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INJECTOR MODELING
AND
ACHIEVEMENT/MAINTENANCE
OF
HIGH BRIGHTNESS

MASTER

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LAWRENCE LIVERMORE NATIONAL LABORATORY

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The presentation has three fundamental parts. In part one the need for numerical calculations is justified and the available computer codes are enumerated. The capabilities and features of the DPC computer code are the focal point in this section. In part two the injector design issues are discussed. These issues include such things as the beam optics and magnetic field profile. In part three the experimental results of two injector designs are compared with DPC predictions.

Injector modeling

I. Numerical injector models

   A. Available computer codes

   B. DPC

      a.) Field equations

      b.) Diagnostics (brightness)

II. Injector design considerations

   A. Beam optics

      a.) Cathode, anode field influence

      b.) Accelerating gradient

   B. Magnetic field profile

III. Comparisons of experiment and computer models

The brightness is defined on this viewgraph. Note that the Livermore definition has a π^2 in the numerator. In order to relate brightness to experimentally measurable quantities the transverse phase space volume V_4 is usually related to emittance. In general, the phase space density profile determines the relationship between V_4 and emittance. In the case of uniform phase space density a brightness formula can be derived.

Brightness definition

$$\mathcal{J} = \pi^2 I_z/((\gamma\beta)^2 V_4)$$

In general the phase space density profile determines the relationship between emittance and V_4.

For uniform phase space,

$$\mathcal{J} = 2 I_z/(\gamma\beta\epsilon)^2$$

where ϵ is edge emittance.

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Why high brightness?

- Operation of an FEL requires a transfer of kinetic energy to radiation

- For an optical amplifier electrons are bunched over an optical wavelength and decelerated

- Useful deceleration over a relevant gain length requires minimum parallel energy spread

- Small parallel energy spread implies small emittance or HIGH BRIGHTNESS

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The requirement of high brightness means particular care must be exercised in designing the accelerator injector. The brightness may be thought of as current divided by the radial or transverse beam temperature. In these terms an appropriate design maximizes current and minimizes temperature or emittance. The complex dependencies of the cathode properties, accelerating gradient and magnetic field profile require numerical calculations.

High brightness ⇒ injector constraints

- Brightness ~ current/radial beam temperature
- Many complex dependencies
  - Cathode
  - Accelerating potential gradient
  - Magnetic field profile (tune)
- Numerical calculations are necessary

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The available computer codes are EGUN, EBQ, DPC and MASK. The EGUN and EBQ codes have similar steady state, ray tracing algorithms. The EBQ code does somewhat better than EGUN in relativistic regimes where the radial self focus approximately balances the space charge divergence. DPC is a time-dependent particle code which solves the relativistic force equation using the Darwin field approximation. MASK is a time-dependent particle code which solves the relativistic force equation using the full electromagnetic fields. The MASK code typically runs two to three times slower than DPC.

Available computer codes

- EGUN, (W. Hermannsfeldt, SLAC)
  - Steady state, particle rays
- EBQ, (A. Paul, LLNL)
  - Steady state, particle rays
- DPC, designs with inductive effects (J. Boyd, LLNL)
  - Time dependent (Darwin), particle code
- MASK, confirmation of final designs (A. Drobot, SAI)
  - Time dependent (full E & M), particle code

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DPC (Darwin Particle Code)

- Capabilities
- Field equations
- Diagnostics (brightness)

DPC capabilities

- Runs on a CRAY 1 computer
- Typically 40,000 particles
- Resolution 80 X 200 radial, axial grid points
- Run time 5 min (preliminary), 25 - 60 min (final)
- Follows the phase space of a beam slice
- Plots of individual particle trajectories including fields and forces along trajectories

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Darwin Particle Code

1. Relativistic particle mover
2. Axisymmetric
3. Rectangular r,z grid
4. "Staircase" electrodes
5. τ_{rise} specified on electrodes
6. B_{vac} from solenoids
7. Space charge limited cathode emission
8. Inductive effects are present

