BNL--42244

DE89 008494

CONF-8810333--2

Institut für Theoretische Physik der Universität Regensburg

Nov. 1988

TPR-88 34

NON-EQUILIBRIUM DYNAMICS IN FINITE-TEMPERATURE QCD*

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Abstract: I give an overview on the status of kinetic theory approaches to non-equilibrium dynamics in finite temperature QCD, and discuss results and difficulties in determining the transport properties and equilibration time scales for a quark gluon plasma produced in nuclear collisions.

* Invited talk given at the International Workshop on Thermal Field Theories and Their Applications, Cleveland, Ohio, October 3-5, 1988

[‡] Work supported in part by Deutsche Forschungsgemeinschaft, grant He1283/3-1, and by the U.S. Department of Energy under contract DE-AC02-76CH00016

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1. Introduction

In the last fifteen years or so, extensive work has been done in finite temperature field theory, mainly motivated by applications in cosmology (early universe) and in relativistic nuclear collisions. While this work has by now led to a rather coherent picture of the finite temperature formalism (both in real and imaginary time) and to a number of very important results, deviations from thermal equilibrium and the dynamics of the approach towards thermal equilibrium have received much less attention. Only recently, spurred by the onset of an experimental program to study quark gluon plasma formation in relativistic nuclear collisions, there have begun serious attempts towards a general non-equilibrium dynamical description of relativistic quantum systems, in particular with non-abelian interactions.

The dynamical behavior of a finite temperature system near equilibrium can be studied by analyzing the response of an equilibrium configuration to specific kinds of (small) external perturbations. These response functions correspond to suitable derivatives of the thermal partition function (with sources) and can be calculated by the rules of finite temperature field theory. (If one uses the imaginary time formalism, an analytic continuation to real time is required at the end to obtain the required dynamical information.) However, for systems which are genuinely out of equilibrium, the thermal partition function does not contain sufficient information, and a proper kinetic treatment is required. The object of interest in kinetic theory is the phase-space distribution function and its equation of motion. In general the latter is highly non-linear, but again for systems near equilibrium it can be linearized around the equilibrium distribution function and should yield the same information as contained in the thermal response functions just mentioned. While this has been shown to be true in some cases, there also remain open questions in other cases, and this program still remains to be completed, if only as a check for the basic correctness of the more general kinetic formulation.

In this contribution I want to review the status of relativistic kinetic theory for systems with gauge interactions and its semiclassical and linear response limits. I will show where we

have established consistency with what we know from finite temperature field theory, but also present a case (the problem of plasmon damping in a gluonic system) where the analyses based on finite temperature field theory have so far not converged to a unique answer and where clarifying information from kinetic theory may still be expected. I will not be able to present any results for truly non-equilibrium situations, simply because the relativistic kinetic theory of QCD has not yet progressed that far.

2. Quark and gluon Wigner functions

The quantum mechanical analogue of the one-particle distribution function in phase space is the Wigner function^{1,2)}, which is therefore the basic object of interest in quantum kinetic theory. In a non-relativistic theory it is essentially the Fourier transform of the density matrix. In a relativistic gauge theory like QCD or QED, the Wigner function is defined as the ensemble expectation value of the Wigner operator, where the latter is again related by Fourier transformation to an object which looks like a density operator, except for some modifications which are required by the gauge invariance of the theory. Specifically, the Wigner operator for fermions (quarks) is defined as^{3,4)} (the minus sign is convention and has to do with the fermion statistics of the quark fields ψ):

$$\hat{W}_{\alpha\beta}^{AB}(x, p) = - \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p \cdot y} \left[e^{-\frac{y}{2} \cdot D(x)} \psi(x) \right]_{\alpha}^A \left[\bar{\psi}(x) e^{\frac{y}{2} \cdot D^+} \right]_{\beta}^B, \quad (2.1)$$

where $D_{\mu}(x) = \frac{\partial}{\partial x^{\mu}} - \frac{ig}{\hbar c} A_{\mu}(x)^{\dagger}$ with $A_{\mu}(x) \equiv A_{\mu}^a(x) \frac{\lambda^a}{2}$ is the covariant derivative for the quark field operators, and $A, B = 1, 2, 3$ and $\alpha, \beta = 1, 2, 3, 4$ denote color and spinor indices. One sees that the partial derivative in the usual shift operator $\exp(\pm \frac{y}{2} \cdot \partial_x)$, which occurs in the density matrix $\psi(x - \frac{y}{2}) \bar{\psi}(x + \frac{y}{2}) = \left[e^{-\frac{y}{2} \cdot \partial_x} \psi(x) \right] \left[\bar{\psi}(x) e^{\frac{y}{2} \cdot \partial_x^{\dagger}} \right]$, here is replaced by the covariant derivative

²We here use the sign conventions for the coupling constant g as in Refs. 3 and 5 and in Itzykson and Zuber, *Quantum Field Theory*, McGraw Hill, 1980. Please note that in Refs. 4,6,7 a different sign convention ($g \rightarrow -g$) had been used.

