x,z,Q^2)$ or $J_\pm(x,z,Q^2)$ is then to be treated as a Lorentz invariant function for that region. Obviously, the four distribution functions so determined have very different dependences on x and z . Taken together they define $G(x,z,q^2)$, which therefore plays the rôle of an interpolating function. The universality rests in the form of $G(r,\beta,Q^2)$ which stresses the geometrical configuration in momentum space and describes the universal features in finding partons in hadrons, and hadrons in quark jets.

The procedure of relating $S_{\pm}$ and $J_{\pm}$ discussed above has a striking parallelism with that of an analytic scattering amplitude in a crossing symmetric theory. Consider a two-body scattering amplitude $A(s,t)$. It is widely accepted that $A(s,t)$ exhibits Regge behaviour at high energies, i.e., in the s channel $A(s,t) \sim s^{\alpha(t)}$ for large s , and in the t channel $A(s,t) \sim t^{\alpha(s)}$ for large t . The universal form for both cases is $G(E, q^2) \sim E^{2\alpha(-q^2)}$ where E is the energy and q^2 the three-momentum transfer squared in the centre-of-mass system (preferred frame) of whichever channel under consideration. To get the Lorentz invariant expressions of the asymptotic behaviours, we make the appropriate substitution of s and t in place of E^2 and q^2 in each channel (similar to our use of the Table). The resultant forms $s^{\alpha(t)}$ and $t^{\alpha(s)}$ had no obvious direct connection before the discovery of the dual amplitude. It therefore illustrates our present status regarding the relationship among $S_{\pm}$ and $J_{\pm}$.

The fact that k_T is limited in many processes observed so far does not imply that the variable z is of no intrinsic interest. In the case of two-body reactions $d\sigma/dt$ is a sharply damped function of t , but it is nevertheless of crucial importance to recognize t as a dynamical variable, having a status equal to that of s . Similarly, we believe, z should be treated on equal footing as x . It is only when Q^2 is large that the phenomenological significance of z will become apparent.

Finally, we emphasize that $G(r, \beta, Q^2)$ has two suppressed labels h and f ; it describes the (conjectured) common features among $S_{\pm}^{f/h}$ and $J_{\pm}^{h/f}$. It then follows that for every hadron h there exist $S_{\pm}^h$ and $\bar{S}_{\pm}^h$. Note that for the sea contribution, (4.23) defines $\bar{S}_{\pm}^p$ for proton and (4.49) gives $\bar{J}_{\pm}^{\pi^{\pm}}$ for $\pi^{\pm}$. The universality conjecture relates the former to $\bar{J}_{\pm}^p$, and the latter to $\bar{S}_{\pm}^{\pi^{\pm}}$, but they need not be related to each other between different hadrons.

6. - PHENOMENOLOGY

The strategy that we shall follow in phenomenology is to determine $G(r, \beta, Q^2)$ by fitting the data on νW_2 in DIS and on $d\sigma/dM_{\mu\mu}$ and $\langle q_T \rangle$ in LPP. Because the dependences on r and β are not factorizable, all three pieces of data must be fitted simultaneously. Only the trivial dependence on Q^2 (due to the dimension of G) will be taken into account. Scaling violation will not be incorporated explicitly in our parametrization so that the question of universality to be examined does not get confused with other unsettled issues such as scaling violation for timelike q^2 . However, the effects of scaling violation are indirectly accounted for by fitting the data for the highest Q^2 available.

To achieve simultaneous fits of various pieces of data on both DIS and LPP as mentioned above is already an affirmation of some aspect of universality. To see what it predicts, we use the G function so determined to calculate W_1 , R , and a host of other quantities. Moreover, on the basis of universality we can also calculate the jet structure in e^+e^- annihilation as well as the proton inclusive cross-section in deep inelastic processes. Unfortunately the results could not be compared with data since the data on protons in quark jets are inadequate. Borrowing the sea component $\bar{S}$ from the proton, we shall construct a phenomenological form for J for the pions and fit $d\sigma/dr$ of LPA. Assuming universality in the β dependence, we then calculate the x_T dependence of $\langle p_T \rangle$ in LPA and the x_F dependence of semi-inclusive cross-sections in deep inelastic processes. Comparison will then be made with the rather limited pion data that are available.

