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Helium Research in Support of Superconducting Power Transmission

Annual Report

October 1, 1976 — September 30, 1977

Published April 1979

Prepared for

U.S. Department of Energy

Division of Electric Energy Systems

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1. Introduction

During FY 77, the NBS Cryogenics Division program of supporting research for Superconducting Power Transmission Line (SPTL) development experienced a change of focus as a result of an ERDA directed realignment of the program management. The NBS work is now subcontracted by the Brookhaven National Laboratory Power Transmission Project who retain management responsibility. The work for FY 77 and beyond was to reflect some more urgent short range problems faced by both the BNL and LASL power transmission projects. As a result, the following four research tasks were undertaken:

- 1) Preparation of computer codes for the computation of cooldown by either single stream or counterstream methods.
- 2) Experimental verification of 1) with emphasis on counterstream cooling.
- 3) Thermal cycling of a length of lead-sheathed model cable destined for testing in the BNL 5th Avenue facility.
- 4) Evaluation of heat flow sensors as a means of non-intrusive vacuum indication for power transmission line vacuum envelopes.

Work in progress in other areas was either shelved for the time being (helium impurity detection) or brought to some conclusion (observations of flow oscillations in single stream cooling; current lead modelling; and microwave pressure transducer development).

In this report we give first, in sections 2 through 5, a series of progress reports on the four new tasks. After this, as a series of appendices, we give final reports in the form of preprints prepared for publication or, in one case, a reprint of a published article.

Appendix 1 is a report of our experimental observations of flow oscillations in a high aspect ratio heated tube with supercritical helium as coolant. We were able to map a stability boundary in fairly generalized coordinates so that the results may be applied to the SPTL cooling problem.

In Appendix 2, we present the results of a study of friction factors measured in the same apparatus as was used in Appendix 1. In addition to essentially isothermal results some pressure drops were also measured in the presence of heating and large overall expansion of the supercritical helium. Agreement with calculated pressure drop in this case was good.

In appendix 3, in a report co-authored with members of the staff of the Krzhizhanovsky Power Institute, Moscow, we give results of a numerical study of

the response of SPTL current leads to overload currents. Features of the study are the establishment of operating conditions on the basis of burn-out current and consideration of the effect of copper purity. It is shown that for expected overloads, current leads for power transmission lines can be safely designed.

A final report on the microwave cavity pressure transducer developed at NBS for SPTL application is given in Appendix 4. Details are given of construction, coupling to the cavity, control and measurement electronics and calibration as a helium temperature absolute or differential transducer.

Appendix 5 is a reprint of a published study of a correlation of heat transfer data to supercritical helium in forced convection. This paper is again co-authored with our Soviet colleagues.

Finally, in Appendix 6, a brief description is given of a pressure regulator designed and used by us in controlling the pressure of cryogenic helium at supercritical pressures. This, we feel, may have application in other experimental work in SPTL development.

2. Cooldown Experiments (D. E. Daney, P. R. Ludtke and M. C. Jones)

The aim of the cooldown experiments is to model the cooldown of counterflow cooled SPTL's such as the ones currently under consideration by BNL and LASL. The two principal goals are to complement the numerical cooldown calculations by providing an empirical test of their validity and to provide an empirical data base for a general theory of counterflow cooldown.

The importance of this latter goal is best understood by noting that not even a crude cooldown model was available for estimating counterflow cooldown times when we first expressed our concern over this problem several years ago. Since that time, we have developed a simple theory of counterflow cooldown and have identified what we believe to be the important dimensionless parameters. Experimental verification of the model and evaluation of empirical constants remains.

Both goals require simple, well defined initial conditions and boundary conditions.

2.1. Theory

As an aid to understanding the counterflow cooldown problem, let us first consider single stream cooldown since solutions to this problem are well known. Shutt [1] has shown that for large values of $\frac{bn}{U}$ the dimensionless cooldown time may be approximated by

$$\tau_c(\eta) = \left[\left(\frac{bn}{U} \right)^{1/2} + 2 \right]^2 + \frac{n}{U} \quad (1)$$

with $\eta = \frac{x}{L}$

L = total length of line (m)

x = current length (m)

$$\tau = Pht / \rho_s C_s A_s$$

P = wetted perimeter

h = heat transfer coefficient

ρ_s = density

C_s = heat capacity

A_s = cross-sectional area

t = time (s)

} of solid

$$U = \frac{\dot{m}}{Ph} \frac{\rho_s C_s A_s}{\rho A L}$$

$\dot{m}$ = mass flow rate

ρ = density

A = cross-sectional area

} of fluid

$$b = \frac{\rho_s C_s A_s}{\rho C_p A}$$

C_p = fluid heat capacity.

The quantity $b/U = \frac{hPL}{\dot{m}C_p}$ is the familiar number of exchanger heat transfer units, NTU [2], so equation (1) may also be written as

$$\tau_c(\eta) = \left[(NTU \eta)^{1/2} + 2 \right]^2 + \frac{NTU}{b} \eta \quad (2)$$

For large b , which is the case for a transmission line, and for large NTU, the dimensionless cooldown time of the complete line becomes

$$\tau_c \approx NTU. \quad (3)$$

This result may also be obtained by inspection of the curves for the single blow-down of a regenerator as obtained by Furnas [3].

Expressed in dimensional time the result becomes

$$t_c \approx \frac{(\rho A L C)}{\dot{m} C_p} \quad (4)$$

This same result may be obtained by a simple energy balance on the system, assuming the gas enters at T_0 and exits at T_1 during the entire cooldown.

The energy balance is

$$\dot{m} C_p (T_0 - T_1) t_c = -(\rho A L C)_s (T_1 - T_0) \quad (5)$$

or

$$t_c = \frac{(\rho A L C)_s}{\dot{m} C_p} \quad (6)$$

Turning our attention now to the cooldown of a two stream device, we observe that we have, essentially, a counterflow heat exchanger, which in the case of the far end expander, is cooled down from both ends at once, rather than one end as in the usual case. If we neglect the $\frac{\rho A}{\dot{m}} \frac{\partial T}{\partial t}$ terms, which are very small [4], and set $(hP) \text{ warm stream} = (hP) \text{ cold stream}$, then the governing differential energy equations become

$$\frac{\partial T_1}{\partial \xi} = T_w - T_1 \quad (7)$$

$$\frac{\partial T_2}{\partial \xi} = T_2 - T_w \quad (8)$$

$$\frac{\partial T_w}{\partial \tau} = T_1 - T_2 \quad (9)$$

where

$$\xi = \frac{hP}{\dot{m} C_p} x = NTU_x \quad (10)$$

and

$$\tau = \frac{hPL}{(\rho A L C)_s} t \quad (11)$$

Here h includes not only the heat transfer coefficient, but the wall resistance also. These dimensionless variables are the same as for the single stream cooldown problem so that we might again expect a solution for the total cooldown time to be of the form

$$t = f(NTU). \quad (12)$$

To find an approximate form of the solution, let us again apply the first law to a control volume around a counterflow heat exchanger.

Expected counterflow temperature profile.

Then

$$\int_{t=0}^{t=t_c} \dot{m} C_p (T_1 - T_0) dt = (\rho A L C)_s (T_2 - T_0) \quad (13)$$

or

$$t_c \approx \frac{(\rho A L C)_s}{\dot{m} C_p} \times \frac{T_2 - T_0}{\overline{T_1 - T_0}} \quad (14)$$

which is the expression for single stream cooldown multiplied by

$$\frac{T_2 - T_0}{\overline{T_1 - T_0}}$$

where $\overline{T_1 - T_0}$ is the time averaged value. Now we know that for the steady state operation of a balanced flow, counterflow heat exchanger

that for high effectiveness or large NTU

$$\epsilon = \frac{T_2 - T_1}{T_2 - T_0} = 1 - \frac{T_1 - T_0}{T_2 - T_0} = 1 - \frac{1}{NTU} \quad (15)$$

or

$$\frac{T_2 - T_0}{T_1 - T_0} = NTU \quad (16)$$

During cooldown, the equality may not hold but we might still expect that

$$\frac{T_2 - T_0}{T_1 - T_0} \sim NTU \quad (17)$$

which gives a cooldown expression of the form

$$t_c \sim \frac{(\rho A L C)_s}{\dot{m} C_p} \times NTU \quad (18)$$

In terms of dimensionless variables this becomes

$$\tau_c \sim (NTU)^2 \quad (19)$$

which, interestingly enough, has the same form as the expression for the cooling of a rod or semi-infinite cylinder, viz:

$$t \sim \frac{x^2}{k} \quad (20)$$

We may find an approximate value for the proportionality constant by rewriting energy equation (13) in differential form

$$\dot{m} C_p (T_1 - T_0) dt = -(\rho A L C)_s \int_{\eta=0}^{\eta=1} dT(\eta, t) d\eta \quad (21)$$

Once the cooldown wave has penetrated to $L(\eta=1)$, we approximate $T(\eta, t)$, the temperature profile of the solid, by a sine wave

$$T(\eta, t) - T_0 = T(\eta) - T_0 = (T^* - T_0) \sin\left(\frac{n\pi}{2}\right) \quad (22)$$

and

$$dT = dT^* \sin\left(\frac{n\pi}{2}\right) \quad (23)$$

Replacing dT in equation (21) and integrating yields

$$\dot{m} C_p (T_1 - T_0) dt = - \frac{2}{\pi} (\rho ALC)_s dT^* \quad (24)$$

As before

$$\frac{T_1 - T_0}{T^* - T_0} \approx \frac{1}{NTU}$$

and equation (24) becomes

$$\frac{\pi}{2} \frac{\dot{m} C_p}{(\rho ALC)_s NTU} dt = - \frac{d(T^* - T_0)}{T^* - T_0} \quad (25)$$

or

$$\frac{T^* - T_0}{T_2 - T_0} = e^{-\alpha t} \quad (26)$$

where

$$\alpha = \frac{\pi}{2} \frac{\dot{m} C_p}{(\rho ALC)_s NTU} \quad (27)$$

After an initial transient of relative short duration, the cooldown occurs as an exponential decay of the cable end temperature (one end cooled) or the mid-point temperature (both ends cooled). For 99 percent cooldown $\alpha t = 4.6$, which gives

$$\tau = 2.93 NTU^2 \quad (28)$$

for single end cooldown. The cooldown time goes as L^2 , so for two ended cooldown we obtain

$$\tau = 0.73 NTU^2 \quad (29)$$

From numerical calculations we note that the initial phase of the cooldown (that time required to give the initial sign wave temperature distribution) is approximately equal to the single stream cooldown time. Thus, equation (28) may be corrected to give a total dimensionless cooldown time of

$$\tau = \text{NTU} (1 + 2.93 \text{ NTU}) \text{ -- one end cooled} \quad (30a)$$

$$\text{or } \tau = \text{NTU} (1 + 0.73 \text{ NTU}) \text{ -- both ends cooled.} \quad (30b)$$

We emphasize that these results are approximate, and could be in error by a factor of 2 due to the approximation in equation (16) for cooldown conditions.

Our cooldown expression may also be written as

$$t_c \frac{V}{L} \sim b (\text{NTU}) \quad (21)$$

where V is the fluid velocity. Thus, in our experiments we plan to construct a model which will have values of b and NTU typical of prototype transmission lines. Our cooldown times will be much different than for the prototype, but the essential features of the process should be the same. Ambient temperature values of NTU range from 30 to 100 for prototype transmission lines, making the transmission line a rather good counterflow heat exchanger at ambient temperature. The poor local thermal conductance between streams is overcome by the great length of the line. The decrease in the heat capacity and thermal conductivity of the line with temperature result in values of only a few NTU at the line operating temperature.

2.2. Experimental

Our approach to modeling a transmission line is to model the appropriate values of b and NTU since these are the important parameters of the cooldown process. In order to obtain a controlled thermal resistance of the desired value between the two streams, we have arrived at the design shown in figure 1. When plated with a 0.002-inch thickness of copper, the webbing between flow passages is designed to give an ambient temperature NTU near 30 for 60 feet of tubing and a flow rate of 0.25 g/s. Various values of NTU may be obtained by varying the flow rate.

Figure 2 is a flow schematic of the model which will be installed in our long line apparatus in place of the present 183 m test section. The pressure and temperature at the inlet to either end of the line is controlled by a cold pressure regulating valve and controlled heater. Thus, the far-end expander arrangement may be modeled.

Radiation heat leak from the model presents a special problem, because the temperature of the model varies both temporally and spatially during the cooldown. The heat leak can be reduced to about 10 percent of the heat removal

Figure 1. Heat exchanger tubing for cooldown experiments.

Figure 2. Flow schematic for cooldown experiments.

rate by 1) reducing the length of the model from 120 feet to 60 feet or 40 feet, 2) gold-plating the model to reduce its emissivity, and 3) starting the cooldown at 200 K rather than 300 K.

Kay Industries of Detroit, Michigan are in the process of fabricating the tubing for us.

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3. Progress on Computing Cooldown with Exact Hydrodynamic Equations. (M.C. Jones)

Since January 1977 we have been developing the analysis and a computer program that will allow the comparison of different approaches to the cooldown of a superconducting transmission line from room temperature. We have now developed the computer program to the point that we feel the calculation can be performed for any realistic boundary conditions. The boundary conditions imply something about the refrigeration scheme to be used, however, and until the design of actual refrigeration schemes are frozen we can only look at certain general considerations with regard to refrigerator capability.

In this report we give a comparison of two cooldown schemes applied to a 15 km length of a 3 phase cable with the geometry of the BNL LILCO III B design. Calculations are also in progress for the LASL design of a dc cable for PECO.

3.1. Comparison of Two Cooldown Schemes

In the first scheme, helium gas is forced through the center cores of the cables and it is assumed that the warm gas emerging from the far end is returned to storage or to the refrigerator by a separate return line. This is, essentially, the usual blow-down approach to cooldown and probably is the fastest of all schemes. However, it implies a much larger refrigerator than is necessary for the steady normal operation, or alternatively, an enormous storage capacity for cold helium.

In the second scheme, the gas emerging from the far end of the cores is expanded down to 6.5 atm and returned in the space between the outside of the cables and the cold envelope walls. The flow path is thus identical to the steady state operation proposed by both BNL and LASL [1].

In the near future we expect to consider other cooldown schemes which would be more practical than either of these and would be expected to give times of cooldown intermediate between them.

In both schemes we have assumed that the helium pressure at inlet to the "go" stream is maintained at 15 atm throughout the process and that the pressure drop is maintained at 1 atm. Also, the refrigerator itself is assumed to be able to supply helium gas at 6 K after an initial refrigerator cooldown period of 3 hours. In the second scheme we have further assumed that the return to the refrigerator is maintained at 6.5 atm.

Except for the equality of mass flow at the exit of the "go" stream and inlet to the "return" stream we have made no other assumptions regarding either the constancy or uniformity of the mass flow in the streams in either scheme.

The equations to be solved are the mass, energy and momentum conservation equation for the helium and an energy conservation equation for the cables. We have treated them here as one-dimensional throughout. While we see no justification for any other approach in the case of the helium streams, we expect to consider the radial dependence of cable temperature in addition at a later date. For now, the cable temperature is calculated as an average temperature having only axial variation.

For the helium streams, we have chosen pressure, enthalpy and axial velocity as the dependent variables (P , H and U). We have generalized the steady state equations of Arp [2] to their time dependent equivalents, taking into account change in elevation, change in cross sectional area, friction, and heat transfer from the cables and vacuum envelope. Of these, only friction and heat transfer from the cable were included in the examples given below for the two cooling schemes mentioned, but the other interactions are trivial to include. Thus, for each of the helium streams we have

$$\frac{\partial P}{\partial t} = -U \frac{\partial P}{\partial x} - \rho c^2 \frac{\partial U}{\partial x} + \frac{\phi \mathcal{P}_h}{A} (T_s - T) + \frac{\phi \rho U^2 |U| f \mathcal{P}}{2A} \quad (1)$$

$$\frac{\partial H}{\partial t} = -U \frac{\partial H}{\partial x} - c^2 \frac{\partial U}{\partial x} + \frac{(1 + \phi) \mathcal{P}_h}{\rho A} (T_s - T) + \frac{(1 + \phi) U^2 |U| f \mathcal{P}}{2A} \quad (2)$$

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} - U \frac{\partial U}{\partial x} - \frac{f U |U| \mathcal{P}}{2A} \quad (3)$$

where $\mathcal{P}$ is the wetted perimeter of the stream; A is the cross sectional area; f is the friction factor. We require equation of state information for the quantities $\rho(P,H)$ = density; $\phi(P,H) = \frac{\rho}{T} \left(\frac{\partial T}{\partial \rho} \right)_s$ (the Gruneisen parameter); $c^2 = \left(\frac{\partial P}{\partial \rho} \right)_s$ (square of the sound velocity); and $T(P,H)$, the temperature.

For the examples given below the equation of state information was available as fast computer routines consisting of polynomial fits to the data derived from McCarty's [3] equation of state. These fits were made in the limited range $3 < P < 20$ atm and $4 < T < 300$ K.

Note that equations (1) and (2) are linear combinations of the more familiar mass, momentum and energy conservation equations while (3) is the momentum equation itself. This form, in which the time derivative of only one of the dependent variables appears in each equation, is more suitable for numerical solution.