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operator. This guarantees gauge invariance (which the usual density matrix does not possess) and ensures through Weyl's correspondence principle that the momentum argument p of the Wigner operator is the kinetic (rather than the canonical) momentum of the particles which classically obeys the mass-shell condition $p^2 = m^2$.

In Ref. 4, it was shown that (2.1) can be rewritten in terms of so-called link-operators

$$U(x', x) = P \exp \left\{ \frac{ig}{\hbar c} \int_x^{x'} dz^\mu A_\mu(z) \right\}, \quad (2.2)$$

where the path from x to x' is the straight line $z(s) = x + s(x' - x)$, $0 \leq s \leq 1$, and P is the path-ordering operator:

$$\tilde{W}(x, p) = - \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-ik \cdot y} U\left(x - \frac{y}{2}, x\right) \psi(x) \bar{\psi}(x) U\left(x, x + \frac{y}{2}\right). \quad (2.3)$$

Note that there is no path ambiguity in (2.3) because the identity of (2.1) with (2.3) holds only for the straight path in the definition of (2.2).

The underlying relationship to the quark density operator is best reflected in the expression for the quark (baryon number) current

$$b_\mu(x) = \int d^4 p \langle \text{Tr} \gamma_\mu \tilde{W}(x, p) \rangle; \quad (2.4)$$

also the quark color current is given by

$$j_\mu^a(x) = \int d^4 p \langle \text{Tr} \frac{\hbar \lambda_a}{2} \gamma_\mu \tilde{W}(x, p) \rangle. \quad (2.5)$$

Please note the formal similarity of (2.4) with the classical expression^{3,5)}

$$\begin{aligned} b_\mu(x) &= \int \frac{d^4 p}{(2\pi\hbar)^3} 2\delta(p^2 - m^2) p_\mu [\theta(p_0) f(x, p) + \theta(-p_0) \bar{f}(x, -p)] \\ &= \int \frac{d^4 p}{(2\pi\hbar)^3} 2\delta(p^2 - m^2) \theta(p_0) p_\mu [f(x, p) - \bar{f}(x, p)]. \end{aligned} \quad (2.6)$$

Here f and $\bar{f}$ are the classical distribution functions for quarks and antiquarks, and the classical mass-shell condition (which in general does not apply to the quantum mechanical quark fields described by $W(x, p)$) has been extracted explicitly; it can be combined with the momentum space integration measure to yield the more familiar expression ($E_p = \sqrt{m^2 + \vec{p}^2}$)

$$b_\mu(x) = \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{p_\mu}{E_p} [f(x, E_p, \vec{p}) - \bar{f}(x, E_p, \vec{p})] . \quad (2.7)$$

Indeed, if (2.4) is evaluated in local thermal equilibrium in the absence of any color background fields (this means that all spin effects due to couplings of the kind $\vec{\sigma} \cdot \vec{B}$ vanish), then, as $g \rightarrow 0$, (2.4) exactly reduces to (2.7).

For the gluons (or more generally the gauge field quanta) the starting point is not the gluon density operator (because gluon number density is not conserved and thus not well defined), but something more related to gluon energy density^{6)†} :

$$\hat{\Gamma}_{\mu\nu}^{ab}(x, p) = \int \frac{d^4 p}{(2\pi\hbar)^4} e^{-ik \cdot p \cdot y} \left[e^{-\frac{\hbar}{2} \cdot \mathcal{D}(x)} F_\mu^\lambda(x) \right]^a \left[e^{\frac{\hbar}{2} \cdot \mathcal{D}(x)} F_{\lambda\nu}(x) \right]^b . \quad (2.8)$$

Now the color indices a, b run from 1 to 8, and again we have used the shorthand notation $F_{\mu\lambda}(x) = F_{\mu\lambda}^a(x) \frac{\hbar\lambda_a}{2}$. The covariant derivatives occurring in the shift operators in this case are those acting on a color octet operator $O(x) = O^a(x) \frac{\hbar\lambda_a}{2}$, namely $\mathcal{D}_\mu(x)O(x) = \partial_\mu(x) - \frac{i g}{\hbar c} [A_\mu(x), O(x)] \equiv [D_\mu(x), O(x)]$. Again (2.8) can be reexpressed by link operators (2.2) as ($x_+ \equiv x + \frac{y}{2}$, $x_- \equiv x - \frac{y}{2}$)

$$\hat{\Gamma}_{\mu\nu}(x, p) = \int \frac{d^4 p}{(2\pi\hbar)^4} e^{-ik \cdot p \cdot y} [U(x, x_-) F_{\mu\lambda}(x_-) U(x_-, x)] \otimes [U(x, x_+) F_\nu^\lambda(x_+) U(x_+, x)] , \quad (2.9)$$

where the tensor product indicates that the color indices are not contracted. This gluon Wigner function is most directly related to the gluon energy-momentum tensor

$$T_{\mu\nu}(x) = \int d^4 p \langle \text{Tr} \hat{\Gamma}_{\mu\nu}(x, p) - \frac{1}{4} g_{\mu\nu} \text{Tr} \hat{\Gamma}^\lambda_\lambda(x, p) \rangle . \quad (2.10)$$

[†]To better see the parallels to the quark Wigner function, we have changed the sign of y under the integral relative to the definition in Ref. 6.