Because G has the dimension of inverse momentum squared, let us define

$$G(r, \beta, Q^2) = \frac{1}{P^2} g(r, \beta) \quad (6.1)$$

where g is dimensionless and P is the momentum of the parent (i.e., hadron for $S_{\pm}$ and quark for $J_{\pm}$). We have ignored scaling violation by dropping q^2 dependence in $g(r, \beta)$, although our formulation of the QPM clearly permits an application that includes explicit scaling violation effects. It should, however, be noted that the independence of $g(r, \beta)$ on Q^2 does not imply Q^2 independence of the measurable quantities, as is evident from the formulae given in Section 4.

It follows from (6.1) that in the cases of DIS and LPP we have

$$\int \frac{d^3k}{k_0} S_{\pm}(x, z, Q^2) = \int dr d\cos\beta r g(r, \beta), \quad (6.2)$$

where the approximation $r_0 \approx r$ for sufficiently large Q^2 has been used. From (4.7) one sees that after expressing the non-trivial integration in terms of $\cos\beta$, the integrand becomes proportional to $r^2 g(r, \beta)$. Hence $g(r, \beta)$ must behave as r^{-2} as $r \rightarrow 0$. We shall therefore adopt the following phenomenological form

$$g(r, \beta) = [s(r) + v(r)] h(r, \beta), \quad (6.3)$$

where

$$s(r) = A r^{-2} (1-r)^{n_s}, \quad (6.4)$$

$$v(r) = B (1-r)^{n_v} (r^{-1} + C), \quad (6.5)$$

$$h(r, \beta) = \frac{2}{\sqrt{\pi} \beta_0(r)} \exp\left[-\beta^2 / \beta_0^2(r)\right], \quad (6.6)$$

$$\beta_0(r) = b_0 + b_1 (1-r)^4. \quad (6.7)$$

Clearly, $s(r)$ and $v(r)$ describe the sea and valence quark distributions, respectively, in the context of a structure function. Owing to (4.12) and (4.22), a sum over the flavours of the quarks has already been taken into account. A similar interpretation can also be given for the jet functions $J_{\pm}(r, \beta)$, but is not necessary.

For protons, we choose $n_v = 3$ and $n_s = 6$. The parameters are determined by fitting : (i) νW_2 , (ii) $d\sigma/dM_{\mu\mu}$, and (iii) $\langle q_T \rangle$ vs. $M_{\mu\mu}$, following the procedure described in the previous section. Because the range of Q^2 in LPP extends beyond $100 (\text{GeV}/c)^2$, we shall attempt to fit νW_2 with emphasis on the highest Q^2 data available. Since the

$15 < Q^2 < 30 \text{ (GeV/c)}^2$ and $30 < Q^2 < 50 \text{ (GeV/c)}^2$ data of Réf. 6) are crude and for small values of x only (see Figs. 4b and 4c), we supplement them with other data for other values of Q^2 and x (see Fig. 4a). The formula used to fit the data is (4.7) with $\mu^2 = 0.2 \text{ (GeV/c)}^2$. Note the Q^2 dependence even with scale invariant $g(r, \beta)$. However, for $10 < Q^2 < 100 \text{ (GeV/c)}^2$ such Q^2 dependence is hardly perceptible. To fit $d\sigma/dM_{\mu\mu}$ and $\langle q_T \rangle$ in LPP, we use (4.25) where

$$\bar{S}_+ = \frac{1}{2} \frac{1}{p^2} s(r) h(r, \beta) , \quad (6.8)$$

the factor $\frac{1}{2}$ being due to the fact that only half the sea is made up of antiquarks. We ignore the difference between neutrons and protons in the nuclear targets. Because the low $M_{\mu\mu}$ portion ($M_{\mu\mu} < 3 \text{ GeV/c}^2$) of the dilepton cross-section cannot be explained by the Drell-Yan mechanism, we do not put any weight to the $\langle q_T \rangle$ data in that $M_{\mu\mu}$ range³⁰⁾. Our fits yield the following values for the parameters :

$$A = 0.226 , \quad B = 1.59 , \quad C = 3.75$$

$$b_0 = 6^\circ , \quad b_1 = 25^\circ . \quad (6.9)$$

The results are plotted in Figs. 4a, 4b, 4c, 5 and 6. The fact that we are able to achieve such good fits of the data lends support to the validity of the Drell-Yan mechanism and the universality of S_+ and S_- via the interpolating function $G(r, \beta, Q^2)$.

