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BY

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Strong Plasma Turbulence and Anomalous Diffusion
in a Magnetic Field

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ABSTRACT

Plasma diffusion in the presence of strong turbulence has been studied by means of analytic theory and numerical simulations. First, diffusion and turbulent fluctuation spectrum in the presence of convective cells are studied using a two-dimensional guiding center model and a two-fluid model keeping the ion inertia. It is shown that for these models, turbulent diffusion which dissipates a density gradient takes place associated with the inverse cascades of the fluctuations transferring the electric field energy towards long wavelength modes. Second, particle diffusion associated with drift wave turbulence using full dynamic ions and Debye shielding electrons is considered. It is shown that diffusion and the inverse cascades are much slower in this model compared with those found for convective cell turbulence. This difference arises from the fact that the mode coupling coefficient for inverse cascade is much smaller for the drift wave turbulence compared with the turbulence associated with the convective cells.

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I. Introduction

Nonlinear plasma diffusion in the presence of strong turbulence has been one of the most challenging problems in the research of controlled nuclear fusion via magnetic confinement of plasmas. Classical¹ or neoclassical² plasma diffusion associated with binary collisions between particles and quasilinear diffusion³ due to small amplitude density and potential fluctuations are well understood. In the presence of strong plasma turbulence, different types of diffusion can take place associated with the turbulent motion of particles. The turbulent diffusion may be considerably greater than the classical or quasilinear diffusion when the fluctuation level is high.

While there are a number of plasma instabilities and hence different types of turbulence, two specific models for plasma turbulence are considered here. In Section II, two-dimensional convective cell turbulence⁴ is first considered in the guiding center limit which is valid in a strong magnetic field.^{5,6} Convective cells are characterized by the presence of potential fluctuations much greater than density fluctuations. Then the model is extended to include the ion inertia using a two-fluid plasma model in Section III. It is shown that the nonlinear equation for the potential fluctuations remains essentially the same as for the guiding center model. Numerical simulations confirm that plasma diffusion which dissipates a density gradient takes place in both models. For a strong magnetic field in which $\Omega_i > \omega_{pi}$ where Ω_i and ω_{pi} are ion gyro and plasma frequencies, both

models exhibit B^{-1} diffusion. Contour plots for the electrostatic potentials indicate the inverse cascade process which shifts electric field energy toward long wavelength fluctuations. Inverse-cascade process takes place on the same time scale as the particle diffusion in two dimensions since both are caused by the nonlinear plasma convections associated with $c\mathbf{E} \times \mathbf{B}/B^2$ drift.

Drift turbulence is considered in Section IV where the electron response is assumed a Boltzmann distribution. In the cold ion limit, the model corresponds to that of Hasegawa and Mima.⁷ It is shown that the diffusion in this model is much smaller than the diffusion associated with the convective cells. The reason for this is that in this model, the mode coupling coefficient for generating long wavelength electric field fluctuations is much smaller than that for the convective cells and, therefore, the diffusion which is primarily caused by the long wavelength modes is much smaller.

II. Guiding Center Model and Convective Cell Turbulence

A. Guiding Center Model

Let us first consider a guiding center model in two dimensions.⁵ In this model, both ions and electrons move by the $\mathbf{v}_E \equiv c\mathbf{E} \times \mathbf{B}/B^2$ drift where $\mathbf{B}$ is the uniform external magnetic field in z-direction. The continuity equation for both ions and electrons is given by

$$\frac{\partial n}{\partial t} + (\nabla n) \cdot \underline{v}_E = 0 \quad (1)$$

where $\nabla \cdot \underline{v}_E = 0$. Subtracting the continuity equation for the electrons from that of ions, and using Poisson's equation,

$$\nabla^2 \phi = -4\pi e(n_i - n_e) \quad (2)$$

we find, for the electrostatic potential,

$$\frac{\partial}{\partial t} \nabla^2 \phi + \frac{c \underline{E} \times \nabla \phi}{B^2} \cdot \nabla (\nabla^2 \phi) = 0. \quad (3)$$