In this case the similarity to the classical expression

$$T_{\mu\nu}(x) = \int \frac{d^4 p}{(2\pi\hbar)^3} 2\delta(p^2 - m^2) \theta(p_0) p_\mu p_\nu f(x, p) \quad (2.11)$$

is less obvious; however, near thermal equilibrium and in the weak coupling limit $g \rightarrow 0$ Eq. (2.10) can be shown⁷⁾ to reduce to (2.11) (with $m = 0$ for gluons).

So far no general expression for the gluon color current in terms of $\Gamma_{\mu\nu}$ has been derived. This has to do with the fact that $\Gamma_{\mu\nu}$ is not related to a number density, but to the energy-momentum density. A limiting expression, which is valid in the semiclassical limit, will be given in section 4.

3. Kinetic equations of motion for the quark and gluon Wigner operators

Using the quark and gluon field equations given by QCD, one may derive equations of motion for the Wigner functions which resemble the Vlasov-Boltzmann equations²⁾ for the one-particle distribution function in classical kinetic theory. The derivation is lengthy due to the occurrence in the definition of the Wigner operator of the link operators (2.2) whose derivatives are not completely trivial⁴⁾:

$$\begin{aligned} \frac{\partial U(b, a)}{\partial x^\mu} = & \frac{ig}{\hbar c} \left\{ \partial_\mu b_\nu \left(A^\nu(b) - (b-a)_\alpha \int_0^1 ds s \left[e^{(s-1)(b-a) \cdot \mathcal{D}(b)} F^{\alpha\nu}(b) \right] \right) U(b, a) \right. \\ & \left. - U(b, a) \left(A^\nu(a) - (b-a)_\alpha \int_0^1 ds (s-1) \left[e^{s(b-a) \cdot \mathcal{D}(a)} F^{\alpha\nu}(a) \right] \right) \partial_\mu a_\nu \right\}. \end{aligned} \quad (3.1)$$

The result for the kinetic equations^{4,6,7)} is even more frightening because at first sight it does not appear to have any similarity with what we are used to from classical kinetic theory. However, further scrutiny then reveals that indeed in the classical limit they reduce to exactly what is

expected. We will show this in section 4, but in order to do so we first have to, unfortunately, present the somewhat lengthy complete and exact result. For quarks it reads⁸⁾

$$\begin{aligned}
p \cdot \mathcal{D}(x) \tilde{W}(x, p) = & \\
& - \frac{g}{2c} p^\mu \partial_p^\alpha \int_0^1 ds \left\{ [e^{-s\Delta} F_{\mu\alpha}(x)] \tilde{W}(x, p) + \tilde{W}(x, p) [e^{s\Delta} F_{\mu\alpha}(x)] \right\} \\
& + \frac{ig}{2\hbar c} \left\{ S^{\mu\nu} [e^{-\Delta} F_{\mu\nu}(x)] \tilde{W}(x, p) - \tilde{W}(x, p) [e^{\Delta} F_{\mu\nu}(x)] S^{\mu\nu} \right\} \\
& + \frac{ig\hbar}{4c} \partial_p^\alpha \int_0^1 ds s \left\{ [e^{-s\Delta} F_{\alpha\mu}(x)] \mathcal{D}^\mu(x) \tilde{W}(x, p) - \mathcal{D}^\mu(x) \tilde{W}(x, p) [e^{s\Delta} F_{\alpha\mu}(x)] \right\} \\
& - \frac{ig^2\hbar}{8c^2} \partial_p^\alpha \partial_p^\beta \int_0^1 ds s \int_0^1 d\bar{s} \left\{ [e^{-s\Delta} F_{\alpha\mu}(x)] \left([e^{-i\Delta} F_{\beta}{}^\mu(x)] \tilde{W}(x, p) + \tilde{W}(x, p) [e^{i\Delta} F_{\beta}{}^\mu(x)] \right) \right. \\
& \quad \left. - \left([e^{-i\Delta} F_{\beta}{}^\mu(x)] \tilde{W}(x, p) + \tilde{W}(x, p) [e^{i\Delta} F_{\beta}{}^\mu(x)] \right) [e^{s\Delta} F_{\alpha\mu}(x)] \right\} .
\end{aligned} \tag{3.2}$$

Here $S^{\mu\nu} = \frac{\hbar}{2} \sigma^{\mu\nu} = \frac{i\hbar}{4} [\gamma^\mu, \gamma^\nu]$ is the quark spin tensor, and the momentum derivative ∂_p always acts on the argument p of $\tilde{W}(x, p)$. The “triangle operator” $\Delta = \frac{i\hbar}{2} \partial_p \cdot \mathcal{D}(x)$ originates from the covariant shift operators $\exp\{\frac{y}{2} \cdot \mathcal{D}(x)\}$ after the replacement $y \rightarrow i\hbar\partial_p$ under the y -integral in (2.3). While the momentum derivative in Δ always acts on $\tilde{W}(x, p)$, the covariant spatial derivative always acts only on the field operator $F_{\mu\nu}$ immediately following it, which is indicated by the square brackets.