For the cable temperature, T_s , neglecting for now axial conduction and heat generation, we have

$$\frac{\partial T_s}{\partial t} = - \frac{h_1 \rho_1}{\rho_s C_s A_s} (T_s - T_1) - \frac{h_2 \rho_2}{\rho_s C_s A_s} (T_s - T_2) \quad (4)$$

where the subscripts 1 and 2 refer to "go" and "return" streams, respectively and, of course, only the "go" stream term is included in the first scheme. ρ_s , C_s and A_s are the density, heat capacity and cross sectional area of the cables respectively. Since the controlling heat transfer resistance is through the cable insulation, the heat transfer coefficients $h_{1,2}$ in the examples below were calculated by

$$h_1 = \frac{k}{R_1} \frac{1}{\ln(R_{\text{mean}}/R_1)}$$

and

$$h_2 = \frac{k}{R_2} \frac{1}{\ln(R_2/R_{\text{mean}})}$$

where $k = k(T_s)$ is the temperature dependent thermal conductivity of the insulation material. For this we took Jelinek's data [4] on polycarbonate. R_1 is the inside radius of the cable core, R_2 the outside radius and R_{mean} , the mean radius of the insulation.

For the heat capacity, C_s , we took a representation of the heat capacity of polycarbonate compiled in [5]. For the temperature range of interest it is a simple linear function of temperature.

3.2. Results of Computations

In figures 3, 4, 5 and 6 we give results for scheme 1 of profiles of pressure, fluid enthalpy, mass flow rate and cable temperature. Successive profiles were computed at 10^5 second intervals after an initial one at 10^2 seconds. As can be seen, the cooldown time is very close to 10^6 seconds, or 11-1/2 days. The run time for this calculation was 450 seconds. These results demonstrate the essential wave nature of this solution, the active region being of the order of 10% of the total length (or 1.5 km) and lasting for about 10^5 seconds (~ 1 day).

In figure 7 we give the cable temperature for the second cooldown scheme. In this example, successive profiles are at 10^6 seconds after an initial one

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Figure 3. Pressure profiles during cooldown - single stream - LILCO-3B, 15 km length.

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Figure 4. Enthalpy profiles during cooldown - single stream - LILCO-3B, 15 km length.

Figure 5. Mass flow profiles during cooldown - single stream - LILCO-3B.

Figure 6. Cable temperature profiles during cooldown - single stream - LILCO-3B.

Figure 7. Cable temperature profiles during cooldown - counterstreams - LILCO-3B.
1 atm. pressure drop

at 1.5×10^4 seconds. The cooldown time is now 8×10^6 seconds, or 93-1/2 days. This run took 600 seconds to compute. The fundamentally different nature of this cooldown process is observed. It is now more of a diffusive process and no wave properties are observed. In figure 8, for the same scheme, we show the cable temperature profiles when the pressure drop in the "go" stream is held at 2 atm. The cooldown time is now between 4 and 5×10^6 seconds.

3.3. Refrigeration Requirements

It is instructive to look at the refrigeration requirements demanded by these two alternatives, bearing in mind that for the LILCO 3B design the estimated compressor horse power is 2668 H.P.* for steady state operating conditions. In the first scheme, the ideal minimum power demanded of the refrigerator varies from 0.21 to 1.27 MW during the cooldown process as the mass flow increases from 0.05 to 0.305 kg/s. Assuming 25% efficiency, this would require an installed compressor power of 6812 H.P. Thus additional refrigeration or liquid storage would be required just for the cooldown process. In addition we need a 15 km return line. The liquid storage would be of the order of 2.7×10^6 liters.

For the second scheme, as the long duration of cooldown might suggest, the compressor power for steady state would be underutilized as only about 1000 H.P. is required when the pressure drop of the "go" stream is held at 1 atm. Increasing this to 2 atm gives a compressor power of 1980 H.P. We see therefore that some improvement in the matching of cooldown refrigeration requirements and installed horse power is possible in the second scheme.

3.4. Future Work

In the next reporting period, we intend to investigate alternative schemes, such as supplying refrigerator gas through one phase and returning the warm gas through the remaining 2 phases with the space between phases evacuated. This should provide a better match between cooldown requirements and installed refrigeration than either of the schemes we report on here. The goal will be to fully utilize installed refrigeration in minimizing cooldown time, without a separate return line only used during cooldown.

3.5. References

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* 1 H.P. = 745.7 W.

Figure 8. Cable temperature profiles during cooldown - counterstreams - 2 atm. pressure drop - LILCO 3B.

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4. Thermal Cycling of the Model Cable. (C. F. Sindt)

NBS has been asked to experimentally characterize the mechanical behavior of a section of model superconducting cable while it is thermally cycled between 80 K and ambient temperature. When the cable cools, thermal contraction takes place in all elements except the lead jacket which is fixed at both ends. All of the other cable elements are also partially or totally fixed at both ends, but they are helically wound; therefore they act as springs which absorb the thermal contraction. Stress beyond yield is expected in the lead jacket, so permanent elongation occurs as a result of the cooling cycles. Since helically wound elements totally close the lead, prediction of the behavior of the lead jacket during yield and during the subsequent warm up is difficult.

To get better acquainted with the cable structure and the desired test procedures we visited BNL to discuss the test program. During this visit we outlined the program objectives and discussed test constraints and procedures.

4.1. Task Objectives

Because the mechanism for securing the cable at the ends will surely affect the stress in the cable, the method for securing the cable in the test apparatus at NBS is to be the same as the actual installation at BNL. Therefore, BNL agreed to design and fabricate two of the cable securing devices for NBS. Other items agreed upon are as follows: 1) the minimum test temperature will be the boiling point of liquid nitrogen which will be the test fluid; 2) the cable will be subjected to twenty cooling cycles and fifteen atmospheres differential pressure; 3) the cable is to be constrained to maintain the length to ± 0.12 mm during cooldown; 4) the cooldown time will be about one hour; and 5) the cooling rate will be nearly uniform on the inside and outside. The cable will be checked visually for deformation; also, it will be checked for gas leakage across the lead jacket.

In addition to the cooling test, NBS will measure the permeability of the dielectric to helium gas while the cable is at 80 K. The lead jacket will be perforated to allow gas to flow through the dielectric. Gas flow will be measured at various pressure differences.

4.2. Experimental Approach

The design of the cryostat and the support structure for thermal cycling the cable is complete. Detailed drawings and parts fabrication are in process. Figure 9 shows the layout of the cryostat and the support structure.

The system is manually operated. The operator observes scribe marks through two view ports -- one at each end of the cable -- and then adjusts the cable length with a screw mechanism at each end of the support structure. The mechanical screws load the cable in tension or compression to maintain a constant length. Visual observation of the scribe marks is enhanced with a 30 power telemicroscope at each of the port holes. Since the cooling and warming cycles are one hour long, the operator has time to observe the cable length and adjust the load at frequent intervals and thereby maintain the cable length within ± 0.12 mm. A strain gage type of load cell senses the axial load on the cable at all times.

The horizontal orientation of the cable in the apparatus was selected for two reasons. First, the cable will be horizontal when in use, and second, easy access to both ends of the first apparatus is essential for observation and adjustment of the axial load. Horizontal orientation combined with the requirement to maintain the cable length during warm up complicates the support system at the load cell end. The loads that are not axial must not be applied to the load cell, therefore, the support structure is designed to carry the tension and bending loads which occur when the cable is in compression. The support structure carries these loads plus the dewar on ball bushings mounted in a rigid frame. Ball bushings also carry the loads at the end opposite the load cell. At this end the tension and bending load do not affect the load cell reading so a single bushing for each carries the dewar and cable end assembly.

4.3. Future Work

We anticipate completion of the fabrication and assembly of the system by early 1978. Cable tests are expected to take place by the end of March 1978.

SHEET 1

5. Non-intrusive Pressure Measurement in the Vacuum Envelope of a Transmission Line. (P. J. Giarratano)

An investigation of the feasibility of using heat flow sensors for pressure measurement in the vacuum envelope of a buried transmission line cryostat is in progress. Since these devices can be installed on the outer wall of the vacuum jacket to detect changes in heat flow (e.g., due to vacuum deterioration) they offer a non-intrusive method of vacuum measurement.

The investigation includes both experimental and analytical studies as described below.

5.1. Experimental Plan

The plans for the experimental study are:

- (1) Commercially available heat flow sensors will be calibrated and evaluated in the laboratory under controlled heat flux and temperature level conditions. The sensors will be installed on a 20 inch (50.8 cm) diameter steel pipe (simulating the actual superconducting power transmission line vacuum jacket) and thus will be calibrated in an "as installed" configuration. Two heat flow sensors from each of four commercial suppliers have been acquired or ordered.
- (2) After laboratory calibration the 20 inch diameter steel pipe with installed sensors will be buried in the soil and sensor outputs will be monitored as they are subject to controlled amounts of heat and existing environmental conditions. Effects, if any, of extraneous heat sources will be noted and performance over a period of time will be evaluated.

Construction of the apparatus has not yet begun, however the major components have been acquired or ordered.

5.2. Analytical Study

Preliminary calculations indicated possible detrimental effects of extraneous heat fluxes (e.g., diurnal solar flux) on the performance of sensitive heat flow sensors installed on a buried transmission line. The calculations were done for typical rock material (diffusivity, $0.01 \text{ cm}^2/\text{s}$) at a depth of one meter. The results indicated that the amplitude of the solar heat flux variation (assumed to be $0 - 0.1 \text{ W/cm}^2$ at the earth's surface) decreased to $1.9 \times 10^{-4} \text{ W/cm}^2$ at

a depth of one meter. The magnitude of the amplitude of this incident heat flux on the surface of the sensor (noise) was considered large enough to be possibly detrimental to a heat flow sensor detection of small changes in heat flux (e.g., 10^{-4} to 10^{-6} W/cm²) resulting from vacuum deterioration (signal we want to detect).

Therefore, we have performed more refined calculations to estimate the signal to noise ratio for the sensor, taking into account the thermal properties and thicknesses of the earth, sensor, and steel pipe to which the sensor would be thermally attached.

The solution is in accordance with that given by Carslaw and Jaeger [1] for the steady periodic temperature in a composite slab shown in figure 10.

Figure 10.

Let v_n and f_n be the temperature and flux at the top face of each slab and v'_n and f'_n the temperature and flux at the bottom face of each slab.

The objective is to calculate the amplitude of the periodic temperature difference across the sensor ($v_2 - v'_2$) and thus the amplitude of the periodic output of the heat flow sensor ($q_{\text{sensor}} = K_{\text{sensor}} \frac{(v_2 - v'_2)}{l_2}$) for the following boundary conditions:

Case 1

$f_1 = A$ (the amplitude of the diurnal solar heat flux incident on the earth's surface)

$f'_3 = 0$ (no vacuum deterioration)

This case gives the "noise" output for the sensor.

Case 2

$$\begin{aligned} f_1 &= A \\ f_3' &= 10^{-4} \text{ W/cm}^2 \quad (\text{This corresponds to a gradual change in heat leak at the inside surface of vacuum jacket over a time period the same as the diurnal flux, i.e., one day}). \end{aligned}$$

This case gives the signal plus noise output for the sensor. We then

$$\begin{aligned} \text{compare: } \frac{\text{signal} + \text{noise}}{\text{noise}} &= \frac{(\text{Case 2})}{(\text{Case 1})} \\ &= \frac{\text{signal}}{\text{noise}} + 1. \end{aligned}$$

For perfect thermal contact between the faces, the following matrix equation is the solution.

$$\begin{pmatrix} v_n' \\ f_n' \end{pmatrix} = \begin{pmatrix} A_n & B_n \\ C_n & D_n \end{pmatrix} \begin{pmatrix} A_{n-1} & B_{n-1} \\ C_{n-1} & D_{n-1} \end{pmatrix} \dots \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} v_1 \\ f_1 \end{pmatrix} \quad (1)$$

where the A_r , B_r , C_r , D_r of each slab are:

$$\begin{aligned} A_r &= \cosh \bar{k}_r \ell_r (1 + i) \\ B_r &= \frac{\sinh \bar{k}_r \ell_r (1 + i)}{K_r \bar{k}_r (1 + i)} \\ C_r &= K_r \bar{k}_r (1 + i) \sinh \bar{k}_r \ell_r (1 + i) \\ D_r &= \cosh \bar{k}_r \ell_r (1 + i) = A_r \\ \bar{k}_r &= (\omega / 2\alpha_r)^{1/2} \\ \alpha_r &= \text{thermal diffusivity of each slab} \\ \omega &= \text{angular frequency} = 2\pi f \\ K_r &= \text{thermal conductivity of each slab} \\ \ell_r &= \text{thickness of each slab.} \end{aligned} \quad (2)$$

For our problem, (1) may be written (noting that $v_1' = v_2$ and $f_1' = f_2$),

$$\begin{aligned}
v_2 &= A_1 v_1 + B_1 f_1 \\
f_2 &= C_1 v_1 + D_1 f_1 \\
v_2' &= X_1 v_1 + X_2 f_1 \\
f_2' &= X_3 v_1 + X_4 f_1 \\
v_3' &= X_5 v_1 + X_6 f_1 \\
F_3' &= X_7 v_1 + X_8 f_1
\end{aligned}
\tag{3}$$

where

$$\begin{pmatrix} X_1 & X_2 \\ X_3 & X_4 \end{pmatrix} = \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}$$

and

$$\begin{pmatrix} X_5 & X_6 \\ X_7 & X_8 \end{pmatrix} = \begin{pmatrix} A_3 & B_3 \\ C_3 & D_3 \end{pmatrix} \begin{pmatrix} X_1 & X_2 \\ X_3 & X_4 \end{pmatrix}$$

Now with f_1 and f_3' boundary conditions known the remaining variables including v_2 and v_2' may be calculated.

A summary of the results of the calculations is given in Table 1.

Table 1. Estimated Signal/Noise Ratios for Typical Heat Flow Sensor.

Earth Material	Earth Thickness ℓ_1	Amplitude of Temperature Difference Across the Sensor (Case 1)	Amplitude of Temperature Difference Across the Sensor (Case 2)	Signal/Noise
Typical Rock	1 meter	7.4×10^{-3} K	3.0×10^{-2} K	3
Typical Rock	1.5 meter	3.6×10^{-4} K	2.5×10^{-2} K	68
Average Soil	1 meter	4×10^{-4} K	2.4×10^{-2} K	59
Average Soil	1.5 meter	4.7×10^{-6} K	2.4×10^{-2} K	5105

Table 2. Data Used in Calculations

Material	Thermal Conductivity (W/cm K)	Thermal Diffusivity (cm ² /s)	Thickness (cm)	Remarks
Rock	1.757×10^{-2}	0.01	100 cm or 150 cm	Ref: Carslaw and Jaeger for thermal properties
Average Soil	9.623×10^{-3}	0.0046	100 cm or 150 cm	Ref: Carslaw and Jaeger for thermal properties
Sensor	2.1×10^{-3}	0.00327	0.635	Ref.: RDF Corp. data on thermal properties and thickness
Steel	4.6×10^{-1}	0.12	1.27	Ref.: Carslaw and Jaeger for thermal properties

Values for f_1 (the amplitude of the incident heat flux at the earth's surface) were calculated from Carslaw and Jaeger's equation for the flux of heat at the surface of a semi-infinite solid with surface temperature a harmonic function of time. The equation is:

$$F = f_1 \cos (\omega t - \epsilon + \pi/4)$$

where

$$f_1 = 2^{1/2} \bar{k}_{\text{earth}} K_{\text{earth}} A_T, \text{ W/cm}^2$$

$$A_T = \text{amplitude of the temperature oscillation, K.}$$

For $A_T = 16.6 \text{ K}$ (30°F) this yields values for f_1 of 0.025 W/cm^2 for rock material and 0.020 W/cm^2 for average soil.

The preliminary calculations mentioned in 5.2 were based on the unattended solar heat flux incident on the earth's surface (0.1 W/cm^2). However, it is felt that the values for f_1 as determined above are more realistic when a reasonable temperature variation, A_T , is used in the calculation.

While recognizing that the model, a composite of 3 semi-infinite slabs, is not highly representative of a 20 inch (50.8 cm) diameter pipe with a finite size sensor attached and buried beneath the earth's surface, it is felt that for the metering area of the sensor the calculated estimates of sensor signal/noise ratio offer a reasonable approximation to the real situation.

We note from Table 1 that for typical rock material comprising the earth's surface and the pipe buried one meter deep, the signal/noise ratio is only three where we would have preferred something greater than 10. But we note further that for increased depth of burial of the pipe in rock material and for average soil thicknesses (or depth of burial) greater than one meter, the signal/noise ratio for the sensor is very much greater than 10 (i.e., 68-5105).

Therefore, based on the calculations, we conclude that the proposed use of heat flow sensors for vacuum level measurement on a buried cable will not be hindered by diurnal heat flux incident on the earth's surface. This is dependent of course on the material comprising the earth and the depth of burial, but for depths approximately 1.5 meters there should be negligible effect.

The experimental program planned will provide actual data on this subject.

5.3. Future Work

We hope to complete the design and construction of the experimental apparatus and to begin the experimental test program.