DPC

FIELD EQUATIONS

Preface to equations

$$\vec{F} = \vec{F}_\ell + \vec{F}_t$$

$\vec{F}_\ell$ — Longitudinal; $\vec{F}_t$ — Transverse

DEFINITIONS:

(solenoidal) $\nabla \cdot \vec{F}_t = 0$

(irrotational) $\nabla \times \vec{F}_\ell = 0$

To find $\vec{F}_\ell$, $\vec{F}_t$ given $\vec{F}$

set $\vec{F}_\ell = -\nabla\chi$

solve $\nabla^2\chi = -\nabla\cdot\vec{F}$

$\vec{F}_t = \vec{F} + \nabla\chi$

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The consequence of the Darwin approximation is that the time derivative of the transverse electric field is deleted from Ampere's law. Consequently, in axisymmetry the magnetic field is obtained by solving two elliptic equations. The magnetic field is strictly solenoidal and thus it is not necessary to solve for an irrotational component.

Magnetic fields

$\vec{B}$ is $\vec{B}_t$ since $\nabla \cdot \vec{B} = 0$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}_\ell}{\partial t}$$

$$+ \cancel{\frac{1}{c}\frac{\partial \vec{E}_t}{\partial t}}$$

Need to solve,

$$\Delta^* \psi_A = -\frac{4\pi}{c} r J_\theta - \cancel{\frac{1}{c}\frac{E_{t\theta}}{\partial t}}$$

$$\Delta^* \psi_B = \frac{4\pi}{c} r \left(\frac{\partial J_z}{\partial r} - \frac{\partial J_r}{\partial z}\right) + \cancel{\frac{1}{c^2}\frac{\partial^2 B_\theta}{\partial t^2}}$$

$\psi_A = r A_\theta$ $\qquad\qquad$ $\psi_B = r B_\theta$

$$\Delta^* = r^2 \nabla \cdot (r^{-2} \nabla)$$

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The irrotational electric field is obtained from a Poisson equation with the usual charge density source. The solenoidal electric field is obtained from an elliptic equation with the time derivative of the solenoidal current as a source. The source term for this equation is given by a moment of the kinetic equation.

Electric fields

$$\vec{E}_\ell = -\nabla\phi,\ \nabla^2\phi = -4\pi\rho$$

$$\nabla \times \vec{E}_t = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$\nabla \times (\ \ )$

$$\nabla^2 \vec{E}_t = \frac{4\pi}{c^2}\frac{\partial \vec{J}_t}{\partial t} + \cancel{\frac{1}{c}\frac{\partial^2 E}{\partial t^2}}$$

E_θ !

Need, $\dfrac{\partial \vec{J}_t}{\partial t}$

$$\frac{\partial \vec{J}}{\partial t} = \vec{D} + \frac{q}{m}\rho\vec{E} + \frac{q}{mc}\vec{J}\times\vec{B}$$

$$\vec{D} = -\nabla\cdot\int q\vec{v}\vec{v}f\,d^3v$$

Solve, $\nabla^2\chi = -\nabla\cdot\left(\dfrac{\partial \vec{J}}{\partial t}\right)$

D P C

DIAGNOSTICS

DPC diagnostics

- Contour plots of $\phi(r,z)$, $\rho(r,z)$
- Enclosed z current $I_z(r,z)$
- $\gamma\beta_\perp$
- Electric field direction
- Brightness $\mathcal{J} = \pi^2 I_z / ((\gamma\beta)^2 V_4)$
  a) V_4 is the $(x, v_x/c, y, v_y/c)$ phase space volume containing current I_z
  b) Plots are made of $\mathcal{J}$ versus I_z, which is then the average phase space density as a function of current enclosed by V_4

The computational brightness is calculated by dividing the current by the enclosing phase space volume, V_4. The phase space volume is calculated by fitting an ellipse to the x, P_x and y, P_y phase space cross sections.