Writing ϕ in $\underline{k}$ -space,

$$\phi(\underline{x}, t) = \sum_{\underline{k}} \phi_{\underline{k}}(t) e^{i \underline{k} \cdot \underline{x}}, \quad (4)$$

Equation (3) becomes

$$\frac{\partial}{\partial t} \phi_{\underline{k}} + \frac{1}{2} \frac{1}{k^2} \sum_{\substack{\underline{k} = \\ \underline{k}_1 + \underline{k}_2}} (\underline{k}_1 \times \underline{k}_2) \cdot \hat{e}_z (k_2^2 - k_1^2) \phi_{\underline{k}_1} \phi_{\underline{k}_2} = 0 \quad (5)$$

where $\phi_{\underline{k}}$, $\underline{k}$ and t now represent $e\phi_{\underline{k}}/T_e$, $k\rho_s$ and $\Omega_i t$. Here $\hat{e}_z$ is the unit vector in z , and $\rho_s \equiv (T_e/m_i \Omega_i^2)^{1/2}$ is the ion gyro-radius at the electron temperature.

Since the two-dimensional guiding center model is identical to the two-dimensional incompressible Euler's equation, one can apply results found in the two-dimensional turbulence to the guiding center model. In particular, it is well-known⁶ that

a statistical theory for the fluctuation spectrum for the electric field predicts the inverse-cascade of the energy spectrum,

$$\langle |E_{\underline{k}}|^2 \rangle \sim \frac{1}{\alpha + \beta k^2} \quad (6)$$

Here α and β are the two temperatures determined from the total energy of the system

$$E = \sum_{\underline{k}} |E_{\underline{k}}|^2$$

and enstrophy

$$\Omega = \sum_{\underline{k}} k^2 |E_{\underline{k}}|^2$$

which are the two rugged constants of motion.

We will study in this Section diffusion of particles in an inhomogeneous density plasma for the guiding center system using a particle simulation code. While there are theoretical calculations on the test particle diffusion⁵, it is not known how the plasma diffusion which dissipates a density gradient takes place in this model. It should be emphasized that the diffusion which dissipates a density gradient is not necessarily the same as the test particle diffusion, since the former involves the macroscopic motions of ions and electrons while the latter computes only the orbits of non-interacting particles. There is no classical or quasilinear diffusion present in this system. The system under

consideration, Eq. (5), is of strong turbulence type since no linear eigenmode exist as far as Eq. (5) is concerned. It will be seen, however, that this guiding center model permits particles to diffuse until the density becomes more or less uniform filling the entire region of plasma simulations. In particular recurrence phenomena such as seen for the formation of solitons⁸ do not seem to exist in the present model. We believe the diffusion in this system is associated with the inverse-cascade process inherent in Eq. (5) which shifts fluctuation field energy toward long wavelength perturbation and hence does not allow the condensation of energy at short wavelengths. It is clear that the inverse-cascade process and particle diffusion take place on the same time scale since the only reason for particle diffusion is the nonlinear $\underline{cE} \times \underline{B}/B^2$ convections which causes the spread of plasma density in an irreversible manner. At the same time nonlinear convections cause the field spectrum to spread toward long wavelength perturbations.

B. Numerical Simulations

Numerical simulations have been carried out using a two-dimensional particle simulation model. Both ions and electrons are pushed using $\underline{v_E} = \underline{cE} \times \underline{B}/B^2$ drift velocity. In order to reduce the numerical noise and hence inaccuracy on the results, sufficiently small time step of integration, typically $\omega_{pi} \Delta t = 0.1$, was used in the predictor-corrector scheme.⁹ Furthermore, cubic interpolations were carried out for the charge and force over the neighboring grid points which significantly improved