The analogous equation for the gluon Wigner operator reads^{6,7)}

$$\begin{aligned}
p \cdot \tilde{\mathcal{D}}(x) \hat{\Gamma}_{\mu\nu}(x, p) = & \\
& - \frac{g}{2c} p^\sigma \partial_p^\alpha \int_0^1 ds \left\{ [e^{-s\Delta} F_{\sigma\alpha}, \hat{\Gamma}_{\mu\nu}]_L + [\hat{\Gamma}_{\mu\nu}, e^{s\Delta} F_{\sigma\alpha}]_R \right\} \\
& - \frac{g}{c} \left\{ [e^{-\Delta} F_{\mu\alpha}, \hat{\Gamma}_\nu^\alpha]_L - [\hat{\Gamma}_\mu^\alpha, e^{\Delta} F_{\alpha\nu}]_R \right\} \\
& + \frac{ig\hbar}{4c} \partial_p^\alpha \int_0^1 ds s \left\{ [e^{-s\Delta} F_{\alpha\sigma}, \tilde{\mathcal{D}}^\sigma \hat{\Gamma}_{\mu\nu}]_L - [\tilde{\mathcal{D}}^\sigma \hat{\Gamma}_{\mu\nu}, e^{s\Delta} F_{\alpha\sigma}]_R \right\} \\
& + \frac{ig^2\hbar}{8c^2} \partial_p^\alpha \partial_p^\beta \int_0^1 ds s \int_0^1 d\bar{s} \left\{ [e^{-s\Delta} F_{\alpha\lambda}, [e^{-\bar{s}\Delta} F_{\beta}{}^\lambda, \hat{\Gamma}_{\mu\nu}]_L + [\hat{\Gamma}_{\mu\nu}, e^{\bar{s}\Delta} F_{\beta}{}^\lambda]_R]_L \right. \\
& \quad \left. - [[e^{-\bar{s}\Delta} F_{\beta}{}^\lambda, \hat{\Gamma}_{\mu\nu}]_L + [\hat{\Gamma}_{\mu\nu}, e^{\bar{s}\Delta} F_{\beta}{}^\lambda]_R, e^{s\Delta} F_{\alpha\lambda}]_R \right\} \\
& + \frac{ig}{2c} \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p \cdot y} \left\{ [e^{-\frac{i}{2} \cdot \mathcal{D}} (\mathcal{D}_\mu j^\lambda - \mathcal{D}^\lambda j_\mu)] \otimes [e^{\frac{i}{2} \cdot \mathcal{D}} F_{\lambda\nu}] \right. \\
& \quad \left. - [e^{-\frac{i}{2} \cdot \mathcal{D}} F_{\mu}{}^\lambda] \otimes [e^{\frac{i}{2} \cdot \mathcal{D}} (\mathcal{D}_\lambda j_\nu - \mathcal{D}_\nu j_\lambda)] \right\}.
\end{aligned} \tag{3.3}$$

Here several shorthand notations have been introduced⁶⁾: $\tilde{\mathcal{D}}$ is the covariant derivative acting on a tensor product of two colored operators $A \otimes B$ (where $A \equiv A_a \frac{\lambda_a}{2}$ etc., see (2.8/9)) and is defined by $\tilde{\mathcal{D}}(A \otimes B) = \mathcal{D}A \otimes B + A \otimes \mathcal{D}B$. Left and right commutators with such a tensor product are defined as $[O, A \otimes B]_L = [O, A] \otimes B$ and $[A \otimes B, O]_R = A \otimes [B, O]$. Finally, to avoid proliferation of square brackets, we define that the spatial derivatives of the triangle operators Δ always act only on the $F_{\mu\nu}$ field immediately following it.

It was noted by Mrówczyński¹⁰⁾ that Eq. (3.3) can be further simplified by switching to the adjoint representation. In this representation all colored operators are 8×8 (rather than 3×3) matrices, and in particular color octet operators are written as $\mathcal{O} = O_c T^c$ with matrix elements $(\mathcal{O})^{ab} = O_c (T^c)^{ab} = -i\hbar O_c f_{cab}$. In this representation the covariant derivative $\mathcal{D}_\mu$ is rewritten as

$$\mathcal{D}_\mu \mathcal{O} = (\partial_\mu - \frac{ig}{\hbar c} A_\mu) \mathcal{O} \tag{3.4}$$

where $\mathcal{A}_\mu = A_\mu^a T_a$. (Please note that the commutator in the fundamental representation (2.8) has in (3.4) been replaced by a simple multiplication.) In contrast, the covariant derivative with a twiddle, $\tilde{\mathcal{D}}_\alpha$, in (3.3) now translates into

