MEASUREMENT OF THE LONGITUDINAL PARAMETERS OF AN ELECTRON BEAM
IN A STORAGE RING*

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1. Introduction

We discuss the determination of the longitudinal parameters of a bunched beam of electrons or positrons circulating in a storage ring. From the analysis of the beam current observed at a fixed azimuthal location, one can learn much about the longitudinal behavior. We present an elementary analysis of the time-dependence of the current. In particular, we discuss the determination of the average current, bunch length, synchrotron oscillation frequency, and the coherent synchrotron oscillation modes associated with longitudinal instabilities. A brief discussion is also given of the incoherent synchrotron oscillations, or Schottky noise.

We review the electromagnetic field traveling with a charge in uniform motion, and introduce some of the most common devices used to detect this field: capacitive pick-up, stripline monitor, and DC beam current transformer. The use of synchrotron radiation as a diagnostic is of the greatest importance, but we do not consider it in this article. It has been discussed by other lecturers at this school [1].

Our paper is organized as follows: In Section 2, we discuss the analysis of the time-dependence of the beam current. In Section 3, the measurement of the current is considered. Finally, in Section 4, we describe some measurements of energy spread and bunch lengthening made recently at SLAC on the SLC damping ring [2].

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2. Analysis of Beam Current Measurements

A. Beam Current Observed at Fixed Azimuth

A great deal of information about the longitudinal behavior of an electron beam circulating in a storage ring can be determined from the measurement of the time-dependence of the beam current $I(t)$ observed at a fixed azimuthal position [3]. We define $I(t)dt$ to be the charge passing the given azimuth during time interval dt . Consider a single bunch containing N electrons circulating with angular revolution frequency $\omega_0 = 2\pi/T_0$, where T_0 is the revolution period. In this case, the beam current $I(t)$ can be expressed as

$$I(t) = eN \sum_{n=-\infty}^{\infty} g(t - nT_0). \quad (2.1)$$

Here, the charge of an individual electron is represented by $e < 0$, and the density profile of the bunch is described by $g(t)$.

At this point in our discussion we are approximating the bunch of discrete electrons by a continuous fluid whose density profile is described by $g(t)$. It is useful to introduce the Fourier transform:

$$\tilde{g}(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} g(t), \quad (2.2)$$

and to employ the normalization

$$\tilde{g}(0) = 1. \quad (2.3)$$

For example, one could consider a Gaussian bunch profile,

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2\sigma^2}, \quad (2.4)$$

whose Fourier transform is

$$\tilde{g}(\omega) = e^{-\omega^2 \sigma^2 / 2}. \quad (2.5)$$

We note that the bunch length $\Delta t = \sigma$, and the width of the Fourier transform $\Delta\omega = 1/\sigma$, hence $\Delta t \Delta\omega = 1$. This equality, of course, does not hold in general, but in the cases of interest to us, $\Delta t \Delta\omega \approx 1$. In particular, as

the bunch length Δt is reduced, the bandwidth $\Delta\omega$ of the Fourier transform of the bunch profile increases.

In the absence of a coherent synchrotron oscillation of the bunch, the beam current $I(t)$ is given by Eq. (2.1). Since in this case $I(t) = I(t + T_0)$ is periodic, it has the Fourier expansion

$$I(t) = \sum_{n=-\infty}^{\infty} I_n e^{in\omega_0 t}, \quad (2.6)$$

where

$$I_n = \frac{1}{T_0} \int_0^{T_0} dt I(t) e^{-in\omega_0 t}. \quad (2.7)$$

From Eqs. (2.1) and (2.7) it is seen that

$$I_n = I_0 \tilde{g}(n\omega_0), \quad (2.8)$$

where the average current I_0 is given by

$$I_0 = Ne/T_0, \quad (2.9)$$

and $\tilde{g}(n\omega_0)$ is the Fourier transform [Eq. (2.2)] of the bunch profile, evaluated at the n th harmonic of the revolution frequency. In the special case of a Gaussian bunch,

$$I_n = I_0 e^{-n^2 \omega_0^2 \sigma^2 / 2}. \quad (2.10)$$

When $n \ll T_0/2\pi\sigma$, we see that $I_n \approx I_0$. Hence, the average current can be determined by measuring I_n for small n . To determine the bunch length σ , one must measure I_n for large $n \sim T_0/2\pi\sigma$.