the conservation of total energy which was typically a few percent at the end of the run.¹⁰ It is believed that the macroscopic structures of diffusion shown here are free from the choice of time step and interpolations, especially at the long wavelength fluctuations which dominate the diffusion in two-dimensions. Time step is normalized by the ion plasma frequency, ω_{pi} , in the code so that the ratio ω_{pi}/Ω_i is an adjustable input parameter, although it is not necessary to introduce ω_{pi} in the guiding center model. 64×64 spatial grid and 128×128 ions and electrons in a periodic system were used. The initial macroscopic density profile was taken to

$$n(x) = \frac{n_0}{(2\pi)^{1/2}\delta} \exp(-x^2/2\delta^2) \quad (7)$$

where δ was typically four grid length. Ions and electrons are more or less randomly distributed according to Eq. (7) so that they do not necessarily form a pair of neutrals in general and at the same time plasma is in quasineutral and therefore no large vortices exist initially. It is the initial electric field energy which determines the evolution of the system. Since there is no thermal energy in the guiding center model, there is no sensible way to represent the level of turbulence in comparison with thermal energy. However, we show later the results of simulations using full dynamics of particles with almost the same initial conditions. The total field energy for both guiding center and full dynamics models discussed in Sect. III is about the same initially. The ratio of field to kinetic energy in

the full dynamics model is approximately 10^{-3} which is not too far from thermal level.

Figure 1 shows one of the simulations for $\omega_p/\Omega = 1/2$ for particle diffusion. These particles represent all the electrons and the observations confirm ions and electrons diffuse almost precisely the same way. At $t = 1000 \omega_{pi}^{-1}$, many small vortices and one or two large vortices have developed and the latter persist throughout the run. Large vortices as well as small vortices move such that they fill the entire space more or less evenly. The average density in x shown in Fig. 2 indicates that the initial peaked density profile spread in time and while one or two bumps persist at $t = 6000 \omega_{pi}^{-1}$, the original density clearly diffused out.

One can estimate the diffusion coefficient from the change of initial density profile. Assuming the particles spread less than one half of the system by $t = 6000 \omega_{pi}^{-1}$, $D_x = \langle (\Delta x)^2 \rangle / 2t \approx 0.036 \Delta^2 \omega_{pi}$ which is reasonably close to the measured test particle diffusion $D_x = 0.04 \Delta^2 \omega_{pi}$. While the density diffusion

coefficient may vary up to 50% depending on the interpretation of the results of simulations, two diffusions are very close. Unfortunately, it is not possible to prove rigorously why the test particle diffusion is almost the same as real diffusion which dissipates a density gradient.

Simulations using $\omega_p/\Omega = 1/4$ but otherwise the same parameters, show the identical results to $\omega_p/\Omega = 1/2$ case if the time scale is

extended by a factor of 2. This observation confirms the theoretical prediction of $D \propto B^{-1}$ for the guiding center model and at the same time guarantees that the effects of numerical errors are indeed negligible for the results.

Figure 3 represents the contour plots for the electrostatic potentials. Contours are closely packed at the center during the early phase of the simulations. As particles move out associated with diffusion, contours also spread so that the long wavelength potential fluctuations are generated at the same time. Since there is no dissipation at the short wavelength, small scale jagged potential contours persist which is in sharp contrast to full dynamics model to be discussed in Sect. III.

We have seen that the guiding center model permits a diffusion process associated with nonlinear $c\mathbf{E} \times \mathbf{B}/B^2$ convective motions despite the absence of collisional or quasi-linear diffusion. Test particle diffusion is reasonably close to net diffusion which dissipates a density gradient. As the plasma particles diffuse in x-space, so do the electric field fluctuations shifting the energy toward the long wavelength modes hence the inverse-cascade. From the particle orbit point of view, each particle moves along the constant potential contours since it moves only through $c\mathbf{E} \times \mathbf{B}/B^2$ drift. With the motion of particles, potential contours are continuously deformed in space and the particles fill the entire space more or less uniformly as the inverse cascade shifts the energy towards long wavelength fluctuations.

