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TIME VARYING EDDY CURRENTS ON A
CONDUCTING SURFACE IN 3-D USING
A NETWORK MESH METHOD

BY

U. R. CHRISTENSEN

**PLASMA PHYSICS
LABORATORY**

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Time-Dependent Electric Currents in a
Conducting Surface in 1-D using
a Network Mesh Method

by H.P. Christensen

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Time Varying Eddy Currents on a Conducting Surface in 3-D using a Network Mesh Method

Abstract

The method presented in this paper was developed for the purpose of analyzing the eddy currents in the TFTR vacuum vessel.

The basic principle in this method lies in representing a conducting surface as a network comprised of a number of branches. Each branch has a resistance and a self-inductance as well as mutuals to all other branches. The resulting branch resistance and branch inductance matrices are transformed into mesh matrices by a conventional network procedure. By using these mesh matrices a set of simultaneous differential equations is then established which, expressed in matrix form, is $[L]\{\dot{I}\} + [R]\{I\} = \{V\}$, where $\{V\}$ is the time dependent voltage vector resulting from time-varying external current sources.

The eddy currents in the above equation are then found by using a standard method for solving simultaneous differential equations.

External current sources, which vary sinusoidally in time will result in complex eddy currents and may be found by solving for I in $[R + j\omega L]\{I\} = \{V\}$.

Introduction

The primary interest of this study was to find the forces on the TFTR vacuum vessel and to determine the field perturbation that the eddy currents would produce.

During regular start-up of the plasma there are three external sources that would induce eddy currents in the vacuum vessel. These sources are the currents in the OH and EF windings and the plasma current itself. It is also of interest to find the eddy currents caused by a sudden plasma disruption.

The results of using this technique on the vacuum vessel of the TFTR are shown in a separate report.¹

Included in this report is also a method to obtain the frequency response of the vacuum vessel at various frequencies when excited by a AC plasma source. This information may be useful to the people in charge of diagnostics.

1. Discretizing the Conducting Surface

The major problem in developing this method, was in devising a scheme that would accurately yield the inductances of the branch elements. This problem was solved by dividing the conducting surface up into two sets of rectangular plates. One of the sets will carry the U-component of the eddy current and the other set will carry the orthogonal V-component.

An example of how the two sets of plates cover a given area is shown in Figure 1. The sides of the plates are chosen to be $1/3$ of the sides of the full area. The plates that carry the current in the U-direction, the U-plates, are arranged as shown in Figure 1a with three plates in the V direction and two plates in the U direction. The arrangement of the V-plates is seen in Figure 1b. When the two systems of plates are superimposed their respective centerlines form a system of meshes as shown in Figure 1c. We will then have four meshes consisting of 12 branches. If one associates a resistance and an inductance with each branch, we will then have a lumped parameter network. Many surfaces with irregular boundaries can be discretized by orthogonal curvilinear coordinates where the boundaries coincide with the curves $u = \text{const.}$ and $v = \text{const.}$

2. Calculating the Branch Inductance and Resistance Matrix

Figure 2a shows four elementary current-carrying plates which overlap to form what shall be termed an "active mesh." Figure 2b shows an arrangement of four plates identical to those described, except that these plates carry no currents and are, therefore, termed a "passive mesh." The respective center lines of the four plates form the cross-hatched rectangular area. The mutual inductance between two meshes is defined as the flux passing through the rectangular area of the four "passive" plates produced by unit current flowing in the four "active" plates. The self-inductance of a mesh may be thought of as a superposition of the "active" and "passive" plates. Thus, the self-inductance of a given mesh may be defined as the flux passing through the common rectangular area, when the four "active" plates each carry one ampere.

Instead of calculating the mesh inductances directly, we chose to calculate the branch inductances because of the resulting simplicity. These branch inductances are actually fragments of the mesh inductances and are obtained by numerically integrating the magnetic vector potential, due to unit current flowing in an active plate, along the center line of a passive plate.

To this end, a subroutine MUPLFL* was written to calculate the partial inductance by supplying six spacial coordinates. Four coordinates define the vertices of the active plate and the remaining two define the end-points of the passive centerline.