If there is more than one bunch in the storage ring, the reader can easily verify that to determine the average current one should measure the Fourier coefficient I_n at an harmonic of the bunch separation frequency. In particular, a measurement of the Fourier coefficient at the RF frequency $\omega_{RF} = h\omega_0$ will yield the average current in the ring quite accurately, with little sensitivity to the bunch length or the number of bunches as long as $h \ll T_0/2\pi\sigma$.

B. Synchrotron Oscillations

Let us now consider a single bunch circulating in the storage ring executing a coherent synchrotron oscillation [4]. We still approximate the bunch of discrete individual electrons by a continuous fluid with profile $g(t)$. In this case, the beam current observed at fixed azimuth is

$$I(t) = e N \sum_{n=-\infty}^{\infty} g\left[t - nT_0 + A \cos(\omega_s t + \phi)\right], \quad (2.11)$$

where ω_s is the synchrotron oscillation frequency, A the amplitude of the oscillation and ϕ the phase. Since I is a periodic function of the variable $t + A \cos(\omega_s t + \phi)$, it follows that

$$I(t) = \sum_{n=-\infty}^{\infty} I_n e^{in\omega_0(t + A \cos(\omega_s t + \phi))}, \quad (2.12)$$

with

$$I_n = I_0 \tilde{g}(n\omega_0). \quad (2.13)$$

Employing the generating function for Bessel functions [5],

$$e^{iz \cos \theta} = \sum_{k=-\infty}^{\infty} i^k J_k(z) e^{ik\theta}, \quad (2.14)$$

one easily derives

$$I(t) = \sum_{nk} I_{nk} e^{i(n\omega_0 + k\omega_s)t}, \quad (2.15)$$

where

$$I_{nk} = I_0 \tilde{g}(n\omega_0) J_k(n\omega_0 A) e^{ik\phi} i^k. \quad (2.16)$$

The Fourier coefficients I_{nk} correspond to the synchrotron sideband frequencies

$$\omega = n\omega_0 + k\omega_s, \quad (2.17)$$

where $n, k = -\infty, \dots, \infty$.

The synchrotron oscillation frequency ω_s is determined by observing the spectrum of a signal proportional to $I(t)$. If the oscillation

amplitude is small, $n\omega_0 A < 1$, then the strength of the first sideband at frequency $\omega = n\omega_0 + \omega_s$ is approximately given by

$$|I_{n1}/I_{n0}| \approx n\omega_0 A/2. \quad (2.18)$$

The sidebands are easily observed in an electron storage ring using a spectrum analyzer. There is generally enough noise so that the amplitude of the coherent synchrotron oscillation is sufficient to yield an easily observable sideband (say 60 dB down from the revolution line), without the need to drive the synchrotron oscillations purposefully.

Note that even though we have considered the oscillation of a rigid bunch, there exist higher-order sidebands at $\omega = n\omega_0 + k\omega_s$ for $|k| \geq 2$. These sidebands do not correspond to bunch shape oscillations. In the next section we consider a bunch comprised of many discrete electrons. We then consider coherent bunch shape oscillations as well as incoherent synchrotron oscillations.

C. Bunch of Discrete Electrons

We now cease to approximate the bunch of electrons by a continuous fluid, and explicitly take into account the discrete nature of the individual electrons. The beam current is

$$I(t) = e \sum_{n=-\infty}^{\infty} \sum_{j=1}^N \delta(t - nT_0 + A_j \cos(\omega_s t + \phi_j)), \quad (2.19)$$

where A_j and ϕ_j are the synchrotron oscillation amplitude and phase of the j th electron in a bunch containing a total of N electrons. We take the synchrotron oscillation frequency ω_s to be the same for all of the electrons, which ignores the effect of the nonlinearity of the RF potential. The discussion leading to Eqs. (2.15) and (2.16) applies to the present case, and we easily derive

$$I(t) = \sum_{nk} I_{nk} e^{i(n\omega_0 + k\omega_s)t}, \quad (2.20)$$

with

$$I_{nk} = \frac{e}{T_0} \sum_{j=1}^N J_k(n\omega_0 A_j) e^{ik\phi_j} i^k. \quad (2.21)$$