We remark that the form used in (6.3) for $g(r, \beta)$ is such that at $r=0$ the value of β_0 is finite, approximately 30° . It implies that k_T is forced to vanish at $k_T=0$. This is, of course, unrealistic. But since the Drell-Yan mechanism is unreliable at small values of $M_{\mu\mu}$, we feel that detailed fitting in that region is unjustified. Hence, the application of (6.3) for the determination of the transverse momentum distribution is not very meaningful for very small r , say $r < 0.1$.

Using the parametrization of G determined in (6.9), we can now calculate a variety of quantities. First, we exhibit the valence and sea components of the structure function νW_2 for proton in Fig. 4d. On the

basis of (4.6), we calculate W_1 which is not simply related to νW_2 because the Callan-Gross relation²⁶⁾ is violated when β is not infinitesimal. The result is shown in Fig. 7. The ranges of Q^2 values for the various data points⁴⁾ are as shown in the figure; clearly, our prediction which represents higher values of Q^2 has the anticipated departure from the data points due to scaling violation. Roughly speaking, our G function represents an average over Q^2 from 10 to 100 (GeV/c)².

Next we calculate R using (4.16). The result is highly sensitive to the dependence of $g(r, \beta)$ on β , and relies heavily on the universality that links DIS with LPP, since the β dependence is determined mainly by the data on the latter process (Fig. 6). We show in Fig. 8 the calculated results for various x values; they are compared with the old data points⁴⁾ as well as the new ones⁹⁾. While the theoretical curves are generally higher than the former, they are lower than the latter. Given the experimental situation, the result should be regarded as satisfactory. Note that the value of R gets quite large at low Q^2 , especially for small x . That is due partly to the effects of quark mass and confinement, which we collectively represent by the choice $\mu^2 = 0.2$ (GeV/c)². The other part is the non-vanishing background, which at asymptotic values of Q^2 is due exclusively to the finiteness of β . This is a possibility that is excluded in the naive QPM.

It is interesting to ask what the difference is, if any, between $\langle k_T \rangle$ for DIS and that for LPP. Recall that although $g(r, \beta)$ is the same for both by hypothesis, the implication for $\langle k_T \rangle$ may be different on account of the Table. For DIS we have for fixed x

$$\langle k_T \rangle = xP \frac{\int_x' d\cos\beta (\tan\beta/\cos^2\beta) g(x/\cos\beta, \beta)}{\int_x' d\cos\beta (1/\cos^2\beta) g(x/\cos\beta, \beta)} \quad (6.10)$$

while for LPP

$$\langle k_T \rangle = xP \frac{\int_0' d\cos\beta \sin\beta g(x, \beta)}{\int_0' d\cos\beta g(x, \beta)} \quad (6.11)$$

where P is the momentum of the hadron in the preferred frames (see Section 3). Since P is the only scale in the problem, $\langle k_T \rangle$ must grow with the hadron energy for fixed x . To see the x dependences we have plotted in Fig. 9 $\langle k_T \rangle$ in units of P as calculated by use of (6.10) and (6.11). Evidently, the values for LPP (solid line) and DIS (broken line) only begin to depart from each other at $x \approx 0.3$; they are expected to become quite different as $x \rightarrow 1$ because for LPP x is the radial variable while for DIS it is the longitudinal fraction. Hence, in the limit $x=1$, $\langle k_T \rangle$ is non-vanishing for LPP but is forced to become zero in the case of DIS for kinematical reasons. We do not plot that part of the curves in Fig. 9 because the extrapolation of (6.3) beyond the range in which the data in Fig. 6 are fitted in the determination of the parameters in (6.9) need not be reliable.