$$\tilde{\mathcal{D}}_\alpha \hat{\Gamma}_{\mu\nu} \equiv \partial_\alpha \hat{\Gamma}_{\mu\nu} - \frac{ig}{\hbar c} [\mathcal{A}_\alpha, \hat{\Gamma}_{\mu\nu}] , \quad (3.5)$$

with a commutator between the two 8×8 matrices $(\mathcal{A}_\alpha)^{mn} = A_\alpha^a (T_a)^{mn}$ and $(\hat{\Gamma}_{\mu\nu})^{ab}$. The left and right commutators in (3.3) between fundamental generators translate into a left and right multiplication with adjoint generators, e.g.,

$$\begin{aligned} [O, \hat{\Gamma}_{\mu\nu}]_L &= \left(\mathcal{O} \hat{\Gamma}_{\mu\nu} \right)^{ab} t_a \otimes t_b , \\ [\hat{\Gamma}_{\mu\nu}, O]_R &= \left(\hat{\Gamma}_{\mu\nu} \mathcal{O} \right)^{ab} t_a \otimes t_b . \end{aligned} \quad (3.6)$$

Noting that also in the triangle operator Δ the covariant derivative $\mathcal{D}_\mu$ has to be interpreted as in (3.4), we thus obtain

$$\begin{aligned} p \cdot \tilde{\mathcal{D}}(x) \hat{\Gamma}_{\mu\nu} = & \\ & - \frac{g}{2c} p^\sigma \partial_p^\alpha \int_0^1 ds \left\{ [e^{-s\Delta} \mathcal{F}_{\sigma\alpha}] \hat{\Gamma}_{\mu\nu} + \hat{\Gamma}_{\mu\nu} [e^{s\Delta} \mathcal{F}_{\sigma\alpha}] \right\} \\ & - \frac{g}{c} \left\{ [e^{-\Delta} \mathcal{F}_{\mu\alpha}] \hat{\Gamma}_\nu^\alpha - \hat{\Gamma}_\mu^\alpha [e^{\Delta} \mathcal{F}_{\alpha\nu}] \right\} \\ & + \frac{ig\hbar}{4c} \partial_p^\alpha \int_0^1 ds s \left\{ [e^{-s\Delta} \mathcal{F}_{\alpha\sigma}] \tilde{\mathcal{D}}^\sigma \hat{\Gamma}_{\mu\nu} - \tilde{\mathcal{D}}^\sigma \hat{\Gamma}_{\mu\nu} [e^{s\Delta} \mathcal{F}_{\alpha\sigma}] \right\} \\ & + \frac{ig^2\hbar}{8c^2} \partial_p^\alpha \partial_p^\beta \int_0^1 ds s \int_0^1 d\bar{s} \left\{ [e^{-s\Delta} \mathcal{F}_{\alpha\lambda}] \left([e^{-\bar{s}\Delta} \mathcal{F}_\beta^\lambda] \hat{\Gamma}_{\mu\nu} + \hat{\Gamma}_{\mu\nu} [e^{\bar{s}\Delta} \mathcal{F}_\beta^\lambda] \right) \right. \\ & \quad \left. - \left([e^{-\bar{s}\Delta} \mathcal{F}_\beta^\lambda] \hat{\Gamma}_{\mu\nu} + \hat{\Gamma}_{\mu\nu} [e^{\bar{s}\Delta} \mathcal{F}_\beta^\lambda] \right) [e^{s\Delta} \mathcal{F}_{\alpha\lambda}] \right\} \\ & + \frac{ig}{2c} \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p \cdot y} \left\{ [e^{-\frac{s}{2}\Delta} \mathcal{D}(\mathcal{D}_\mu j^\lambda - \mathcal{D}^\lambda j_\mu)]^a \otimes [e^{\frac{s}{2}\Delta} \mathcal{F}_{\lambda\nu}]^b \right. \\ & \quad \left. - [e^{-\frac{s}{2}\Delta} \mathcal{F}_\mu^\lambda]^a \otimes [e^{\frac{s}{2}\Delta} (\mathcal{D}_\lambda j_\nu - \mathcal{D}_\nu j_\lambda)]^b \right\} . \end{aligned} \quad (3.7)$$

This is an 8×8 dimensional matrix equation with components (ab) ; in the last term it is explicitly indicated that its (ab) component is obtained by considering both factors of the tensor product

as 8-dimensional color vectors and taking the a 'th component of the first vector times the b 'th component of the second one.

One now clearly sees the formal similarity of the quark and gluon equations (3.2) and (3.7). The only difference is the last line of Eq. (3.7) which couples the gluon Wigner operator explicitly to the quark color current j_μ and thus (via (2.5)) to the quark Wigner operator. A similar coupling of the quark kinetic equation (3.2) to the gluon Wigner operator is not yet obvious and has to emerge at a later stage from a more detailed analysis of the structure of these equations.

