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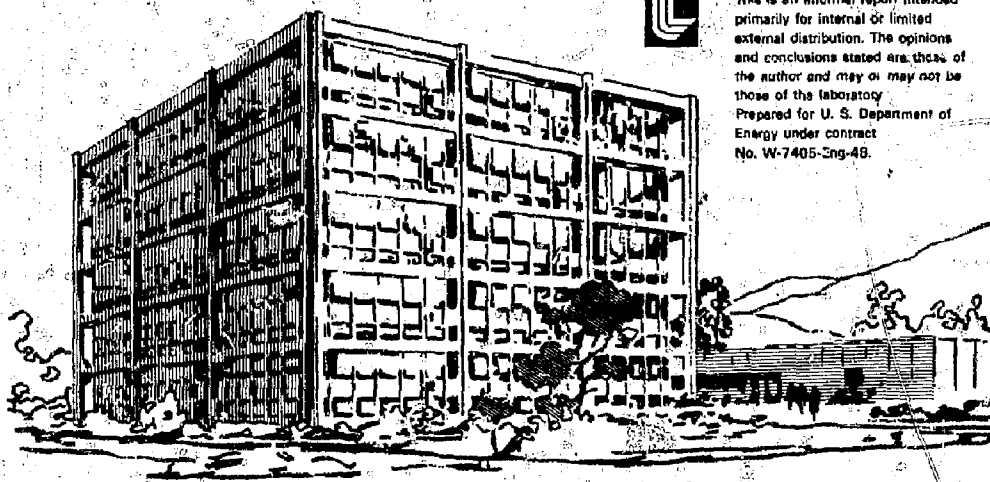
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# Lawrence Livermore Laboratory

COMPUTER CODE FOR THE TRANSFER FUNCTION  
OF SEISMIC SYSTEMS

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## COMPUTER CODE FOR THE TRANSFER FUNCTION OF SEISMIC SYSTEMS

### ABSTRACT

A computer code has been written to compute amplitude and phase transfer functions for various mechanical and electromagnetic seismic systems using analytical functions found in the literature. It can be used to compute transfer functions for the World-wide Standard Seismic Network (WWSSN) system, the Seismic Research Observatory (SRO) system, the Lawrence Livermore Laboratory (LLL) system, the Wiechert system, the Wood-Anderson system, and other user-specified systems.

### INTRODUCTION

It is necessary to know the seismometer transfer function when attempting to determine the actual ground displacement from a seismogram, when converting synthetic ground displacements into synthetic seismograms, and when comparing observations made on instruments with different characteristics.

We define the seismometer transfer function  $T(\omega)$  by  
(Spectrum of actual ground displacement)  $\cdot T(\omega)$  = (Spectrum of trace on seismogram).  
The amplitude  $A$  and phase  $\phi$  of the transfer function are defined by

$$T(\omega) = A(\omega) \exp(i\phi(\omega)) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt ,$$

where  $f(t)$  is the impulse response of the instrument.

An interactive subroutine has been written to compute the transfer function for the World-Wide Standard Seismic Network (WWSSN), Lawrence Livermore Laboratory (LLL), Seismic Research Observatory (SRO), and general mechanical and electromagnetic (E/M) systems. The source code for this subroutine is in:

.640250:C:INST

## DESCRIPTION OF THE SOURCE CODE

The subroutine consists of four parts, corresponding to each of the four major systems. Since the instrument parameters are known for the WWSSN, SRO, and LLL systems, these parameters are set in the subroutine. A nonstandard system can be used by specifying all of the appropriate parameters in the input. Table 1 lists the seismic systems for which the transfer function can be computed by the subroutine and gives the sources<sup>1-7</sup> of the various response functions and seismometer constants used. The transfer function plots referred to in Table 1 are those contained in this report.

### MECHANICAL INSTRUMENTS

The transfer function for either the Wiechert or the Wood-Anderson torsional seismometer can be computed by the subroutine. The transfer functions for these instruments are written from Eqs. (82) and (98) in Ref. 1 as:

$$T(\omega) = \frac{Ms^2}{s^2 + 2h\omega_0 s + \omega_0^2}$$

where

M = magnification

s =  $j\omega$

$\omega_0$  = the natural frequency of the seismometer

h = the damping factor.

The instrument parameters for the two mechanical seismometers are listed in Table 2, and the transfer functions are plotted in Figs. 1 and 2.