In Figure 3 we see a single plate, whose corners are labeled with a's, and a center line whose end points are p_1 and p_2 of another plate. The first plate, referred to by the index "j", is active, i.e., it carries a current of one ampere. The center line of the passive plate is indexed with "i". Using the nomenclature shown in Figure 3, the partial mutual inductance or the branch inductance is given by:

$$L_b(i,j) = \int_{p_1}^{p_2} \vec{A}(\vec{p}) \cdot d\vec{l} \quad (1)$$

where $\vec{A}(\vec{p})$ is the magnetic vector potential. (See Appendix I.)

In the case of self-inductance the center line of the passive plate coincides with the center line of the active plate, i.e.,

$$\vec{p}_1 = (\vec{a}_1 + \vec{a}_4)/2 \quad \text{and} \quad \vec{p}_2 = (\vec{a}_2 + \vec{a}_3)/2. \quad (2)$$

* Mutual inductance between a Plate and Filament.

If the two plates have a distance and an angle between them where their contribution to the mutual inductance may be predictably smaller than a given number, then the branch inductance may then be calculated by the much simpler formula

$$L_b(i,j) = 10^{-7} \frac{(\vec{p}_2 - \vec{p}_1) \cdot (\vec{a}_2 - \vec{a}_1)}{0.5|(\vec{p}_2 + \vec{p}_1) - (\vec{a}_2 + \vec{a}_1)|} \quad (3)$$

The branch resistance in all cases is simply

$$R_b(i,j) = \frac{\rho}{t} \frac{|\vec{a}_2 - \vec{a}_1|}{|\vec{a}_3 - \vec{a}_2|} \delta_{jk} \quad (4)$$

The matrix defined by Equation (4) is a diagonal matrix and is, in the actual calculations, stored as a vector for space saving purposes.

In problems where the stream pattern has symmetries, the size of the inductance matrix can be reduced considerably. This advantage will be discussed later.

It must be pointed out that the branch inductance matrix obtained by this procedure is asymmetric. The reason for this is that we are not dealing with mutuals between filamentary currents but with mutuals between plates with distributed currents.

3. Calculating the Mesh Eddy Currents

The transient eddy currents are found in terms of mesh currents. These are obtained by solving a set of simultaneous differential equations by conventional computer codes such as, the Runge-Kutta or the Predictor-Corrector methods.

The differential equation to be solved in matrix notation is

$$[L]\{\dot{I}\} + [R]\{I\} = \{V\} \quad (5)$$

where $\{V\}$ is the induced mesh voltage vector from external current sources. These voltages can be found by

$$\{V\} = -[L_x]\{\dot{i}_x\} \quad (6)$$

where $[L_x]$ is a matrix containing the mutuals of the external currents to all the meshes and $\{\dot{i}_x\}$ is the time varying external current vector.

Various techniques for solving differential equations all require the time derivative of the mesh currents and they are determined by solving for $\{\dot{I}\}$ in equation (5), i.e.

$$\{\dot{I}\} = [L]^{-1}\{\{V\} - [R]\{I\}\}. \quad (7)$$

Before proceeding to calculate the mesh currents, we must first transform the two branch matrices $[L_b]$ and $[R_b]$ into the mesh matrices $[L]$ and $[R]$ by

$$[L] = [M][L_b][M]^T \quad (8)$$

and

$$[R] = [M][L_b][M]^T \quad (9)$$

where $[M]$ is the mesh incidence matrix.²

Since the mesh incidence matrix is extremely sparse, an unfortunate waste of computer storage space will result. An algorithm has been developed which avoids this problem by constructing an especially dense form of the incidence matrix. (See Appendix II.)

In equation (6) the elements in the inductance matrix $[L_x]$ are the mutuals between the external circuits and all the eddy current meshes. Since these mutuals may be difficult to calculate the following expression for $[L_x]$ may be useful

$$[L_x] = [M][L_{bx}]. \quad (10)$$

Here again $[M]$ is the incidence matrix and the elements in the matrix $[L_{bx}]$ are the mutuals between the external circuits and all the branches of the conducting surface.

4. Calculating Power, Energy and Field Perturbation

The branch currents are the actual eddy currents flowing in the plates and they are found by

$$\{I_b\} = [M]^T \{I\}. \quad (11)$$

There will be two sets of branch currents, those flowing in the u-plates and those flowing in the v-plates.

The dissipative power, the stored magnetic energy, and the field perturbation which are all due to the eddy currents may be determined in the following manner.

