COMPARISON OF COLLISION OPERATORS FOR DRIFT AND MHD-INTERCHANGE
MODES IN UNSHEARED SLAB GEOMETRY

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ABSTRACT

The general procedure for the kinetic analysis of low-frequency electrostatic and electromagnetic modes in toroidal geometry is now well known. In the collisionless limit, the relevant dynamics (e.g., trapped particles, resonances, etc.) can be treated appropriately. However, with the introduction of collisional effects, it is customary, for tractability, to employ model collision operators which do not rigorously satisfy all conservation properties of more exact collision operators. Insight into the essential required features of such operators can be gained by studying models with increasing levels of completeness for a simpler, unsheared slab geometry. The results presented here for this simpler geometry can provide guidance in choosing model collision operators for toroidal-geometry kinetic calculations.

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I. INTRODUCTION

Kinetic calculations of low-frequency modes such as trapped-electron drift modes, ion-temperature-gradient modes, and MHD ballooning modes in toroidal geometry are normally carried out using approximate or model collision operators of various sorts, because more accurate collision operators, like the Lorentz operator, make the linearized gyrokinetic equation too difficult to solve analytically. However, by calculating the eigenfrequencies of the corresponding modes (e.g., the electron drift mode, the ion-temperature-gradient mode, and the MHD interchange mode) in a simpler geometry, namely unsheared slab geometry, it is possible to investigate the accuracy of different model electron collision operators in comparison with a Lorentz operator, and thereby determine directly which properties of a collision operator are important for these modes. These results in unsheared slab geometry can then provide guidance in choosing model collision operators in toroidal geometry.

II. UNSHEARED SLAB GEOMETRY CALCULATION

The linearized gyrokinetic or drift-kinetic equation for electrons is:

$$(\omega - \omega_{de} - k_{\parallel} v_{\parallel}) g_e - iC(g_e) = - \frac{|e|}{T_e} F_{me} (\omega - \omega_{*e}^T) \left(\phi - \frac{v_{\parallel}}{c} A_{\parallel} \right), \quad (1)$$

where $f_e \equiv g_e + (|e|\phi/T_e)F_{me}$ is the perturbed electron distribution function, $\omega_{*j}^T \equiv \omega_{*j}[1 + \eta_j(E/T_j - 3/2)]$ for species $j (= e, i)$, $\eta_j \equiv (d \ln T_j / dr) / (d \ln n_j / dr)$, $\omega_{dj} \equiv$ constant magnetic drift frequency (gravity), and the rest of the notation is standard. We have taken $k_{\perp} \rho_e \ll 1$ and we do not include an equilibrium electric field. We have also taken $A_{\perp} = 0$ for $\beta < 1$, since our goal here is the comparison of different electron collision operators, rather

than the absolute accuracy of the mode eigenfrequencies.

We will compare the following electron collision operators, where the electron collision frequency $\nu_e = \text{constant}$:

(a) $C(g_e) = -\nu_e g_e$, Krook operator with no conservation terms.

(b) $C(g_e) = -\nu_e g_e + \nu_e \frac{F_{me}}{n_e} \int d^3v g_e$, Krook operator with a number conservation term.

(c) $C(g_e) = -\nu_e g_e + \nu_e \frac{F_{me}}{n_e} \int d^3v g_e$
 $+ \nu_e (E - \frac{3}{2} T_e) \frac{2F_{me}}{3n_e T_e^2} \int d^3v (E - \frac{3}{2} T_e) g_e$,

Krook operator with number and energy conservation terms.

(d) $C(g_e) = \nu_e \frac{\partial}{\partial x} (1 - x^2) \frac{\partial}{\partial x} g_e$, $x \equiv v_{\parallel}/v$,

Lorentz operator (conserves number and energy automatically).

(e) $C(g_e) = \nu_e(E) \frac{\partial}{\partial x} (1 - x^2) \frac{\partial}{\partial x} g_e$,

$$\nu_e(E) \equiv \nu_e [1 + H(\hat{v})]/(\hat{v})^3 ,$$

$$\hat{v} \equiv (E/T_e)^{1/2} ,$$

$$H(\hat{v}) \equiv \exp(-\hat{v}^2)/(\pi^{1/2}\hat{v}) + [1 - 1/(2\hat{v}^2)]\text{erf}(\hat{v}) ,$$

Lorentz operator with an energy-dependent collision frequency.

