1512

185
2/13/19

4. 2188

JANUARY 1979

PPPL-1512

UC-20g

ANALYTICAL THEORY OF DRIFT WAVES
AND DRIFT ALFVÉN WAVES
IN TOKAMAKS

BY

Y. C. LEE AND L. CHEN

**PLASMA PHYSICS
LABORATORY**

MASTER

**PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY**

This work was supported by the U. S. Department of Energy
Contract No. EY-76-C-02-3073. Reproduction, translation,
publication, use and disposal, in whole or in part, by or
for the United States Government is permitted.

The recent discovery^{1,2} that the collisionless electrostatic drift wave in a sheared magnetic field in slab geometry is absolutely stable has been reinforced by the elegant analytical proof of Antonsen.³ More recently, Tsang et al.⁴ have indicated through extensive numerical calculations that the finite- β modified drift modes as well as the shear Alfvén modes are similarly not unstable in the absolute sense. The analytical treatments of finite- β modified drift waves have been attempted by Catto et al.⁵ and by Chen et al.⁶ Their treatments, however, are basically perturbative in nature and, hence, cannot determine the stability definitively. As to the shear Alfvén branch, only approximate analytical solutions exist.⁷ In this Letter we first extend Antonsen's method to both finite- β modified drift modes and shear Alfvén modes. We demonstrate absolute stability in both cases; within the approximation of collisionless kinetic electrons and cold-fluid ions.

In slab geometry ignoring the compressional component of the perturbed magnetic field, we can write the finite- β radial eigenmode equations^{5,6} as

$$\rho_s^2 (d^2/dx^2 - k_y^2) \phi = F(x) (\phi - \Omega x_c A_{||}/x), \quad (1)$$

and

$$(v_A \rho_s / c)^2 (d^2/dx^2 - k_y^2) A_{||} = (\Omega x_c / x) F(x) (\phi - \Omega x_c A_{||}/x); \quad (2)$$

where $F(x) = (1 - 1/\Omega) (1 + \xi_e z_e) - (x/\Omega x_s)^2$, $\Omega = \omega/\omega_{*e}$, $\xi_e = \Omega x_e/|x|$, $x_e = \omega_{*e}/k_{||}' v_e$, $k_{||}' = k_y/L_s$, $x_s = \omega_{*e}/k_{||}' c_s$, $c_s^2 = T_e/m_i$, $\rho_s = c_s/\omega_{ci}$, $x_c = \omega_{*e}/k_{||}' c$, $z_e = Z(\xi_e)$, Z is the plasma dispersion function and

the rest of the notations are standard. We wish to solve Eqs. (1) and (2) subject to the outgoing/evanescent boundary conditions. Thus, the asymptotic ($|x| \rightarrow \infty$) behaviors of both ϕ and $A_{||}$ will be described by linear combinations of $\exp(-ix^2/2\Omega x_S \rho_S)$ and $\exp(-k_y |x|)$.

Following Antonsen,³ we first assume that the unstable eigenmodes with $\text{Im } \Omega > 0$ exist. It follows that both ϕ and $A_{||}$ are subdominant (i.e., $|\phi|, |A_{||}| \rightarrow 0$ as $|x| \rightarrow \infty$) along the real- x axis. Now if we allow x to be analytically continued into the complex x plane, then for the case $\text{Im } \Omega, \text{Re } \Omega > 0$, the anti-Stokes lines lie in the second and fourth quadrants as shown in Fig. 1(a). Specifically, in the fourth quadrant, the asymptotic anti-Stokes lines are $\arg x = -\pi/2$ and $\arg x = -(\pi - \arg \Omega)/2$, which correspond to the asymptotic behavior $\exp(-k_y x)$ and $\exp(-ix^2/2\Omega x_S \rho_S)$, respectively; whereas in the second quadrant the asymptotic anti-Stokes lines are $\arg x = \pi/2$ and $\arg x = (\pi + \arg \Omega)/2$. Since $|\pi - \arg \Omega|/2 < \pi/2$, the regions where both ϕ and $A_{||}$ are subdominant are $0 > \arg x > (\arg \Omega - \pi)/2$ and $\pi > \arg x > (\arg \Omega + \pi)/2$. When $\text{Im } \Omega > 0$ but $\text{Re } \Omega < 0$, the subdominant regions are $(\pi + \arg \Omega)/2 > \arg x > 0$ and $(\arg \Omega - \pi)/2 > \arg x > -\pi$ [see Fig. 1(b)]. Therefore, the line $x = -i\Omega\eta$, where η is real, lies entirely within the subdominant regions as long as $\text{Im } \Omega > 0$, regardless of the sign of $\text{Re } \Omega$.³ Hence, upon making the transformation³ $x = -i\Omega\eta$, Eqs. (1) and (2) become