4. Semiclassical and mean-field approximation

The analysis of the kinetic equations from the previous section is still in its infancy. Guided by the original classical approach^{3,5)}, most of the presently available work has concentrated on a combination of a semiclassical expansion with the mean-field approximation. In the mean-field approximation all gluon fields are replaced by their mean-field values in the quark kinetic equation (3.2), and in the gluon kinetic equation (3.7) the gluon Wigner operator is split into a mean-field part $\bar{\Gamma}_{\mu\nu}$ and a fluctuating part $\hat{G}_{\mu\nu}$, where again in the equation of motion for $\hat{G}_{\mu\nu}$ all gluon fields influencing the dynamics of $\hat{G}_{\mu\nu}$ are replaced by their mean-field values. It is important to note that this procedure breaks the gauge invariance of the formalism: Elitzur's theorem forbids the appearance of a gauge-invariant vacuum expectation value $\langle A_\mu \rangle$, hence the separation $\hat{A}_\mu = \langle A_\mu \rangle + \hat{a}_\mu$ has to be gauge dependent. Since transformations of $\langle A_\mu \rangle$ from one gauge to another involve the coupling constant g , results in the lowest order of g (linear response approximation) can be expected to be still gauge invariant. However, at higher orders in g , keeping gauge invariance becomes a non-trivial problem also in the kinetic formulation. We will discuss below how this will affect the issue of the damping rate for QCD plasmons.

Writing $\hat{A}_\mu = \bar{A}_\mu + \hat{a}_\mu$ (where $A_\mu = \langle \bar{A}_\mu \rangle$) and denoting by $\bar{F}_{\mu\nu}$ and $\bar{\Gamma}_{\mu\nu}$ the contributions to the field strength tensor and to the gluon Wigner function coming entirely from the mean field $\bar{A}_\mu$ (i.e., all $\hat{A}$'s are replaced by $\bar{A}$'s), we can define the fluctuating part of the gluon Wigner operator $\hat{G}_{\mu\nu}$ by

$$\hat{G}_{\mu\nu} = \hat{\Gamma}_{\mu\nu} - \bar{\Gamma}_{\mu\nu} .$$

This has a non-vanishing expectation value $\langle \hat{G}_{\mu\nu} \rangle$ (because $\langle \hat{\Gamma}_{\mu\nu} \rangle \neq \bar{\Gamma}_{\mu\nu}$) which we call $G_{\mu\nu}$, the gluon Wigner function.

The definition of the mean field reads

$$\bar{D}_\mu \bar{F}^{\mu\nu} = \frac{g}{\hbar c} (\langle j^\nu \rangle_q + \langle j^\nu \rangle_g) , \quad (4.1)$$

where $\langle j^\nu \rangle_q$ is the average quark color current given by Eq. (2.5), and $\langle j^\nu \rangle_g$ is the average color current generated by the gluon fluctuations.

As mentioned before, writing $\langle j^\nu \rangle_g$ in terms of $G_{\mu\nu}$ is not trivial, and a general expression has not yet been found. Elze⁷⁾ has given arguments that near thermodynamic equilibrium $G_{\mu\nu}$ should have the structure

$$G_{\mu\nu}^{ab}(x, p) = p_\mu p_\nu G^{ab}(x, p) \quad (4.2)$$

where $G^{ab}(x, p)$ is a Lorentz scalar. Indeed, (4.2) is explicitly satisfied in thermodynamic equilibrium. In this case

$$\begin{aligned} \langle j_a^\nu \rangle_g &= \frac{g}{\hbar c} \int d^4 p \, p^\nu \text{tr}(T_a G(x, p)) \\ &= -\frac{ig}{c} \int d^4 p \, p^\nu f_{abc} G^{cb}(x, p) . \end{aligned} \quad (4.3)$$

The presence of spin effects may change this expression. Elze⁷⁾ showed that in the semiclassical limit (to be discussed below) the expression (4.3) is consistent with (4.1), i.e., $\bar{D}_\nu (\langle j^\nu \rangle_q + \langle j^\nu \rangle_g) = 0$.

The semiclassical expansion is an expansion of (3.2), (3.7) in powers of $\hbar$. Since we have consistently kept all factors of $\hbar$ up to here, we can do this step in a systematic way. It is

important to note that each power of the operator $\Delta = \frac{i\hbar}{2}\partial_p \cdot D(x)$ contributes one factor of $\hbar$. This is true, although the covariant derivative $D_\mu = \partial_\mu - \frac{ig}{\hbar c} [A_\mu, \cdot]$ at first sight appears to be troublesome: the factor $\frac{1}{\hbar}$ in the non-Abelian term is cancelled by a factor $\hbar$ from the commutator, because we defined color operator always in terms of the physical color generators, $t_a = \frac{\lambda_a}{2}$ and $(T_a)_{bc} = -i\hbar f_{abc} T_c$ whose commutators $[t_a, t_b] = i\hbar f_{abc} t_c$ and $[T_a, T_b] = i\hbar f_{abc} T_c$ contribute a factor $\hbar$ and thus vanish in the classical limit $\hbar \rightarrow 0$, keeping $\langle t_a \rangle$, $\langle T_a \rangle$ finite⁵⁾.