Calculations will be done for an unburied pipe to determine the effectiveness of insulations for shielding the sensors from extraneous environmental heat fluxes.

A further analytical study is planned wherein we hope to determine the temperature field (and heat flux field) on the surface of the buried cylindrical pipe when subject to a periodic heat flux at the surface of the earth.

The model (representative of the actual situation) is shown in figure

Figure 11.

A knowledge of the temperature and heat flux field on the surface of the pipe provides for the optimum location of heat flux sensors on the pipe surface (i.e., where they are least affected by the solar heat flux).

5.4. Reference

- [1] H. S. Carslaw and J. C. Jaeger, Conduction of heat in solids, Oxford Univ. Press, London, 1959.

APPENDIX 1.

An Experimental Study of Thermally-Induced Flow
Oscillations in Supercritical Helium

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AN EXPERIMENTAL STUDY OF THERMALLY-INDUCED
FLOW OSCILLATIONS IN SUPERCRITICAL HELIUM *

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Abstract

The density wave stability boundary has been experimentally determined for supercritical helium flowing in a long ($L=185$ m), heated channel of high aspect ratio ($L/d = 4.6 (10)^4$). A pressure drop ratio and the fluid expansion ratio correlate the oscillation inception point data.

The growth of enthalpy (temperature) perturbations in a heated channel has been experimentally verified. During the density wave oscillation, the channel exit temperature and inlet mass flow were observed to be in phase, and the oscillation period was close to twice the fluid transit time. All three observations agree with a simple incompressible flow model. Oscillation amplitudes as great as 11 K and 100 percent of inlet flow were observed.

AN EXPERIMENTAL STUDY OF THERMALLY-INDUCED FLOW OSCILLATIONS IN SUPERCRITICAL HELIUM

D. E. Daney, P. R. Ludtke and M. C. Jones

Introduction

This study of the stability of supercritical helium flowing in a heated channel was prompted by an interest in superconducting power transmission lines [1]. These lines will typically be cooled by supercritical helium over the temperature range of 6K to 10K and at pressures up to 15 bar. They are characterized by a high aspect ratio ($L/d \approx 10^5$) and by a low heat flux ($q \approx 10^{-5}$ W/cm²). Of the various flow instabilities described by Boure et al. [2], the density wave instability has been considered the most common and likely to occur. It is this instability that was observed and studied in our work.

In the density wave instability, temperature (density) disturbances move down the heated channel at the fluid velocity, producing flow and temperature oscillations with a frequency of approximately $1/2 \tau$, where τ is the fluid transit time. The amplitude of these temperature oscillations can be several kelvin or more, making operation of the high current superconductor hazardous, if not impossible. This hazard results because if the superconductor warms to its transition temperature (the temperature

at which it becomes normal or nonsuperconducting), large quantities of electrical energy will be dumped into the conductor due to the discontinuous jump in its resistivity. At best, an unacceptable loss of service would occur. Rupture of the coolant channel due to rapid warming of the helium might also occur.

Although cooling of superconducting power transmission lines was the primary motivation for this study, the results should be applicable to other helium-cooled devices such as superconducting magnets. In addition, our data should provide a useful empirical base for the general theory of density wave oscillations since our operating conditions allowed us to observe these oscillations in close to their simplest form. These experiments were characterized by 1) negligible wall heat capacity, 2) nearly perfect heat transfer, 3) fluid expansion primarily due to heat transfer rather than pressure drop, and 4) well-controlled and defined boundary conditions.

At ambient temperature and above, density wave oscillations have been studied extensively in recent years [2-4], but relatively little experimental work has been reported for cryogenic fluids [5,6]. Jones and Petersen [7] have recently reported on a numerical analysis of supercritical helium flow stability using an exact equation of state, and Zuber [8] and Friedly et al [9] have analyzed supercritical systems using an approximate two-zone equation of state. The effort reported here is, to our knowledge, the first experimental work on density wave oscillations in supercritical helium.

One point that becomes clear in reviewing these studies is that the density wave instability is a system instability, that is, it is characterized not only by the conditions in the heated section, but by the boundary conditions as well. Thus control and measurement of the supply pressure and temperature and of the inlet and exit flow impedances are essential to a meaningful study. Adequate control of the boundary conditions during oscillatory flow is not a trivial problem, and some effort was required before the desired level of stability was achieved in the fluid supply.

The primary goals of this work were to observe the nature of density wave oscillations in supercritical helium and to define the stability boundary so that the safe operating region for supercritical helium might be known. Propagation of heat pulses or thermal waves (local temperature spikes or disturbances) was also studied, both as an aid to understanding the nature of density wave oscillations, and because of an interest in the waves themselves. Analysis of superconducting-to-normal zone propagation in helium cooled superconductors requires detailed knowledge of such thermal wave propagation. As a by-product of this study, the hydraulic resistance of supercritical helium was also measured [10].

Experimental

Figure 1 is a schematic of the apparatus which consists of 3 vessels: precooler, subcooler, and experimental chamber (containing the final

Figure 1. Flow schematic.

sub-cooler and test section). The experiment is supplied with helium gas which is passed through an ambient temperature purifier and regulated over the range of 0.8 to 2.0 bar. Cooling to 4.5 K occurs through counterflow heat exchange with the precooler and subcooler boil-off gas, and trace impurities are removed by an 80 K charcoal purifier in the precooler. Reduction to the final experimental pressure is accomplished by a liquid helium temperature, dome-loaded, pressure regulating valve [11]. Final pressure regulation at this point is necessary in order to eliminate supply (point a, figure 1) pressure variations resulting from the variable pressure drop in the heat exchangers that occurs during oscillatory flow. The temperature at the inlet to the plenum is regulated by a direct current heater in combination with a controller and carbon resistance thermometer.

The 185 m long test section, which is wound in a helical coil and held by fiberglass spacers, is suspended in a 914 mm diameter vacuum chamber and is shielded by both an outer radiation shield cooled by liquid nitrogen and by an inner shield cooled by the helium exhausted from the test section. The resulting heat leak to the test section is less than 1 watt. It is heated by passing current through the tube wall, giving uniform wall heat flux. Stray current paths are eliminated by dielectric joints (which use silica filled epoxy) situated at either end and along the pressure taps. Battery supplied direct current to the test section is used to prevent inductive pick-up in the thermometer circuits.

The test section is fabricated from 5 lengths of 3.99 mm I.D. x 0.41 mm wall, type 304 stainless steel tubing joined by copper couplings which are machined to match the inside tube diameter and which serve as thermometer wells and pressure taps. The distance between inlet and exit valves is 184.6 m, giving an aspect ratio (L/d) of $4.6 (10)^4$. The helix diameter is 775 mm, giving a helix to tube diameter ratio of 194. The resulting increase in hydraulic resistance due to the curvature is about 10 percent [10,12].

The operating range in these experiments was:

P_o	2.7 - 15 bar
T_o	4.6 - 6.0 K
$\dot{m}_1$	0.32 - 0.58 g/s
u_1	21 - 38 cm/s
Re_1	$3.2(10)^4 - 6.2 (10)^4$
R	0 - 15

choked flow at exit (position e , figure 1)

For reference, the helium critical point is 2.22 bar and 5.2 K. The ratio of the fluid to wall heat capacity, $(\rho A C_p)_{He}/(\rho A C)_{w}$, is about 450 at 4K and 140 at 10 K.

Instrumentation. Pressures are measured with variable reluctance and capacitance pressure transducers operating at ambient temperature. Temperatures are measured with germanium resistance thermometers which

measure the temperature of the copper couplings between tubing lengths. As a result of the low test section heat flux ($q \approx 10^{-3} \text{ W/cm}^2$), and because the coupling is unheated, the coupling-to-helium temperature difference is estimated to be 0.01 K or less. The installed thermometer response time is estimated to be one second or less. The inlet flow is measured by a sharp-edged orifice (diameter = 1.3 mm) which was calibrated in situ against the ambient temperature thermal type mass flowmeter during steady flow. This commercial meter determines the mass flow rate from the temperature rise of the flow due to a calibrated heat input. The total uncertainty in the measurements is estimated to be: P_1 , 0.5 percent; ΔP_{0-1} , 3 percent; ΔP_{1-6} , 200 Pa; test section temperature, 0.02 K; inlet mass flow rate, 3 percent; ambient (discharge) flow rate 0.5 percent.

Data are collected with a mini computer which gives "real time" conversion of all instrument channels to physical parameters (P,T,m, etc.). A strip chart records the wave form.

Procedure. Because stable inlet boundary conditions were considered essential to a meaningful study, we held the conditions at point a (figure 1) constant during a particular test. This is equivalent to a near zero supply impedance. Typically, the pressure (controlled by the cold PRV) was constant to within a few thousandths of a bar, and the temperature (controlled by the heater-controller), was constant to within a few hundredths K for all but the most severe oscillations.

Density wave oscillations were induced by two methods. The usual technique was to gradually increase the test section heating in a series of steps while holding the inlet valve position fixed, and adjusting the exit valve to maintain a constant inlet mass flow rate. In the alternative technique, which was used to obtain high values of the fluid expansion ratio, R , and the pressure drop ratio, ψ , the heating was increased to give the desired fluid expansion ratio. Then the inlet valve was opened (reducing the supply impedance) until oscillations occurred.

Selection of a stability criterion, i.e., defining the oscillation inception point, was something of a problem. Because of the low frequency of the oscillations, typically 1.5 to 3.5 mHz, more than an hour was required to follow as few as 6 or 10 cycles. As an unstable operating point was approached, damped or weak oscillations were induced (and observed for 5-10 cycles) by perturbations associated with changing the operating conditions. Thus 5 or 6 hours were required to obtain each stability point, and the time required to obtain true steady-state would have been at least several times as long — truly prohibitive in both helium and manpower costs. Instead of defining the inception of flow instability as the break in the steady state curve of flow fluctuation vs. heating rate [3], we settled on similar criteria which include a qualitative judgment about the growth or decay of an oscillation. We

define the following three flow classifications, quantified from our original intuitive judgments of the inception of instability.

<u>Classification</u>	<u>Amplitude</u>	
	$\frac{(\Delta\dot{m}/\dot{m})_0}{}$	$\frac{\Delta T_6}{}$
Stable	< 4 percent	< 0.2 K
Marginal	> 3 percent and $\leq$ 10 percent	> 0.15 K and $\leq$ 0.26 K
Unstable	> 9 percent	> 0.27 K

In case of growth or decay of the oscillation, the amplitude listed is for the end of the observation period, usually 6 to 10 cycles. The apparent overlap of the criteria is eliminated when both flow and temperature are considered. It is interesting to note that our criteria agree well with inception criteria for Freon 113 [3], where inception typically occurred at flow disturbances of about 5 percent of the mean flow.

Results and Discussion

General Observations. Initial difficulties experienced in obtaining steady inlet conditions emphasized the importance of the boundary conditions to the system stability; they suggested that a supply system which allows variation of the temperature of the fluid arriving at point a (figure 1) can have a destabilizing effect. For example, in flow studies which use a helium refrigerator as the supply, Hoffer and Dean [13] report multiple "reflections" of thermal waves off the final re-refrigerator heat exchanger. Because such a boundary condition may be typical of many applications, its effect should receive further study.

The period required for the heated line to achieve steady-state after a change in operating conditions appeared to be many fluid transit times on some occasions, and on one occasion a combination of inlet flow drift and a slowly damped oscillation resulted in a severe excursion to zero inlet flow. It is not clear whether or not this was a true excursive (Ledinegg [2]) instability since calculations by Arp [14] indicate that this instability is not possible under the conditions of our experiments. In any case, the temperature perturbations which accompany such flow disturbances are unacceptable for superconducting applications, and their occurrence points to the need for active flow control in some cases.

Because the response time of large aspect ratio lines is much greater than normally encountered, care is necessary regarding assumptions of steady-state or assumptions of equal flow in and out. This is especially true following a change in operating conditions, e.g. an increase in the heat load. Expansion of the resident fluid can cause a large temporary imbalance in the inlet and exit flows, and failure to maintain the inlet flow by adjustment of the exit valve (or inlet restriction if appropriate) can result in high amplitude density wave oscillations — even under conditions which are stable for sustained oscillations. Although these oscillations may be damped, they are none the less damaging if the initial temperature excursion exceeds the safe operating limit.

In view of this difficulty in changing the heating rate without inducing a significant flow ($\dot{m}_0$) and temperature (T_0) perturbation, the disturbance required to trigger an oscillation was always present. Thus, an experiment generally comprised a series of progressively less damped oscillations with the final oscillations being either sustained or growing. The mode of operation usually gave only a single large sustained disturbance which set the system oscillating at its characteristic frequency. Disturbances of short duration gave spikes superimposed on the basic wave form.

Heat Pulse Behavior. There are two principle features of the density wave oscillations we observed in supercritical helium. One feature concerns the effect of the exit density on the flow and the resulting phase lag between the inlet and exit mass flow rates. The other feature is the tendency of enthalpy (temperature) perturbations to grow [7] in a heated channel. We first consider this latter feature, which is also of interest in its own right.

The growth of heat pulses, or enthalpy perturbations, occurs because an element of fluid that is less dense (warmer) than average undergoes greater than average heating per unit mass (for constant wall heat flux) as it travels down a heated channel, making it even less dense and subject to even greater heating, etc. The phenomenon can be analyzed by considering a simple linear perturbation of enthalpy and specific volume (due to a small perturbation in the heat flux, for example) in a small element of fluid in a stream with steady flow previous to the perturbation. A constant area duct and uniform heat flux, q , are assumed. For the case of negligible wall heat capacity (less than one percent of the fluid heat capacity in these experiments) and negligible axial wall conduction, the instantaneous wall temperature of a resistance-heated tube will adjust itself so as to maintain constant q for both the perturbed and unperturbed elements of

the fluid. If we choose a Lagrangian method of description so that we follow a small fixed mass of perturbed fluid, the problem is greatly simplified since we need consider only the perturbed element rather than all elements. This approach was suggested by the experiment, during which we could observe warm slugs of fluid flow slowly past the thermometers along the 183 meter test section, while maintaining their identity and traveling at the fluid velocity.

For the Lagrangian method, the one-dimensional continuity and energy equations are

$$\rho A_c \Delta x = \Delta m = \text{constant} \quad (1)$$

$$\rho A_c \frac{dh}{dt} = qP \quad (2)$$

where Δx and Δm are the length of a small element, q is heat flux, P heated perimeter, A_c flow area, h specific enthalpy, and ρ density. To obtain these equations in terms of perturbed variables we write the variables h , ρ and Δx , as the sum of a steady part and a time dependent part. For example, $h = \bar{h} + h'(t)$ where $\bar{h}$ is the unperturbed value and $h'(t)$ is the perturbed value. Substituting these expressions into (1) and (2), then subtracting the unperturbed expressions yields the continuity and energy equations for the perturbed variables, which, for convenience, are expressed in terms of the specific volume perturbation, v' , rather than the density perturbation.

$$\Delta x' = v' \frac{\Delta m}{A_c} \quad (3)$$

$$dh' = \frac{qP}{A_c} v' dt \quad (4)$$

In contrast to the perturbations in the other variables, we estimate that the velocity perturbation, u' , is second order small by considering an idealized case with the flow impedance concentrated either at the inlet or the exit. If

it is at the inlet, the velocity of the upstream boundary is unperturbed. If it is at the exit, then the velocity of the downstream boundary of the perturbed element is unperturbed. For either extreme, the magnitude of the velocity perturbation is $u' = \partial \Delta x' / \partial t$, which becomes $u' = (qP/A_c^2) (\partial v / \partial h)_p v' \Delta m$ after manipulation using (3) and (4). For both v' and Δm small, u' is second order small, being the product of two first order small terms.

The expression for heat pulse growth is obtained by dividing (4) by h' and setting $v'/h' = (\partial v / \partial h)_p$. This assumes that pressure effects are absent and that the local equation of state is identical with that for stable equilibrium. Integration of the resulting equation yields the time domain expression for the growth of an enthalpy perturbation in a heated channel in the absence of pressure effects.

$$h' = h_1' e^{\int \Omega dt} \quad (5)$$

where $\Omega \equiv qP/A_c (\partial v / \partial h)_p$ is the rate of fluid expansion and is the so-called reaction frequency introduced by Zuber [8] and used by Friedly [9] and Jones [7]. Equation (5) which is for the characteristic line, $dt = dx/u(x,t)$, indicates an enthalpy perturbation will grow exponentially with time as the fluid travels down a heated line. Because the velocity increases along the channel, the growth with respect to length will not be exponential.

Enthalpy perturbation growth may be expressed in terms of specific volume by noting in (4) that $qPdt/A_c$ is the enthalpy change per unit volume dh/v . Making this substitution gives

$$dh' = \frac{dh}{v} v'.$$

Dividing by h' , setting $v'/h' = (\partial v / \partial h)_p$ and integrating results in

$$h' = h_1' \left(\frac{v}{v_1} \right). \quad (6)$$

Thus, for uniform heating, the growth of an enthalpy perturbation will go as $v = v(h)$, which is more nearly linear than exponential.

Expressed in terms of temperature, equation (5) becomes

$$T' = (C_{p1}/C_p) T_1' e^{\int \Omega dt} \quad (7)$$

where C_p is the specific heat at constant pressure.