$$-\rho_S^2 (\Omega^{-2} d^2/d\eta^2 + k_y^2) \phi = F(\eta) (\phi - ix_c A_{||}/\eta), \quad (3)$$

and

$$-(v_A \rho_S/c)^2 (\Omega^{-2} d^2/d\eta^2 + k_y^2) A_{||} = (ix_c/\eta) F(\eta) \times (\phi - ix_c A_{||}/\eta); \quad (4)$$

where $F(\eta) = (1 - 1/\Omega)G(\eta) + \eta^2/x_s^2$ and $G(\eta) = 1 + i(x_e/\eta)Z(ix_e/\eta) \leq 0$.
 Multiplying Eqs. (3) and (4) by $\Omega^2 \phi^*$ and $\Omega^2 A_{||}^*$, respectively,
 and integrating from $\eta = -\infty$ to $\eta = \infty$ (where both ϕ and $A_{||}$ vanish)
 yields a quadratic equation for Ω

$$\Omega^2 A_1 - \Omega A_2 - A_3 = 0. \quad (5)$$

Here, $A_1, A_3 > 0$ and $A_2 \geq 0$ are Ω -independent quadratic forms
 given by $A_1 = k_y^2 \rho_s^2 \langle |\phi|^2 + |v_A A_{||}/c|^2 + [G(\eta) + \eta^2/x_s^2] |\phi - ix_c A_{||}/\eta|^2 \rangle$,
 $A_2 = \langle G(\eta) |\phi - ix_c A_{||}/\eta|^2 \rangle$, $A_3 = \rho_s^2 \langle |d\phi/d\eta|^2 + (v_A/c)^2 |dA_{||}/d\eta|^2 \rangle$, and
 $\langle \dots \rangle$ denotes $\int_{-\infty}^{\infty} d\eta (\dots)$. Equation (5) can be formally solved to
 give $\Omega = [A_2 \pm (A_2^2 + 4A_1 A_3)^{1/2}] / 2A_1$. Hence, $\text{Im } \Omega = 0$ which contradicts
 the assumption that the mode is unstable and thereby disproves the
 existence of such modes. We have found, moreover, that the stable
 nature of the collisionless finite- β drift modes and shear Alfvén
 modes is not changed by the presence of electron temperature gra-
 dients.

The preceding analysis does not rule out the possibility of
 marginally stable modes ($\text{Im } \Omega = 0$). From a practical standpoint,
 marginally stable modes can cause enhanced transport just as unstable
 modes do and therefore could be almost as dangerous to the plasma
 confinement. Furthermore, it is possible that other non-ideal effects
 such as toroidal curvature, current drift and trapped particles
 which are not considered here may destabilize a marginally stable
 mode more easily than a damped mode. Theoretically, the question
 of marginal stability is also interesting because there seem to be
 some numerical indications^{1,2,8} that for sufficiently weak shear
 and finite $k_y^2 \rho_s^2$, the electrostatic drift wave becomes marginally

because of the numerical uncertainties inherent in the code, this question must be answered analytically.