### ELECTROMAGNETIC SYSTEMS

The "standard" seismometer/galvanometer systems of the WWSSN-LP (long-period) and WWSSN-SP (short-period) instruments are electromagnetic systems.

Table 1. Seismic systems for which the transfer function can be computed by means of our computer code.

Instrument	Response function	Seismometer constants	Transfer function plot
Wiechert	Ref. 1	Ref. 2	Fig. 1
Wood-Anderson	Ref. 1	Ref. 3	Fig. 2
WSSN (long period)	Ref. 4	Ref. 5	Fig. 3
Any electromagnetic instrument	Ref. 4	User specifies	--
LLL permanent stations	Ref. 6	Ref. 6	Fig. 4
LLL type	Ref. 6	User specifies	--
SRO (broadband)	Ref. 7	Ref. 7	Fig. 5
SRO (short period)	Ref. 7	Ref. 7	Fig. 6
SRO (long period)	Ref. 7	Ref. 7	Fig. 7

Table 2. Instrument parameters for two seismometers.

	Magnification M	Natural frequency $\omega_0$ , rad/s	Damping factor h
Wiechert	188.5	$2\pi/9.65$	0.4037
Wood-Anderson	2800	$2\pi/0.8$	0.8

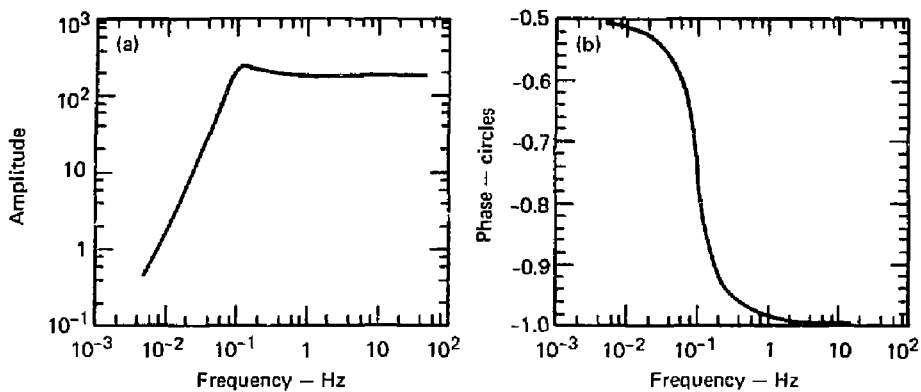


FIG. 1. Amplitude transfer function (a) and phase transfer function (b) for the Wiechert seismometer.

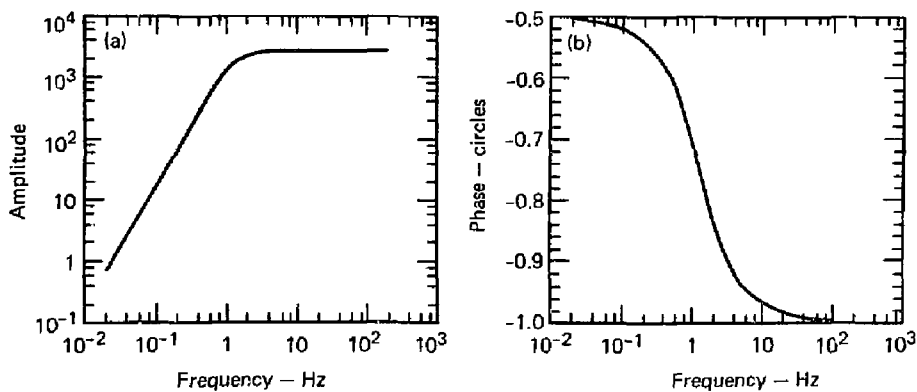


FIG. 2. Amplitude transfer function (a) and phase transfer function (b) for the Wood-Anderson torsional seismometer.

For these systems the transfer function is calculated from Eqs. (28) and (29) of Ref. 4:

$$\begin{aligned} \text{Re}(T) &= 2(h_1 + ph_2) - 2(ph_1 + h_2)pu_1^2, \\ \text{Im}(T) &= -u_1^{-1} + [(1 + p^2) + 4h_1h_2p(1 - \sigma^2)]u_1 - p^2u_1^3, \end{aligned}$$

where

- $p$  = free period of the transducer/free period of the galvanometer,
- $u_1$  = period/free period of the transducer,
- $h_1$  = damping factor for the transducer,
- $h_2$  = damping factor for the galvanometer,
- $\sigma^2$  = coupling factor.