Brightness

$$\mathcal{J} = \pi^2 I_z/(\gamma^2 V_4)$$

Four space ellipsoid

$$\frac{x^2}{c_1^2} + \frac{y^2}{c_2^2} + \frac{P_x^2}{\gamma^2 c_3^2} + \frac{P_y^2}{\gamma^2 c_4^2} \leq 1$$

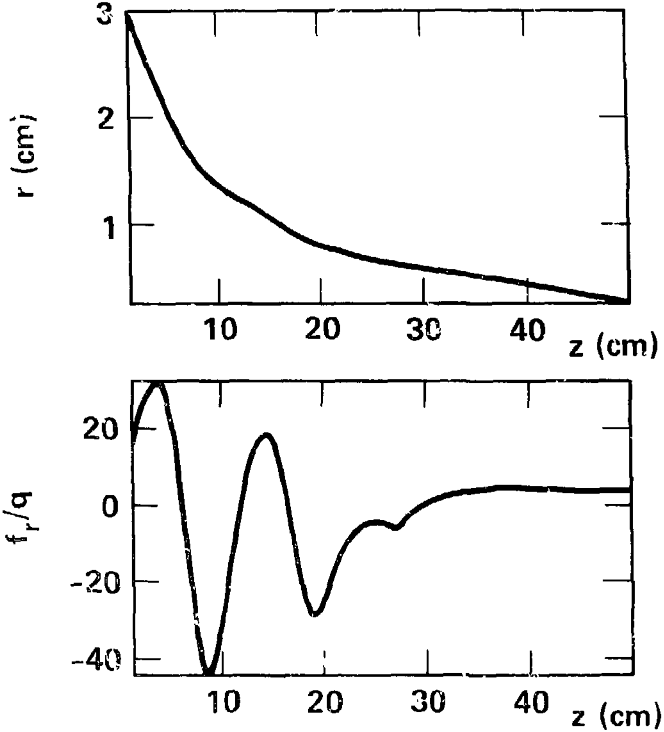
$$c_2/c_1 = \langle y^2 \rangle^{1/2} / \langle x^2 \rangle^{1/2}$$

$$c_3/c_1 = \langle (P_x/\gamma)^2 \rangle^{1/2} / \langle x^2 \rangle^{1/2}$$

$$c_4/c_1 = \langle (P_y/\gamma)^2 \rangle^{1/2} / \langle x^2 \rangle^{1/2}$$

$$V_4 = \frac{\pi^2}{2} c_1^4 \left(\frac{c_2}{c_1}\right) \left(\frac{c_3}{c_1}\right) \left(\frac{c_4}{c_1}\right)$$

The top plot shows the radius of a particle as a function of z propagation distance. This is a particle launched from a concave cathode in a triode design. The bottom plot illustrates the radial force per charge along the trajectory. Positive is focusing and negative is defocusing. This plot permits the study of the balance between the effect of the concave surface focusing and the cathode-grid and anode entrance defocusing.



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INJECTOR DESIGN CONSIDERATIONS

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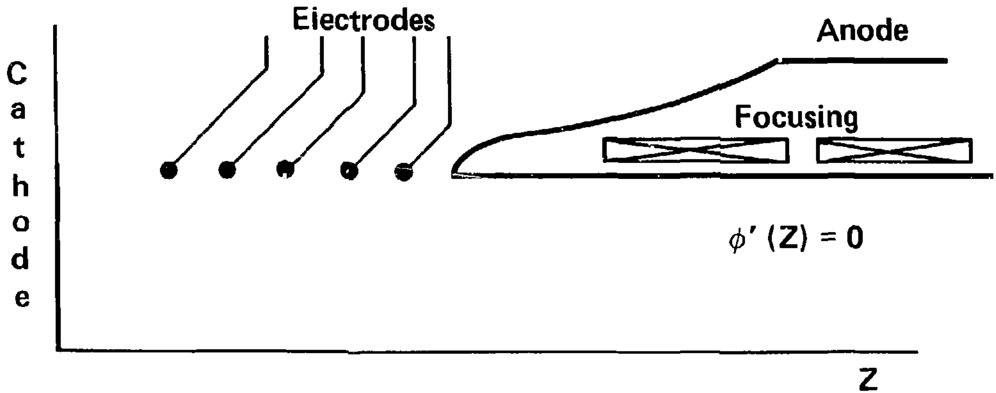
Design considerations