If marginally stable modes ($\text{Im } \Omega = 0$) do exist, we can only construct from Eqs. (3) and (4) the following asymptotic solution

$$\left(\frac{v_A}{c}\right)^2 \left[A_{||} \frac{dA_{||}^*}{d\eta} - A_{||}^* \frac{dA_{||}}{d\eta} \right] + \phi \frac{d\phi^*}{d\eta} - \phi^* \frac{d\phi}{d\eta} = 0. \quad (6)$$

valid everywhere along the real η axis, it is also valid for $|\eta| \rightarrow \infty$. Defining $\Gamma(\eta) = i \left\{ \left(\frac{v_A}{c} \right)^2 \left(A_{||} \frac{dA_{||}^*}{d\eta} - A_{||}^* \frac{dA_{||}}{d\eta} \right) - \phi \frac{d\phi^*}{d\eta} + \phi^* \frac{d\phi}{d\eta} \right\}$, from Eq. (6) we see that $\Gamma(\eta)$ is a constant; i.e., the wave flux is conserved. Asymptotically, the solution should consist of subdominant solutions only, i.e.,

$$\alpha_{\pm} \eta^{-1/2} \exp(i\Omega \eta^2 / 2x_S \rho_S) + i\beta_{\pm} (x_C / \eta) \times \exp(\pm i\Omega k_y \eta), \quad (7a)$$

$$\beta_{\pm} \exp(\pm i\Omega k_y \eta) + i\alpha_{\pm} (c/v_A)^2 x_C \eta^{-3/2} \times \exp(i\Omega \eta^2 / 2x_S \rho_S); \quad (7b)$$

that in Eqs. (7a) and (7b) we have kept terms of order $1/\eta$ higher than the leading terms in the asymptotic expansion so as to emphasize that the coupling between the modes is rapid with $|\eta|$. These higher-order terms do not contribute to the wave flux $\Gamma(\eta)$ in the asymptotic limit,

$$\Gamma(\pm\infty) = \pm 2[(v_A/c)^2 \Omega k_Y |E_\tau|^2 + (\Omega/x_S \rho_S) |\alpha_\pm|^2] \quad (8)$$

Equation (8) clearly contradicts the constancy of the wave flux Γ unless $\alpha_\pm$ and $\beta_\pm$ are all exactly equal to zero. From the asymptotic analysis, this implies that both ϕ and $A_{||}$ vanish exactly, which in turn means that ϕ and $A_{||}$ vanish everywhere along η (using the uniqueness theorem of ordinary differential equation). Thus, we have proved that, within the approximations implied by Eqs. (1) and (2), exact marginally stable eigenmodes cannot exist.

We note that, while Antonsen's method is rather novel, it works only in the case of cold-fluid ions. We now present a more general technique⁹ which has much wider applicability. For simplicity let us consider the electrostatic case but with warm-kinetic ions. Here, Eq. (1), with $A_{||} = 0$, retains the same form except $F(x)$ is replaced by $G(x, \lambda_i)$, where

$$G(x, \lambda_i) = - \frac{(\tau/\lambda_i^2)(1+\lambda_i/\Omega\tau)(1+\xi_i Z_i) + (1-1/\Omega)(1+\xi_e Z_e)}{(1+\lambda_i/\Omega\tau)\xi_i Z_i} \quad (9)$$

Here, $\tau = T_e/T_i$, $\xi_i = \Omega x_i/|x|\lambda_i$, $x_i = \omega_{*e}/k_{||} v_i$, $Z_i = Z(\xi_i)$ and $\lambda_i (0 \leq \lambda_i \leq 1)$ is the kinetic-ion parameter. Thus, $\lambda_i = 0$ and 1 correspond to the cases of cold-fluid ions and warm-kinetic ions, respectively. As λ_i is increased from 0 to 1, the eigenmode frequencies, $\Omega(\lambda_i)$, would then evolve continuously from the $\lambda_i \rightarrow 0^+$ limit. As discussed above, we can use Antonsen's method to prove that, in the $\lambda_i \rightarrow 0^+$ limit, the eigenmodes are damped; i.e., $\text{Im } \Omega(\lambda_i \rightarrow 0^+) < 0$.¹⁰ Furthermore, the preceding analysis on

stable. However, because of the numerical uncertainties inherent in any numerical code, this question must be answered analytically.