Operators b, c, d, and e conserve electron particle number, e.g., $\int d^3v C(g_e) = 0$. Operators c, d, and e conserve energy as well, e.g., $\int d^3v EC(g_e) = 0$ [we treat the ions as collisionless, $C(g_i) = 0$]. Note that none of these operators conserve momentum. We do not consider the question of momentum conservation because the associated time scales are much longer than the characteristic time scales for kinetic instabilities of interest here.

For the Krook operators a, b, and c, the quasineutrality equation and the component of Ampere's law along the equilibrium magnetic field lines can be evaluated exactly in terms of the usual plasma dispersion function. For the ions, we take the collision frequency $\nu_i = 0$, but include full finite Larmor radius effects, as well as the magnetic drift frequency $\omega_{di} = \text{constant}$ (gravity). Since the methods used to evaluate the two equations for these three Krook operators are standard,¹ we only present the results here. The quasineutrality equation is:

$$0 = \left(1 + \frac{T_e}{T_i}\right)\phi + \frac{(T_e/T_i)\omega + \omega_{*e}[1 - (3/2)\eta_i]}{k_{\parallel}v_i} r_0 \left(\phi Z_0^i - \frac{v_i}{c} A_{\parallel} Z_1^i\right) + \frac{\eta_i \omega_{*e}}{k_{\parallel}v_i} \left\{ r_0 \left(\phi Z_2^i - \frac{v_i}{c} A_{\parallel} Z_3^i\right) + \left[r_0 + b_i(r_1 - r_0)\right] \left(\phi Z_0^i - \frac{v_i}{c} A_{\parallel} Z_1^i\right) \right\} + X, \quad (2)$$

and the component of Ampere's law along the equilibrium magnetic field lines is:

$$0 = k^2 \lambda_{De}^2 A_{\parallel} + \frac{v_i}{c} \frac{(T_e/T_i)\omega + \omega_{*e}[1 - (3/2)\eta_i]}{k_{\parallel}v_i} r_0 \left(\phi Z_1^i - \frac{v_i}{c} A_{\parallel} Z_2^i\right) + \frac{v_i}{c} \frac{\eta_i \omega_{*e}}{k_{\parallel}v_i} \left\{ r_0 \left(\phi Z_3^i - \frac{v_i}{c} A_{\parallel} Z_4^i\right) + \left[r_0 + b_i(r_1 - r_0)\right] \left(\phi Z_1^i - \frac{v_i}{c} A_{\parallel} Z_2^i\right) \right\} + Y. \quad (3)$$

Here, the particular forms of X and Y depend on the electron collision operator employed. Also, $I_{0,1} \equiv I_{0,1}(b_1)\exp(-b_1)$, $b_1 \equiv k_1^2 \rho_1^2/2$, $I_{0,1}(b_1)$ are the usual modified Bessel functions, $\lambda_{De}^2 \equiv T_e/(4\pi n_e e^2)$, $\hat{v}_1 \equiv v_{||}/v_j$, $v_j \equiv (2T_j/m_j)^{1/2}$, $\zeta_j \equiv (\omega - \omega_{dj})/k_{||}v_j$, and

$$Z_j^j \equiv \pi^{-1/2} \int_{-\infty}^{\infty} d\hat{v}_{||} \frac{(\hat{v}_{||})^n \exp(-\hat{v}_{||}^2)}{\hat{v}_{||} - \zeta_j} \quad (4)$$

for $\gamma \equiv \text{Im}(\omega) > 0$ and the analytic continuation for $\gamma \leq 0$. $Z_j^j = Z(\zeta_j)$ is the usual plasma dispersion function. In terms of Z_j^j , we find that

$Z_1^j = 1 + \zeta_j Z_0^j$, $Z_2^j = \zeta_j + \zeta_j^2 Z_0^j$, $Z_3^j = 1/2 + \zeta_j + \zeta_j^3 Z_0^j$, $Z_4^j = \zeta_j/2 + \zeta_j^3 + \zeta_j^4 Z_0^j$, and $Z_5^j = 3/4 + \zeta_j^2/2 + \zeta_j^4 + \zeta_j^5 Z_0^j$. When Eqs. (2) and (3) are solved numerically, the frequency $\omega \equiv \omega_r + i\gamma$ is iterated by Newton's method from some initial guess.