It should be noted here that a two-dimensional plasma model can exhibit a peculiar type of diffusion via a formation of

neutral particles. When the gyroradius of plasma particles is smaller than the inter-particle spacing, $n^{-1/2}$, a pair of ion and electrons can form a neutral particle and drifts by $cE \times B/B^2$ motion across magnetic field. Diffusion coefficient caused by neutral particles may be estimated in the following manner. Using the formula $D = v^2 \tau$ and $v = cE/B$ where $E = e/r_0$ is the electric field between an ion and an electron at the separation of r_0 and $\tau = n^{-1/2}/v$ is the collision time, we find

$$D = \frac{ce}{Br_0} \frac{1}{\sqrt{n}} \quad (8)$$

r_0 may be taken as the gyroradius of the ions, $\rho_i = (T/m_i)^{1/2}/\Omega_i$, for a full dynamics plasma model. Then the diffusion given by Eq. (8) becomes independent of B , $D = e^2/(nm_i T)^{1/2}$. In a guiding center plasma on the other hand, r_0 may be given by $n^{-1/2}$ so that $D = ce/B$. These diffusion coefficients associated with neutral particles are, however, much smaller than the diffusion associated with the macroscopic fluctuations in the numerical simulations presented here. The main reason for this is that the electric field between two particles, e/r_0 , is reduced to a much smaller value by the use of finite size particles when the particle size is larger than r_0 which is the present case.¹¹ Therefore, what we have seen in Fig. 1 is the diffusion caused by the macroscopic, long wavelength fluctuations as discussed in the theory.

III. Two-Fluid Model and Convective Cell Turbulence

A. Two-Fluid Model

The guiding center model considered in the previous section does not differentiate the motions of ions and electrons. We know, however, that the finite inertia effects of the ions cause them to move differently from the electron motion which in turn generates a class of low frequency oscillations called drift waves. To include such effects in the calculations, it is necessary to use a kinetic theory for the motion of ions. Here we consider a two-fluid model and examine it for any significant changes which may appear in the nonlinear dynamics of the system.

Consider an inhomogeneous system in a magnetic field. Electrons are assumed to move by the $\underline{v}_E = c \underline{E} \times \underline{B} / B^2$ drift while for the ion perpendicular motion, we use

$$\underline{v}_\perp = \underline{v}_E - \frac{c}{\Omega_i B} \left[\frac{\partial}{\partial t} \nabla \phi + (\underline{v}_E \cdot \nabla) \nabla \phi \right] \quad (9)$$

where the second and the third term of the R.H.S of (9) represent the linear and nonlinear polarization drifts. The nonlinear polarization drift arises from the convective derivative term in the equation of motion so that Eq. (9) is different from the previous guiding center model in which the nonlinear term in the equation of motion was neglected. Using Eq. (9) in the ion continuity (1), we find

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \underline{v}_E) - \frac{cn_i}{\Omega_i B} \left[\frac{\partial}{\partial t} \nabla^2 \phi + \nabla \cdot (\underline{v}_E \cdot \nabla) \nabla \phi \right] = 0 \quad (10)$$

Subtracting the electron continuity equation from (10), and using Poisson's equation, we find for $k \gg |d\ln n_0/dx|$

$$\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) \frac{\partial \phi_k}{\partial t} + \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) \frac{1}{2} \frac{1}{k^2} \sum_{\substack{\mathbf{k} = \\ \mathbf{k}_1 + \mathbf{k}_2}} (\mathbf{k}_1 \times \mathbf{k}_2) \cdot \hat{\mathbf{e}}_z (k_2^2 - k_1^2) \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} = 0. \quad (11)$$

After dividing by $(1 + \omega_{pi}^2/\Omega_i^2)$, the above equation is identical to Eq. (5) where the guiding center dynamics was used for the ions and electrons. This somewhat surprising result stems from the fact that both the linear and nonlinear polarization drifts for the ions are larger than the corresponding terms in the guiding center model by ω_{pi}^2/Ω_i^2 and they cancel out.