Assuming that marginally stable modes ($\text{Im } \Omega = 0$) do exist, one can immediately construct from Eqs. (3) and (4) the following Wronskian identity

$$\frac{d}{d\eta} \left[\left(\frac{v_A}{c} \right)^2 \left\{ A_{||} \frac{dA_{||}^*}{d\eta} - A_{||}^* \frac{dA_{||}}{d\eta} \right\} + \phi \frac{d\phi^*}{d\eta} - \phi^* \frac{d\phi}{d\eta} \right] = 0. \quad (6)$$

Since Eq. (6) is valid everywhere along the real η axis, it is also valid asymptotically, $|\eta| \rightarrow \infty$. Defining $\Gamma(\eta) = i \left[(v_A/c)^2 (A_{||} dA_{||}^*/d\eta - A_{||}^* dA_{||}/d\eta) + d\phi^*/d\eta - \phi^* d\phi/d\eta \right]$, from Eq. (6) we see that $\Gamma(\eta)$ must be a real constant; i.e., the wave flux is conserved. Asymptotically, ϕ and $A_{||}$ should consist of subdominant solutions only, i.e.,

$$\begin{aligned} \phi = & \alpha_{\pm} \eta^{-1/2} \exp(i\Omega\eta^2/2x_S\rho_S) + i\beta_{\pm}(x_C/\eta) \\ & \times \exp(\pm i\Omega k_y \eta), \end{aligned} \quad (7a)$$

and

$$\begin{aligned} A_{||} = & \beta_{\pm} \exp(\pm i\Omega k_y \eta) + i\alpha_{\pm}(c/v_A)^2 x_C \eta^{-3/2} \\ & \times \exp(i\Omega\eta^2/2x_S\rho_S) \quad ; \end{aligned} \quad (7b)$$

for $\eta \rightarrow \pm\infty$. Note that in Eqs. (7a) and (7b) we have kept terms which are of order $1/\eta$ higher than the leading terms in the asymptotic series so as to emphasize that the coupling between ϕ and $A_{||}$ decreases rapidly with $|\eta|$. These higher-order terms do not contribute to the wave flux $\Gamma(\eta)$ in the asymptotic limit, where we find

$$\Gamma(\pm\infty) \approx \pm 2[(v_A/c)^2 \Omega k_y |\beta_{\pm}|^2 + (\Omega/x_S c_S) |\alpha_{\pm}|^2] \quad (8)$$

Equation (8) clearly contradicts the constancy of the wave flux Γ unless $\alpha_{\pm}$ and $\beta_{\pm}$ are all exactly equal to zero. From the asymptotic analysis, this implies that both ϕ and $A_{||}$ vanish exactly, which in turn means that ϕ and $A_{||}$ vanish everywhere along η (using the uniqueness theorem of ordinary differential equation). Thus, we have proved that, within the approximations implied by Eqs. (1) and (2), exact marginally stable eigenmodes cannot exist.

We note that, while Antonsen's method is rather novel, it works only in the case of cold-fluid ions. We now present a more general technique⁹ which has much wider applicability. For simplicity let us consider the electrostatic case but with warm-kinetic ions. Here, Eq. (1), with $A_{||} = 0$, retains the same form except $F(x)$ is replaced by $G(x, \lambda_i)$, where

$$G(x, \lambda_i) = - \frac{(\tau/\lambda_i^2) (1+\lambda_i/\Omega\tau) (1+\xi_i Z_i) + (1-1/\Omega) (1+\xi_e Z_e)}{(1+\lambda_i/\Omega\tau) \xi_i Z_i} \quad (9)$$