Let us introduce a phase space distribution $f(A, \phi)$ of the electrons according to:

$$Nf(A, \phi) AdA \frac{d\phi}{2\pi} = \text{number of electrons in } AdA \frac{d\phi}{2\pi} . \quad (2.22)$$

Averaging over phase space, we find

$$\langle I_{nk} \rangle = \frac{eN}{T_0} \int_0^\infty AdA J_k(n\omega_0 A) i^k \int_0^{2\pi} \frac{d\phi}{2\pi} e^{ik\phi} f(A, \phi) . \quad (2.23)$$

We now employ the Fourier expansion

$$f(A, \phi) = \sum_{k=-\infty}^{\infty} R_k(A) e^{-ik\phi} , \quad (2.24)$$

and obtain

$$\langle I_{nk} \rangle = \frac{eN}{T_0} i^k \int_0^\infty AdA J_k(n\omega_0 A) R_k(A) . \quad (2.25)$$

For a given harmonic n of the revolution frequency, Eq. (2.25) gives the average strength of the revolution line ($k=0$), the coherent dipole mode ($k=1$), the coherent quadrupole mode ($k=2$),

When discussing coherent synchrotron oscillation modes [6] it is conventional to use a time-dependent phase space distribution $\psi(A, \theta, t)$, rather than the time-independent distribution $f(A, \phi)$ which we have introduced in Eq. (2.22). The relationship between these distribution functions is easily determined. Let $\tau_j(t)$ be the time interval by which the j th electron precedes the synchronous electron [4]. Then the harmonic synchrotron oscillation of the j th electron is described by

$$\tau_j(t) = A_j \cos \theta_j, \quad \dot{\tau}_j = -\omega_s A_j \sin \theta_j , \quad (2.26)$$

where

$$\theta_j = \omega_s t + \phi_j . \quad (2.27)$$

Hence, the phase space distribution simply rotates as time passes, and

$$\psi(A, \theta, t) = f(A, \theta - \omega_s t)$$

$$= \sum_{k=-\infty}^{\infty} R_k(A) e^{-ik(\theta - \omega_s t)} \quad (2.28)$$

As an example, let us consider a quadrupole mode oscillation

$$\psi(A, \theta, t) = R_0(A) + 2R_2(A) \cos(2\theta - 2\omega_s t). \quad (2.29)$$

In Fig. 1, we illustrate the rotation in phase space of an elliptical distribution. Recall [4] that $\dot{r} = -\alpha\epsilon/E_0$, where $\epsilon = E - E_0$ is the deviation of the energy from the synchronous energy E_0 , and α is the momentum compaction. It is easily seen that the bunch length and energy spread oscillate with frequency $2\omega_s$.

Figure 1

D. Incoherent Synchrotron Oscillations: Schottky Noise

Schottky noise refers to effects resulting from the discrete point-like nature of individual electrons [7]. These effects are absent in a continuous fluid. In order to discuss the incoherent synchrotron oscillations, let us consider the phase space distribution of Eq. (2.22) to be independent of ϕ , i.e. $f = f(A)$, hence there are no coherent synchrotron oscillations of the bunch. From Eq. (2.23) we see that

$$\langle I_{nk} \rangle = \begin{cases} \frac{eN}{T_0} \int_0^{\infty} A dA f(A) J_0(n\omega_0 A) & (k=0) \\ 0 & (k \neq 0) \end{cases} \quad (2.30)$$