For the nonconserving Krook operator a , the results for X and Y are

$$X_a = \frac{\omega - \omega_{*e}(1 - \eta_e/2)}{k_{||}v_e} (\phi Z_0^e - \frac{v_e}{c} A_{||} Z_1^e) - \frac{\eta_e \omega_{*e}}{k_{||}v_e} (\phi Z_2^e - \frac{v_e}{c} A_{||} Z_3^e) \quad (5)$$

and

$$Y_a = \frac{v_e}{c} \left[\frac{\omega - \omega_{*e}(1 - \eta_e/2)}{k_{||}v_e} (\phi Z_1^e - \frac{v_e}{c} A_{||} Z_2^e) - \frac{\eta_e \omega_{*e}}{k_{||}v_e} (\phi Z_3^e - \frac{v_e}{c} A_{||} Z_4^e) \right] \quad (6)$$

For the number-conserving Krook operation b , the results for X and Y are:

$$X_b = X_a \left(1 + i \frac{v_e}{k_{||}v_e} Z_0^e \right)^{-1}, \quad (7)$$

and

$$\begin{aligned}
Y_b = & \frac{v_e}{c} \left[\frac{\omega - \omega_{*e} (1 - \eta_e/2)}{k_{\parallel} v_e} \left(\phi Z_1^e - \frac{v_e}{c} A_{\parallel} \{ Z_2^e + i \frac{v_e}{k_{\parallel} v_e} [Z_2^e Z_0^e - (Z_1^e)^2] \} \right) \right. \\
& - \frac{\eta_e \omega_{*e}}{k_{\parallel} v_e} \{ \phi [Z_3^e + i \frac{v_e}{k_{\parallel} v_e} (Z_3^e Z_0^e - Z_1^e Z_2^e)] - \frac{v_e}{c} A_{\parallel} [Z_4^e \\
& \left. + i \frac{v_e}{k_{\parallel} v_e} (Z_4^e Z_0^e - Z_1^e Z_3^e)] \} \right] (1 + i \frac{v_e}{k_{\parallel} v_e} Z_0^e)^{-1} \quad (8)
\end{aligned}$$

For the number and energy-conserving Krook operator c , the results for X and Y are:

$$\begin{aligned}
X_c = & \left(\left[\frac{\omega - \omega_{*e} (1 - \eta_e/2)}{k_{\parallel} v_e} (\phi Z_0^e - \frac{v_e}{c} A_{\parallel} Z_1^e) - \frac{\eta_e \omega_{*e}}{k_{\parallel} v_e} (\phi Z_2^e - \frac{v_e}{c} A_{\parallel} Z_3^e) \right] \right. \\
& \cdot \left\{ 1 + i \frac{2v_e}{3k_{\parallel} v_e} (Z_4^e - Z_2^e + \frac{5}{4} Z_0^e) \right\} - i \frac{2v_e}{3k_{\parallel} v_e} (Z_2^e - \frac{1}{2} Z_0^e) \\
& \cdot \left\{ \frac{\omega - \omega_{*e} (1 - 3\eta_e/2)}{k_{\parallel} v_e} \left[\phi (Z_2^e - \frac{1}{2} Z_0^e) - \frac{v_e}{c} A_{\parallel} (Z_3^e - \frac{1}{2} Z_1^e) \right] \right. \\
& - \frac{\eta_e \omega_{*e}}{k_{\parallel} v_e} \left\{ \phi (Z_4^e + \frac{1}{2} Z_2^e + \frac{1}{2} Z_0^e) - \frac{v_e}{c} A_{\parallel} (Z_5^e + \frac{1}{2} Z_3^e \right. \\
& \left. \left. + \frac{1}{2} Z_1^e) \right\} \right\} \cdot \left[(1 + i \frac{v_e}{k_{\parallel} v_e} Z_0^e) \left\{ 1 + i \frac{2v_e}{3k_{\parallel} v_e} (Z_4^e - Z_2^e \right. \right. \\
& \left. \left. + \frac{5}{4} Z_0^e) \right\} + \frac{2}{3} \left(\frac{v_e}{k_{\parallel} v_e} \right)^2 (Z_2^e - \frac{1}{2} Z_0^e)^2 \right]^{-1} \quad (9)
\end{aligned}$$