The validity of the semiclassical limit thus requires

$$\langle \Delta F \hat{W} \rangle = \langle DF \cdot \partial_p W \rangle \ll \langle F \hat{W} \rangle, \quad (4.4)$$

and similarly for $\hat{\Gamma}$, i.e., the color fields $F_{\mu\nu}$ have to be slowly varying in space, the non-Abelian terms from the covariant derivative have to be small ($g \ll 1$ or nearly Abelian fields), and the Wigner function has to have a smooth momentum dependence. Unless these requirements are fulfilled, the full equations (3.2), (3.7) have to be solved, and we have gained nothing. It is an unresolved question how the expansion in powers of Δ and the usual perturbation expansion in QCD in powers of g are related; this is presently being investigated¹¹⁾.

Were it not for the mean-field approximation, an expansion in powers of Δ would be gauge invariant order by order. The gauge-symmetry breaking by the mean-field approximation is particularly crucial for the gluonic sector, because the very definition of the fluctuating “gluon” part (as opposed to the average “mean-field” contribution) is affected by it. To obtain gauge invariant results, the expansion should therefore be performed on the operator level, before ensemble averages are taken and the mean field is separated off.

Up to now, only the lowest order of this expansion has been studied. Taking expectation values, neglecting correlations by writing, e.g., $\langle F \hat{W} \rangle = \bar{F}(\hat{W})$ etc., and separating the Wigner

function $W = \langle \tilde{W} \rangle$ into its positive frequency ("quark") and negative frequency ("antiquark") part^{2,3)}, we find at this order in the quark sector^{3,4)}

$$p^\mu \mathcal{D}_\mu(x) f(x, p) + \frac{g}{2c} p^\mu \partial_p^\nu \left\{ F_{\mu\nu}(x), f(x, p) \right\} - \frac{ig}{2\hbar c} \left[S^{\mu\nu} F_{\mu\nu}(x), f(x, p) \right] - \frac{g}{4c} \left\{ S^{\mu\nu} D_\sigma(x), F_{\mu\nu}(x) \partial_p^\sigma f(x, p) \right\} = C ; \quad (4.5a)$$

$$p^\mu \mathcal{D}_\mu(x) \tilde{f}(x, p) - \frac{g}{2c} p^\mu \partial_p^\nu \left\{ F_{\mu\nu}(x), \tilde{f}(x, p) \right\} + \frac{ig}{2\hbar c} \left[S^{\mu\nu} F_{\mu\nu}(x), \tilde{f}(x, p) \right] + \frac{g}{4c} \left\{ S^{\mu\nu} D_\sigma(x), F_{\mu\nu}(x) \partial_p^\sigma \tilde{f}(x, p) \right\} = \tilde{C} , \quad (4.5b)$$

where f and $\tilde{f}$ are the quark and anti-quark Wigner functions. The "collisions terms" C and $\tilde{C}$ stand for everything that has been left out in this approximation, i.e., all terms of higher order in $\hbar$ and all correlations. These terms still have to be studied¹¹⁾. Please note that in the second line of (3.2), $e^{-\Delta}$ had to be expanded up to the linear term to collect all terms of order $\hbar^0$. (The $1/\hbar$ in the coefficient of the third term of eqs. (4.5a,b) is cancelled by an $\hbar$ coming from the color commutator between $F_{\mu\nu}(x)$ and $f(x, p)$.) It is not hard to check that (4.5a,b) are fully consistent with the classical equations of motion for particles with color and spin derived in Refs. 5, 12.

For the gluons, the procedure is similar, although complicated by the separation of the mean-field contribution. Again neglecting all correlations (in particular the terms in the last line of (3.7) which couple to the quark current drop out in this way) one arrives at the following equation^{6,7,10)} (at lowest order in $\hbar$):

$$p^\sigma \tilde{\mathcal{D}}_\sigma G_{\mu\nu} + \frac{g}{2c} p^\sigma \partial_p^\tau \left\{ \mathcal{F}_{\sigma\tau}, G_{\mu\nu} \right\} + \frac{g}{c} (\mathcal{F}_{\mu\alpha} G_\nu^\alpha - G_\mu^\alpha \mathcal{F}_{\alpha\nu}) = C_g .$$

If one assumes that the Lorentz-structure of $G_{\mu\nu}$ is given by Eq. (4.2) (near equilibrium this assumption is justified), we can contract the Lorentz indices $(\mu\nu)$ and obtain an equation¹⁰⁾ for the Lorentz scalar function $G^{at}(x, p)$:

$$p^\sigma \tilde{\mathcal{D}}_\sigma(x) G(x, p) + \frac{g}{2c} p^\sigma \partial_p^\tau \left\{ \mathcal{F}_{\sigma\tau}(x), G(x, p) \right\} = C_g . \quad (4.5c)$$

If one neglects the spin terms in (4.5a,b) one notices that the semiclassical gluon kinetic equation (4.5c) has exactly the same structure as the semiclassical quark equations (4.5a,b). Possible contributions analogous to the last two terms in eqs. (4.5a,b), involving the gluon spin, have been eliminated by the ansatz (4.2). The gluon spin sector requires more study.