We have included in Fig. 9 also the curve for $\langle q_T \rangle / \sqrt{2}P$ as a function of $M_{\mu\mu} \sqrt{s}$, the same curve as in Fig. 6. In the limit $k_T \rightarrow 0$, x can be identified with $M_{\mu\mu} \sqrt{s}$, and $\langle k_T \rangle$ with $\langle q_T \rangle / \sqrt{2}$. Clearly, the curves in Fig. 9 indicate unequivocally that such an identification cannot be made for $x \lesssim 0.4$. Indeed, it is for that range of x that the mean angle β_0 is greater than 10° . To understand the origin of the failure of the usual connection

$$\langle q_T^2 \rangle = 2 \langle k_T^2 \rangle \quad (6.12)$$

we note that (6.12) follows from (4.25) only in the limit of negligible k_T when the phase space $d^2\vec{k}$ is reduced to a sphere and the integrals over $\vec{k}$ and $\vec{l}$ become factorizable. However, for values of $M_{\mu\mu}$ and q_T shown in Fig. 10, the ellipsoidal phase space (see Fig. 3) is far from spherical and the integrals in (4.25) are no longer factorizable; in that case (6.12) is then false.

There is another interesting feature about the difference between $\langle q_T \rangle$ and $\langle k_T \rangle$ in Fig. 9. While $\langle q_T \rangle$ stays essentially flat, $\langle k_T \rangle$ increases monotonically over the same range of x . The behaviour of $\langle k_T \rangle$ vs. x is just what was obtained in the calculations ¹⁰⁾⁻¹²⁾ based on asymptotically free gauge theory ^{13),14)}. Because the naive relation (6.12) is used in Refs. 10)-12) to infer the $\langle q_T \rangle$ behaviour, their predictions do not agree with data. Figure 9 shows that a flatter $\langle q_T \rangle$

behaviour is the more correct inference if the large k_T problem is properly treated. We stress again that by "large k_T " we mean k_T not small compared to k_L , which is just the case in the range of x under discussion.

Another interesting investigation in LPP is to calculate the q_T distribution of the dimuon inclusive cross-section. We do this at rapidity $y=0$ for which there are data ⁷⁾ that are compatible with the Drell-Yan picture. Based on the distribution function $g(r,\beta)$ already determined and without any more adjustable parameters, the results of our calculations are shown and compared with the data ⁷⁾ in Fig. 10. The agreement is striking. This renders further credibility to the use of the spherical scaling variable r and β .

A simple extension of our calculation is to predict the s dependence of $\langle q_T \rangle$. We have plotted $\langle q_T \rangle$ against $M_{\mu\mu}/\sqrt{s}$ in Fig. 11 for various values of $\sqrt{s}$. There is a plateau in each case; the value of $\langle q_T \rangle$ at the plateau is directly proportional to $\sqrt{s}$ and is found to satisfy

$$\langle q_T \rangle_{\text{plateau}} = 0.042 \sqrt{s} \quad (6.13)$$

This is a scaling result that is expected in our model.

We now consider LPA and DIF, and the associated jet structure. Since the structure function that we have determined above is for the proton only, it can at best be applied to describe proton distributions in quark jets, if universality can be extended to include reciprocity. However, data on such distributions are scanty. We shall therefore only consider pions in quark jets, beginning with the determination of $G(r,\beta,Q^2)$ in LPA. There is a dilemma one must face in the choice of data for that determination. In e^+e^- annihilation ²⁷⁾ the jet structure becomes increasingly more prominent at higher initial energy Q . But above 4 GeV there is a sizeable fraction of charm production which leads to additional pions in the final state due to jets associated with the charm quark. They are responsible for the jump in the ratio R (hadron to $\mu^+\mu^-$ cross-sections) in LPA at around $Q=4$ GeV. Since the charm quarks have not been included in our consideration of $S_{\pm}$

or $J_{\perp}$, to include them now will surely lead to a mismatch in normalizations. We are therefore forced to consider the data below $Q = 4$ GeV despite the drawback of less pronounced jet structure.

The SPEAR data ²⁷⁾ are presented in two different ways : $s \, d\sigma/dx_p$ or $s \, d\sigma/dx_{||}$ where $x_p = 2p/Q$ and $x_{||} = 2p_{||}/Q$. Our considerations in Section 4 allow us either to fit the former with (4.30) or the latter with (4.33). However, the only data available from SPEAR for $Q < 4$ GeV are at 3.0 GeV. The data from the PLUTO experiment ³¹⁾, on the other hand, start at $Q = 3.6$ GeV, and have inclusive distributions that are in agreement with those of DASP ³²⁾, but not with those of SPEAR ^{27), 33)}. Because the $x_{||}$ distribution is obtained only after applying a procedure of reconstructing the jet axis by means of Monte Carlo simulation, we have chosen to use the more direct momentum distribution in x_p at 3.6 GeV from PLUTO. We therefore use (4.30) with

$$J_+^{\pi}(p, k, q) = \frac{4}{Q^2} g'(r, \beta) \quad (6.14)$$

where $g'(r, \beta)$ is the scaling function for pions. By integrating out the angular variables, (4.30) leads to

$$\frac{1}{\sigma_T} \frac{d\sigma}{dr} = 3\pi \frac{r^2}{r_0} \int_{-1}^1 d\cos\beta \, g'(r, \beta) \quad (6.15)$$