- Emission surface temperature
- Surface roughness
- Uniformity of emission
- Aberrations, non-linearities
- Typical injector configuration
- Effects of magnetic field tune
- Relationship of brightness to phase space

A typical injector design consists of three parts; a cathode emission surface which may be convex, flat or concave, intervening electrodes, and an anode. The cathode may have a shroud with various kinds of geometric shaping. There may be many or no intervening electrodes. The anode typically has focusing magnets to transport the beam and prevent current loss to the wall.

Typical configuration



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Effects of the magnetic field tune

- Focusing

  1) Single particle (radial force term $v_\theta B_z$)

  2) Envelope equation

- Rotation due to P_θ conservation

  1) $B_z = r^{-1} \partial r A_\theta / \partial r$

  2) $P_\theta = \gamma m r v_\theta + q r A_\theta / c$

- Equivalent emittance

  1) Emission with non-zero B limits brightness

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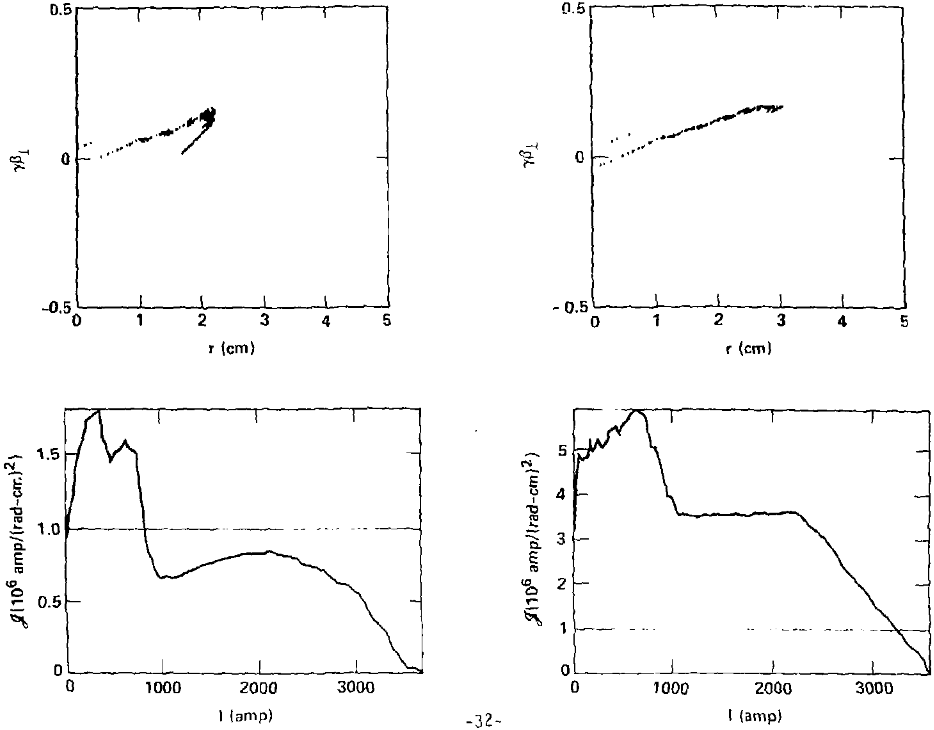
This plot shows $\gamma\beta_{\perp}$ as a function of radius and the resulting brightness curves. Note that the brightness is highest when $\gamma\beta_{\perp}$ is undistorted. This plot thus shows the degree to which edge effects are degrading brightness.