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The only coherent lines in this case are the revolution lines at multiples of the revolution frequency. Very precise measurements would show, however, the existence of incoherent sidebands. The power spectrum of the incoherent sidebands ($k \neq 0$) can be estimated by considering:

$$\langle I_{nk} I_{nk}^* \rangle = \left(\frac{e}{T_0} \right)^2 \sum_{j=1}^N \sum_{\ell=1}^N J_k(n\omega_0 A_j) J_k(n\omega_0 A_\ell) e^{ik(\phi_j - \phi_\ell)} \quad (2.31)$$

For $k \neq 0$, it is clear that $\langle \exp ik(\phi_j - \phi_\ell) \rangle = 0$ unless $j = \ell$, therefore,

$$\langle I_{nk} I_{nk}^* \rangle = \left(\frac{e}{T_0} \right)^2 \sum_{j=1}^N J_k^2(n\omega_0 A_j) \quad (2.32)$$

which is well-approximated by

$$\langle I_{nk} I_{nk}^* \rangle = \left(\frac{e}{T_0} \right)^2 N \int_0^\infty A dA f(A) J_k^2(n\omega_0 A) \quad (2.33)$$

employing the distribution function $f(A)$. As an example consider

$$f(A) = \frac{1}{\sigma} e^{-A^2/2\sigma^2} \quad (2.34)$$

Using Eq. (2.34) in Eq. (2.30), one finds the power in the coherent rotation line $k=0$ is:

$$\langle I_{n0} \rangle^2 = \left(\frac{eN}{T_0} \right)^2 e^{-n^2 \omega_0^2 \sigma^2} \quad (2.35)$$

The coherent power $\sim N^2$, i.e. varies as the square of the number of electrons in the bunch, and the power spectrum has a high-frequency falloff $\sim \exp(-n^2 \omega_0^2 \sigma^2)$, dependent upon the bunch length σ .

The power in the incoherent sidebands is obtained by inserting Eq. (2.34) into Eq. (2.33), and one finds ($k \neq 0$):

$$\langle I_{nk} I_{nk}^* \rangle = \left(\frac{e}{T_0} \right)^2 N e^{-n^2 \omega_0^2 \sigma^2} I_k^2(n^2 \omega_0^2 \sigma^2) \quad (2.36)$$

We see that the incoherent power $\sim N$, i.e. varies linearly with the number of electrons in the bunch. Since N is typically on the order of 10^{10} or more, the incoherent power is very small compared to the coherent power.

Schottky signals are not usually observable in electron storage rings where they are dominated by coherent signals, but are of great importance in proton storage rings, where they are used for diagnostics and stochastic cooling [8].

Note that the high-frequency falloff of the incoherent power spectrum of Eq. (2.36) is algebraic, $\sim 1/n$, as can be seen by recalling the asymptotic expression [5] for the Bessel function of imaginary argument $I_k(x)$:

$$I_k(x) \sim \frac{e^x}{\sqrt{2\pi x}} \cdot (x \rightarrow \infty). \quad (2.37)$$

Hence the Schottky spectrum for high frequency is an example of the very common $1/f$ noise and is not characterized by the bunch length.

3. Measurement of Beam Current

A. Electromagnetic Field

In Section 2, we discussed some of the information which can be obtained from knowledge of the beam current observed at fixed azimuth. The existence of the electromagnetic fields associated with moving charges allows one to obtain signals proportional to the electron beam current at a given azimuth. To describe these electromagnetic fields, we follow the approach of Cuperus [9], who employed a Lorentz transformation. In the Lab frame F, we assume there exists a charge density,

$$\rho = \rho(\vec{r}_{\perp}, z-vt) , \quad (3.1)$$

where $\vec{r}_{\perp}$ are the coordinates perpendicular to the electron velocity $\vec{v}$, which we take to be in the z-direction. One now makes a Lorentz transformation to the frame F',

$$z' = \gamma(z-vt) , \quad (3.2a)$$

$$t' = \gamma(t-\beta z/c) , \quad (3.2b)$$

$$\vec{r}_{\perp}' = \vec{r}_{\perp} , \quad (3.2c)$$

where $\vec{\beta} = \vec{v}/c$ and $\gamma = \sqrt{1-\beta^2}$. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ in the Lab frame F are related to the fields in F' by

$$E_z = E_z' , \quad \vec{E}_{\perp} = \gamma(\vec{E}_{\perp}' - c\vec{\beta} \times \vec{B}_{\perp}') , \quad (3.3a)$$

$$B_z = B_z' , \quad \vec{B}_{\perp} = \gamma(\vec{B}_{\perp}' + \frac{\vec{\beta}}{c} \times \vec{E}_{\perp}') , \quad (3.3b)$$