The time, Δt , required for a square wave to pass a fixed point is easily obtained from the continuity equation. Rearranging (1) gives

$$\Delta x = \frac{v}{v_1} \Delta x_1$$

where the subscript 1 refers to the initial perturbation. Using the definition of velocity, $\Delta x = u \Delta t$ gives

$$\Delta t = \Delta t_1 \left(\frac{u_1}{u} \right) \left(\frac{v}{v_1} \right),$$

and the steady flow Eulerian continuity equation is substituted for u to obtain the final form

$$\Delta t = \Delta t_1 \left(\frac{\bar{v}_1}{\bar{v}} \right) \left(\frac{v}{v_1} \right), \quad (8)$$

where $\bar{v}$ is the unperturbed (steady state) specific volume and v is the specific volume of the perturbed fluid. Depending on the equation of state and the operating conditions, equation (8) can give either a positive or negative change in Δt .

Figure 2 shows the progression down the unheated test section of a thermal wave which was generated by a 20 second activation of the pulse heater. Although there is some rounding of the wave due to diffusion and wall heat transfer, there is no significant growth before position 6. A small anomalous heat leak to the end of the test sections produces a slight broadening at this point. Note that the wave front is on the left side of the wave since figure 2 shows the response of a fixed thermometer to moving wave.

Figure 2. Propagation of a heat pulse in unheated test section;
 $P_1 = 2.87$ bar, $T_1 = 4.37$ K, $\dot{m} = 0.493$ g/s, $U_1 = 30.2$ cm/s.

In contrast, a heat pulse traveling down a heated line, figure 3, exhibits rapid growth in both amplitude and time required to pass a fixed point as predicted by equations (7) and (9). The growth in Δt predicted by equation (8) for the experimental conditions is 33 percent compared to about 50 percent observed in figure 3. The agreement is probably as good as can be expected considering the inexact theoretical model, the irregular wave form, and the non-zero wall heat capacity. The irregular shape of the original pulse is due to flow perturbations induced by the heat pulse in the more compressible heated helium. The decay of the wave as it passes thermometer 6 results because the front of the wave arrives at the exit valve, accelerating the flow, before the remainder of the wave has passed position 6.

Density wave oscillations. The second feature of density wave oscillations which we consider is the phase or space lag between the inlet and exit mass flow rates (and other system variables as well) which is required for an oscillatory instability [8]. This feature is most easily understood by considering a simple incompressible flow model (which considers thermal expansion, but not pressure effects) with a large exit flow impedance such as was present in the experiments. Focusing our attention on the exit restriction, we note that the velocity at this point is given by

$$u_d \sim \sqrt{\frac{\Delta P_{d-f}}{\rho_d}} \quad (9)$$

When warmer than average fluid (lower density) arrives at the exit restriction, the velocity increases, and the fluid in the line accelerates to satisfy continuity. The residence time of the fluid in the heated section decreases, resulting in less heating per unit mass and, hence, in cooler fluid delivered to the exit restriction after some time lag. As the cooler fluid flows through

Figure 3. Propagation of a heat pulse in heated test section; $P_1 = 3.93$ bar, $T_1 = 5.40$ K, $\dot{m} = .429$ g/s, $U_1 = 30.4$ cm/s, $Q = 11.1$ W.

the exit restriction the fluid in the heated section decelerates, giving it an increased residence time and greater than average heating. Thus, a positive perturbation in the exit temperature induces a negative perturbation or reflection which arrives at the exit after the transit time of the fluid in the line. The period of the oscillation is twice the transit time, and the mass flow rate at the inlet and exit are 180 degrees out of phase. This phase difference results because in the absence of pressure effects, the inlet and exit velocities are in phase, making the inlet mass flow, $\dot{m}_b$, proportional to $1/\sqrt{\rho_d}$, i.e., $\dot{m}_b \sim u_b \sim u_d \sim \sqrt{1/\rho_d}$. According to (9), however, the exit mass flow, $\dot{m}_d$, is proportional to $\sqrt{\rho_d}$, giving the 180 degree phase shift in inlet and exit mass flow rates.

Figure 4 shows a typical density wave oscillation, composed from a strip chart trace of orifice pressure drop and thermometer resistance. In this case the instability was induced by reduction of the inlet impedance. The exit valve was adjusted at the same time so as to maintain the average flow rate constant. Note that the exit temperature and inlet flow are in phase, implying that inlet and exit mass flow rates are 180 degrees out of phase -- in agreement with the simple qualitative model discussed above. Although continuous traces of the exit flow were not recorded, discrete simultaneous readings of $\dot{m}_a$ and $\dot{m}_f$ were made, and these also confirm the 180 degree phase shift. The density wave oscillation shown in figure 5 was induced and then damped by changes in the power level. This trace shows both the growth and phase relation of the thermal wave as it progresses down the heated channel. As before, the inlet mass flow and exit temperature are nearly in phase.

Development of a stability (inception point) map for density wave oscillations in supercritical helium requires a set of scaling or correlating parameters, for which we have relied on the work of Zuber [8]. In his simplified stability

Figure 4. Density wave oscillation induced by reducing inlet impedance, $Q = 37.2$ W.

Figure 5. Density wave oscillation, $P_0 = 6.0$ bar, $T_0 = 5.80$ K, $\psi = 0.008$. Power level reduction causes an unstable oscillation to become damped or marginally stable.

analysis, Zuber assumes a two zone equation of state for a supercritical fluid. In the first (fluid) zone the specific volume is independent of enthalpy, and in the second (gas) zone the specific volume is proportional to enthalpy. The stability criterion he derives with this model [equation VII-29 in reference 8], expressed in terms more convenient to our purposes here is

$$\frac{2\Delta P_{ac}}{\Delta P_{cf}} > \left[1 + \frac{\Delta h_{bc}}{\Delta h_{cd}} \right] \left[\frac{R}{1 + R} \right] \quad (10)$$

for stability.

A similar expression with an essentially linear dependence on the expansion ratio has been used by Serov and Smirov [15].

Because (10) is based on a rather crude equation of state, it should not be expected to give a highly accurate description of the stability boundary. It does suggest, however, the use of $\Delta P_{ac}/\Delta P_{cf}$ and expansion ratio, R , as correlating parameters, taking the zone transition point c to be the transposed critical point (the locus of the specific heat maxima).

The dimensionless position of the transposed critical in the channel is determined from $\zeta = (h_c - h_b)/(h_d - h_b)$, where h_c is obtained from a plot of the transposed critical enthalpy vs. pressure, and the pressure drop ratio, ψ , is given by

$$\psi \equiv \frac{\Delta P_{ab} + \Delta P_{bc}}{\Delta P_{cd} + \Delta P_{de}} = \frac{\Delta P_{ab} + \zeta \Delta P_u}{\Delta P_h - \zeta \Delta P_u + \Delta P_{de}} \quad (11)$$

The unheated line pressure drop, ΔP_u , is that calculated for the measured conditions at point b with $Q = 0$, whereas the heated pressure drop, ΔP_h , is the mean measured value. Because the exit flow is choked, the pressure drop to choked conditions rather than the total pressure drop is used to evaluate ΔP_{de} . Except for very low values of ψ (obtained by removing the inlet valve) ΔP_{bc} is generally less than 25 percent of ΔP_{ab} .

Even though ψ is derived from a somewhat inaccurate and crude equation of state, its use gives a rather good correlation of the stability data as shown in figure 6. The curve defining the stability boundary is estimated from the experimental data. The points shown are those observed closest to the stability boundary for a particular experimental run, either the last stable operating point or the first unstable point. The points marked marginal are for weakly damped or low amplitude oscillations. Thus, two or three points are shown for each run. A summary of the experimental data is given in the Appendix. Numerical results of Jones and Peterson for supercritical helium [7] are also given in figure 6 for comparison. These points, which were calculated for typical design and off-design conditions, are not at the stability boundary, but they do seem to indicate that the numerical calculations give conservative results, i.e., the numerical method predicts unstable conditions at lower values of fluid expansion. Two likely reasons for this trend are the neglect of the wall specific heat in the numerical method and differing definitions of stability. The latter is probably the more important. The numerical method requires that the stability boundary be defined as the point at which infinitesimal disturbances grow, whereas the experimental method classifies mass flow oscillation amplitudes of less than 4 percent as stable (see procedure section for details).

Figure 7 compares the measured oscillation frequency to the calculated fluid transit time as a function of the degree of instability. The abscissa, $2\tau f$, is the ratio of the measured frequency to the frequency predicted by the simple incompressible flow model, $1/2\tau$. The fluid transit time, τ , is calculated from the mean measured inlet conditions (P_1 , T_1 and $\dot{m}_1$) and from the heat input, $Q = \dot{m}_1 (h_6 - h_1)$. The correlating parameter, R_m/R_i is the ratio of the measured fluid expansion to the fluid expansion at the oscillation inception point (stability

Figure 6. Density wave stability map for supercritical helium. The curve is estimated from the experimental results.

Figure 7. Normalized frequency as a function of the degree of instability.

boundary in figure 6). This parameter is a measure of the relative degree of instability. A value less than one denotes stable conditions, and a value greater than one denotes unstable conditions.

The principal feature of figure 7 is the clustering of $2\tau f$ about a value of 1.0 as predicted by the simple incompressible flow model. At the stability boundary, $R_m/R_i = 1.0$, the frequency ratio $2\tau f$ is 1.0 within the experimental error. Although R_m/R_i is a somewhat imprecise parameter because of the uncertainty in the location of the stability boundary in figure 6, the trend towards a higher frequency ratio at greater degrees of instability is clear. Because the oscillation amplitude increases with the degree of instability, linearly time averaged properties may over-estimate the actual transit time, contributing to the trend in figure 7. Compressibility effects may also be present and contribute to this trend, since a pressure dependence may also be indicated.

Correlation of the frequency ratio vs. both the position of the transposed critical point, ζ , and the pressure drop ratio, ψ , proved inconclusive.

Summary

This experimental study of flow stability of supercritical helium flowing in a long ($L = 185$ m, $L/d = 4.6 (10)^4$) heated channel has defined the stability boundary of density wave oscillations for the case of a choked exit restriction. The pressure drop ratio, ψ , and fluid expansion ratio, R , correlate the oscillation inception point data.

The growth of enthalpy (temperature) perturbations in a heated channel has been experimentally verified. During density wave oscillations, the channel exit temperature and inlet mass flow were observed to be in phase, and the oscillation period was close to twice the fluid transit time. All three observations agree with a simple incompressible flow model.

For superconducting power transmission lines, the application which prompted this study, density wave oscillations of the amplitude observed in this study would make operation hazardous if not impossible. Fortunately, however, the anticipated operating conditions are generally such that the fluid expansion ratios are expected to be less than 2.0 — a stable condition for even low inlet impedances. Off design operating conditions which give high expansion ratios should be avoided though, and consideration should be given to active control of the helium temperature at the inlet to the transmission line so as to avoid "reflection" of temperature perturbations from the refrigerator heat exchangers.

If operation of a system under unstable conditions is anticipated, then active control of the inlet flow will be required. Uncoupling the flow rate from the exit conditions should give stable operation even for severe conditions.

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Nomenclature

A_c	cross sectional
A_s	channel surface area
C	specific heat
C_p	specific heat at constant pressure
d	channel diameter
f	oscillation frequency
h	specific enthalpy
L	channel length
$\dot{m}$	mass flow rate
p	pressure
P	wetted perimeter of channel
q	wall heat flux
Q	total channel heat transfer rate
R	expansion ratio, $\Delta v_{1-6}/v_1 = \Delta v_{bd}/v_b$
Re	Reynolds number, $\frac{\rho u d}{\mu}$
t	time
T	temperature
u	velocity
v	specific volume
x	longitudinal position along channel

Greek

μ	viscosity
ζ	dimensionless position of the transposed critical, equation (11)
τ	fluid transit time
ρ	density
ψ	pressure drop ratio, $\frac{\Delta P_{ab} + \Delta P_{bc}}{\Delta P_{cd} + \Delta P_{de}}$
Ω	rate of fluid expansion, $\frac{qP}{A} \left(\frac{\partial v}{\partial h} \right)_p$

Subscripts

a - f	positions indicated in figure 1
c	position of the transposed or pseudo-critical point - Location varies with operating conditions.
h	heated
H _e	helium
i	inception point of oscillation (stability boundary)
m	measured
t	transposed critical point
u	unheated
w	wall
0 - 6	instrument positions indicated in figure 1.

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APPENDIX. Oscillation Data

Run No.	P ₁ (bar)	T ₁ (K)	ΔP ₀₁ (bar)	ΔP ₁₆ (bar)	T ₆ (K)	P _e ** (bar)	$\dot{m}_1$ (g/s)	Q (W)	f (mHz)	oscillation amplitude $\Delta \dot{m}_1 / \dot{m}_1$ (percent)	ΔT ₆	ρ ₁ g/cm ³	Rex10 ⁻⁴	ζ	ψ	R	classification
5-3	2.87	4.85	0.033	.085	7.79	1.35	0.317	10.9	1.34	12.3	0.27	0.1212	3.18	.287	0.026	4.73	unstable
5-5	3.96	5.42	0.040	.118	9.93	1.84	0.384	16.3	1.87	excursion to zero flow*		0.1123	4.04	.214	0.024	4.20	"
5-5	3.90	5.40	0.076	.184	11.23	1.79	0.430	21.8	2.36	34.0	1.83	0.1122	4.54	.177	0.041	5.35	"
5-5	3.94	5.39	0.065	.147	10.57	1.82	0.417	19.5	2.16	9.8	0.49	0.1132	4.39	.199	0.035	4.79	"
5-11	3.78	5.00	0.182	.259	15.58	1.70	0.418	32.7	2.90	35.0	2.18	0.1231	4.02	.151	0.091	9.84	"
5-12	5.99	6.03	0.042	.193	17.06	2.78	0.421	32.3	2.71	37.4	2.70	0.1121	4.20	.129	0.015	5.50	"
5-25	2.45	4.63	0.197	.285	13.26	1.05	0.397	28.2	2.71	100.0	10.91	0.1243	3.86	.139	0.145	14.68	"
5-26	3.20	5.00	0.274	.366	18.78	1.38	0.435	41.8	3.44	9.6	0.84	0.1191	4.40	.099	0.153	15.11	"
6-14	2.95	5.00	0.015	.139	7.56	1.36	0.470	15.1	1.85	29.4	0.68	0.1171	4.89	.284	0.021	4.22	"
6-15	3.99	5.21	0.012	.098	8.71	1.87	0.399	14.3	.60	21.3	0.78	0.1188	4.01	.293	0.012	3.43	"
6-15	5.96	5.80	0.014	.140	12.54	2.78	0.448	24.0	2.12	22.0	0.90	0.1176	4.35	.221	0.008	3.68	"
4-20	2.87	4.86	0.083	.119	7.07	1.33	0.444	13.3	1.46	8.3	0.21	0.1211	4.50	.335	0.066	3.93	marginally stable
4-21	2.91	4.85	0.066	.223	8.45	1.30	0.527	20.8	2.32	5.9	0.19	0.1215	5.31	.257	0.053	5.79	"
5-12	6.01	5.92	0.040	.183	16.76	2.80	0.433	33.5	2.81	26.2 [†]	2.07 [†]	0.1149	4.39	.146	0.015	5.53	"
5-25	2.44	4.60	0.216	.272	10.72	1.04	0.457	25.2	2.22	3.3	0.26	0.1250	4.47	.178	0.164	11.05	"
6-14	2.98	5.00	0.162	.120	6.99	1.37	0.461	12.9	1.59	10.0	0.21	0.1173	4.93	.332	0.022	3.42	"
6-15	5.94	5.80	0.016	.139	11.09	2.78	0.480	21.4	2.07	4.2	0.17	0.1174	4.79	.269	0.010	2.99	"
4-20	2.87	4.83	0.082	.098	6.49	1.37	0.446	11.1		strongly damped		0.1218	4.53	.405	0.066	2.91	stable
4-21	2.89	4.84	0.080	.211	7.44	1.28	0.576	19.0		2.0	0.09	0.1218	6.16	.309	0.067	4.64	"
5-3	2.87	4.85	0.037	.074	6.79	1.34	0.350	9.6		6.4	0.11	0.1211	3.74	.365	0.033	3.45	"
5-5	3.93	5.39	0.072	.131	9.25	1.35	0.419	17.0		3.3	0.15	0.1129	4.52	.226	0.040	3.79	"
5-11	3.67	5.00	0.296	.267	15.46	1.56	0.427	33.1		2.7	0.05	0.1224	4.28	.142	0.150	10.14	"
5-12	5.84	5.99	0.040	.154	14.17	2.83	0.426	26.3		1.2	0.07	0.1132	4.38	.176	0.015	4.27	"
5-13	14.81	6.00	0.058	.116	15.77	6.74	0.501	31.3		0.0	0.0	0.1431	3.64	.390	0.010	1.95	"
5-25	2.39	4.59	0.252	.261	9.20	1.03	0.496	23.1		2.4	0.13	0.1249	4.98	.316	0.197	9.24	"
5-26	3.17	5.00	0.299	.370	18.27	1.36	0.439	41.0		1.5	0.17	0.1189	4.60	.105	0.168	14.85	"
6-14	2.98	4.98	0.017	.113	6.41	1.40	0.498	11.5		1.8	0.05	0.1182	5.32	.409	0.028	2.64	"
6-15	3.92	5.21	0.015	.099	7.76	1.84	0.437	12.8		3.0	0.07	0.1185	4.50	.360	0.016	2.69	"

* Combined excursive and oscillatory instability.