We parametrize $g'(r, \beta)$ by a form similar to (6.3) ; in particular, we assume that the β dependence is identical to that expressed in (6.6) and (6.7). Moreover, on the basis of universality we assume that the sea quark distribution $s(r)$ is the same for the pion function $g'(r, \beta)$ as it is for the proton function $g(r, \beta)$; i.e., we use (6.4) with the same normalization. It is for this identification that we have chosen to exclude the charm production. Thus the only parameters left to fix for $g'(r, \beta)$ are those in (6.5). By fitting the PLUTO data ³¹⁾ shown in Fig. 12, we find that the best values are

$$B = 7.71 \quad , \quad C = 0 \quad (6.16)$$

with $n_V = 2$. (We have not attempted to fit with fractional values of n_V .) To sum up, we obtain

$$g'(r, \beta) = \left[0.226(1-r)^4 + 7.71 r \right] \frac{(1-r)^2}{r^2} h(r, \beta), \quad (6.17)$$

where $h(r, \beta)$ is given by (6.6), (6.7) and (6.9). The fitted curve is shown in Fig. 12 together with the separations into the $s(r)$ and $v(r)$ components.

A comparison of Fig. 12 with Fig. 4d indicates that a common sea distribution for $S_{\pm}^p$ and $J_{\pm}^{\pi}$ certainly provides a reasonable fit, but that the over-all valence to sea ratio is higher for $J_{\pm}^{\pi}$ than for $S_{\pm}^p$. The former lends support to the possibility of universality, while the latter bears no implications on universality for two reasons. First, the r dependences are different because the proton and pion have different numbers of valence quarks. Secondly, the normalizations are different because the jet function for pions includes not only the direct fragmentation of a quark into a pion but also the cascade decay of a quark into a vector meson (like ρ) and then into one or more pions. Owing to the three-spin states of a vector particle its contribution is quite significant in spite of its higher mass. For this reason the over-all normalization of $g'(r, \beta)$ is expected to be greater than that of $g(r, \beta)$. Strictly, the vector particle contribution should be subtracted out in a proper determination of the function $G^{\pi}(r, \beta, Q^2)$. But this is not possible at present.

Proceeding to the angular dependence of the jet, we note that (6.17) enables us to calculate $\langle p_T \rangle$ as a function of x_L (defined as $2p_L/Q$) for the pions in a jet in LPA. From (4.33) we obtain for every fixed x_L

$$\langle p_T \rangle = \frac{Q x_L}{2} \cdot \frac{\int d\cos\beta \cos^{-1}\beta \left[r^2 g'(r, \beta)/r_0 \right]_{x_L} \tan\beta}{\int d\cos\beta \cos^{-1}\beta \left[r^2 g'(r, \beta)/r_0 \right]_{x_L}} \quad (6.18)$$

The results of the calculation for two initial energies ($Q=7.5$ and 15 GeV) are shown in Fig. 13. It is evident from (6.18) that our scaling distribution function implies a linear dependence of $\langle p_T \rangle$ on Q . Hanson has preliminary results ³⁴⁾ of an analysis of the SPEAR data on jets at $Q=7.5$ GeV ; the $\langle p_T \rangle$ values for $x_L < 0.5$ are shown in Fig. 13 for comparison. Clearly the experimental points are much higher than

predicted, although the x_L dependences are similar. Two remarks can be made to account to a certain extent for the discrepancy. The data contain a significant contribution from charm production, whose decay distribution may well be less collimated than that of the jet. The theoretical prediction, on the other hand, is based on a scaling form whose validity is not expected at small jet momenta. The experimental reality and the theoretical limitation work against each other and could possibly contribute to the sizeable differences at $Q = 7.5$ GeV. We anticipate the experimental $\langle p_T \rangle$ to increase with Q . Whether an agreement with the predictions of universality will prevail at higher Q remains to be seen.