and

$$\begin{aligned}
Y_c = & \frac{v_e}{c} \left(\frac{\omega - \omega_{*e} (1 - \eta_e/2)}{k_{\parallel} v_e} (\phi Z_1^e - \frac{v_e}{c} A_{\parallel} Z_2^e) - \frac{\eta_e \omega_{*e}}{k_{\parallel} v_e} (\phi Z_3^e - \frac{v_e}{c} A_{\parallel} Z_4^e) \right. \\
& - i \frac{2v_e}{3k_{\parallel} v_e} (Z_3^e - \frac{1}{2} Z_1^e) [1 + i \frac{2v_e}{3k_{\parallel} v_e} (Z_4^e - Z_2^e + \frac{5}{4} Z_0^e)]^{-1} \\
& \cdot \left\{ \frac{\omega - \omega_{*e} (1 - 3\eta_e/2)}{k_{\parallel} v_e} \cdot [\phi (Z_2^e - \frac{1}{2} Z_0^e) - \frac{v_e}{c} A_{\parallel} (Z_3^e - \frac{1}{2} Z_1^e)] - \frac{\eta_e \omega_{*e}}{k_{\parallel} v_e} \right. \\
& \cdot \left. \left[\phi (Z_4^e + \frac{1}{2} Z_2^e + \frac{1}{2} Z_0^e) - \frac{v_e}{c} A_{\parallel} (Z_5^e + \frac{1}{2} Z_3^e + \frac{1}{2} Z_1^e) \right] \right\} \\
& + \frac{v_e}{c} \left\{ -i \frac{v_e}{k_{\parallel} v_e} Z_1^e - \frac{2}{3} \left(\frac{v_e}{k_{\parallel} v_e} \right)^2 (Z_3^e - \frac{1}{2} Z_1^e) (Z_2^e - \frac{1}{2} Z_0^e) [1 + i \frac{2v_e}{3k_{\parallel} v_e} (Z_4^e \right. \right. \\
& \left. \left. - Z_2^e + \frac{5}{4} Z_0^e)]^{-1} \right\} X_c. \quad (10)
\end{aligned}$$

For the Lorentz collision operator d , a continued fraction solution using an expansion of g_e in Legendre polynomials is necessary. This approach has been used before in related contexts for sheared slab geometry by R. A. Koch and W. Horton, Jr.,² by N. T. Gladd, et al.,³ and by C. L. Chang, et al.⁴ Specifically, we write

$$g_e(E, \chi) = \sum_{n=0}^{\infty} g_n(E) P_n(\chi), \quad (11)$$

where $\chi \equiv v_{\parallel}/v$ and $P_n(\chi)$ is the Legendre polynomial of order n . Then, since

$$\frac{\partial}{\partial \chi} (1 - \chi^2) \frac{\partial}{\partial \chi} P_n(\chi) = -n(n+1) P_n(\chi), \quad (12)$$

and by using the recursion and orthogonality relations for Legendre polynomials, the linearized gyrokinetic equation with the Lorentz collision operator d becomes:

$$\begin{aligned}
 n = \left[\omega - \omega_{de} + i v_e n(n+1) \right] g_n - k_{\parallel} v \left(\frac{n}{2n-1} g_{n-1} + \frac{n+1}{2n+3} g_{n+1} \right) \\
 + \frac{|e|}{T_e} F_{me} (\omega - \omega_{*e}^T) \left(\delta_{n0} \phi - \delta_{n1} \frac{v}{c} A_{\parallel} \right) , \quad (13)
 \end{aligned}$$

where $\delta_{nm} = 1$ for $n = m$ and $\delta_{nm} = 0$ for $n \neq m$. We solve recursively for g_0 , g_1 , g_2 , ... in terms of the higher g_n . This gives:

$$g_n = \frac{n+1}{2n+3} g_{n+1} + \frac{n! \alpha^n W}{1 \cdot 3 \cdot 5 \cdots (2n-1)} (1/a_1/a_2/a_3 \cdots /a_n) , \quad (14)$$

where

$$\alpha \equiv \frac{k_{\parallel} v}{\omega - \omega_{de}} , \quad (15)$$

$$a_1 \equiv 1 + 2i \frac{v_e}{\omega - \omega_{de}} - \frac{\alpha^2}{3} , \quad (16)$$

$$a_2 \equiv 1 + 6i \frac{v_e}{\omega - \omega_{de}} - \frac{2}{3} \frac{2}{5} \frac{\alpha^2}{a_1} , \quad (17)$$

$$a_3 \equiv 1 + 12i \frac{v_e}{\omega - \omega_{de}} - \frac{3}{5} \frac{3}{7} \frac{\alpha^2}{a_2} , \quad (18)$$