Brightness and $\gamma\beta_\perp$ versus r phase space



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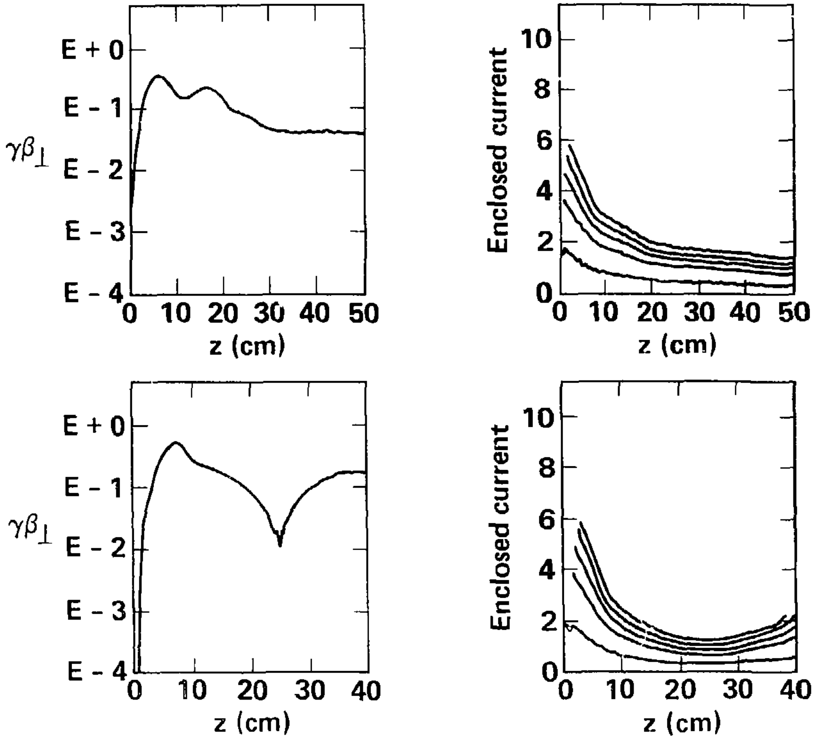
This viewgraph shows the relationship of the $\gamma\beta_\perp$ diagnostic to the behavior of the enclosed current. In the top two plots a well matched beam is seen to relax to a slow rate of convergence. The motion becomes nearly parallel in a 2 cm pipe. The bottom two plots illustrate a beam which is poorly matched. The minimum in $\gamma\beta_\perp$ indicates the radial convergence stops and divergence begins at 25 cm. This behavior is consistent with the enclosed current plot.


E + 0
E - 1
E - 2
E - 3
E - 4
$\gamma\beta_\perp$
0 10 20 30 40 50
z (cm)
Enclosed current
10
8
6
4
2
0
0 10 20 30 40 50
z (cm)
E + 0
E - 1
E - 2
E - 3
E - 4
$\gamma\beta_\perp$
0 10 20 30 40
z (cm)
Enclosed current
10
8
6
4
2
0
0 10 20 30 40
z (cm)