In frame F', the charge density

$$\rho' = \frac{1}{\gamma} \rho(\vec{r}_{\perp}, z'/\gamma) , \quad (3.4)$$

and assuming the vacuum chamber to have constant cross-section, the determination of the electromagnetic fields in the lab frame F reduces to the solution of an electrostatic problem in F'.

$$\vec{E}' = -\nabla\phi , \quad \vec{B}' = 0 \quad (3.5)$$

$$\nabla^2 \phi = -\rho' / \epsilon_0 . \quad (3.6)$$

As an example, let us consider a filament electron beam with charge density in the lab frame F given by

$$\rho = \rho_k \cos k(z-vt) \frac{\delta(r)}{2\pi r} . \quad (3.7)$$

Then in frame F', the charge density becomes

$$\rho' = \frac{\rho_k}{\gamma} \cos(kz'/\gamma) \frac{\delta(r)}{2\pi r} . \quad (3.8)$$

Writing the scalar potential

$$\phi = \frac{\rho_k}{\gamma \epsilon_0} \Phi(r) \cos(kz'/\gamma) , \quad (3.9)$$

one derives the equation

$$(\nabla_{\perp}^2 - \frac{k^2}{\gamma^2}) \Phi = - \frac{\delta(r)}{2\pi r} . \quad (3.10)$$

Here, $r = |\vec{r}_{\perp}|$, and $\nabla_{\perp}^2$ is the two-dimensional transverse Laplacian. Upon solving Eq. (3.10) subject to the boundary conditions corresponding to the surrounding vacuum chamber, one obtains the electromagnetic field observed in the lab frame F: (cylindrical coordinates)

$$E_r = \gamma E_r' = - \frac{\rho_k}{\epsilon_0} \frac{d\Phi}{dr} \cos k(z-vt) , \quad (3.11)$$

$$B_{\theta} = \frac{v}{c} E_r , \quad (3.12)$$

$$E_z = E_z' = \frac{1}{\gamma} \frac{\rho_k}{\epsilon_0} \Phi(r) \sin k(z-vt) . \quad (3.13)$$

From Eq. (3.13) we see that $E_z \approx 0$ for $\gamma \gg 1$, so the electromagnetic field is approximately a TEM wave.

For a cylindrical vacuum chamber with infinite conductivity and circular cross section of radius R, one has

$$\Phi = 0 \text{ for } r = R, \quad (3.14)$$

and one finds from Eq. (3.10) that

$$\Phi = \frac{-1}{2\pi\epsilon_0} \frac{1}{I_0(kR/\gamma)} \left[K_0(kR/\gamma) I_0(kr/\gamma) - I_0(kR/\gamma) K_0(kr/\gamma) \right] \quad (3.15)$$

The surface charge density induced on the inside of the circular chamber wall is

$$\Sigma = -\epsilon_0 E_r(r=R) = -\frac{\rho_k}{2\pi R} \frac{1}{I_0(kR/\gamma)} \cos k(z-vt) \quad (3.16)$$

The charge induced in a surface element subtending angle from θ_1 to θ_2 , and length from $z = -L/2$ to $z = L/2$, is

$$\begin{aligned} q &= \int_{\theta_1}^{\theta_2} d\theta \int_{-L/2}^{L/2} dz \Sigma \\ &= -\rho_k \frac{\Delta\theta}{2\pi} \frac{1}{I_0(kR/\gamma)} \frac{2 \sin(kL/2)}{k} \cos(kvt), \end{aligned} \quad (3.17)$$

where $\Delta\theta = \theta_2 - \theta_1$.