The dissipative power can be found by either

$$P = \sum_{k=1}^{Nb} i_b^2(k) r_b(k) \text{ or } P = \{I\}^T [R] \{I\}, \quad (12)$$

where Nb is the number of branches and $r_b(k)$ is the resistance of an element in the branch matrix.

The magnetic energy can be found by

$$U = \frac{1}{2} \{I\}^T [L] \{I\}. \quad (13)$$

The field perturbation at a given point is found by summing, at the point in question, the field contribution from all of the U-Y plates. In general, for points that are more than a few 'plate sizes' away from the surface, it is sufficient to have the branch currents flow in finite length filaments that are positioned at the center lines of the plates.

On the conducting surface the flux density normal to the surface, averaged over each mesh, is

$$\{B\} = [1/s][L]\{I\} \quad (14)$$

where $[1/s]$ is a diagonal matrix whose elements are the inverse areas of the meshes. These flux densities are due to eddy currents only. The flux density vector resulting from the external current sources is

$$\{B_x\} = [1/s][L_x]\{I_x\} \quad (15)$$

5. Plotting the Stream Lines

It is interesting to note that the mesh currents are invariant in respect to the size of the meshes, and that they are numerically equal to the electric scalar potential T ,^{3, 4}. Once these potentials are known the stream lines can be plotted as equipotentials of T or along constant mesh current values. The linear current density, $\vec{k}$, is related to the potential by

$$\vec{k} = \vec{n} \times \vec{\nabla} T \quad (16)$$

and in our discretized model the U and V components of k are

$$k_u = I_b(u)/b_v \quad \text{and} \quad k_v = I_b(v)/b_u \quad (17)$$

where b_v is the width of a plate carrying current in the v direction and similarly for b_u .

As mentioned above, the branch currents are given by $I_b = M^T I$ which is simply the difference in currents between the two adjacent meshes that form the branch. This difference in mesh currents is the equivalent to taking the gradient of T .

In 3-D problems T becomes a vector and has the same physical meaning as the magnetization vector M . It can be visualized as a host of small circulating currents that partially cancel each other at their interface.

In this case T is defined by $\vec{j} = \vec{\nabla} \times \vec{T}_v$ where the v stands for volume. The dimension of T is (A) and of T_v (A/m). T_v , when multiplied by the thickness of the surface, becomes T .

6. Symmetry Condition

The size of the matrices can be reduced by taking advantage of symmetry. To achieve a symmetry in the stream-line pattern of eddy currents three conditions of symmetry must be satisfied:

There must be at least 1) one plane of symmetry of the conducting surface that is in common with 2) a plane of symmetry of the externally imposed field pattern which in turn must be in common with 3) a plane of symmetry of the resistivity pattern.

Figure 4a illustrates an end view of a conducting plate and the field distribution of an externally imposed B-field. Note that the $Y = 0$ plane is a common plane of symmetry for both the conducting plate and the flux pattern.

Figure 4b illustrates a front view of a conducting plate with two sets of eddy current stream-lines. It can be seen that the $Y = 0$ plane and the $X = 0$ plane both serve as common planes of symmetry for the conducting plate and the stream-line pattern. Figure 4b also reveals that there are two types of lines of symmetry:

- 1) The Y-axis, in addition to being an obvious line of symmetry, is also an equipotential line because the stream-lines are all perpendicular to it.
- 2) The line of symmetry formed by the X-axis is different in nature since the coincident stream-lines are tangent to it. A cut along the X-axis will, therefore, not alter the flow pattern.

For surfaces in three dimensions, the lines of symmetry become planes of symmetry or mirrors. Similar to the two types of symmetry lines, there are two types of symmetry planes or mirrors. We designate the plane of symmetry where the stream-line flows tangentially as an s-mirror, and the equipotential plane where the stream-lines are perpendicular as an e-mirror. Figures 4c and 4d illustrate that an s-mirror produces a true image since, upon reflection, both the spatial distribution as well as the direction of a stream-line are preserved. Reflections produced by an e-mirror preserve the spatial distribution but reverse the stream-line direction.

