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Kinetic Theory and Boundary Conditions for
Flows of Highly Inelastic Spheres.

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I. SUMMARY

Our second quarter milestone was to reparameterize the solutions obtained by Richman and Marciniec (J. Appl. Mech., Vol. 57, No. 4, pp. 1036-1043, 1990) for steady, fully developed, gravity-driven, granular flows down bumpy inclines. The purpose of the reparameterization was to follow the results presented by Johnson, et. al. (J. Fluid. Mech., Vol. 210, pp. 501-535, 1990) by predicting the dependence of mass flow rate on mass hold-up. With this dependence established, it would be possible to compare, in a qualitative manner, the theoretical results obtained by Richman and Marciniec [1990], to the experimental results obtained by Johnson et. al. [1990].

In this quarter, we completed this milestone and discuss a specific case below. In addition, we have begun to modify existing boundary conditions to account for the effects of boundary vibration on the granular flows with which they interact. A preliminary calculation will be completed next quarter, and will be described in that report.

II. STEADY, FULLY DEVELOPED, GRAVITY-DRIVEN, GRANULAR FLOWS DOWN BUMPY INCLINES: REPARAMETERIZATION

Of interest here are steady, fully developed, gravity-driven flows of smooth spheres of diameter σ and coefficient of restitution e down bumpy inclined surfaces. The vertical acceleration due to gravity is g , the angle between the incline and the horizontal is ϕ , and the perpendicular distance from the incline to the free surface is L . We introduce an x_1 - x_2 - x_3 Cartesian coordinate system such that x_1 measures parallel distances along the incline, x_2 measures perpendicular distances above the incline, and x_3 is the direction in which the flows are infinitely extended. In these flows, the solid fraction ν , the dimensionless velocity $u = u_1/(\sigma g)^{1/2}$, and the dimensionless measure $w = (T/\sigma g)^{1/2}$ of the granular temperature T depend only on the dimensionless coordinate $y = (L - x_2)/\sigma$. The inclined boundary is a flat surface to which smooth hemispherical particles of diameter d are randomly spaced at a distance s apart. The coefficient of restitution between the boundary and the flow particles is e_w .

The parameters that crudely characterize these flows are the dimensionless mass flow rate m ,

$$\dot{m} = \int_0^\beta \nu u dy \quad , \quad (1)$$

the depth-averaged solid fraction $\bar{\nu}$,

$$\bar{\nu} = \frac{1}{\beta} \int_0^{\beta} \nu dy \quad , \quad (2)$$

the dimensionless depth-averaged velocity $\bar{u}$,

$$\bar{u} = \frac{1}{\beta} \int_0^{\beta} u dy \quad , \quad (3)$$

and the dimensionless depth $\beta=L/\sigma$. The mass hold-up M_T is simply the product $\beta\bar{\nu}$.

In their analysis of these flows, Richman and Marciniec [1990] replaced $\nu(y)$ by $\bar{\nu}$ wherever it occurred in the balance equations. In this manner, they obtained a closed form expression for $w(y)$ that was an approximate solution of the energy equation. The profile $w(y)$ was expressed in terms of modified Bessel functions ("I-solutions") when the boundary supplied fluctuation energy to the flow, and in terms of Bessel function ("J-solutions") when the boundary absorbed energy from the flow. For fixed values of e , e_w , σ/d , s/d , and ϕ , if the value $\bar{\nu}$ is prescribed, then the solution procedure employed by Richman and Marciniec [1990] determines the profiles of $\nu(y)$, $u(y)$, and $w(y)$, and therefore the values of β , $\bar{u}$, M_T , and $\dot{m}$.

Here, we use parameter values used by Richman and Marciniec [1990]: $e=.8$, $e_w=.95$, $\sigma/d=1/2$, $s/d=-1+\sqrt{2}$, and $\phi=20.04^\circ$. For these values, exactly one steady, fully developed solution exists for each value of $\bar{\nu}$ between 0 and .6. Because, over this range of depth-averaged solid fraction, the boundary must supply energy to the flow, each of these solutions is an "I-solution." To each value of $\bar{\nu}$ in the range, there corresponds a value of β , M_T , $\dot{m}$, and $\bar{u}$. It is then possible to study the dependence of any one of these parameters on any other.

In Figure 1, we follow the manner in which Johnson et. al. [1990] presented their experimental results and show the variation of $\dot{m}$ with M_T . In Figures 2, 3, and 4 we show the corresponding variations of $\bar{\nu}$, β , and $\bar{u}$. For small values of $\bar{\nu}$, the value of β varies with $1/\bar{\nu}$, the values of $\bar{u}$ and $\dot{m}$ vary with $1/\bar{\nu}^{1/2}$, and the value of M_T is nonzero but finite. According to Figure 2, as $\bar{\nu}$ increases from zero to .04, M_T increases from .047 to .10, and according to Figures 1, 3, and 4, each of $\dot{m}$, β , and $\bar{u}$ decrease dramatically and begin to increase more gradually. As $\bar{\nu}$

continues increases from .04 to .27, the mass hold-up M_T increases to its maximum value of 3.89, while $\dot{m}$, β , and $\bar{u}$ each increase to their maximum values (of 50.0, 14.3, and 13.6) and then begin to decrease. As $\bar{v}$ increases beyond .27, the mass hold-up decreases to 3.27 (at $\bar{v}=.50$), and then increases slightly to 3.37 (at $\bar{v}=.6$). Simultaneously, $\dot{m}$, β , and $\bar{u}$ monotonically decrease to 4.55, 5.61 and 1.40.

Interestingly, for values of $\dot{m}$ between 4.55 and 50.0, an extremely dilute ($\bar{v}<.04$), a moderately dilute, and a relatively dense flow are possible. As the flow rates increase within this range of $\dot{m}$, the moderately dilute flow becomes denser, the relatively dense flow becomes more dilute, and both become deeper and faster. Typically, for the same flow rate, the moderately dilute flow is both deeper and faster than the relatively dense flow that also might occur.

FIGURES

Figure 1: $\dot{m}$ vs. M_T for $e=.8$, $e_w=.95$, $\sigma/d=1/2$, $s/d=-1+\sqrt{2}$, and $\phi=20.04^\circ$.

Figure 2: $\bar{\nu}$ vs. M_T for $e=.8$, $e_w=.95$, $\sigma/d=1/2$, $s/d=-1+\sqrt{2}$, and $\phi=20.04^\circ$.

Figure 3: β vs. M_T for $e=.8$, $e_w=.95$, $\sigma/d=1/2$, $s/d=-1+\sqrt{2}$, and $\phi=20.04^\circ$.

Figure 4: $\bar{u}$ vs. M_T for $e=.8$, $e_w=.95$, $\sigma/d=1/2$, $s/d=-1+\sqrt{2}$, and $\phi=20.04^\circ$.

I solution $\phi=20.04^\circ$

Figure 1

Figure 2

Figure 3

Figure 4

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