† Grew to amplitude given and then decayed to near zero amplitude.

** Pressure at choked condition.

APPENDIX 2.

Friction Factors for Flow of Near-Critical Helium in Curved Tubes

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FRICION FACTORS FOR FLOW OF NEAR-CRITICAL HELIUM
IN CURVED TUBES*

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FRICTION FACTORS FOR FLOW OF NEAR-CRITICAL HELIUM IN CURVED TUBES

D. E. Daney and P. R. Ludtke

Abstract

Friction factors have been measured for turbulent flow, $1.9(10)^4 < Re < 1.1(10)^5$, of near critical helium in a helically wound ($D/d = 194$) circular, 4-mm I.D. tube of high aspect ratio ($L/d = 4.6(10)^4$). These measurements are in close agreement with the classical results of Ito for curved pipes. Measured pressure drops in a heated test section with large fluid expansion ratios ($\Delta v/v$ up to 13) agreed well with calculated pressure drops. Results are also given for a 1.32 mm diameter sharp edge orifice flow meter operating in near-critical helium.

INTRODUCTION

During the course of an experimental investigation of thermally-induced flow oscillations in supercritical helium¹, we measured the turbulent flow resistance ($1.9(10)^4 < Re < 1.1(10)^5$) of near-critical helium in a circular pipe of high aspect ratio ($L/d = 4.6(10)^4$) for both heated and unheated conditions. There is no a priori reason to suspect that the friction factor of near-critical helium should differ from that of other fluids. In fact, classical results obtained for the forced flow heat transfer coefficient² support this view. Recent measurements have raised some question, however, about the value of the

liquid and near-critical helium friction factor. Vécsey³ reports an anomalously high hydraulic resistance in a helically wound 720 m long x 5 mm I.D. magnet cooling channel, and Dean, Stewart and Hoffer⁴ report friction factors 45 percent greater than for smooth straight tubes in a test section 504 m long x 4.8 mm I.D. constructed of straight tubes connected by long radius bends ($D/d = 104$). In contrast, the measurements reported here are in excellent agreement with friction factors reported by Ito⁵ for turbulent flow in curved tubes. Neither the Vécsey or Dean measurements were made in apparatus, or with instrumentation, intended specifically for friction factor measurements. Consequently, the total effect of piping arrangements and the magnitude of experimental errors are not clear for those results^{3,4}.

Because of the current interest in liquid and near critical helium flow measurement, we also report calibration results for a small ($d_{or} = 1.32$ mm), sharp-edged orifice meter which was used in the flow stability studies.

Experimental

Figure 1 is a schematic of the apparatus which consists of 3 vessels: precooler, subcooler, and experimental chamber (containing the final subcooler and test section). The experiment is supplied with grade A helium gas passed through an ambient temperature purifier and regulated to 8 to 20 bar. Cooling to 4.5K occurs through counterflow heat exchange with the precooler and subcooler boil-off gas, and trace impurities are removed by an 80K charcoal purifier in the precooler.

Figure 1. Schematic of experimental apparatus.

Reduction to the final experimental pressure is accomplished by a liquid helium temperature, dome-loaded, pressure regulating valve, which was required for the flow stability tests. The temperature at the inlet to the plenum is regulated by a d.c. heater in combination with a controller and carbon resistance thermometer.

Test Section

The test section, which is wound in a helical coil and held by fiberglass spacers, is suspended in a 914-mm diameter vacuum chamber. It is shielded by both a liquid nitrogen-cooled radiation shield and by the helium exhausted from the test section. The resulting heat leak to the test section is less than one watt. It may be heated by passing current through the tube wall, giving uniform wall heat flux; stray current paths are eliminated by dielectric joints (which use silica-filled epoxy) situated at either end and along the pressure taps.

The test section is fabricated from 5 lengths of $3.99 \pm .01$ mm I.D. x 0.41 mm wall, type 304 stainless steel seamless tubing (0.7 μ m surface finish) joined by special copper couplings which are machined to match the tubing inside-diameter to within 0.02 mm, and which serve as both thermometer wells and pressure taps. Extra care was taken to prevent solder penetration (and the resulting flow disturbance) into the flow passage of these couplings.

The distance between pressure taps is 182.3 m giving an aspect ratio (L/d) of $4.6 (10)^4$; the helix diameter is 775 mm, giving a helix to tube diameter ratio of 194. The tubing diameter was determined from the mean value of air guage measurements made on samples cut from each

end of the tubing lengths. The test section length was determined from the mean diameter of the helix as measured by a steel rule. Both measurements were corrected for thermal contraction to the operating temperature.

Orifice Meter

The orifice meter, figure 2, used corner taps and had a diameter ratio, β , of 0.33. Some irregularity in the bore is indicated in the shadowgraph insert. The rounding of the edge (as measured by a shadowgraph of the sectioned orifice) is 0.06 mm - radius or less, and the width of cylindrical face is also 0.06 mm or less. Because it was difficult to install the meter with significant lengths of straight inlet and exit tubing, and because our original intention was not meter calibration, the meter was connected to tubing with a radius of curvature of approximately 30 cm.

Instrumentation

Pressure P_1 and the orifice meter pressure drop were measured with variable reluctance pressure transducers operating at ambient temperature, whereas the test section pressure drop ΔP_{1-6} was measured with a high precision capacitance pressure transducer, also operating at ambient temperature. The pressure taps, 1.1 mm diameter holes drilled in the couplings normal to the channel diameter on the inside of the helix, are deburred to assure a smooth flow passage. To inhibit thermal-acoustic oscillations in the gauge lines, these 0.5 mm I.D. stainless steel tubes are filled with 0.25 mm wire for most of their length.

Figure 2. Sharp edge orifice meter with corner taps (cold dimensions shown).

Because of a 50 cm difference in elevation between the taps for P_1 and P_6 , it was necessary to apply a hydrostatic pressure correction to account for the density difference between the test channel and the lower gauge line. Ignorance of the gauge line temperature gradient prevented accurate calculation of the correction and required an experimental determination of the correction. A combination of factors such as fluid expansion due to heat leak, leakage through the cold PRV, and gauge line response made zero flow determination of this correction difficult. Consequently we relied upon extrapolations of ΔP vs p_v and p_v^2 to zero flow rate. Our mean estimate of this correction, $4.3(10)^{-3} \pm 2.0(10)^{-3}$ bar, compares to a maximum possible value of $6.05(10)^{-3}$ bar, assuming zero density in the gauge line. The uncertainty in this correction is the major source of error at low Reynolds number.

Temperatures were measured by germanium resistance thermometers inserted into wells (nonprotruding into the flow stream) in the couplings. For all but the first 4 data points (for which a dry diaphragm meter was used), the flow was measured by an ambient-temperature, thermal-type flow meter calibrated against a 0.3 m^3 (10ft^3) bell prover using helium gas. This commercial meter measures the mass flow rate by sensing the temperature rise due to a calibrated heat input. Data were collected with a minicomputer system which gave "real time" conversion of all instrument channels to physical parameters.

The total uncertainty in the measurements is estimated to be:

P_1	± 0.5 percent
ΔP_{or}	± 3 percent

ΔP_{1-6}	$\pm 2 (10)^{-3}$ bar
T	$\pm 0.02K$
$\frac{Q}{m}$	± 0.5 percent

Procedure

Because of the long transit time (approximately 10 minutes) of the fluid in the line, extra care and time were required to assure that steady state had been attained and that the flow rate measured by the meter was characteristic of that in the test channel. Typically, three to ten readings of the parameters were recorded over about a five minute period, during which the test section inlet temperature and pressure were held constant to within $\pm 0.02K$ and ± 0.002 bar. The friction factor was then obtained by averaging properties both spatially and temporally with the pressure drop averaged by the expression

$$\Delta P = \left(\frac{1}{n} \sum (\Delta P)^{1/2} \right)^2$$

to account for the V^2 dependence of WP.

RESULTS AND DISCUSSION

Friction Factor

Figure 3 summarizes the friction factor measurements for the unheated channel, with the friction factor defined by

Figure 3. Friction factor for near-critical helium in curved cylindrical tubes: $d = 3.99$ mm, $D/d = 194$ and $L/d = 4.6 (10)^4$.

$$\Delta P = 4 f \left(\frac{L}{d} \right) \rho \frac{V^2}{2} \quad (1)$$

The range of conditions is:

P_1	2.2 to 15 bar
T_1	4.3 to 10K
$\dot{Q}_m$	0.21 to 1.24g/s
V	13 to 80 cm/s
ΔP	0.016 to .33 bar

The thermophysical properties of helium were evaluated from the correlations of McCarty⁶. Test section dimensions were corrected for thermal contraction to operating temperature.

No significant temperature or pressure dependence was observed. The line for a curved tube is taken from the work of Ito⁵ using his expression (Ito equation 2)

$$f = 0.076 (Re)^{-0.25} + 0.00725 \left(\frac{d}{D} \right)^{0.5} \quad (2)$$

and the curve for the smooth straight tube is taken from Moody⁷. Curve fitting our data in the form

$$\ln(f) = -5.4860 + 1.8034 (10)^{+4} Re^{-1} - 1.9644(10)^{+8} Re^{-2} \quad (3)$$

gives a standard deviation of 3.3 percent. The estimated systematic error in f ranges from 15 percent at the lowest Reynolds number to

3 percent at the highest Reynolds number. At mid-range the estimate is about 5 percent. The major source of this relatively large uncertainty at low Reynolds number is the uncertainty in the gauge line hydrostatic head correction. The tail-off of the data from the Ito curve at low Reynolds number probably results from the systematic error rather than from a retarded turbulent transition.

The exceptional agreement of these results with those of Ito at higher Reynolds number indicates that the near-critical helium hydraulic resistance may be evaluated by classical expressions.

Heated Channel Hydraulic Resistance

Figure 4 compares the pressure drop calculated with our unheated channel friction factors to the measured pressure drop in the heated channel. The calculated pressure drop is evaluated by the one-dimensional, steady flow computer program of Arp⁸, using the measured inlet pressure and temperature, mass flow rate, and calculated heat input, ($q = \dot{m} (h_6 - h_1)$). The acceleration pressure drop is less than one percent of the friction pressure drop. The data scatter at low values of fluid expansion ($\Delta v_{1-6}/v_1$) results from exit conditions near the transposed critical line (locus of specific heat maxima) where measurement errors have a pronounced effect on the calculated exit density. The agreement between measured electrical heat input and calculated heat input was poor for these points. In general, however, the agreement between calculated and measured pressure drops is excellent, which is to be expected since the boundary layer temperature gradient is small (.02K or less) as a result of the large surface area.

Figure 4. Hydraulic resistance of near-critical helium in a heated channel.

Orifice Meter

The sharp-edge orifice discharge coefficient measurements, summarized in figure 5, are defined by the expression

$$C_d = \frac{CA_o}{\sqrt{1-\beta^4}} \sqrt{2\rho\Delta P_{or}} \quad (4)$$

where A_o is evaluated at the operating temperature. The measured orifice discharge coefficient is 0.565 with a standard deviation of 0.0089. There is no statistically significant dependence on Reynolds number. The error due to the assumption of incompressible flow is less than 0.3 percent because of the low orifice pressure drop (0.067 bar or less) and moderate compressibility of helium in the range of interest.

The 6 percent negative deviation of our results from values expected for large orifice meters and common fluids⁹ is probably due to a number of factors such as orifice irregularity, pressure tap details, size, and curved approach tube rather than any anomalous behavior of near-critical helium.

Although small orifice meters evidently require calibration to account for geometry and installation details, the 1.5 percent standard deviation of this calibration indicates that these meters offer a reasonably accurate means of determining the flow of liquid and near-critical helium.

Figure 5. Orifice discharge coefficient.

SUMMARY

Friction factors for turbulent flow, $1.9(10)^4 < Re < 1.1(10)^5$, of near-critical helium in a circular tube of high aspect ratio ($L/d = 4.6(10)^4$) have been measured. Their close agreement with the classical results of Ito⁵ for curved pipes indicates that the hydraulic resistance of near-critical helium may be evaluated using standard expressions. This assumption is further confirmed by the agreement we obtain between calculated and measured pressure drops in a heated test section with large fluid expansion ratios ($\Delta v_{1-6}/v_1$) up to 13).

The discharge coefficient for a sharp-edge orifice flow meter was found to be about 6 percent below the expected value for large orifices, but this deviation is believed to be due to geometry effects rather than any anomalous behavior of near-critical helium.

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NOMENCLATURE

A	flow area at operating temperature.
C	orifice discharge coefficient defined by equation (4).
d	tube inside diameter
d _{or}	orifice diameter
D	test section helix diameter
f	friction factor defined by equation 1
h	specific enthalpy
L	test section length
$\dot{m}$	mass flow rate
P	pressure
Re	Reynolds Number, $\frac{\rho V D}{\mu}$
q	test section heating rate
T	temperature
v	specific volume
V	fluid velocity

Greek

β	orifice diameter ratio d_{or}/d
ϵ	tube surface finish
μ	viscosity
ρ	density

Subscripts

or	orifice
o-6	locations in figure 1.

APPENDIX 3.

Transient Behavior of Helium-Cooled Current Leads for Superconducting Power Transmission

M. C. Jones, V. M. Yeroshenko, A. Starostin and L. A. Yaskin

TRANSIENT BEHAVIOR OF HELIUM-COOLED CURRENT LEADS FOR
SUPERCONDUCTING POWER TRANSMISSION*

M. C. Jones, V. M. Yeroshenko, A. Starostin and L. A. Yaskin

Abstract

Based upon time dependent differential equations, numerical calculations are presented which allow, first, the determination of burn-out conditions for a helium cooled current lead. Secondly, the response of the lead to an overload current may be examined, both during the overload and during subsequent time when the normal current again persists. Representing the temperature dependent resistivity and thermal conductivity of the copper conductor as a function of its purity (residual resistivity ratio) the results are parameterized by dimensionless mass flow rate, current and residual resistivity ratio. Defining the burn-out temperature as 500 K and an operating current as 85% of the burn-out current, it is shown that although all residual resistivity ratios between 5 and 100 are suitable, there is a limit on the mass flow rate number which can be utilized. This results from secondary temperature surges occurring after the overload, the conditions for which are rather complicated. It is concluded in particular that, within the permissible range of mass flow rates, no damage should result from a 10X current overload for periods up to 0.2 seconds.

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TRANSIENT BEHAVIOR OF HELIUM-COOLED CURRENT LEADS FOR SUPERCONDUCTING POWER TRANSMISSION

M. C. Jones, V. M. Yeroshenko, A. Starostin and L. A. Yaskin

Introduction

Current leads for superconducting power transmission will differ from typical magnet leads in several respects. The leads will be cooled by supercritical helium at anywhere from 0.3 to 1.5 MPa; they will not be self-sustained leads, but will most likely be cooled by a bleed from the cable coolant. Because of the elevated pressure, flow rate can be controlled independently. The refrigeration for the leads will only be a small fraction of the total cable refrigeration except in short prototypes or laboratory tests of relatively short lengths of cable. Finally, the peculiarities of electric utilities networks are such that, in addition to a steady sustained current, the leads as well as the cables must be able to stand overload currents perhaps as high as 10X normal operation currents for short periods. Current thinking is that these periods may be as short as 0.05 seconds in the era of superconducting power transmission, although state-of-the-art for circuit breaker operation is about 0.1 seconds. It may also be necessary to withstand a second overload immediately afterward in the event the fault has not cleared when the breakers are again closed. Thus, the combined effect would be equivalent to a 10X overload for from 0.1 to 0.2 seconds. This study was prompted by the need to answer the question: what protection must be built into the current lead design in order to withstand these network faults without damage? We also deal with the question of what time is available to cut

off the current in the event a lead loses its coolant flow. For the cables themselves the time is measured in hours, but this is not true for the current leads. Although the calculations reported below were done with these questions in mind they are obviously applicable to all uniform leads cooled with a regulated helium gas flow.

Before we can answer the questions posed above, we must first decide on the operating conditions for a given application. It is customary in magnet applications to try to operate at an optimum operating condition. For a uniform lead with a large amount of heat transfer area such that the gas temperature is everywhere close to the lead conductor temperature, thermodynamic optimization leads to fixed values for the quantities IL/A and m/I , where I is the current, L the length, A the conductor cross sectional area and m the mass flow rate of helium gas. The precise values depend upon the temperatures at which the warm end and cold end are held and on the particular copper that is used as conductor. For these purposes the copper may be characterized by its helium temperature residual resistance or, equivalently, the residual resistivity ratio, RRR, which we define here as the ratio of the resistivity at the ice point (273 K) to its value in a helium bath. (As is well known since the resistivity is independent of temperature for most coppers of interest up to at least 10 K, the lower temperature need not be specified precisely.)