Here, $\tau = T_e/T_i$, $\xi_i = \Omega x_i / |x| \lambda_i$, $x_i = \omega_{*e} / k_{||} v_i$, $Z_i = Z(\xi_i)$ and $\lambda_i (0 \leq \lambda_i \leq 1)$ is the kinetic-ion parameter. Thus, $\lambda_i = 0$ and 1 correspond to the cases of cold-fluid ions and warm-kinetic ions, respectively. As λ_i is increased from 0 to 1, the eigenmode frequencies, $\Omega(\lambda_i)$, would then evolve continuously from the $\lambda_i \rightarrow 0^+$ limit. As discussed above, we can use Antonsen's method to prove that, in the $\lambda_i \rightarrow 0^+$ limit, the eigenmodes are damped; i.e., $\text{Im } \Omega(\lambda_i \rightarrow 0^+) < 0$.¹⁰ Furthermore, the preceding analysis on

the absence of marginally stable modes in the cold-ion limit can be readily extended to show that, for the present case and the entire range of $\lambda_i (0 \leq \lambda_i \leq 1)$, there again exists no eigenmode with $\text{Im } \Omega = 0$. Thus, $\Omega(\lambda_i)$ never crosses the real- Ω axis and the eigenmodes remain damped.

We now address the problem of convective instabilities. For simplicity, we will consider only the purely electrostatic case and assume cold ions. The problem has also been discussed briefly by Antonsen.³ Although he did not obtain an explicit solution, he stated that there will be contributions from the $\Omega = 0$ singularity to the long-time behavior of the drift mode. However, since Eq. (1), in which we now set $A_{||} = 0$, is actually not valid in the zero Ω limit, it is desirable to avoid dealing with this fake singularity. We accomplish this by formulating the problem as a scattering one. Hence, we look for solutions with the following boundary conditions:

$$\phi = x^{-1/2} \exp(ix^2/2\Omega\rho_s x_s) + R(\Omega)x^{-1/2} \exp(-ix^2/2\Omega x_s \rho_s) ,$$

$$x \rightarrow \infty$$

and

$$\phi = T(\Omega)x^{-1/2} \exp(-ix^2/2\Omega x_s \rho_s) ; \quad x \rightarrow -\infty \quad (10)$$

where $R(\Omega)$ and $T(\Omega)$ would be interpreted as reflection and transmission coefficients, respectively, and we consider only real values for Ω . It is possible to prove¹¹ that in a sectorial region bounded by two adjacent anti-Stokes lines there is a uniform asymptotic expansion, whose leading terms is a linear combination of the two independent solutions $\phi_{1,2}$ of the equation

$$[\rho_s^2 d^2/dz^2 - k_y^2 \rho_s^2 - 1 + 1/\Omega + (z/\Omega x_s)^2] \phi = 0 \quad (11)$$

Notice that Eq. (11) differs from Eq. (1) only by dropping the terms $\xi_e Z_e \phi$ which goes to zero as $|z| \rightarrow \infty$. Using the relation between parabolic cylinder functions, we obtain

$$\phi_1(a, z) = \phi_1(-a, -iz) \exp[i(\theta + \pi/4)] \quad (12a)$$

$$\phi_2(a, z) = (2 \cosh \pi a)^{-1/2} [\phi_2(-a, iz) \exp(\pi a + i\pi/4 + i\theta) + \exp(i3\pi/4) \phi_1(-a, -iz)] \quad (12b)$$

where $a = (\Omega x_s / 2\rho_s) (1/\Omega - 1 - k_y^2 \rho_s^2)$ and $\theta = \arg[\Gamma(1/2 - ia)]$. ϕ_1 and ϕ_2 are the subdominant and dominant solutions to Eq. (11), respectively. We have assumed $\Omega > 0$, but similar relations can be obtained when $\Omega < 0$. Equations (12a) and (12b) enable us to relate the scattering problem along the real- x axis to that along the real- η axis for $\Omega > 0$ ($z = -i\Omega\eta$). If we define the scattering boundary conditions along η to be

$$\phi = \eta^{-1/2} \exp(-i\Omega\eta^2/2x_s\rho_s) + r(\Omega)\eta^{-1/2} \exp(i\Omega\eta^2/2x_s\rho_s) \quad (13a)$$