The induced charge density on the vacuum chamber wall is seen to be inversely proportional to the Bessel function $I_0(kR/\gamma)$. The physical interpretation of this result is well-known and has been clearly stated by Shafer [10]. Consider a point charge at rest on the axis of the circular vacuum chamber of radius R . The wall charge is spread out longitudinally over about $\pm 0.3R$. When a Lorentz transformation is made to a moving frame along the vacuum chamber axis, the wall charge is contracted to a length $\pm 0.3R/\gamma$. For $\gamma \gg kR$, the Bessel function $I_0(kR/\gamma)$ is unity, and the longitudinal distribution of the charge induced on the wall coincides with the line charge distribution of the electron beam. On the other hand, when $\gamma \leq kR$, the Bessel function $I_0(kR/\gamma)$ becomes greater than unity, and the high-frequency components of the wall current are seen to be reduced relative to the high-frequency components of the electron beam current.

B. Capacitive Pickup

Following Lambertson [11], let us consider the voltage induced on a pickup electrode capacitively coupled to the electron beam. Such an electrode is pictured schematically in Fig. 2. We suppose the length of the electrode L is short compared to the wavelength of the electromagnetic field detected, i.e. $kL \ll 1$, so the charge induced on the electrode is

$$q = -\kappa L I_B / v, \quad (3.18)$$

where I_B is the electron beam current, v the electron velocity, and κ a capacitive coupling constant dependent on the geometry of the vacuum chamber and pickup electrode. For the circular geometry employed in the derivation of Eq. (3.17), $\kappa = \Delta\theta/2\pi$, when $kR \ll \gamma$.

We consider the pickup electrode to be electrically isolated from the vacuum chamber, which we assume to be grounded. The capacitance between the electrode and the vacuum chamber is denoted by C , and the resistance between the electrode and ground is denoted R . The voltage on the electrode can be determined from the simple analog circuit of Fig. 3, and one finds

$$V_\omega = \frac{-j\omega q R}{1+j\omega C R} = I_B j\kappa \frac{\omega L}{v} \frac{R}{1+j\omega R C}, \quad (3.19)$$

where we use the electrical engineering convention of assuming the quantities of interest have time-dependence $\exp(j\omega t)$. At low frequencies, the denominator in Eq. (3.19) can be approximated by unity, and the voltage induced is seen to be proportional to the time derivative of the electron beam current.

Figure 2

Figure 3

C. Stripline Monitor

The short capacitive pickup electrode considered in the previous section was grounded, and the problem could be treated quasistatically, ignoring inductance and the propagation of electromagnetic waves. In the case of a stripline monitor, the electrode inside the vacuum chamber is not grounded. It is connected to the outside world through transmission lines, so propagation effects are essential. We shall summarize some of the properties of stripline monitors following the discussion of Shafer [10].

A stripline monitor is sketched in Figs. 4a,b,c. In Fig. 4a, a point charge q is shown approaching the monitor with velocity v_B . Next, in Fig. 4b, the charge q is shown entering the monitor. One also sees an induced charge $-\kappa q/2$ traveling along the stripline with velocity v_s in the direction of motion of charge q , and a second induced charge $\kappa q/2$ leaving the upstream port along a transmission line with velocity v_s . Because the stripline electrode is not grounded, no net charge can be induced upon it. The boundary conditions at the entrance to the stripline are satisfied by the two induced charges of equal magnitude, but opposite sign, traveling in opposite directions. This provides for charge conservation, and current continuity. In Fig. 4c, the charge q is pictured leaving the stripline. In order to satisfy the boundary conditions at the exit, an induced charge $-\kappa q/2$ is sent traveling back along the stripline with velocity v_s , and an induced charge $\kappa q/2$ leaves the downstream port traveling with velocity v_s along a transmission line.