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COMPARISONS OF EXPERIMENT AND COMPUTER MODELS

Results

- HBTS

  A. 2 cm radius pentode

  B. Mark II triode

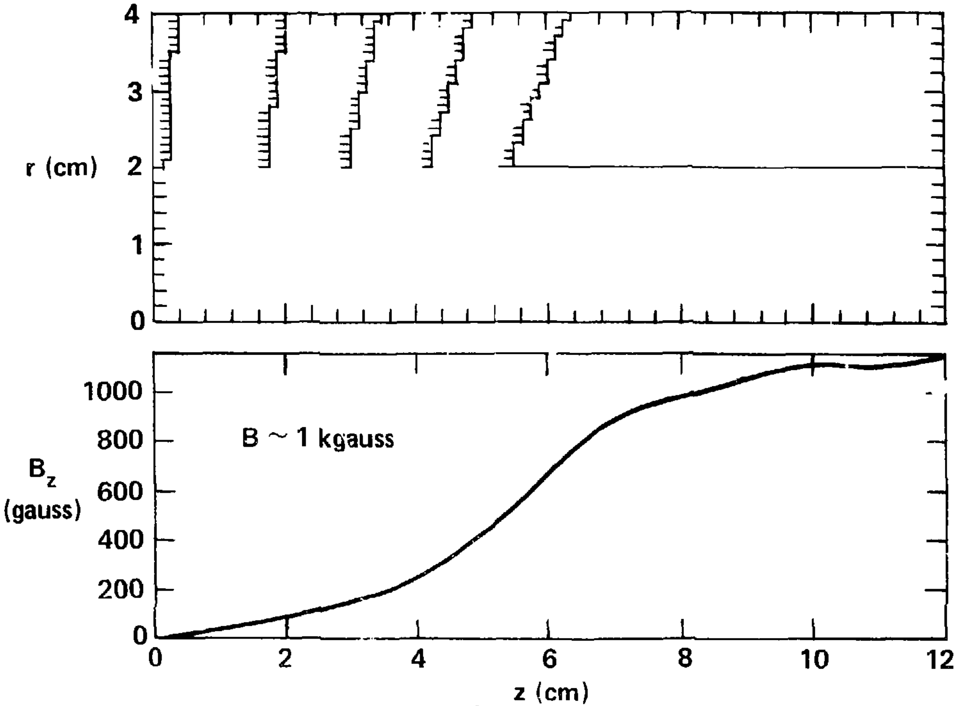
Goal:

Determine the effect of anode voltage on brightness.

Method:

Fix the electrode positions, then set $\phi(z)$ according to the acclerating potential solution.

HBTS 2 cm radius



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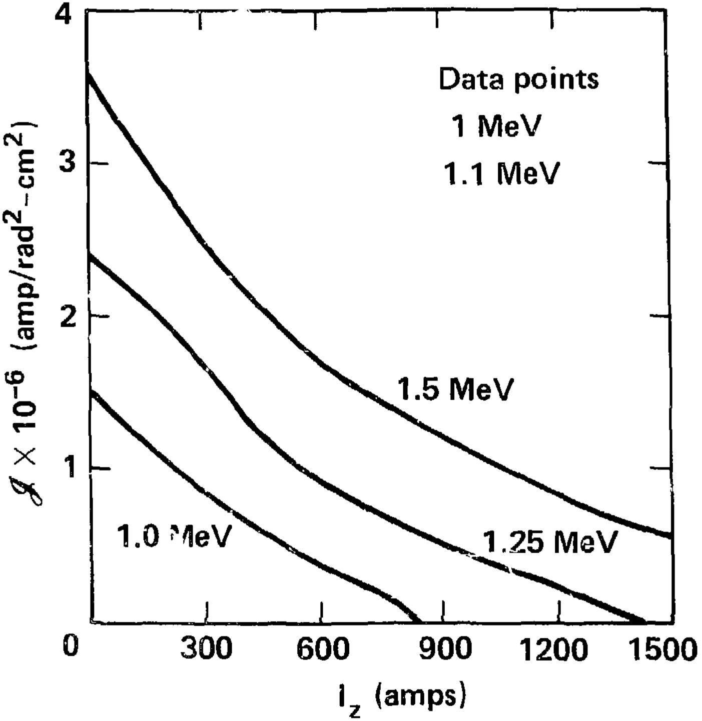
HBTS brightness