A problem such as the one described by Figure 4, requires that the U and V plates, or in this case the X and Y plates, be constructed in the first quadrant only. The mutual inductance between two plates is then arrived and its three symmetrical plates, and then summing their four contributions to the inductance along the centerline of the second plate in the first quadrant. The same procedure is used for the self-inductances. From the above symmetry consideration, the signs of the four contributions must first be altered. Figure 5a shows the signs of the inductances that the subroutine MUPLLEL returns. Figure 5c shows the signs that the inductances must have in accordance with Figure 4d. To obtain these signs Figure 5b indicates the necessary multiplier. As indicated the signs are reversed when the images are formed by an s-mirror and remain unchanged for an e-mirror.

The rule by which a conducting surface is discretized is illustrated in Figure 6. If the surface is a strip bounded by two s-lines then the widths of the plates are arranged as in Figure 6a. The s-line could be either an edge of the surface or a line of symmetry. If the strip is bounded by an e-line and an s-line then the widths are arranged as in Figure 6b. The length of the plates is established by the rule set forth in Figures 6c and 6d.

Two examples will show in detail how to handle problems with planes of symmetry. Figure 7a shows a rectangular sheet with a hole. If the external field normal to the sheet is such that

$$B(-X,Y) = B(-X,-Y) = B(X,-Y) = B(X,Y) \quad (18)$$

then the principle axes are equipotential lines or e-lines. Such a field could be produced by a number of coils co-axial with the Z-axis. The edges of the sheet form natural stream-line boundaries and are therefore as indicated, s-lines. In following the rule illustrated in Figure 6, the X and Y plates will be distributed as shown in Figure 7b and 7c. Figure 7d shows a network model of the quarter sheet. Note that the hole counts as an additional mesh and that branches 9 and 18 carry a current equal to the net circular flow. From this network we can now write the incidence mesh matrix. The conventional form for such a matrix is

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	-1	0	0	1	0	0	0	0	0	-1	1	0	0	0	0	0	0	0
2	0	-1	0	0	1	0	0	0	0	0	-1	1	0	0	0	0	0	0
3	0	0	-1	0	0	1	0	0	0	0	0	-1	1	0	0	0	0	0
4	0	0	0	0	0	-1	1	0	0	0	0	0	0	-1	1	0	0	0
5	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	-1	1	0
6	0	0	0	-1	-1	0	0	0	1	0	0	0	0	1	0	1	0	-1

Since this matrix becomes extremely sparse when dealing with large numbers of branches and meshes, a very dense type of incidence matrix was developed.

In the dense matrix, the elements are the branch numbers that line up with a '1' or '-1' of the original incidence mesh matrix. All signs, i.e. (+) or (-), appearing in the incidence mesh matrix are transferred without change. By inspection one can see that the matrix below contains the same information as the conventional incidence matrix.

1	-1	4	-10	11	0	0
2	-2	5	-11	12	0	0
3	-3	6	-12	13	0	0
4	-6	7	-14	15	0	0
5	-7	8	-16	17	0	0
6	-4	-5	9	14	16	-18

An algorithm using the dense matrix for calculating the mesh inductance is shown in Appendix II.

The plate shown in Figure 6a need not be a plane surface but could just as well be a part of a spherical, a cylindrical or any other surface that obeys the following condition of symmetry

$$z(-x, -y) = z(x, -y) = z(-x, y) = z(x, y). \quad (19)$$

After calculating and storing the vertices of the plates the elements of the branch inductance matrix can be found by using subroutine MUPLFL. The structure of this matrix will be as follows:

		<div style="display: flex; justify-content: space-around; align-items: center;"><div style="border: 1px solid black; padding: 2px;">→</div><div>ACTIVE</div><div style="border: 1px solid black; padding: 2px;">↑</div></div>																	
	j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
<div style="display: flex; flex-direction: column; align-items: center;"><div style="border: 1px solid black; padding: 2px;">→</div><div>PASSIVE</div><div style="border: 1px solid black; padding: 2px;">↑</div></div>	i	1							0	0	0								0
	2								0	0	0								0
	3								0	0	0								0
	4								0	0	0								0
	5				A				0	0	0				B				0
	6								0	0	0								0
	7								0	0	0								0
	8								0	0	0								0
	9								0	0	0								0
	10								0	0	0								0
	11								0	0	0								0
	12								0	0	0								0
	13								0	0	0								0
	14				C				0	0	0				D				0
	15								0	0	0								0
	16								0	0	0								0
	17								0	0	0								0
	18								0	0	0								0
		E																	

The row index (j) of the 18 by 18 matrix represents the active plates, and column index (i) represents the passive plates. The matrix is divided into four sub-matrices. The sub-matrices A and B define, respectively, the mutuals between the X-plate and X center lines and between the Y-plates and X-lines, and similarly for sub-matrices C and D. This arrangement is caused by the way the branches were numbered in the network. If the conducting surface was a plane figure then the elements in sub-matrices B and C would be zero. The elements in columns 8, 9, 10, and 11 are all zero and represent the branches that make up the two equipotential lines. The resistance for these branches is also zero.