Finally, we consider lepton-hadron semi-inclusive reactions. Data are woefully lacking in the deep inelastic region, so phenomenology using the QPM is at present meaningless. Nevertheless, theoretical predictions can be made on the basis of the cross-section formulae developed in Section 4 and the distribution functions given by (6.3) and (6.17). We have considered two options for the jet function : no strange quarks or SU(3) symmetric sea. The semi-inclusive cross-sections for $\pi^+ + \pi^-$ are given respectively by (4.48) and (4.53). The results of our calculation of the pion distributions (in terms of the experimental quantity $z' = E_\pi/\nu$ defined in the lab frame) are shown in Fig. 14 for two sets of kinematical parameters : (a) $W = 3$ GeV and $Q^2 = 1$ (GeV/c)² corresponding to $\nu = 4.85$ GeV and $x = 0.11$, and (b) $W = 3$ GeV and $Q^2 = 10$ (GeV/c)² corresponding to $\nu = 9.64$ GeV and $x = 0.55$. Case (a) is hardly deep inelastic, but it is chosen because it corresponds to the experimental parameter of the data available ³⁵⁾. Clearly, the QPM cannot account for those data points, and should not be expected to. Recent Fermilab data ³⁶⁾ are also inapplicable for our purpose since the Q^2 range begins at 0.3 (GeV/c)² only. Case (b) is given as an example with a typical set of parameters for which the QPM should be relevant. Obviously, meaningful tests of these predictions must await better data in the deep inelastic region.

7. - CONCLUSION

We have presented the formulation and phenomenology of the quark parton model with no theoretical restrictions on the parton k_T . Although high energy phenomena at low Q^2 are consistent with the notion that k_T is limited, it has been our view that there is no evidence to support that notion at high values of Q^2 . In lepton-pair production processes, QCD calculations^{16),17)} have shown that $\langle q_T \rangle$ of the dimuons increases linearly with s for fixed $M_{\mu\mu}^2/s$. Thus, if QCD is consistent with the Drell-Yan picture²¹⁾, then the latter is useful only if the quark-parton model is generalized to allow for arbitrary large k_T . We have done this generalization without any reference to QCD so that it is valid as long as the parton picture is valid.

In formulating the large k_T problem in a Lorentz invariant way, we have introduced a new scaling variable. Together with the familiar Bjorken variable, we have found a unified kinematical description of the four related virtual-photon processes. A unified dynamical description is, of course, much more difficult to establish. Our suggestion of a universal function arises mainly out of a desire for economy in the phenomenological parametrization of those processes. The analogy with the crossing symmetric scattering amplitude mentioned in Section 5 is only suggestive, and the validity of the generalized QPM is, of course, independent of whether a simple universal function that interpolates among the four processes can be found.

There are two orthogonal aspects to the unification of the four processes: one is between timelike and spacelike processes, and the other is between the structure function and the jet function. The former has been quite satisfactorily established by our phenomenological analysis, while the latter is essentially untested because of the lack of appropriate data.

One important result of this work is the realization that the flat $M_{\mu\mu}$ dependence of $\langle q_T \rangle$ does not imply constant parton $\langle k_T \rangle$ as a function of x . We have shown that the parton $\langle k_T \rangle$ in a proton increases with x in both the spacelike and the timelike cases, but in different ways. The specific dependences of $\langle k_T \rangle$ on x are consequences of phenomenology in which the $\langle q_T \rangle$ data are an input. However, the

general conclusion that $\langle k_T \rangle$ and $\langle q_T \rangle$ behave differently follows from our more careful treatment of the large k_T problem, and is of interest independent of phenomenology.

A specific phenomenological result that may have general utility is our parametrization of the structure function for the proton at high Q^2 with not only a separation of the valence and sea components but also the angular dependence of the parton momentum. It is consistent with a wide variety of data on photon-mediated processes.

Our general conclusion is that the parton k_T is not small at high Q^2 in deep inelastic scattering or lepton-pair production processes. If the quark-parton model is to be used to describe such processes, one must abandon many of the familiar notions established for low k_T , such as the identification of Bjorken's x variable always with the longitudinal momentum fraction. We have attempted to formulate the large k_T problem in the general framework of the parton model. It provides a vehicle for further investigations on the subject complementary to the interesting studies made in the context of quantum chromodynamics (QCD).