When the finite ion temperature or viscosity μ_i is retained in the ion equation of motion, Eq. (11) will contain a linear term in ϕ corresponding to either ion drift mode ω_{ik}^* or convective cells $ik^2\mu_i$. Since $\omega_{ik}^* \gg k^2\mu_i$, we consider ion drift mode here so that a definite comparison of the nonlinear term with the linear term can be made. A full kinetic theory treatment will be given elsewhere.¹² Then the model equation becomes

$$\frac{\partial \phi_k}{\partial t} + i\omega_{ik}^* \phi_k + \frac{1}{2} \frac{1}{k^2} \sum_{\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2} (\mathbf{k}_1 \times \mathbf{k}_2) \cdot \hat{\mathbf{e}}_z (k_1^2 - k_2^2) \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} = 0 \quad (12)$$

where

$$\omega_{ik}^* = -\frac{ek_y T_i}{m_i \Omega_i^2} \frac{1}{n_0} \frac{dn_0}{dx} \bigg/ \left(1 + \frac{\Omega_i^2}{\omega_{pi}^2}\right) \quad (13)$$

is the normalized ion drift frequency.

Equation (12) exhibits weak turbulence properties when the nonlinear term is smaller than the linear term so that each Fourier mode has its eigen frequency. In this case, the nonlinear term may be treated by perturbation methods.³ When the nonlinear term becomes larger than the linear one, then Eq. (12) becomes of strong turbulence type since the nonlinear mode coupling determines the evolution of each Fourier mode. In fact, owing to a large mode coupling coefficient, even a very small amplitude of fluctuations can cause the equation of strong turbulence type.

Figure 4 shows the numerical evaluations of the ratio of the nonlinear term to the linear term in Eq. (12) for several different values of ϕ_k and system size where $e\phi_k/T_e = 10^{-2} \sim 10^{-4}$ and $\rho_i/L_n = 10^{-1} \sim 10^{-2}$ have been assumed. $L_n = (d\ln n/dx)$ is the density scale length. Since we are interested in the long wavelength modes, $k_{\max} = \rho_i^{-1}$ is used in accordance with the use of fluid theory. We see even for a small system such as 10×10 modes in (k_x, k_y) which corresponds to $k_{\min}\rho_i = 0.1$ and $\Delta k\rho_i = 0.1$, nonlinear term dominates over the linear term with $e\phi_k/T_e = 10^{-2}$ for $k < 1$, hence, the equation is of strong turbulence type. Under such conditions, fluctuation spectrum with respect to both wave number and frequency must indicate a monotonic decay peaked at $\omega \approx 0$ and $k = k_{\min}$, characteristic to strong turbulence. Even much smaller amplitude of fluctuations can cause the nonlinear term to dominate over the linear one for larger systems. Figure 4(b) shows fluctuations as small as $e\phi_k/T_e = 10^{-4}$ are sufficient for $k < 1$ for a system of 40×40 modes where $k_{\min}\rho_i = 0.025$. For a

larger system such as 100×100 modes, the nonlinear term can exceed the linear one by two orders of magnitude even for $e\phi_k/T_e = 10^{-4}$ as indicated in (c). This is because the nonlinear term in Eq. (12) approaches $\sum (k_{1y}^2 - k_{1x}^2) \phi_{k_1} \phi_{k_1}^*$ while the linear term goes to zero with k .

B. Numerical Simulations

Let us compare the results of numerical simulations using the full dynamics for both the ions and electrons with the guiding center dynamics model reported in Sect. II. We will use the same set of plasma parameters except now the model includes thermal motion of particles associated with gyration. Since we are not interested in linear drift waves, we shall assume $m_i = m_e$ in the simulations reported here. This assumption removes the linear drift waves from Eq. (12) in agreement in the guiding center model. Physically speaking, if the gyroradii of ions and electrons are the same there will be no phase difference between the $c\mathbf{E} \times \mathbf{B}/B^2$ drifts so that there will be no drift waves. Incidentally this is why there are no drift waves in the guiding center model.