$\vdots$

$$a_n \equiv 1 + n(n+1)i \frac{v_e}{\omega - \omega_{de}} - \frac{n}{2n-1} \frac{n}{2n+1} \frac{\alpha^2}{a_{n-1}} , \quad (19)$$

and

$$W \equiv - \frac{\omega - \omega_{*e}^T}{\omega - \omega_{de}} \frac{|e|}{T_e} F_{me} \left(\phi - \frac{\omega - \omega_{de}}{k_{\parallel} c} A_{\parallel} \right) . \quad (20)$$

Then we take $g_n = 0$ for $n > N$ ($N = 10$ or 20 , typically) and compute g_N from Eq. (14). The lower g_n are given by the downward recursion relation

$$g_{n-1} = \frac{2n-1}{na} \left\{ \left[1 + n(n+1)i \frac{v_e}{\omega - \omega_{de}} \right] g_n - \alpha \frac{n+1}{2n+3} g_{n+1} \right\} \quad (21)$$

The quasineutrality condition, which involves only g_0 , and the parallel component of Ampere's law, which involves only g_1 , can be constructed. The results for X and Y in Eqs. (2) and (3) are:

$$X_d = -2(\pi)^{-1/2} \int_0^\infty d\hat{E} \hat{E}^{1/2} \exp(-\hat{E}) \{ \omega - \omega_{*e} [1 + \eta_e (\hat{E} - \frac{3}{2})] \} \\ \cdot \left[\frac{\phi}{\omega - \omega_{de}} + \frac{\alpha^2}{3} \frac{1 + (2/5)g_2/W}{a_1} \left(\frac{\phi}{\omega - \omega_{de}} - \frac{A_{||}}{k_{||}c} \right) \right] \quad (22)$$

and

$$Y_d = - \frac{v_e}{c} \frac{2}{3} \pi^{-1/2} \int_0^\infty d\hat{E} \hat{E} \exp(-\hat{E}) \{ \omega - \omega_{*e} [1 + \eta_e (\hat{E} - \frac{3}{2})] \} \\ \cdot \alpha \frac{1 + (2/5)g_2/W}{a_1} \left(\frac{\phi}{\omega - \omega_{de}} - \frac{A_{||}}{k_{||}c} \right) \quad (23)$$

where $\hat{E} \equiv E/T_e$. When X_d and Y_d are evaluated, the integrations over energy are done numerically using a 60-point Gauss-Hermite integration scheme. For the Lorentz collision operator e with the energy-dependent collision frequency $v_e(E)$, v_e is replaced by $v_e(E)$ in Eqs. (13) - (23).

III. UNSHEARED SLAB GEOMETRY RESULTS

We first look at results for the usual collisionless and collisional drift waves in the electrostatic limit with the magnetic drift frequencies suppressed, $\omega_{dj} = 0$. The growth rate γ is shown as a function of $v_e/k_{\parallel}v_e$ in Fig. 1. All of the operators give the same collisionless limit ($v_e = 0$). However, the choice of operator strongly influences the result for the collisional drift wave. The Krook operator with no conservation terms (curve a) does not produce the collisional drift wave instability at all. Including the number conservation term (curve b) produces the collisional drift wave with γ peaking at $v_e/k_{\parallel}v_e = 70$ for the parameters given in Fig. 1. Including the energy-conserving term as well (curve c) affects γ moderately in the range $v_e/k_{\parallel}v_e = 0.1$ to 10, but the peak value of γ is identical, even though there are non-zero temperature gradients. The Lorentz operator with $v_e = \text{constant}$ (curve d) gives a growth rate curve of about the same height but peaking at $v_e/k_{\parallel}v_e = 3$, perhaps because of the stronger effect of small- $(v_{\parallel}/v)$ particles with this operator. Including the energy dependence of $v_e(E) = v_e[1 + H(\hat{v})]/\hat{v}^3$ (curve e) lowers the peak value of γ slightly and moves it to smaller $v_e/k_{\parallel}v_e$. Also, the shape of the curve for $v_e/k_{\parallel}v_e < 1$ is different, because of the effect of small- $\hat{v}$ particles. Thus, the results of a conserving Krook operator and a Lorentz operator are seen to be significantly different quantitatively. Also, number conservation is seen to be necessary to produce the collisional drift wave instability at all.