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Processes	DIS	DIF	LPP	LPA
Distribution function	S_-	J_-	S_+	J_+
r	$x z$	$x^{-1} z$	x	x^{-1}
$\cos \beta$	z^{-1}	z^{-1}	z	z

TABLE - Relationships between (r, β) and (x, z)
for the four virtual photon processes.

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FIGURE CAPTIONS

Figure 1 Diagrams for the four virtual photon processes.

- (a) Deep inelastic scattering (DIS).
- (b) Lepton pair production (LPP).
- (c) Lepton pair annihilation (LPA).
- (d) Deep inelastic fragmentation (DIF).

Figure 2 Physical region of the four virtual photon processes.

Figure 3 Ellipsoidal phase space for lepton-pair production with $M_{\mu\mu}$ and q_T fixed and $q_L = 0$.

Figure 4 νW_2 structure function of deep inelastic scattering. $\bullet$ and $\circ$ denote data of Anderson et al. ⁶⁾, Δ denote data of Riordan et al. ⁴⁾. Curves correspond to the parametrization (6.3) to (6.7) with values of the parameters given by (6.9). In (d) the two components of νW_2 are shown.

Figure 5 Mass distribution for $\mu^+\mu^-$ production at 400 GeV/c. Data of Kaplan et al. ⁷⁾. The curve corresponds to the parametrization (6.3) to (6.7) with values of the parameters given by (6.9).

Figure 6 Average transverse momentum for $\mu^+\mu^-$ versus the mass of μ pair at 400 GeV/c. Data of Kaplan et al. ⁷⁾, and Branson et al. ³⁰⁾. The curve corresponds to the parametrization (6.3) to (6.7) with values of the parameters given by (6.9).

Figure 7 $2M W_1$ structure function of deep inelastic scattering. Data of Riordan et al. ⁴⁾. The curve is the prediction resulting from fitting the data of Figs. 4, 5, 6.

Figure 8 Ratio $R = \sigma_{\text{I}}/\sigma_{\text{T}}$ for deep inelastic scattering for various x regions. $\bullet$ denotes data of Riordan et al. ⁴⁾, $\circ$ denotes data reported by Hand ⁹⁾. The curves are the predictions resulting from fitting the data of Figs. 4, 5, 6.

Figure 9 Average transverse momentum of the parton in units of parent hadron momentum vs. x for deep inelastic scattering and lepton pair production, according to (6.10) and (6.11). Also plotted is $\langle q_T \rangle / \sqrt{2}P$ ($P = \sqrt{s}/2$) against $M_{\mu\mu}/\sqrt{s}$ for comparison.

- Figure 10 Transverse momentum distributions for $\mu^+\mu^-$ production for various $M_{\mu\mu}$ intervals at 400 GeV/c. Data from Kaplan et al. ⁷⁾. The curves are predictions resulting from the fit to the data shown in Figs. 4, 5, 6.
- Figure 11 Predictions for the behaviour of the average transverse momentum for $\mu^+\mu^-$ production as a function of $M_{\mu\mu}/\sqrt{s}$.
- Figure 12 Inclusive hadron distribution for $e^+e^- \rightarrow h^\pm X$ at $\sqrt{s} = Q = 3.6$ GeV. Data from Ref. 31). The curve corresponds to the parametrization (6.17). The two components $v(r)$ and $s(r)$ are shown.
- Figure 13 Average transverse momentum of hadrons in $e^+e^- \rightarrow h^\pm X$ vs. x_L at $Q = 7.5$ GeV. Data from Ref. 34). The curves at $Q = 7.5, 15$ GeV correspond to the parametrization (6.17).
- Figure 14 $z' = E_\pi/\nu$ distribution for pion production in deep inelastic fragmentation. Data of Berger et al. ³⁵⁾. The solid curves correspond to the assumption of no strangeness excitation. The dashed curves correspond to assuming SU(3) symmetric sea. The curves (a) correspond to $Q^2 = 1$ (GeV/c)², $W = 3$ GeV, $\nu = 4.85$ GeV and $x = 0.11$. The curves (b) correspond to $Q^2 = 10$ (GeV/c)², $W = 3$ GeV, $\nu = 9.64$ GeV and $x = 0.55$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig.8

FIG. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14