Figure 5 shows the results from numerical simulations using the same parameters shown in Fig. 1 where $\lambda_D = \Delta$ and $\rho = \Delta/2$ are used. Here λ_D is the Debye length. The initial particle density is taken about the same as before and the initial total electric field energy is also about the same. The ratio of field to thermal energy is about 10^{-3} and is found to increase by a factor of 2 toward the end of the run as the particles spread out associated with diffusion. The way particles diffuse is more or less the same as the previous results. Many small and a few large

vortices are excited as the particles spread out to fill the box. A careful comparison of Fig. 5 with Fig. 1 suggests that the density profile in Fig. 5 is considerably smoother compared with Fig. 1 as seen more clearly in the potential contour plots in Fig. 6. Overall diffusion found in Fig. 5 is about 60% larger which may be explained by noting the difference in the total field energy. In fact, the time averaged field energy in Fig. 5 is 60% more than the field energy for the guiding center model. Furthermore collisional diffusion cannot be totally ignored in the full dynamics model. Using the familiar $D_C = \rho^2 v_C$ formula with $\rho = \Delta/2$ and $v_C = 0.01 \omega_p$ for the present model,¹¹ $D_C \approx 0.01 \Delta^2 \omega_p$ is found which is close to one fourth of the total diffusion. This collisional diffusion can smooth out the local small scale density and potential discontinuities seen in the guiding center model.

Figure 7 shows the simulation results for $\Omega/\omega_p = 4$ while keeping the other parameters fixed. Again similar vortices are formed as the particles diffuse. The overall diffusion coefficient is smaller than but close to a factor of 2 compared with the previous case. This is partly because the collisional diffusion is negligibly small here. Also, the total field energy started at about the same value as the previous case but did not increase as much as it did which also contributes to the reduction of the diffusion coefficient. In contrast to guiding center model, vortex structures for two different values of ω_p/Ω did not agree well with each other by shifting the time scale by a

factor of 2. This suggests the real plasmas are much more complicated than the guiding center model.

In a full dynamics model, diffusion coefficient depends on whether ω_p/Ω is greater or smaller than unity.⁴ Clearly ω_p/Ω changes considerably in the present simulations as the plasma density depends on position. Therefore near the center of the plasma where $\omega_p/\Omega > 1$, $D \sim (T/n)^{1/2}$ should be expected while for the major part, $D \sim B^{-1}$ is expected to hold before significant particle diffusion smooth out the density gradient.

IV. Drift Wave Turbulence

Let us now consider a model equation for drift instabilities in which density and potential fluctuations are about the same magnitude, $\delta n/n \sim e\phi/T_e$. The new feature in this model is that the electrons are assumed to take the Boltzmann distribution in the potential fluctuations set up by the ions,

$$n_e = n_0 \exp(e\phi/T_e). \quad (14)$$

Clearly it is assumed that the electron thermal speed is greater than the phase speed of the potential fluctuations along magnetic field ($\omega/k_{\parallel} < v_e$). It is also assumed that each field line does not have any net charges or, equivalently, zero order potential of each line is assumed to be the same. Linearizing (14) and assuming $n^i \approx n^e$ in (10), we find⁷

$$\frac{\partial \phi_k}{\partial t} + i\omega_e^* \phi_k + \frac{1}{2} \frac{1}{1+k^2} \sum (k_1 \times k_2) \cdot \hat{e}_z (k_1^2 - k_2^2) \phi_{k_1} \phi_{k_2} = 0 \quad (15)$$

where

$$\omega_e^* = e k_y T_e d \ln n_0 / dx / (1+k^2) m_i \Omega_i^2 \quad (16)$$

Equation (15) is similar to Eq. (12) but is different in an essential way in that the mode-coupling coefficient in Eq. (15) is much smaller for $k < 1$. This is because the Boltzmann distribution for the electrons causes the shielding of the electric field so that the mode coupling arising from the nonlinear $c\mathbf{E} \times \mathbf{B}/B^2$ motion is much weaker. We expect, therefore, that this model requires much larger amplitude of potential fluctuations at long wavelength, $k < 1$, for the nonlinear term to dominate over the linear term.