Next, we look at results for the ion-temperature-gradient (η_i) mode in the same limit as for the collisional drift wave. The growth rate γ is shown versus $v_e/k_{\parallel}v_e$ in Fig. 2. The Krook operator with no conservation terms (curve a) has a spuriously large growth rate for $v_e/k_{\parallel}v_e > 1$. For a Krook operator with number conservation (curve b), the growth drops off for

$v_e/k_{\parallel}v_e > 1$. Including an energy-conserving term also changes the values of γ only moderately in the range $v_e/k_{\parallel}v_e = 0.1$ to 10, as with the collisional drift wave. For the Lorentz operator with $v_e = \text{constant}$ (curve d), the collisional stabilization starts at smaller $v_e/k_{\parallel}v_e$ than for the number and energy-conserving Krook operator (curve c). For the Lorentz operator with the energy-dependent collision frequency $v_e(E)$, the collisional stabilization starts at still smaller $v_e/k_{\parallel}v_e$ and the shape of the curve is changed. These differences are analogous to those for the collisional drift wave.

Finally, we look at results for the MHD-interchange (shear-Alfvén) mode with a nonzero magnetic drift frequency ω_{dj} for β well above the critical value. The growth rate is shown versus $v_e/k_{\parallel}v_e$ in Fig. 3. Number conservation is seen to be essential for any $v_e/k_{\parallel}v_e \geq 10^{-5}$ (cf. curve a, without any conserving terms). When the number-conserving term is included, the absence (curve b) or presence (curve c) of the energy-conserving term makes no difference. The Lorentz operator gives a result which is essentially independent of v_e , without (curve d) or with (curve e) the energy dependence of $v_e(E)$, and is close to that of the conserving Krook operators. Thus, with the most accurate collision operator, the growth rate is independent of the collision frequency.

IV. CONCLUSIONS

The results here may be summarized as follows: In unsheared slab geometry, from the results for the modes considered here, it is seen that a number-conserving term is a necessity when employing a Krook operator. However, an energy-conserving term has only a moderate additional effect, even when temperature gradients are included. For the electrostatic collisional drift wave, using a Lorentz operator shifts the growth rate peak to lower

$v_e/k_{||}v_e$ values compared to a number-conserving Krook operator. For the ion-temperature-gradient (n_i) mode, the effect of collisions is stabilizing when the collision frequency is sufficiently high. For the MHD-interchange mode, on the other hand, the collisional results are almost the same as the collisionless result, using either the conserving Krook operators or the Lorentz operators.

These results for a simpler geometry can provide some guidance in constructing collision operators for use in toroidal-geometry calculations. The property of number conservation appears to be vital, except in certain particular limits. However, the property of energy conservation appears to be considerably less important. The pitch-angle variation of the perturbed distribution function, which is important with a Lorentz operator, makes a significant quantitative difference. It appears desirable to try to model this effect in toroidal geometry when using a Krook operator.⁵ The energy dependence of the collision frequency also makes a significant quantitative difference, and it is important to retain it. These "lessons" are applied in the construction of a model collision operator for toroidal eigenmode-eigenvalues calculations in a separate paper.⁶

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FIGURE CAPTIONS

FIG. 1. Growth rate versus collision frequency for the collisionless and collisional drift waves in unsheared slab geometry. The parameters are: $\beta = 0$, $n_e = n_i = -1$, $T_e/T_i = 1$, $k_\theta \rho_i = 0.5$, $\omega_{de} = \omega_{di} = 0$, $k_\parallel r_n = 0.067$, $m_i/m_e = 3672$, and $\nu_i = 0$. The curves represent: (a) a Krook operator with no conservation terms; (b) a Krook operator with number conservation term only; (c) a Krook operator with number and energy conservation terms; (d) a Lorentz operator; (e) a Lorentz operator with an energy-dependent collision frequency.

FIG. 2. Growth rate versus collision frequency for the ion-temperature-gradient (η_i) mode. The parameters are $\beta = 0$, $n_i = n_e = 10$, $T_e/T_i = 1$, $k_\theta \rho_i = 0.5$, $\omega_{de} = \omega_{di} = 0$, $k_\parallel r_n = 0.067$, $m_i/m_e = 3672$, and $\nu_i = 0$. The curves are labeled as in Fig. 1.

FIG. 3. Growth rate versus collision frequency for the MHD-interchange mode. The parameters are: $\beta = 0.25$, $\omega_{dj}/\omega_{sj} = 0.13$, $n_e = -1$, $n_i = 1$, $T_e/T_i = 1$, $k_\theta \rho_i = 0.1$, $k_\parallel r_n = 0.087$, $m_i/m_e = 3672$, and $\nu_i = 0$. The curves are labeled as in Fig. 1.

Fig. 1

Fig. 2

Fig. 3

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