In the second problem we use the same rectangular sheet as in Figure 6a and change only the condition of symmetry of the external field, e.g.,

$$B(-x,y) = -B(-x,-y) = -B(x,-y) = B(x,y) . \quad (20)$$

This type of field could be produced by a coil in the $y = 0$ plane with a diameter on the z -axis. With this excitation the y -axis is an equipotential line and the x -axis a stream-line boundary. The center lines of the plates are drawn in the quarter sheet shown in Figure 8a in accordance with the rule stated earlier. Figures 8b and 8c show the x and y plates and Figure 8d shows the resulting network. By inspection one can see that the net current crossing any cut from the outside to the hole is zero. The dense incidence matrix for the problem is:

1	-1	4	-8	9
2	-2	5	-9	10
3	-3	6	-10	11
4	-6	7	-12	13

and the structure of the resulting branch inductance matrix can be seen below

		i												
		<div>→ Active</div> <div>↑</div>												
i		1	2	3	4	5	6	7	8	9	10	11	12	13
1									0					
2									0					
3									0					
4									0					
5				A					0		B			
6									0					
7									0					
8									0					
9									0					
10									0					
11					C				0			D		
12									0					
13									0					

Passive

→

↑

7. Steady State AC Currents

In many eddy current problems, if not most, the external sources are alternating currents. The resistance and inductance mesh matrices are first found as in the transient case and are thereafter used in the following matrix equation

$$[R + j\omega L]\{\underline{I}\} = \{\underline{V}\} \quad (21)$$

when $\{\underline{I}\}$ and $\{\underline{V}\}$ are phasor vectors.

Without using complex variables Equation (21) can be written as

$$\begin{bmatrix} R & -\omega L \\ -\omega L & R \end{bmatrix} \begin{Bmatrix} I_{re} \\ I_{im} \end{Bmatrix} = \begin{Bmatrix} V_{re} \\ V_{im} \end{Bmatrix} \quad (22)$$

By solving for "I" in this matrix equation one obtains the mesh eddy currents and their phase relation to the voltage phasors $\{\underline{V}\}$. The real and the imaginary parts of $\{V\}$ are

$$\begin{aligned} \{V_{re}\} &= [\omega L(x)]\{I(x)_{im}\} \\ \{V_{im}\} &= -[\omega L(x)]\{I(x)_{re}\} \end{aligned} \quad (23)$$

8. Conclusion

By modeling the conducting surface by an electrical network it is possible to utilize established methods of network analysis. The eddy current profiles for transients are easily obtained by a simple matrix multiplication at each time-step and solution to problems with A.C. sources are readily obtained.

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Appendix I

The Vector Potential from a Rectangular Plate at a given Point

If the field point is at the origin and r is the radius-vector to an element on the plate ($r^2 = x^2 + y^2 + z_0^2$) then the vector potential

$$\vec{A} = \frac{1}{4\pi} \mu_0 \vec{k} \int_S \frac{ds}{r} = 10^{-7} \vec{k} \iint \frac{dxdy}{r} = 10^{-7} \frac{\hat{x} I}{y_2 - y_1} a$$

For the integral 'a' we find

$$a = \int_{y_1}^{y_2} \int_{x_1}^{x_2} \frac{dxdy}{(x^2 + y^2 + z_0^2)^{1/2}} =$$

$$\sum_{n=1}^4 (-1)^{n+1} [x_n \ln(y_n + r_n) + y_n \ln(x_n + r_n) - z_0 \arctan(\frac{y_n x_n}{z_0 r_n})]$$

See figure 9 for the definition of the variables.

Appendix II

Algorithm for using the dense incident matrix.

The conventional incident matrix is used in the four matrix equations (8), (9), (10), and (11). The algebra in performing these multiplications is simple and straightforward, this is however not the case when the conventional incident matrix is replaced by the dense incident matrix. The following algorithm indicates how these operations are to be performed.