Figure 4 indicates this in which the ratio of the nonlinear to the linear terms in Eq. (15) is shown. We see the ratio exceeds 1 only near $k\rho_s \sim 1$ and it is much smaller than the ratio for convective cell turbulence for $k\rho_s < 1$. It is natural to expect then in this model a weak turbulence theory may be applied for $k < 1$ unless the fluctuation level is very high. For $k \sim 1$, the mode coupling coefficients for both models become comparable, however, in this region, finite gyroradius corrections may be necessary.¹²

We show next the diffusion in this model is much smaller owing to the fact that the mode coupling coefficients for small k 's are very small so that it takes a long time to excite long

wavelength perturbations which dominate in particle diffusion. Therefore one expects particle diffusion in this model is much smaller. Note also in Eq. (15), plasma diffusion caused by the quasilinear effects does not exist since the Boltzmann distribution electrons, Eq. (14), excludes any phase-shift between potential and density fluctuations.

In order to simulate a system where the electrons follow Boltzmann distribution, a technique of quasi-neutral simulation was used.¹³ In this model ions are pushed according to full dynamics and hence are fully nonlinear. Electrons are assumed to reach a Boltzmann distribution by following the ions in the homogeneous y -direction. Since we consider small amplitude fluctuations, $e\phi_k/T_e \ll 1$, linearized form of Eq. (14) was used for the electron density $n_e(x,y) = n_o(x)(1 + e\phi/T_e)$ where $n_o(x)$ is the density averaged in y which is taken to be equal to the average ion density. Physically a small $k_{||}$ is assumed to exist in y -direction which is large enough so that the electrons move freely along the ions but small enough so that the ions remain two-dimensional.

Since the density is non-uniform, Poisson's equation takes the form of

$$\nabla^2 \phi(x,y) = -4\pi e [n_i(x,y) - n_o(x)] + \frac{4\pi e^2 n_o(x)}{T_e} \phi(x,y) \quad (17)$$

This equation can be solved iteratively by expanding $n_o(x)$ about its mean value in the nonlinear term. However, for simplicity

here we assume $n_0(x)/T_e(x)$ is constant everywhere so that Poisson's equation may be solved by Fourier transform method.

Figure 8 shows the results of simulations using the similar initial conditions as before. In this model, electron inertia is ignored and hence no mass ratio appears. Here $\omega_{pi}/\Omega_i = 0.5$, $\rho_i/\Delta = 0.5$ and 128×128 ions are used with the same initial density distribution as before. Initial electric field energy in this example is about a factor 2 smaller than the corresponding simulations for convective cells. We see the diffusion of particles is quite slow and no clear vortex structure is visible. Diffusion here is much smaller than the previous cases where large vortices smeared out the original peaked density distribution.

Equi-potential contours for this model are also shown. Most of the contours are small scale fluctuations and confined near the center of the plasma column and they do not evolve into large vortices in contrast to the convective cell case.

Similar simulations, but using $\omega_{pi}/\Omega_i = 0.25$, indicates further reduced diffusion in this model, again much smaller than the corresponding convective cell turbulence case.