$\eta \rightarrow \infty$

and

$$\phi = \tau(\Omega)\eta^{-1/2} \exp(i\Omega\eta^2/2x_s\rho_s) \quad (13b)$$

$\eta \rightarrow -\infty$

then $r(\Omega)$ and $\tau(\Omega)$ are related to $R(\Omega)$ and $T(\Omega)$ by

$$R(\Omega) = [r(\Omega) + i] \exp(\pi a) \quad (14a)$$

and

$$T(\Omega) = \tau(\Omega) \exp(\pi a) \quad (14b)$$

Along the real- η axis the scattering problem is greatly simplified, now that the potential is real. For $k_{y0s}^2 < 1/\Omega - 1$, there exist two pairs of real turning points.⁸ The inner turning points at $\eta = \pm E$ are determined primarily by the electron Z function and the other ones at $\pm P$ are the usual Pearlstein-Berk ion-sound turning points determined quite accurately as $\pm \Omega x_s (1/\Omega - 1 - k_{y0s}^2)^{1/2}$. If we ignore the outer turning points, then, since the wave is propagating inside the inner turning points, the eigenvalue is determined by the quantization conditions

$$\int_{-E}^E Q^{1/2}(\Omega, \eta) d\eta = (n+1/2)\pi \quad ; \quad n = 0, 1, 2, \dots \quad (15)$$

where $Q(\Omega, \eta)$ is the real potential. Due to the presence of outer turning points, the wave cannot remain trapped and, therefore, no real- Ω eigensolution is possible. However, if the tunneling factor $\exp(-\int_E^P Q^{1/2} d\eta)$ is small, then it is possible to consider the eigenvalue Ω as determined by Eq. (15) to be a resonance. For real Ω sufficiently far from resonance, $r(\Omega) + i = 0[\exp(-\int_E^P Q^{1/2} d\eta)]$ and so $R(\Omega) = 0(1)$; in this case there is no significant convective amplification. On the otherhand, for Ω very close to resonance, $|r(\Omega) + i| = 0(1)$ and both the reflection and transmission coefficients are of order $\exp(\pi a)$ which is assumed to be large. Thus, we can identify $\exp(\pi a)$ as the convective amplification factor, which can be estimated using the data of Ref. 8. Taking $m_i/m_e = 1837$, $L_s/L_n = 50$ and $k_{y0s}^2 = 0.5$, the resonance frequency is found to be $\Omega = 0.55$. We thus have $\pi a = 13.74$ and the amplification

factor is exponentially large; of $O(10^6)^{12}$

In conclusion we have shown that there are no unstable drift modes and drift Alfvén modes of either parity and that the convective amplification of the electrostatic drift modes can become exponentially large. The obvious implication of the latter result is that we may expect anomalous transport of a plasma confined in a sheared magnetic field due to the convectively unstable drift modes.

ACKNOWLEDGMENT

The authors wish to thank Mr. Jim Van Dam for his careful reading of the manuscript.

This work was supported by NSF Grant # PHY-77-12873, the United States Department of Energy contract Nos. EY-76-C-02-3073, EY-76-S-03-0034-PA 128 , and W-7405-ENG-48.

REFERENCES

1. D.W. Ross and S.M. Mahajan, Phys. Rev. Lett. 40, 325 (1978).
2. K.T. Tsang, P.J. Catto, J.C. Whitson, and T. Smith, Phys. Rev. Lett. 40, 327 (1978).
3. T.M. Antonsen, Phys. Rev. Lett. 41, 33 (1978).
4. K.T. Tsang, J.C. Whitson, J.D. Callen, and J. Smith, Phys. Rev. Lett. 41, 557 (1978).
5. P.J. Catto, J.A. El Nadi, C.S. Liu, and M.N. Rosenbluth, Nucl. Fusion 14, 405 (1974).
6. L. Chen, J. Hsu, P.K. Kaw, and P.H. Rutherford, Nucl. Fusion 18, 1371 (1978).
7. G. Laval, R. Pellat, and M. Vuillemin, in Plasma Physics and Controlled Nucl. Fusion Research (IAEA Vienna, 1965), Vol. II, p. 259; J.F. Drake and Y.C. Lee, Phys. Fluids 20, 1341 (1977); L. Chen, P.H. Rutherford, and W.M. Tang, Phys. Rev. Lett. 39, 460 (1977); D.W. Ross, H.R. Strauss, S.M. Mahajan, R.D. Hazeltine, F.L. Hinton, B. Coppi, J. Filreis, and J. W-K Mark, paper IAEA-CN-37-W-4, presented at 7th Internat. Conf. on Plasma Physics and Controlled Nucl. Fusion Research, Innsbruck, 1978.
8. L. Chen, P.N. Guzdar, R.B. White, P.K. Kaw, and C. Oberman, Phys. Rev. Lett. 41, 649 (1978).
9. Y.C. Lee, Bull. Am. Phys. Soc. 23, 798 (1978).
10. It is necessary to consider two branches of eigenmodes with frequencies $\Omega_i(\lambda_i)$ and $\Omega_2(\lambda_i)$ such that, as $\lambda_i \rightarrow 0^+$,