5. Linear response theory, plasma oscillations, and their damping

Equations (4.5) can be solved in linear approximation around global thermal equilibrium^{13,7,10)}:

$$\begin{aligned} f(x, p) &= n(p)1 + \delta f(x, p) ; \\ \bar{f}(x, p) &= \bar{n}(p)1 + \delta \bar{f}(x, p) ; \\ G(x, p) &= n_g(p)1 + \delta G(x, p) ; \end{aligned} \tag{5.1}$$

where $n(p), \bar{n}(p), n_g(p)$ are the usual Fermi–Dirac and Bose–Einstein distributions for quarks, antiquarks, and gluons. Since in global thermal equilibrium there are no color fields and thus a gauge can be chosen such that also the vector potentials vanish in equilibrium, $F_{\mu\nu}$ and A_μ can be considered as small quantities, too. The procedure is outlined in more detail in Mrówczyński's contribution to this volume¹⁰⁾, so I will not repeat it here.

Perturbing the equilibrium by a small external $F_{\mu\nu}$ and leaving out the collision terms in the kinetic equations, one finds response functions for both the longitudinal and transverse components¹³⁾ which are completely real in the region of timelike momenta $\omega^2 > k^2$ and have an imaginary part for $\omega^2 < k^2$ which is due to Landau (mean-field) damping. Both response functions have a pole in the timelike region, namely at $\omega_L^2 = \omega_0^2 + \frac{3}{5}k^2 + \mathcal{O}(k^4)$ and $\omega_T^2 = \omega_0^2 + \frac{6}{5}k^2 + \mathcal{O}(k^4)$, where $\omega_0^2 = g^2 T^2 (N_f + 2N)/18$ is the QCD plasma frequency known from one-loop calculations in FT-QCD. These poles correspond to collective longitudinal and transverse plasma oscillations,

and in the absence of collision terms they are completely undamped. The result is independent of the gauge (due to using a gauge invariant kinetic framework as a starting point and using an approximation scheme which to this order in the perturbation preserves the gauge invariance — see comments in the previous section). The above dispersion relations agree with the leading term from a high-temperature expansion for the poles of the one-loop FT-QCD gluon propagator, which is also known to be gauge invariant.

Of course, there exist physical mechanisms which can contribute an imaginary part to the dispersion relation, like pair decay of the perturbation into massless quark-antiquark or gluon pairs, or higher order scattering processes. Depending upon the sign of this imaginary part, the plasma oscillations will be either damped or unstable. The underlying mechanisms are contained in the (so far neglected) collision terms which unfortunately have not yet been sufficiently analyzed. In the framework of FT-QCD, the imaginary part resides in the non-leading terms of a high-temperature expansion for the poles of the response function, which have recently been analyzed in a series of papers (see a separate contribution to this volume for references). Unfortunately, the response function defining plasma oscillation has so far not been written down in a generally accepted, gauge invariant way, and as a result its imaginary part (and thus the plasmon damping rate) is plagued by gauge dependence. (This gauge dependence does not show up in the leading term of a high-temperature expansion, but crucially affects the next-to-leading term.) The different results obtained up to now do not even agree in sign!

Clearly this shows that there is a severe problem in our understanding the nature of QCD plasma oscillations within FT-QCD. The different approaches tried so far include both old-fashioned linear response theory^{14,15)} and the method of the thermal partition function with sources (“effective action”)^{16,17)} mentioned in the introduction. Within FT-QCD these methods have not reached mutual consistency, i.e., it is not known how they translate into each other (which might

shed some light onto the origin of the discrepancies). I believe that starting from the gauge invariant kinetic framework outlined above, the discrepancies may be eventually resolved. For this to happen, the kinetic equations have to be analyzed beyond the classical limit; in particular, the effect of correlations has to be investigated, because they will be responsible for entropy generation and thus also for plasmon damping. The major stumbling block to preserving gauge invariance in this approach is the separation of gluon fluctuations from the mean field, and more work is needed to more clearly understand this step. However, I am confident that these problems can be overcome, thereby clarifying the unsatisfactory present situation.

As a final goal, I have the vision of a complete set of translation rules between finite temperature field theory (FTFT) concepts and the kinetic transport theory near equilibrium, thereby casting some of the abstract notions of FTFT into the more intuitively understandable language of classical plasma physics. Having such a dictionary at hand, one may then try to generalize FTFT away from equilibrium, using non-equilibrium kinetic theory as an instructive guiding line.

Acknowledgement

I would like to thank the organizers of the Workshop of Thermal Field Theories and Their Applications, Cleveland, Oct. 3-5, 1988, in particular Ken Kowalski, for their support, and Brookhaven National Laboratory, where I prepared this contribution, for its hospitality.

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