Since the initial electric field energy is within a factor of 2 for the present model and the previous model for the convective cells, it appears a paradox why the diffusion in these two models is so much different. In fact, if one uses a conventional expression for test particle diffusion,⁵

$$D = \frac{c}{B} \left(\sum_k \frac{E_k^2}{k^2} \right)^{1/2}, \quad (18)$$

diffusion is the same as long as $\sum_k (E_k^2/k^2)$ remains the same. It should be emphasized, however, that certain approximations went into the derivation of Eq. (18). Clearly, the fluctuation spectrum, E_k^2 , should not change on the same time scale of particle diffusion in Eq. (18), because if it does, the evolution of E_k^2 must be calculated self-consistently using (15). A more rigorous expression of test particle diffusion will be given by¹⁴

$$D = \frac{c^2}{B^2} \int_0^\infty dt \sum_{k,k'} \langle \underline{E}(k) \underline{E}(k') \exp[ik \cdot \underline{\Delta R}(t)] \rangle \quad (19)$$

where $\underline{\Delta R}(t)$ is the displacement vector of the test particle at time t and the bracket $\langle \rangle$ represents an ensemble average. In arriving at Eq. (18) from Eq. (19), several fundamental assumptions are made. One of them is the decomposition of the ensemble average in Eq. (19) neglecting the correlation of electric field and the orbit function $\exp(i\mathbf{k} \cdot \mathbf{\Delta R}(t))$. Clearly this assumption is invalid for the diffusion problems under consideration. As the localized density diffuses as the result of inverse-cascade process, E_k^2 also changes since the more and more energy will go into the long wavelength perturbations which enhance particle diffusion. The inverse-cascade process of the electric field is governed by Eq. (15) for drift waves and in particular by the nonlinear mode-coupling term which is much smaller than the corresponding convective cell turbulence. Therefore for drift wave model it takes a very long time to shift the electric field energy to the longest wavelength fluctuations and therefore particle diffusion will be much smaller in this model since the

diffusion depends on the large scale fluctuations sensitivity.

V. Concluding Remarks

We have examined particle diffusion associated with convective cell turbulence and drift wave turbulence by means of analytic theory and numerical simulations. It is shown that the diffusion in both cases is associated with the inverse-cascade properties of the electric field energy. Diffusion associated with the drift wave turbulence is considerably smaller than the convective cell induced diffusion. This is interpreted because of the difference of mode-coupling coefficient and hence the inverse-cascade process for both cases.

While we discussed two kinds of turbulence independently, they tend to couple very strongly in three dimensions.¹⁵ Because the drift instabilities are strongly unstable at $kp_i \sim 1$, coupling to convective cells is perhaps most dangerous for the confinement of plasmas since the long wavelength turbulence can be generated which will enhance particle diffusion. Furthermore, for a system of magnetic shear or mirroring, it is not always clear to separate these two kinds of strong turbulence. These problems will be treated in the future.

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FIGURE CAPTIONS

FIG. 1. Electron orbits in the two-dimensional guiding center model. Note the development of vortex structures along with density diffusion.

FIG. 2. Density profile averaged in y .

FIG. 3. Equi-potential contours corresponding to Fig. 1. Note the persistence of jagged, small scale contours.

FIG. 4. Ratio of the nonlinear to the linear terms in Eq. (12) for convective cell turbulence and for drift wave turbulence in Eq. (15).

FIG. 5. Electron orbits in the two-dimensional full dynamics model. $\omega_p/\Omega = 0.5$

FIG. 6. Potential contours corresponding to Fig. 5.

FIG. 7. Electron orbit in the two-dimensional full dynamics model. $\omega_p/\Omega = 0.25$.

FIG. 8. Ion orbits for the drift wave turbulence model and potential contours. $\omega_{pi}/\Omega_i = 0.5$

FIG. 9. Ion orbits and potential contours for drift wave turbulence. $\omega_{pi}/\Omega_i = 0.25$.

Fig. 1. 792066

Fig. 2. 792061

Fig. 3. 792059

FIG. 4. 792064

Fig. 5. 792062

Fig. 6. 792063

Fig. 7. 792065

Fig. 8. 792067

Fig. 9. 792056

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