$\Omega_1(\lambda_1) \rightarrow 0(1)$ and $\Omega_2(\lambda_1) \rightarrow 0(\lambda_1)$. A detailed presentation of this more general s-matrix technique will be published elsewhere.

11. F.W.J. Olver, Proc. Roy. Soc. London A250, 479 (1958) and private communications.
12. This enormous amplification indicates that the system has eigenmodes with negligibly small damping rates, which cannot be distinguished from marginally stable modes by numerical and WKBJ analyses.

FIGURE CAPTION

Fig. 1: Asymptotic anti-Stokes lines (---) for (a) $\text{Re } \Omega < 0$ and (b) $\text{Re } \Omega > 0$. Also shown are the real η axis (-.-) and the subdominant regions(s).

Fig. 1. 782368

ALL CATEGORIES

R. Askew, Auburn University, Alabama
 S. T. Wu, Univ. of Alabama
 Geophysical Institute, Univ. of Alaska
 G.L. Johnston, Sonoma State Univ., California
 H. H. Kuehl, Univ. of S. California
 Institute for Energy Studies, Stanford University
 H. D. Campbell, University of Florida
 N. L. Oleson, University of South Florida
 W. M. Stacey, Georgia Institute of Technology
 Benjamin Ma, Iowa State University
 Magne Kristiansen, Texas Tech. University
 W. L. Wiese, Nat'l Bureau of Standards, Wash., D.C.
 Australian National University, Canberra
 C.N. Watson-Munro, Univ. of Sydney, Australia
 F. Cap, Inst. for Theo. Physics, Austria
 Ecole Royale Militaire, Bruxelles, Belgium
 D. Paulinbo, Comm. of Eur. Comm., Belgium
 P.H. Sakunaka, Instituto de Fisica, Campinas, Brazil
 C. R. James, University of Alberta, Canada
 T.W. Johnston, INRS-Energie, Varceenes, Quebec
 H. M. Skarsgard, Univ. of Saskatchewan, Canada
 Librarian, Culham Laboratory, Abingdon, England
 A.M. Dupas Library, C.E.N.-G, Grenoble, France
 Central Res. Inst. for Physics, Hungary
 R. Shingal, Meerut College, India
 M. Naraghi, Atomic Energy Org. of Iran
 Biblioteca, Frascati, Italy
 Biblioteca, Milano, Italy
 G. Rostagni, Univ. Di Padova, Padova, Italy
 Preprint Library, Inst. de Fisica, Pisa, Italy
 Library, Plasma Physics Lab., Gokasho, Uji, Japan
 S. Mori, Japan Atomic Energy Res. Inst., Tokai-Mura
 Research Information Center, Nagoya Univ., Japan
 S. Shoda, Tokyo Inst. of Tech., Japan
 Inst. of Space & Aero. Sci., Univ. of Tokyo
 T. Uchida, Univ. of Tokyo, Japan
 H. Yamato, Toshiba R. & D. Center, Japan
 M. Yoshikawa, TAERI, Tokai Res. Est., Japan
 N. Yajima, Kyushu Univ., Japan
 R. England, Univ. Nacional Auto-noma de Mexico
 B. S. Liley, Univ. of Waikato, New Zealand
 S. A. Moss, Saab Univas Norge, Norway
 J.A.C. Cabral, Univ. de Lisboa, Portugal
 O. Petrus, A.L.I. CUZA Univ., Romania
 J. de Villiers, Atomic Energy Bd., South Africa
 A. Maurech, Comisaria De La Energy y Recursos
 Minerales, Spain
 Library, Royal Institute of Technology, Sweden
 Cen. de Res. En Phys.Des Plasmas, Switzerland
 Librarian, Fom-Instituut Voor Plasma-Fysica, The
 Netherlands
 V. E. Golant, A.F. Ioffe Physical-Tech. Inst., USSR
 B.B. Kadomtsev, Kurchatov Inst. of Atomic Energy,
 USSR
 The Kharkov Physical-Tech. Inst., USSR
 M. S. Rabinovich, Academy of Sci, USSR
 Bibliothek, Stuttgart, West Germany
 R.D. Buhler, Univ. of Stuttgart, West Germany
 Max-Planck-Inst. fur Plasma-physik, W. German
 Nucl. Res. Estab., Julich, West Germany
 K. Schindler, Inst. Fur Theo. Physik, W. Germany