The definitions of the variables used in the algorithm are:

A	-	Mesh current
AB	-	Branch current
L	-	Mesh inductance (real)
LB	-	Branch inductance (real)
LX	-	Mesh mutual to external circuit (real)
LBX	-	Branch mutual to external circuit (real)
NM	-	Number of meshes
NB	-	Number of branches
NX	-	Number of external circuits
ND	-	Number of branch numbers in the dense incident matrix
IDMX	-	Dense incident mesh matrix

1. Transform external branch mutual to
external mesh mutual.

For 'IDMX' in conventional form

$$[LX] = [IDMX][LBX]$$

NM×NX NM×NB NB×NX

For 'IDMX' in dense form

```

      REAL LX, IBX
C
      DO 100 I=1,NM
      DO 100 J=1,NX
      LX(I,J)=0.0
      DO 100 N=1,NB
      IDN=IABS(IDMX(I,N))
      IF(IDN.EQ.0) GC TC 100
      IDS=ISIGN(1,IDMX(I,N))
100  LX(I,J)=LX(I,J)+IDS*LBX(IDN,J)

```

2. Transform mesh current to branch current.

For 'IDMX' in conventional form

$$\{AB\} = [IDMX]^T \{A\}$$

NB×1 NB×NM NM×1

For 'IDMX' in dense form

```

      DATA AB/C.0/
C
      DO 200 I=1,NM
      DO 200 N=1,NB
      IDN = IABS(IDMX(I,N))
      IF(IDN.EQ.0) GC TC 200
      IDS = ISIGN(1,IDMX(I,N))
200  AB(IDN)=AB(IDN)+IDS*A(I)

```

3. Transform branch resistance to mesh resistance.

For 'IDMX' in conventional form

$$[R] = [IDMX][RB][IDMX]^T$$

NM×NM NM×NB NB×NB NB×NB

For 'IDMX' in dense form

```

DO 330 J=1,NM
DO 330 I=1,J
R(J,I)=0.0
DO 330 N=1,ND
IDNI=IABS(IDMX(I,N))
IF(IDN.EQ.0) GO TO 330
DO 310 NN=1,ND
IDNJ=IABS(IDMX(J,NN))
IF(IDNJ-IDNI) 310, 300, 330
300 IDSJ=ISIGN(1,IDMX(J,NN))
GO TO 320
310 CONTINUE
GO TO 330
320 IDSI=ISIGN(1,IDMX(I,N))
R(J,I)=R(J,I)+IDSI*RB(IDNI)*IDSJ
IF(I.NE.J) GO TO 330
330 CONTINUE
C   FILL IN UPPER HALF
MF=NM-1
DO 340 I=1,MF
MB=I+1
DO 340 J=MB,NM
340 R(I,J)=R(J,I)

```

4. Transform branch inductance to mesh inductance.

For 'IDMX' in conventional form

$$[L] = [IDMX][LB][IDMX]^T$$

NM×NM NM×NB NB×NB NB×NM

For 'IDMX' in dense form

```

      REAL LB,L
C
      DO 410 IM=1,NM
      DO 410 JM=1,NM
      SUML=0.0
      DO 400 I=1,NE
      KI=IDMX(IM,I)
      DO 400 J=1,NE
      KJ=IDMX(JM,J)
      IF (KI.EQ.0.OR.KJ.EQ.0) GO TO 400
      SIG=ISIGN(1,KI*KJ)
      SUML=SUML+SIG*LB(IABS(KI),IABS(KJ))
400 CONTINUE
410 L(IM,JM)=SUML

```

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Figure Captions

The figure captions are referenced within the text.

(a) U - Plates

(b) V - Plates

(c) Meshes

Fig. 1. 794024

Fig. 2. 794026

Fig. 3. 794025

TWO TYPES OF SYMMETRIES

Fig. 4. 794023

- (a) Signs as Returned by Subroutine
- (b) Multiplier
- (c) Desired Signs of Inductances

Fig. 5. 784427

Fig. 6. 784405

Fig. 7. 784406

(a)

(b)

(c)

(d)

Fig. 8. 784407

Legend Used In Calculating The
Vector Potential.

Fig. 9. 794020

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 Res. Institut. of Elect. App., USSR

EXPERIMENTAL

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