As examples of optimization calculations, Güsewell and Haebel [1] give $IL/A = 2.6 \times 10^5$ A/cm and $m/I = 4.9 \times 10^{-5}$ g/A.s for a slotted-finned rod with RRR = 57 operating between 4.4 K and 300 K. In an earlier report [2] we gave $IL/A = 2.25 \times 10^5$ A/cm and $m/I = 4.62 \times 10^{-5}$ g/A.s for a braided lead with RRR = 49 operating between 6 K and 300 K. The differences are not too significant. We could choose operating conditions close to these were it not for the fact that such points

also turn out to coincide with the burn-out condition for the lead. Let us therefore define what we mean by burn-out.

From a practical standpoint, a lead has an upper operating temperature limit corresponding, usually, to the melting temperature of one of the materials of which it is constructed. We cannot operate above this temperature without irreversible change or damage. Mathematically, we find that such a temperature may be reached in one of two ways. In the first, it may be a steady state solution to the governing differential equations for a given choice of current and gas flow rate. In the second, we may not be able to find such a steady temperature for any choice of operating parameters; instead, the temperature is passed only in the non-steady state as the lead runs away indefinitely, or at least to much higher temperatures. In this case we call the lead unstable. However, if we use the practical approach of selecting a temperature limit, it will define an upper operating current for a given mass flow rate unambiguously, for the unstable lead will ultimately reach this temperature also. In the computations we report below, we have selected 500 K as the burn-out temperature since many leads contain lead/tin solders or even polymers for which irreversible change can be expected if the temperature exceeds this value. We shall thus define a lead to have burned out if at any time this temperature has been exceeded.

It is clear that the establishment of burn-out conditions for this class of lead is of paramount importance.

We must distinguish the process of burn-out under consideration here from that discussed by Thullen, et al. [3]. These authors identified an instability -- leading to burn-out -- where the lead pressure drop increases with reduction of gas flow rate resulting in choking off of the gas flow. This, however, is only applicable to leads where the gas pressure drop is predominantly through

the current lead. For the class of lead we are discussing, where we are operating at elevated pressures, there is no need for this restriction and the gas flow can be held constant even through burn-out.

In the next section we obtain equations describing the unsteady or transient behavior of gas-cooled current leads. We then describe and give the results of numerical calculations which establish burn-out conditions, forming a basis for the selection of normal operating conditions. These results are compared with some experimental results. Finally, we give the results of numerical calculations of lead temperatures as a function of time in response to overload currents. These enable us to answer the questions originally posed.

Theory

The unsteady state, energy conservation equation for the conductor, neglecting transverse temperature gradients, is

$$\rho AC \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(kA \frac{\partial T}{\partial x} \right) - \lambda + \frac{\rho^* I^2}{A} \quad (1)$$

The thermal conductivity k , heat capacity C , and resistivity ρ^* are known to be strong functions of temperature, while the density ρ may be assumed to be effectively constant by comparison. We limit this discussion to uniform leads, $A =$ constant, although it is noted that in performing numerical integrations of these equations a variable cross section is no more bothersome than a temperature dependent thermal conductivity. In equation (1) λ is the lateral heat transfer to the helium per unit length in the x (axial) direction.

An equivalent equation for the helium coolant may be written for low velocity flow at essentially constant pressure.

$$\rho' C_p' A' \frac{\partial T'}{\partial t} = \lambda - \dot{m} C_p' \frac{\partial T'}{\partial x} \quad (2)$$

In order to narrow the discussion to within reasonable bounds for this paper, we shall only consider leads with large heat transfer surfaces per unit length so that, approximately, $T' = T$ throughout. Again, for the numerical methods we have used, this is not a necessary approximation. Equations (1) and (2) may therefore be combined by eliminating λ , and we get

$$(\rho C A + \rho' C_p' A') \frac{\partial T}{\partial t} = A \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) - m C_p' \frac{\partial T}{\partial x} + \frac{\rho^* I^2}{A}.$$

For almost the entire length of the lead C_p' is constant and will be so assumed throughout. We define reference values k_0 , C_0 and ρ_0^* for the properties of the conductor at the warm end temperature, T_{ref} . We also non-dimensionalize time and distance by the definitions $\tau = \frac{k_0 t}{\rho C_0 L^2} = \alpha_0 t/L^2$ and $\eta = x/L$, where L is the active length of the lead, to obtain

$$\frac{\partial T}{\partial \tau} = \left[\frac{k}{k_0} \frac{\partial^2 T}{\partial \eta^2} + \frac{1}{k_0} \left(\frac{dk}{dT} \right) \left(\frac{\partial T}{\partial \eta} \right)^2 - M \frac{\partial T}{\partial \eta} + \frac{\rho^*}{\rho_0^*} J^2 T_{ref} \right] / \left[\frac{C}{C_0} + \frac{\rho' C_p' A'}{\rho C_0 A} \right] \quad (3)$$

We have not made the popular Wiedemann-Franz approximation for the conductor, but have used a reference temperature Lorenz constant $L_0 = k_0 \rho_0^* / T_{ref}$ and have represented the dimensionless groups $m C_p' L / k_0 A$ and $L_0 I^2 L^2 / k_0^2 A^2$ by M and J^2 , respectively. Thus M and J have their familiar meanings but are evaluated unambiguously at reference temperature conditions. This equation was used in the numerical studies reported below. We note that M and J are the only temperature independent coefficients and that k/k_0 , ρ^*/ρ_0^* and $1/k_0 (dk/dT)$ are all dependent on the particular material of the conductor. The transient behavior can therefore be characterized for these high heat transfer area leads by the three dimensionless constants M , J , and RRR .

Some useful approximate results can be obtained quite simply. If we reduce equation (3) to the constant conductivity-linear resistivity case, using reference temperature values, we can first determine the familiar burn-out relationship from the singularity in the resulting steady state temperature profile.

$$J^2 - (M/2)^2 = \pi^2 \quad (4)$$

This could be used as a rough guide for the choice of operating conditions.

Secondly, if we also neglect heat transfer to the helium gas and thermal conduction along the conductor during the period of a current overload, we can solve for temperature as a function of time and get the adiabatic heating result:

$$T(\eta, \tau) = T(\eta, 0) \left[1 + \frac{n^2 - 1}{n^2} \left(e^{n^2 J^2 \tau} - 1 \right) \right]$$

$$\text{or } T(\eta, t) = T(\eta, 0) \left[1 + \frac{n^2 - 1}{n^2} \left(e^{n^2 J^2 \alpha_0 t / L^2} - 1 \right) \right] \quad (5)$$

where $T(\eta, 0)$ is the steady state temperature profile prior to overload when the current parameter was J , and nJ is the overload value. This formula might be expected to give a reasonable upper limit to the temperature at the warm end of the lead except at the very end; the neglect of heat conduction prevents us from describing the thermal anchoring of the lead at the ends. The error should not be great for distances from the warm end greater than $(\alpha_0 t)^{1/2}$, and for times up to 1 second this is less than about 1 centimeter.

Within the same framework, we can obtain the adiabatic result for loss of coolant, i.e., $M \rightarrow 0$. We again neglect conduction and obtain

$$T(\eta, t) = T(\eta, 0) + \frac{M}{J^2} \frac{\partial T(\eta, 0)}{\partial \eta} \left(e^{J^2 \alpha_0 t / L^2} - 1 \right). \quad (6)$$

In this equation M has the value which was in effect before loss of coolant.

We can already see that this will always be a less severe disturbance than any overload condition because the exponent is always smaller.

Other closer approximations may be obtained by Laplace transform methods, but we have found more accurate numerical solutions of equation (3) to be no more difficult.

Properties of Copper

In this section we summarize the mathematical representations of the resistivity, thermal conductivity and heat capacity which we used in the integration of equation (3).

For ρ^* we used Matthiessen's rule, $\rho^* = \rho_R^* + \rho_I^*$, and calculated the residual resistivity ρ_R^* from the specified RRR and the ideal lattice resistivity ρ_I^* from the data given by White and Woods [4]. Our curve, fitted to those data, is

$$\rho_I^* = 10^{-8} \cdot \exp \sum_0^4 C_i (\ln T)^i \quad (\text{ohm.m.})$$

where

$$C_0 = -9.600976$$

$$C_1 = -12.52445$$

$$C_2 = 8.309361$$

$$C_3 = -1.583458$$

$$\text{and } C_4 = 0.0993132.$$

The deviation from the data is no more than two percent. This equation is used up to 300 K, after which ρ_I^* is assumed proportional to T.

For the conductor thermal conductivity k we used the approach followed by Lock [5]. In this approach an analogue of Matthiessen's rule is used. Thus we take

$$\frac{1}{k} = \frac{1}{k_R} + \frac{1}{k_I}.$$

This is a compromise between accuracy and computational efficiency. The procedure is known not to be accurate above 10 K (the error can reach 10 percent) for the thermal resistivity, but for engineering purposes the possibility of computing the thermal conductivity from a single property - RRR - is very appealing. We calculate k_R from $L_0 T / \rho_R^*$ using an average experimental value for L_0 of $2.1 \times 10^{-8} \text{ V}^2/\text{K}^2$. The lattice contribution to thermal resistivity $1/k_I$ was assumed to be independent of purity and calculated from the data of Powell, et al.[6] for a sample of RRR = 110 using a value for L_0 appropriate to those data. Thus we computed k_I from the expression

$$k_I = T \exp \sum_{i=1}^5 C_i / T^i \quad (\text{W/m.K})$$

with

$$\begin{aligned} C_1 &= 146.587, \\ C_2 &= 269.117, \\ C_3 &= -16956.6, \\ C_4 &= 101324.0, \\ C_5 &= -171156.0. \end{aligned}$$

This expression was used up to 150 K after which the thermal conductivity was assumed to be constant at its 150 K value. The fit generally predicts the thermal conductivity data of Powell to better than five percent but is low by 14 percent at 40 K. Nevertheless, considering the complex nature of the temperature dependence of the thermal conductivity and its variation by orders of magnitude at low temperatures, this procedure gives a very realistic and economical representation. This is illustrated in figure 1 where we have compared some of our computed curves with experimental data for which RRR was known and was of comparable magnitude.

Finally, for the heat capacity of copper, we used the following expression above 10 K fit to the smooth curve of Miller and Brockhouse [7]

Figure 1. Comparison of computed thermal conductivity of copper with some experimental data. $\square$, data of reference [6]; $\circ, \Delta$, data of reference [4].

$$C = \frac{10^3}{15.186} \exp -T^3 / \sum_0^5 C_i T^i \quad (\text{J/kg.K})$$

where

$$C_0 = -0.158278 \times 10^4$$

$$C_1 = -0.292746 \times 10^3$$

$$C_2 = 0.115882 \times 10^3$$

$$C_3 = -0.268058 \times 10$$

$$C_4 = 0.249819 \times 10^{-2}$$

$$C_5 = -0.242886 \times 10^{-5}$$

The error is at most 3.8% between 10 and 500 K.

For temperatures below 10 K we used the expression [8]

$$C = 10.8 \times 10^{-3} T + 7.4834 \times 10^{-4} T^3 \quad (\text{J/kg.K})$$

Establishment of Burn-out Conditions

Having defined burn-out above, establishing the condition was quite straightforward. We simply fixed values for M and RRR and, starting from some arbitrary temperature profile, raised J by small increments allowing integration of (3) to proceed to the steady state after each change. When we found the highest value of J for which a steady state profile existed and for which no more than a subsequent, say, 1% increase led to a temperature exceeding 500 K (either as a steady state profile or one "running away"), then we could say that we had found the burn-out J to within 1% for the given M and RRR.

This procedure was followed using an integration computer package PDECOL [9] which was designed for fast and stable integration of non-linear partial differential equations of the type (3). In this manner we produced the curves shown in figure 2 of J vs. M with RRR as a parameter. For these calculations and all calculations reported below, the cold end temperature was 7 K and the warm end 300 K.

Figure 2. Burn-out conditions for helium cooled current leads with residual resistivity ratio (RRR) as parameter. The dashed line is computed from the constant conductivity, linear resistivity model.

We also plotted on the same graph the relationship (4) for comparison. In figure 3 we compare these results with some experimental burn-out data on some model leads reported earlier [2]. The experimental determination of burn-out corresponded to our definition of burn-out above. Lead #2 (using the earlier numbering system) was a 200A braided lead where the copper braids were compressed into the annular space between a stainless steel cylinder and phenolic core. Lead #4, an 800A lead, was made of a piece of heat exchanger tubing with coiled spiral ribbons between an inner rod and an outer copper tube which acted as the conductor. These leads had RRR's of 49 and 5 respectively and gave high heat transfer rates as indicated by the gas outlet temperature. Now we certainly don't expect leads with poor heat transfer (poor design?) to be accurately predicted by these curves. Future work should be aimed at establishing burn-out curves for less than perfect heat transfer. But, judging by our computed curves, there does not appear to be more than a possible 10% degradation in the burn-out J for #2 and no degradation for #4 within the precision of the experiments. We conclude that it should not be difficult to attain these burn-out levels in any well designed lead. Incidentally, Bejan and Cluss [10] have made calculations of burn-out with imperfect heat transfer based on the constant conductivity-linear resistivity model. Degradation of burn-out J was parameterized by a constant heat transfer factor $H = hPL^2/kA$, but this appears to overpredict the degradation, probably because h and k are quite variable over the length in the real case.

In figure 4 we show some of the steady temperature profiles obtained just prior to burn-out. The significance of these profiles is that about a 1% increase in J over values given will send the maximum temperature significantly over 500 K. (Due to the high sensitivity of temperature to J at this threshold, some of the curves in figure 4 are already a shade over 500 K. However, no

Figure 3. Comparison of calculated burn-out conditions with experimental data. ●, ○ and ▽, braided lead at 0.4, 1.0 and 1.5 MPa respectively; ⊗, actual burn-out of braided lead at 1.0 MPa pressure; I, lead constructed of heat exchanger tubing.

Figure 4. Temperature profiles at the threshold of burn-out.

significant correction in J would result from recomputing these curves to be actually below 500 K.) It can be seen in 4a, as is well known, that for the higher purity coppers an extreme sensitivity to current occurs on the threshold of burn-out -- some 200 K for a 1% increase in J . Figures 4a and b also show, respectively, that burn-out threshold temperature profiles for different RRR and different M are non-similar.

One unexpected aspect of burn-out discovered in our computations needs further discussion. It is the fact that for given stable operating conditions it is possible to compute, sometimes, two steady state temperature profiles. The one may have a modest maximum; the second will always have a much higher maximum temperature. Which one is arrived at by allowing integration to reach steady state depends upon the assumed starting profile and the rate at which J is increased to its preselected value. This phenomenon has also been observed by Hoffer [11]. In every case our burn-out J 's were arrived at by gently increasing J . A sudden increase of J from zero to its near burn-out value often resulted in the higher temperature profile which, by our definition would in most cases have meant burn-out. Our starting profile was always taken to be linear (approximately what one would get for zero J and M). This appears to say that a lead may or may not burn-out depending on the manner in which current and gas flow are set to their prescribed values. In normal practice one would set gas flow rate before bringing current up to its design value, however, and this should not result in burn-out.

This possibility also has ramifications for studies of response to current pulses. If the equations were linear we could be certain that temperature profiles would resume their prepulse steady values after the pulse has passed.

But there is no guarantee here and the existence of an alternate steady state at high temperatures is worrying to say the least. We therefore investigated relaxation after the current pulse also and this is discussed below.

Response to Current Overload

Equation (5) already indicates to us that the rate at which the lead temperature rises during a current overload is dependent on the J value for normal operation. Therefore, to investigate the time response to an overload for a range of M and RRR values we must first select the operating J . We did this on the basis of the burn-out results above. We arbitrarily selected the operating J as 85% of the burn-out value.

Since our earlier calculations [2] had shown that for an RRR of 48, the thermodynamically optimum M value was about 14 and $J/M = 0.65$, we decided to investigate the three values, 7, 14, and 28. In table 1 we give the corresponding operating J -- i.e., 85% of the burn-out value -- for each M , and RRR 's of 5, 10, 20, 40 and 100.

The time development of a lead temperature profile in response to a ten-fold increase in current was obtained, using PDECOL, by first integrating to steady state at the operating J followed by a step increase to $10 \times J$. Integration was then carried out in small increments of time until, somewhere in the lead, the temperature hit 500 K. At this point, or somewhat beyond, the J value was stepped back to its original value and integration continued for sufficient time for the temperature profile to have relaxed back to its steady state starting value. We note here that for all cases tested -- table 1 -- the original temperature profile was recovered, but not, in some cases without further rise in temperature above 500 K.

Table 1. Operating current parameter, J, based on 85% of burn-out threshold values.

M	RRR				
	5	10	20	40	100
7	3.59	3.85	4.13	4.51	5.04
14	5.40	6.04	6.59	7.31	8.42
28	7.28	9.01	9.61	11.9	14.2

In figure 5 we show the time development of temperature profiles for two values of RRR for $M = 14$ showing the completely different shape of the profiles and the faster rise for the higher RRR by virtue of its higher operating J . In figure 6 a similar comparison is made for a fixed RRR and the extreme M values. A large M value has an effect similar to a high RRR and, again, gives rise to the faster temperature rise by virtue of the higher operating J value.

Finally, in figure 7 we compare rates of temperature rise for different RRR's. Plotted is the maximum temperature vs. time; M values are as indicated and operating J values as in table 1.