EXPERIMENTAL
THEORETICAL

M. H. Brennan, Flinders Univ. Australia
 H. Barnard, Univ. of British Columbia, Canada
 S. Screenivasan, Univ. of Calgary, Canada
 J. Radet, C.E.N.-B.P., Fontenay-aux-Roses, France
 Prof. Schatzman, Observatoire de Nice, France
 S. C. Sharma, Univ. of Cape Coast, Ghana
 R. N. Aiyer, Laser Section, India
 B. Buti, Physical Res. Lab., India
 L. K. Chayda, S. Gujarat Univ., India
 I.M. Las Das, Banaras Hindu Univ., India
 S. Cuperman, Tel Aviv Univ., Israel
 E. Greenspan, Nuc. Res. Center, Israel
 P. Rosenau, Israel Inst. of Tech., Israel
 Int'l. Center for Theo. Physics, Trieste, Italy
 I. Kawakami, Nihon University, Japan
 T. Nakayama, Ritsumeikan Univ., Japan
 S. Nagao, Tohoku Univ., Japan
 J.I. Sakai, Toyama Univ., Japan
 S. Tjotta, Univ. I Bergen, Norway
 M.A. Hellberg, Univ. of Natal, South Africa
 H. Wilhelmson, Chalmers Univ. of Tech., Sweden
 Astro. Inst., Sonnenborgh Obs., The Netherlands
 N.G. Tsintsadze, Academy of Sci USSR, USSR
 T. J. Boyd, Univ. College of North Wales
 K. Hubner, Univ. Heidelberg, W. Germany
 H. J. Kaeppler, Univ. of Stuttgart, West Germany
 K. H. Spatschek, Univ. Essen, West Germany

EXPERIMENTAL
ENGINEERING

B. Grik, Univ. de Quebec, Canada
 P. Lukac, Komenského Univ., Czechoslovakia
 G. Horikoshi, Nat'l Lab for High Energy Physics,
 Tsukuba-Gun, Japan
 V. A. Glukhikh, D.V. Efremov Sci.
 Res. Institut. of Elect. App., USSR

EXPERIMENTAL

F. J. Paoloni, Univ. of Wollongong, Australia
 J. Kistemaker, Fom Inst. for Atomic
 & Molec. Physics, The Netherlands

THEORETICAL

F. Verheest, Inst. Vor Theo. Mech., Belgium
 J. Teichmann, Univ. of Montreal, Canada
 T. Kahan, Univ. Paris VII, France
 R. K. Chhajlani, India
 S. K. Trehan, Panjab Univ., India
 T. Namikawa, Osaka City Univ., Japan
 H. Narumi, Univ. of Hiroshima, Japan
 Korea Atomic Energy Res. Inst., Korea
 E. T. Karlson, Uppsala Univ., Sweden
 L. Stenflo, Univ. of UMEA, Sweden
 J. R. Saraf, New Univ., United Kingdom