From these considerations, it is seen that if an electron beam current $I_B(t)$ passes through the stripline, the signal leaving the upstream port is

$$I(t) = \frac{\kappa}{2} [I_B(t) - I_B(t-\tau)] , \quad (3.20a)$$

$$V(t) = Z_0 I(t), \quad (3.21a)$$

where

$$\tau = \frac{L}{v_B} + \frac{L}{v_s} , \quad (3.21)$$

L is the length of the stripline, and Z_0 is the characteristic impedance of the stripline and the transmission lines. The signal leaving the downstream port is

$$I(t) = \frac{\kappa}{2} [I_B(t - \frac{L}{v_B}) - I_B(t - \frac{L}{v_s})], \quad (3.22)$$

$$V(t) = -Z_0 I(t).$$

The signal at the downstream port vanishes if $v_B = v_s$ (directional coupler). Note that in this case, if electron and positron beams are traveling in opposite directions through the stripline monitor, then the signal leaving one port would be exclusively due to the electron beam, and the signal from the other port exclusively due to the positron beam.

Figure 4

D. D.C. Beam Current Transformer

A toroidal D.C. beam current transformer has been developed by Unser [12]. In order to observe a signal in the secondary winding of a transformer, proportional to a D.C. current in the primary, it is necessary to modulate the permeability of the cores of the transformer. This is illustrated in Fig. 5. The bias current drives both cores into partial saturation. The currents in the two bias windings flow in opposite directions, so that in the absence of the electron beam there is no signal induced in the secondary. The D.C. component of the electron beam current produces in the secondary winding an alternating voltage at twice the frequency of the bias current.

Figure 5

The effect of the electron beam at one instant of time is to increase the flux in one ring, driving it closer to saturation, and to decrease the flux in the other ring, reducing its saturation. Because of saturation, the increase of flux in one ring will be less than the decrease in the other ring, and a net current flows through the secondary winding. One half cycle later, the roles of the two rings are interchanged, hence the induced signal in the secondary has twice the frequency of the bias current, see Fig. 6.

Figure 6

4. Energy Spread and Bunch Lengthening

The measurement of the longitudinal emittance of the electron beam using synchrotron radiation has been discussed in this school by Hofmann [1]. Therefore, we shall not enter into a detailed treatment of this important subject. Instead, we shall review some interesting measurements [2] made recently at SLAC on the bunch lengthening in the SLC e^- damping ring.

The electron beam was extracted into the ring-to-linac transport line. Special optics were used to obtain a high dispersion η and low β function on the fluorescent screen at which measurements of the transverse profile were made. In this case, the chromatic size of the beam dominated the betatron size,

$$\sigma_x = \sqrt{\epsilon_x \beta_x + \eta^2 (\sigma_\epsilon / E)^2} \approx \eta \sigma_\epsilon / E, \quad (4.1)$$

where ϵ_x is the horizontal betatron emittance and σ_ϵ is the energy spread. Digitizing the video signal from the profile monitor TV camera, the intensity on the screen was measured as a function of horizontal position. Because of the validity of Eq. (4.1), the transverse position of the electrons was correlated with their energy. Therefore, the plot of the intensity vs. horizontal position yielded the energy distribution function. Measurements for different values of the electron bunch current I showed that the width of the energy distribution remained constant below a current threshold, but increased $\sim I^{1/3}$ above threshold. Also, it was found that the shape of the energy distribution remained Gaussian for all currents.

Using a bunch length compression scheme implemented in the ring-to-linac transport line, the longitudinal current distribution was also measured. A correlation between the energy and the longitudinal position within the bunch was introduced using an S-band accelerator section immediately downstream of the damping ring. This was followed by a high dispersion section in which the high energy electrons traversed a longer path than the low energy electrons. Since the electrons at the front of the bunch had been increased in energy relative to those in the rear by the acceleration section, the bunch performed a rotation in longitudinal phase space, arriving at the linac with the required short bunch length.

In the high dispersion region, the chromatic size of the beam again dominated over the betatron size, so Eq. (4.1) held. In this case, however, the horizontal position x of the electrons hitting the screen was correlated with the longitudinal position the electrons occupied when they first entered the compressor. Therefore, measurement of the screen intensity as a function of the horizontal position yielded the longitudinal profile of the bunch as it was leaving the damping ring. The bunch distribution was observed to lengthen as current increased, and while the distribution was Gaussian at low currents, it became asymmetric at high currents in agreement with a theoretical calculation.

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