Temperature Decay after Overload

In studying the relaxation of the temperature profiles back to their initial values it was found that for $M = 28$ they would not do so without a subsequent secondary rise above 500 K again. An example of this unusual behavior is shown in figure 8. It appears that the initial temperature peak diffuses along the lead and subsequent joule heating at the operating J is sufficient to cause the new rise in temperature. This effect is more pronounced the lower the RRR. We note that in these cases the resistivity is higher and the thermal conductivity at the cold end poorer.

A few spot calculations were performed with J at its threshold value for burn-out. In some cases ($M = 14$, $RRR = 40$ and $M = 14$ and 28 at $RRR = 100$), the temperature profile did not return to its pre-pulse value after the operating J had been restored, but increased until the calculation was terminated at several thousand kelvins. It appears that such a run-away condition was happily avoided by our choice of an 85% operating J and that somewhere between 85 and 100% leads may not return to their original steady profile. The unusual behavior noted for $M = 28$ and the double temperature profiles noted above would appear to be related

Figure 5. Comparison of temperature profile time development for $RRR = 5$ and 20 , $M = 14.0$, following a ten-fold step increase in current.

Figure 6. Comparison of temperature profile time development for $M = 7.0$ and 28.0 , $RRR = 20$, following a ten-fold step increase in current.

Figure 7. Temperature-time graphs for different M and RRR following the ten-fold increase in current. The temperature plotted is the maximum temperature.

Figure 8. Temperature profiles at indicated times after the return of the current to its normal operating value.

effects. Such behavior is the result of the peculiar temperature dependence of the properties of copper. If there were no such dependence we would have linear differential equations and could be assured of the return of the temperature profiles to their original values after the passage of the current pulse. Then too, there is only one solution for the temperature profile for given J and M.

Conclusions

1. Operating conditions for superconducting power line current leads have been established based on the burn-out criteria and an upper operating temperature of 500 K.
2. In all cases studied, no damage should result from a 10X current overload for expected periods of up to 0.2 seconds (or consecutive overloads of 0.1 seconds in quick succession) using these operating conditions.
3. To be safe, M values should not be set above 14 to avoid the phenomenon of a secondary higher temperature rise in relaxing from a current overload condition.
4. From these results none of the RRR values can be eliminated from consideration. The higher values have the advantage of higher current carrying capacity, and although their rate of rise of temperature during overload is higher, they are, nevertheless, still acceptable for power transmission application.

Acknowledgement

Milton L. Loeb is thanked for performing the tedious task of running the computer program.

Notation

A	Cross sectional area of conductor
A'	Flow cross sectional area for helium
C	Heat capacity of conductor
C' _p	Constant pressure heat capacity of helium
h	Heat transfer coefficient
H	dimensionless heat transfer factor, defined in text
I	Current through lead
J	Dimensionless current parameter, defined in text
k	Thermal conductivity of conductor
L	Length of conductor
L ₀	Lorenz constant at reference temperature, defined in text
n	Multiplier for overload current (= overload/normal)
P	Wetted perimeter
RRR	Residual resistivity ratio
t	Time
T	Temperature
x	Axial coordinate of conductor
α	Thermal diffusivity
ρ	Conductor density
ρ'	Helium density
ρ^*	Conductor resistivity
λ	Heat transfer to helium per unit length of conductor
τ	Dimensionless time, defined in text
η	Dimensionless axial coordinate of conductor (= x/L)

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APPENDIX 4.

A Microwave Cavity Pressure Transducer for Liquid Helium Temperatures

N. V. Frederick and V. D. Arp

A Microwave Cavity Pressure Transducer
for Liquid Helium Temperatures

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Abstract

This paper outlines the design, assembly, and performance of a 1.3 GHz cavity slaved oscillator pair which functions as a sensitive transducer for low temperature pressure measurement.

Salient features are; small size, insensitivity to electromagnetic noise, absence of gasketed seals, a high-level analog voltage output, and high sensitivity as a differential pressure gage, though the calibration is complicated by a dielectric correction for the working fluid, and non-linear terms. A precision of 1 part in 5000 (or better) of the working range is obtainable. In the form reported here, the accuracy is limited by thermal drift in the analog output voltage, but this can be greatly reduced if one can afford the expense of a suitable counter. The device should be useful for experiments on superconducting power lines and magnets.

Introduction

Pressure measurement in helium cooling systems may be accomplished in either of two ways (1) by bringing pressure taps out from the low temperature region to room temperature where conventional instrumentation may be used, or (2) by placing a transducer at low temperature, in the helium system, and bringing an electrical signal out to room temperature. It is common to experience inconveniences or trouble with method (1), e.g., the temperature gradients in the pressure taps promote thermoacoustic oscillations, variable thermal gradients show up as drift in the output, and long pressure lines may easily reduce the frequency response below acceptable limits. The problem with method (2) is that few if any commercial transducers can do the job if one demands in addition that the transducer be insensitive to applied magnetic fields and electromagnetic noise, be operable as a differential pressure transducer at high sensitivity, and that the readout instrument be inexpensive while providing a continuous high-level analog voltage output.

We have built and operated a simple microwave cavity stabilized oscillator which meets these requirements as a pressure transducer. The cavity itself is somewhat similar to that developed independently by Van Degriфт (1974),¹ who has since that time continued his development to achieve a highly accurate ($1:10^8$) laboratory instrument at the NBS Gaithersburg laboratories. Ours differs from his in that its ultimate

accuracy is far less, though still adequate for most pressure measurement applications and uses relatively simple electronics. Van Degriift incorporates a tunnel diode into the cavity structure to achieve his greater stability, but then the unit must be supplied with precisely controlled low-level d.c. power, noise on which could be a problem in field installations. Ours operates at 1300 MHz so that a small cavity can be used.

The Microwave Cavity

The heart of this device is a microwave cavity, one wall of which deflects elastically when subjected to external pressure, causing a measureable shift in the cavity's resonant frequency. The nominal operating frequency is in the range from 1100 to 1400 MHz, where amplifiers, oscillators, etc. are available from commercial sources. At this high frequency, and by using a highly reentrant cavity design, the external cavity dimensions can be made acceptably small, nominally 2 cm diam x 3 cm long. The cavity, with its enclosing walls, is seen in figure 1.

It is important to note several features. First, the transducer can be made entirely of metal, usually copper or beryllium copper, except for the insulator in the connecting coaxial electrical lead. This design has eliminated the usual problems with differential thermal

contraction, O-ring seals, electrically insulated pieces, etc. Secondly, below about 20 K, where the thermal expansion coefficient goes essentially to zero and the elastic moduli are constant, the resonant frequency of the empty cavity is independent of temperature (though when used as a differential pressure gage, there is an important frequency shift based on fluid helium properties, as discussed below). Thirdly, the cavity resonance is essentially unaffected by ambient electromagnetic fields at power line frequencies; similarly, any related electromagnetic pickup on the coaxial line from the cavity to the external electronics has no discernable effect on the output.

Though other cavity geometries and dimensions are possible, the simple cylindrical geometry illustrated in figures 1 and 2 is a reasonable compromise between small size and ease of fabrication and assembly. The cavity may be thought of as a simple LC resonant circuit, with the cavity volume which contains the magnetic field determining L,

$$L = (\mu_0/2\pi) \ell \ln(b/a)$$

and the spacing between the reentrant piece and the opposite wall determining C

$$C = \epsilon_0(\pi d^2/4)/g,$$

Figure 1. Assembly of the microwave cavity pressure transducer.

- $a = 3.81 \text{ mm}$
- $b = 11.91 \text{ mm}$
- $d = 8.38 \text{ mm}$
- $g = 0.07 \text{ mm}$
- $l = 10.67 \text{ mm}$
- $t = 0.2 \text{ to } 1.0 \text{ mm, depending on desired sensitivity}$

Figure 2. Geometrical parameters of the microwave cavity.

With the cavity machined in separate pieces, as in figure 1, the resonant frequency is finally determined by the capacitance spacing, g. During fabrication this is best set by observing the resonance while sliding the two machined pieces relative to each other until the resonance occurs at the desired frequency; the two pieces can then be soldered together. If the cavity is to be used at variable temperatures around 4 K, care should be taken that the solder does not measurably extend onto the internal cavity walls where it will cause a discontinuous frequency shift (with temperature) when it goes superconducting. The effects of possible misalignments between these pieces require good but not exceptional machining accuracy during fabrication. Note that the diaphragm (flexible wall) is an integral machined part of the cavity. Since the diaphragm must be accurately perpendicular to the centerline of the piece, we found that the machining operation gave a higher success rate than did soldered assembly of a separate diaphragm and cylindrical wall.

Mechanical Design

For a given differential pressure, P, the deflection, Z of the center of the diaphragm is

$$Z = \frac{3}{256} (1-\nu^2) \left(\frac{b}{t}\right)^3 b \frac{P}{E}$$

where E is Young's modulus and ν is Poisson's ratio. The maximum stress in the diaphragm, which occurs at its outside radius, is

$$\sigma_{\max} = \frac{3}{16} \left(\frac{b}{t} \right)^2 p$$

For copper at room temperature $E \approx 130$ GPa. The elastic limit of OFHC copper is a very strong function of the material condition, ranging from about 320 MPa for material which has been cold drawn 60% down to 1.4 MPa (or less, depending on the sensitivity of the measurement technique) in the annealed condition. If one chooses to use OFHC copper, it must be cold-drawn, and one should adopt a conservative value, $\sigma_{\max} \approx 20$ MPa, for the design, including reasonable allowance for overpressure. However, without extensive testing, one would never be assured of freedom from non-linear or fatigue effects.

Beryllium copper is a much better choice for the diaphragm material. Its mechanical properties depend somewhat on anneal conditions, etc., but typical values (Handbook of Materials for Superconducting Machinery, 1977) ^{2,3} are:

$E \approx 115$ to 130 GPa at room temperature, increasing about 10% at 4 K

$\sigma_{\text{yield}} \approx 500$ to 1100 MPa.

Since the yield stress is generally defined to occur when the strain offset (hysteresis when the stress is removed) is 0.2%, the design $\sigma_{\max}$ should be much less than σ_{yield} . A design $\sigma_{\max} \approx 100$ MPa is reasonable. The machining of the diaphragm must be performed giving proper attention to its support when in thin section.

For general use the cavity is packaged with a mechanical support structure as seen in Fig. 1 so that mounting forces and arbitrary motion of the hydraulic or electrical lines cannot distort the cavity and cause a spurious signal. If high frequency pressure waves must be measured, the mechanical shield could be replaced with a flange which seals the cavity directly into the fluid system.

Coupling to the Cavity

Capacitive coupling of the external circuitry to the cavity is achieved by simple termination of a short length of the commercial coaxial lead in the region of high E-field, i.e., in the gap region, within the cavity, as seen in figure 1. This has the advantage of maintaining cylindrical symmetry in the construction, and a theoretical disadvantage in that the coupling, which is determined by the gap separating the center conductor and opposite wall, varies with the diaphragm deflection. In practice this variation in coupling strength with pressure creates no operational problems provided that the overcoupling does not occur.

During fabrication the coupling is adjusted by sliding the coaxial conductor, indicated in figure 1, until the reflected power is small but not zero at resonance; 10-20% is a reasonable range. When the proper coupling is obtained, Wood's metal is applied to fix the axial position of the conductor with respect to the cavity body. The exterior end of the coaxial conductor is provided with slip fittings so that any axial forces from the external lead are not transmitted to it.

A commercial hermetic seal prevents vacuum or pressure leaks along the coaxial line which extends from the cavity to the external circuitry.

We have been using a commercially available 2.18 mm diameter coaxial conductor having a characteristic impedance of 50 ohms, and commercial connectors of various configurations, including the hermetic seal mentioned above. The electromagnetic reflections from the various connectors, etc., tend to create standing waves and spurious resonances in the GHz range which can confuse the detection and lock-on circuits especially if the cavity Q is low. These are illustrated in figure 3. In particular, these spurious resonances become closely spaced together (in frequency) as the cable length is increased, and they become very sharp if superconducting coax is used to compensate for the increased attenuation in long cable lengths. This has limited our cable lengths to about 2 meters of copper coax, which has been adequate for purposes

Figure 3. Copy of an oscilloscope trace of power reflected from the sensor and coaxial line connectors versus frequency. The effect of weak coupling to the sensor is illustrated by the small upward deflection at the sensor resonance.

so far. It is possible that this limiting cable length could be increased if necessary; however, our attempt to use 250 meters of superconducting coax failed completely due to closely-spaced high-Q resonances between the cable and the detection circuits.

Control Electronics

The control electronics locks the 1 GHz_z voltage tunable oscillator (VTO) to the peak of the resonance where the slope of the resonance response with frequency is zero. This frequency is located by frequency modulating the 1 GHz oscillator at 50 kHz. Figure 4 illustrates the process whereby the oscillator is locked to the cavity. Whenever the oscillator is unlocked from the cavity, e.g., during start up, the control circuit reverts to a search mode, finds the cavity and locks the oscillator to it. The auto-search circuit is illustrated in Figure 5. The auto-search circuit is also used to provide a sweep frequency signal to the sensor during sensor fabrication and testing. This arrangement permits adjustment of the cavity's resonant frequency and the degree of matching to the transmission line using only a conventional oscilloscope. No other test equipment is required. The beryllium copper cavities have demonstrated only small changes in Q and line matching, over the 4 K to 300 K temperature range; consequently, the circuit has no difficulty remaining locked during cool downs. The lock-on circuit is sufficiently quick to permit tracking pressure oscillations which are (10%) of full scale pressure up to over 100 Hz.

Figure 4. Schematic outline of the electrical circuits and the technique for locking the oscillator to the cavity.

Figure 5. The automatic search circuit. The comparator, diode D_1 , resistors R_c and R_d , capacitor C_c , integrator, and VTO driver comprise a relaxation oscillator which sweeps the VTO tuning voltage whenever the VTO is not locked to the sensor. The "lock" signal from the double-balanced mixer amplifier overwhelms and disables the relaxation oscillator.

Figure 6. The complete electrical circuit.

The Auto-Search Circuit

The auto-search circuit is shown in figure 5. When the VTO is locked to the sensor cavity the voltage at the comparator's inverting input is less than the voltage at the non-inverting input and diode, D_1 disconnects the comparator from the charging resistor R_C . This permits the input from the double balanced mixer amplifier to drive the lock-on integrator. When the VTO is not locked to the sensor cavity the integrator bias causes the voltage at the comparator's inverting input to exceed the voltage at the non-inverting input and the comparator output goes negative and quickly charges C_C through D_1 and R_C . The voltage on C_C supplies the integrator with sufficient input through R_d to overcome the fixed bias. The integrator ramps the VCO driver and the comparator inverting input. The comparator output soon goes positive and diode D_1 disconnects the comparator from C_C . Before C_C is completely discharged the VCO will have been tuned to the sensors' resonant frequency and the double balanced mixer amplifier will overwhelm the input from C_C and the VTO will be locked to the sensor.

Calibration

The calibration of the system may be divided into two parts: 1) the pressure vs. frequency calibration of the cavity itself, and 2) the frequency vs. voltage calibration of the external electronics. Both

turn out to be slightly non-linear over the 5% frequency range which typifies the transducers which we have tested. In practice, one separately calibrates pressure vs. voltage of the complete system and frequency vs. voltage of the electronics, and then calculates the pressure vs. frequency characteristic of a given transducer. We have shown that, within experimental error, the same transducer characteristic is obtained by this process using two different electronics packages.

Differential Pressure Measurement: The Dielectric Correction

For absolute pressure measurement, the cavity is evacuated, and no correction for the dielectric constant of the high pressure helium is necessary. However, for differential pressure measurement, the cavity contains either the low or the high pressure helium, and a dielectric correction is necessary. This dielectric effect manifests in two ways:

(1) if the density of helium in the cavity is constant while the measured pressure varies, the dielectric effect shifts the resonant frequency to lower values $f = f_0/\sqrt{\epsilon_r}$ without changing the calibration curve

$$\frac{f(P) - f(P=0)}{f(P=0)} \text{ vs } P$$

Note however, that if the frequency to voltage conversion is non-linear (as is the case with our 1 GHz_z oscillator) an apparent change in the sensitivity dV/dP will result;

(2) If the density of the helium in the cavity changes as ΔP changes, there is an additional direct change in the sensitivity. The dielectric constant ε is determined from the density by the equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = 1.2910 \times 10^{-4} \rho - 1.5 \times 10^{-9} \rho^2 \text{ (SI units)}$$

Thus, for differential pressure measurements, it is necessary to know approximate values of the absolute pressure and temperature within the cavity so that the density can be determined and the dielectric correction can be made.

In practice the dielectric correction can result in a sizable frequency shift (10 to 30% of the full scale pressure shift), which is treated operationally as a zero-shift in the instrument. The differential sensitivity to pressure is much less affected by the dielectric correction. For helium at 1 atmosphere pressure and 4K, $\epsilon_r \approx 1.05$.

Figure 7. Calibration of the 0.2 mm (diaphragm thickness) cavity at 4.0 K.