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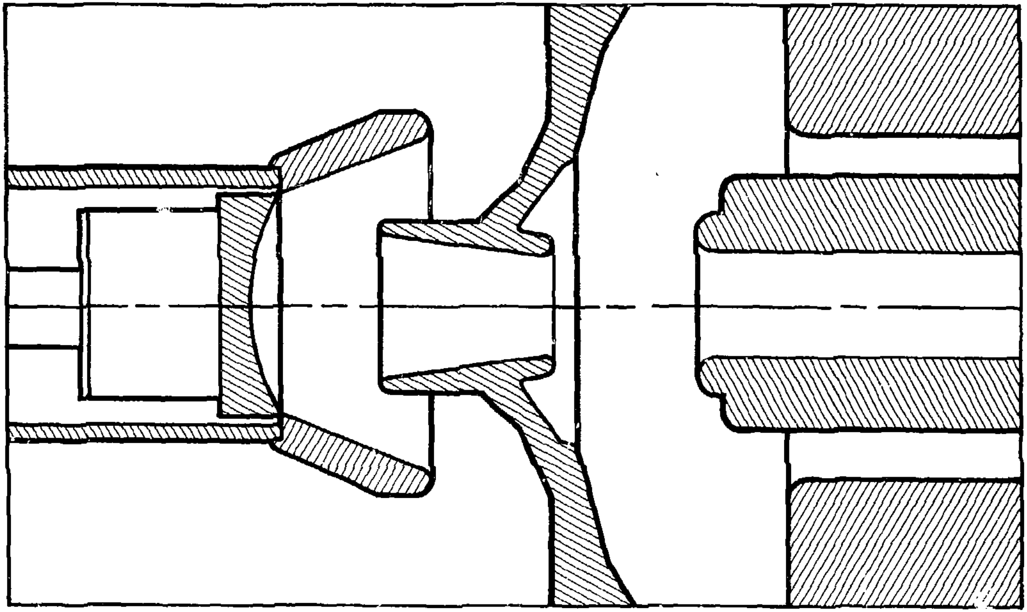
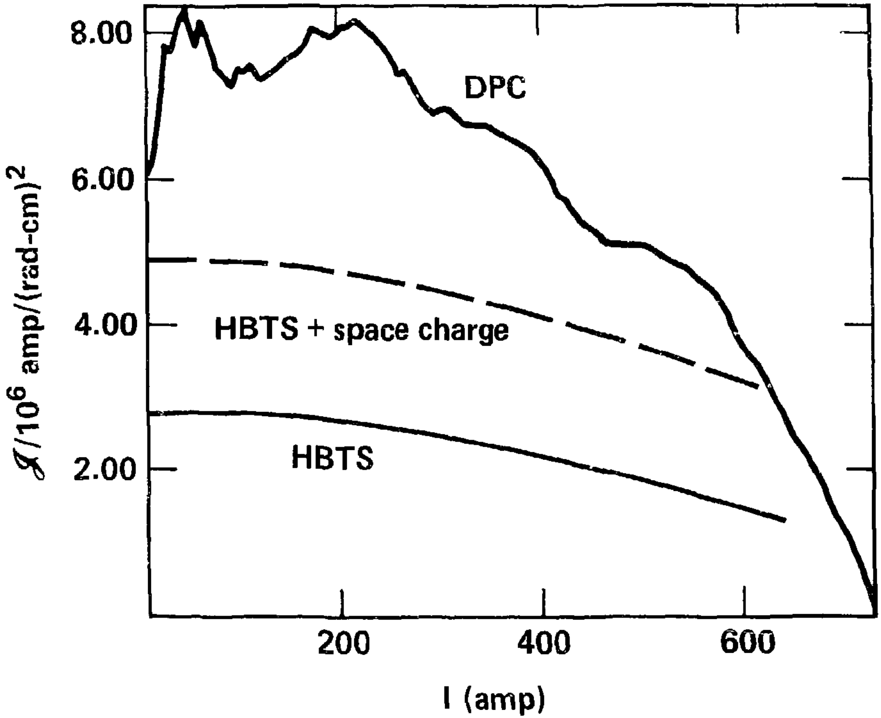
Mark II triode



Preliminary results: 3.5″ diameter dispenser cathode



8.00
6.00
4.00
2.00
$\mathcal{J}/10^6$ amp/(rad-cm)^2
DPC
HBTS + space charge
HBTS
200
400
600
I (amp)

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