Accuracy and Precision

By far the greatest uncertainty in the measurement system stems from temperature drifts in the voltage tuned oscillator (VTO). For a constant cavity frequency (determined by a precise laboratory-standard calibrated resonator), the drive voltage, which is the analog output, varies by roughly .02 V/K near room temperature. This is equivalent to an apparent frequency shift of about 1 part in 4000 per Kelvin at the operating frequency. An output system based on a 1 to 2 GHz counter may be worth the extra cost to remove the effects of drift where high accuracy is demanded.

When the effects of this oscillator drift are minimized, by taking rapid readings and/or thermally shielding the electronics, we find no evidence of a zero shift as a function of maximum pressure applied and released between the "zero" readings, to an accuracy of about 1 part in 3000 of full scale. This is evidence for lack of hysteresis to this level.

Additional factors influence the accuracy specification. Foremost is the VTO temperature dependence, referred to above. A 1 K room temperature variation would lead to an output shift of about .05 % of full scale. It is possible that this temperature variation could be included in the system calibration; alternatively, if it is possible to zero the pressure difference, a correction can be evaluated from the observed output voltage under this condition.

A small frequency drop occurs as the cavity is cooled down from room temperature to liquid nitrogen temperature, due primarily to the thermal contraction of the cavity. A typical value for this shift is 4 MHz, but this value is reproducible only to within about 10% upon thermal cycling. We believe that this latter irregularity is probably related to tiny distortions or irregular contractions in the insulating dielectric for the capacitive coupling coax during cooldown. The additional thermal contraction from 76 to 4 K has negligible additional effect. Thus, if it is not possible to recheck the zero setting (i.e. the analog output voltage at zero ΔP) after cooldown, the output will be uncertain by approximately 0.2% of full scale.

The output frequency will be fairly sensitive to thermal gradients along the axis of the transducer, though we have not made any measurements of this effect. If, for example, the reentrant piece is a few K different in temperature from the surrounding cavity wall and diaphragm, the differential thermal expansion between these two pieces will be magnified by approximately the ratio of the reentrant piece length to the gap--a factor of roughly 500. With the design as shown in fig. 1, thermal gradients in the cavity itself (as contrasted with the surrounding enclosure) should be small, but nevertheless, due care should be used in mounting the transducer in an apparatus. It is not reasonable to try to estimate a "typical" error from this cause.

The cavities have generally been insensitive to mechanical forces, though not invariably so. One was accidentally dropped 1 meter onto a tiled concrete floor with no apparent change in performance. Another, however, would show occasional discontinuous frequency shifts of about 1 MHz when it was deliberately subjected to sharp blows. We suspect that the source of this instability is the thermal contraction of the insulator in the coaxial electrical lead, leading to a slightly loose fit inside the outer conductor of the coaxial line at low temperature. Thus, if zeroing of the output meter is not possible after installation, one must allow for a contribution of this magnitude in estimating the possible inaccuracy of the measurement system.

Conclusion

The microwave cavity described here serves well as a pressure transducer operating at liquid helium temperature. Its principle virtues are small size, insensitivity to electromagnetic noise and magnetic fields, lack of need for expensive or difficult pressure seals, off-the-shelf availability of components, a high-level analog output voltage, and a capability for high-precision measurements, both in differential and absolute modes. Fabrication and assembly should be within the capability of good mechanical and electronics technicians. When used as a differential pressure gage, readings must be corrected for the dielectric constant of the helium in the cavity, and the accuracy of the output will be questioned unless the meter output at zero differential pressure can be measured in place.

Acknowledgement

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APPENDIX 5.

A Correlation for Heat Transfer to Supercritical Helium in Turbulent Flow in Small Channels

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Fig. 1 Heat transfer as a function of helium dimensionless temperature (experimental data of¹) $q = 8750 \text{ W m}^{-2}$, $G = 180 \text{ Kg m}^{-2} \text{ s}^{-1}$, $\circ - P = 0.8 \text{ MPa}$, $\square - P = 0.5 \text{ MPa}$, $\triangle - P = 0.3 \text{ MPa}$

analogy between heat transfer to a fluid with temperature dependent properties and heat transfer in the presence of injection through a permeable surface. According to this thesis, the essential effect of non-isothermal conditions is a reduction in the fluid density in the boundary layer which gives rise to displacement of the flow lines. There is thus a mass cross flow near the wall, which would have the same effect as injection at the wall. The first step, therefore, in our approach is to identify the mass flux perpendicular to the wall in the non-isothermal case.

In the non-isothermal case, we can apply the equation of continuity for the mean flow

$$\bar{\rho} u = \text{const.} \quad (1)$$

to show that a heat flux q at the wall gives the same acceleration of the mean flow as an injection mass flux j in the isothermal case if the two are related by

$$j = - \frac{q}{C_p} \frac{1}{\bar{\rho}} \left[\frac{\partial \bar{\rho}}{\partial T} \right]_p \quad (2)$$

Here, $\bar{\rho}$ and $\bar{C}_p$ are mean flow values of density and heat capacity, respectively. On the other hand, if we apply conservation of mass and energy to the fluid in a layer adjacent to the wall we find that there is a perpendicular component of mass flux which varies from zero at the wall to a value which can be at most

$$j = q \rho_g \left\langle \frac{1}{\rho C_p} \frac{1}{\bar{\rho}} \left[\frac{\partial \bar{\rho}}{\partial T} \right] \right\rangle_p \quad (3)$$

In the case of strong deterioration this value is approached. In this case ρ_g is evaluated at core conditions and the

quantity in angular brackets is averaged over the layer. Now although C_p and $1/\rho [(\partial \rho / \partial T)]_p = -\beta$ change quite sharply near the pseudo-critical temperature, the quantity $\beta/\rho C_p$ does not, and is in fact a weak function of temperature. We may therefore replace the average by the value calculated at the core temperature and get in place of (3)

$$j \approx \frac{q \beta_g}{C_{p,g}} \quad (4)$$

Further, properties at the core temperature are very close to those at the mixed mean temperature. It is clear there is very little difference between (2) and (4). In future, we shall write ρ , C_p and β without subscript, it being understood that such properties are to be evaluated at the mixed mean temperature of the stream.

Thus, while in isothermal flow with injection we have a constant value of j across the boundary layer, in the flow of a gas with heating, j is position dependent. Nevertheless, each can give rise to a similar overall acceleration of the flow. Little progress can be expected in further analyzing the turbulent flow problem for the non-isothermal case, but let us assume the equivalence given by (4) and apply results from injection.

We now introduce the dimensionless injection parameter.⁷

$$b = j/\rho u St_0 = q\beta/\rho u C_p St_0 \quad (5)$$

According to the work by Kutateladze et al⁷ the available data on injection are satisfactorily represented by

$$Nu/Nu_0 = (1 - b/b_K)^2 \quad (6)$$

where b_K is the critical value of the injection parameter, ie the value at which the boundary layer is blown off the wall and Nu/Nu_0 becomes zero. Further, for the range of Re from 10^4 to 10^5 , $b_K \approx 5$. Thus, if we follow this approach, the experimental data on deteriorated heat transfer should be generalized as a function of the parameter b . We note that both the analytical model and the experimental results suggest that as $q/\rho u$ increases, and the nearer P_R and T_R are to unity (ie the greater β becomes), the more pronounced will be the deterioration in heat transfer.

We would like to mention in passing that a parameter similar to b was arrived at by a different method,⁸ where it was theorized that deterioration in heat transfer may be due to a reduction in eddy diffusivity, caused by the presence of strong density gradients in the boundary layer.

The parameter b may be transformed further

$$b = \frac{\alpha \Delta T \beta \rho u C_p}{\rho u C_p \alpha_0} = \beta \Delta T \frac{Nu}{Nu_0} \quad (7)$$

Then on the basis of (6) and (7) we obtain

$$Nu/Nu_0 = (1 - 0.2(Nu/Nu_0) \beta \Delta T)^2 \quad (8)$$

Thus, according to (8) the deterioration in heat transfer expressed as Nu/Nu_0 is determined solely by a parameter $\beta \Delta T$.

Fig. 2 Heat transfer deterioration for near ideal gas states of helium (experimental data of reference 5) as a function of the correlating parameter ψ . Curve 1 — equation (8); Curve 2 — equation (9)

The relationship (8) or its equivalent should satisfy two extreme cases:

1. for a supercritical fluid where, for small temperature gradients, large density gradients occur (large values of β);
2. for an ideal gas, in which the same density gradients may arise at considerably greater temperature gradients.

It would therefore be desirable to derive a universal relationship for this phenomenon, independent of thermodynamic state. Now, for ideal monatomic and diatomic gases it is well known that the experimental dependence for the heat transfer coefficient as a function of heating rate is⁹

$$Nu/Nu_0 = \psi^{-0.5} \quad (9)$$

where ψ is the temperature ratio $T_w/\bar{T}$. However, in this case $\beta = 1/T$, so we have the alternative expression for ψ :

$$\psi = 1 + \beta \Delta T \quad (10)$$

Thus, for this particular case of the ideal gas we again confirm the importance of the parameter $\beta \Delta T$ in correlating data on heat transfer deterioration; furthermore, we know the functional form in this case. We adopt this latter definition of ψ to obtain $Nu/Nu_0(\psi)$ from (8) as the suggested correlation.

Comparison with experimental results

Helium as an ideal gas. First it should be noted that in the range $1 < \psi < 10$ the discrepancy between (8) and (9) (which may be regarded as an experimental test of (8)) does not exceed 13%. We may also compare experimental results of Giarratano et al (1971) with equations (8) and (9) because in this work values of ψ significantly greater than 1.0 were achieved, mostly at helium temperatures well

above the pseudocritical value. The range was $1.3 < T_R < 9.5$. In Fig. 2 these data are plotted as Nu/Nu_0 vs ψ and it is seen that the relationships (8) or (9) account quite well for the trend of the data. We may therefore conclude that experimental data for cryogenic helium in thermodynamic states approximating the ideal gas with large ΔT 's are consistent with the gas injection analogy.

Supercritical helium. The experimental results documented in references 1–4 are plotted again as Nu/Nu_0 vs ψ in Fig. 3. The range of experimental parameters is given in Table 1. In plotting this figure, as for Fig. 2, thermodynamic and transport properties were taken from McCarley's work.¹⁰ The results of Ogata¹ and Giarratano² are seen to be in good agreement, but 15–30% lower than those of Pronko et al.³ Brassington's data,⁴ when plotted, were selected for maximum heat flux and proximity to $P_R = 1.0$, ie maximum ψ . If we look at values of ψ in excess of two where significant deterioration occurs we see excellent agreement between the data, reported by Ogata and Giarratano^{1,2} and equation (8) (curve 1). For this range of ψ , the experimental deviation does not exceed $\pm 20\%$, with deterioration in heat transfer varying by an order of magnitude. For the data reported by Pronko,³ the trend is correct, but the data are displaced by an average of almost 50%, while for Brassington,⁴ even the data selected do not extend to high enough ψ to show a conclusive trend.

This approach therefore has not only yielded a dimensionless criterion, $\beta \Delta T$, for the deterioration in heat transfer but it indicates, apparently, that we may use the established relationship for injection with success when we interpret the mass cross flow as in (4). Furthermore, the relationship (8) may be used for all thermodynamic states.

In the report by Petukhov et al,⁸ results for the initiation of deterioration in heat transfer to H_2O and CO_2 are generalized. On this basis a value for $\psi = 1.9$ may be predicted for the initiation of deterioration in helium. Referring to Fig 3, this value is consistent with the data for helium.

Fig. 3 Heat transfer deterioration for supercritical helium as a function of ψ . $\circ$ — data of Ogata;¹ $\triangle$ — data of Giarratano;² $+$ — data of Pronko;³ $\bullet$ — data of Brassington.⁴ Curve 1 — equation (8); Curve 2 — equation (9)

Table 1. Range of experimental parameters corresponding to figures 1, 2, and 3.

Reference	[1]	[2]	[3]	[4]	[5]
D (mm)	1.09	2.13	1.04	17.8	2.13
P (MPa)	0.3; 0.5; 0.8	0.25	0.3; 0.4	0.22–0.29	0.3–1.0
$P_R = P/P_c$	1.32; 2.24; 3.59	1.11	1.34; 1.79	1.0–1.3	1.3–4.5
$T_R = T/T_{Pc}$	0.7–1.6	0.76–0.99	0.92–1.34	0.91–1.2	0.8–5.9
Re	(0.5–1.0) $(10)^5$	(0.5–2.0) $(10)^5$	(0.35–0.5) $(10)^5$	(2.4–7.9) $(10)^5$	(0.5–3.8) $(10)^5$
q ($W\ m^{-2}$)	8750	80–7130	4500–5100	255–1990	1770–21560
G ($kg\ m^{-2}\ s^{-1}$)	178–183	70–220	83.5–91.5	25–144	56–760
Flow direction	upward	downward	upward	downward	downward

For $\psi < 1.9$, there is evidence in Fig. 3 of some enhancement of heat transfer for data of Giarratano² although the data cluster satisfactorily about $Nu/Nu_0 = 1.0$ for the smallest values of ψ . The cause of this enhancement, which is also reported by Brassington,⁴ is not understood, but we note, as has been pointed out before,⁸ that axial temperature profiles are often ambiguous in this regime of heat transfer. Hence, a fairly large ($\pm 20\%$) scatter of the data is to be expected in any case.

We emphasize that the correlation in terms of the parameter ψ is understood to apply to pure viscous-inertial (forced convection) flow regimes. The special effects of buoyancy forces, particularly associated with upward flow in large diameter tubes, are not taken into account here. Temperature peaks associated with the latter have in fact been observed for helium⁴ for 17.8 mm diameter tubes with upward flow, but are not the subject of this paper.

Approximate criteria for the onset of the effects of buoyancy have been given. Thus, we have $Gr/Re^2 \approx 10^{-2}$,¹¹ or $Gr/Re^{2.7} \approx 1.2 \times 10^{-5}$.¹² Both confirm that, for the data presented, buoyancy forces should not have been a factor. The good agreement found between the data of Ogata

(upward flow) and Giarratano (downward flow) are taken as confirmation of this fact.

In practical applications, for example, in the cooling of superconductors, we have to deal with large pipe diameters. Then, since $Gr \sim \text{diameter}$,³ buoyancy forces could become important in the steady state. However, in these applications heat transfer often only becomes important under transient conditions when the mixed (forced-free convection) flow regime is not yet established. Deterioration in heat transfer under these conditions should again be predictable from the present correlation.

Conclusions

On the basis of an analogy with the heat transfer process accompanying gas injection at a heated wall, the available data on heat transfer to supercritical helium in a purely forced convection flow regime can be correlated. The same correlation reveals the continuity of the process of deterioration from helium states close to critical up to ideal gas states. If the mass cross flow is interpreted as in (4), one may with good accuracy predict the deterioration with the same equation as for injection, namely (8).

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APPENDIX 6.

A Cryogenic Pressure Regulator

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A CRYOGENIC PRESSURE REGULATOR

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During an experimental study of supercritical helium flow oscillations, we found it necessary to regulate the pressure at low temperature in order to eliminate pressure variations induced by variable pressure drop (with variable flow) in the heat exchangers connecting the experiment to the ambient temperature regulator. Realizing that there are probably a number of applications in which superior pressure control can be achieved with a low temperature pressure regulator, and encouraged by the easy success we experienced with our regulator, we give the following brief description of our dome-load, helium temperature, pressure regulating valve. To our knowledge, the only previous PRV employing a cold actuator is one assembled with bolted flanges and which delivered a fixed discharge pressure [1].

The regulator, figure 1, is based on the conventional scheme, but the construction details are compatible with hermetic seals in both a cryogenic and a high vacuum environment. The dome pressure (control pressure), which is conventionally transmitted through an elastomeric diaphragm, is transmitted through a brass bellows in this case. The bellows position is determined by the balance of control pressure (top side of bellows), controlled pressure (bottom side of bellows), and the spring forces of the bellows and valve plug spring. The motion of the bellows is transferred to the valve plug by the push rod. Thus, a decrease in the outlet (controlled), pressure causes a downward movement of the bellows, push rod, and valve plug, opening the valve and increasing the flow.

All joints, both internal and external, are soldered -- the various solder melting temperatures allowing ease of construction. For ease of machining and soldering, all parts were machined from brass, except the push rod, which is of type 304 stainless steel for greater strength. External joints are minimal to reduce the possibility of leaks -- making the regulator compatible with a high vacuum environment, and the Woods metal seal of the valve plug screw allows valve plug changes with relative ease. A flat is filed on the push rod to prevent flow shut off when the push rod is in its fully extended position.

Sizing the orifice diameter according to simple incompressible flow theory was adequate for our purpose, and choice of the valve plug angle is determined from the valve plug spring and bellows spring constants together with the allowable change of regulated pressure with flow. Our regulator has a 1.51 mm diameter orifice and a 12° half-angle on the plug. The nominal flow rate is 1 g/s of liquid helium for a 1 bar pressure differential across the regulator. Under dynamic conditions, a change in flow from 0.1 g/s to 0.6 g/s caused a change of 0.1 bar or less in the regulated pressure. For steady state flow, typical pressure variations were a few thousandths of a bar.

A stop to prevent excessive bellows movement would improve the design shown in figure 1.

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Figure 1. Regulator assembly.

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