Parameters for the WWSSN-LP systems are based on values published in Ref. 5 and are listed in Table 3. The code allows one to specify either a 15- or a 30-s seismometer at magnifications of 375, 750, 1500, 3000, and 6000. The free period of the galvanometer is assumed to be 100 s. Instruments with other parameters can also be specified. Values of the free periods of the seismometer and galvanometer, damping ratios of the seismometer and galvanometer, and the coupling factor are needed. The value of the peak in the amplitude transfer function is arbitrary, and should be normalized to the specified magnification before proceeding further in computations. The transfer function for a 15-100 WWSSN-LP seismometer at a gain of 1500 is plotted in Fig. 3.

#### LLL SYSTEM

This system consists of the LLL permanent seismic monitoring stations at Elko, Nevada; Kanab, Utah; Mina, Nevada; and Landers, California. Each station has one vertical and two horizontal components. The instrument parameters are listed in Table 4 of Ref. 6. A different natural frequency and damping factor may also be specified if desired. The transfer function is obtained from Eq. (A-8) of Ref. 6. In addition, there is a transfer function due to the data transmission system. The transfer function for the complete system can be represented by a rational function with zeros at

$$s = 0, 0$$

Table 3. Instrument parameters for WSSN-LP systems.

Magnification M	Coupling factor $\sigma^2$
$T_1 = 15 \text{ s}, T_2 = 100 \text{ s}, h_1 = 0.93, h_2 = 1.0$	
374	0.003
750	0.013
1500	0.347
3000	0.204
6000	0.805
$T_1 = 30 \text{ s}, T_2 = 100 \text{ s}, h_1 = 1.5, h_2 = 1.0$	
375	0.003
750	0.012
1500	0.044
3000	0.195
6000	0.767

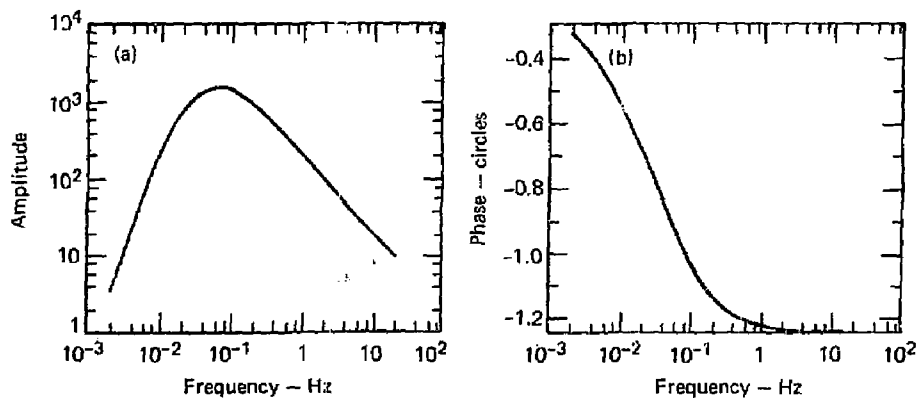


FIG. 3. Amplitude transfer function (a) and phase transfer function (b) for the WSSN-LP seismometer (15-100 at a magnification of 1500).



and poles at

$$s = \pm \omega_0 \pm \omega_0 \sqrt{h^2 - 1}, -114.28 \pm 23.317j, -100.48 \pm 70.650j, -67.677 \pm 120.85j.$$

The phase of the transfer function is such that a positive velocity corresponds to an upward ground displacement. The transfer function for the vertical component at Mina is plotted in Fig. 4.

#### SRO SYSTEM

The analytic form of the transfer functions for this system is given in Refs. 7 and 8. For the broadband (BB) system, the transfer function is represented by a rational function with zeros at

$$s = -0.125, -50, 0, 0;$$

poles at

$$s = -0.13, -6.02, -8.66, -35.2;$$

and a constant multiplier of -394. This transfer function is plotted in Fig. 5. For the short-period (SP) system, the transfer function is represented by a rational function with zeros at

$$s = -50, 0, 0, 0, 0;$$

poles at

$$s = -0.13, -6.02, -8.66, -35.2, -100, -17.97, -17.97, -63.29, -63.29;$$

and a constant multiplier of  $3.94 \times 10^7 \times 322 \times 4006$ . This transfer function is plotted in Fig. 6. For the long-period (LP) system, the transfer function is represented by a rational function with zeros at

$$s = -50, 0, 0, 0, \pm 1.05j, 0, 0;$$

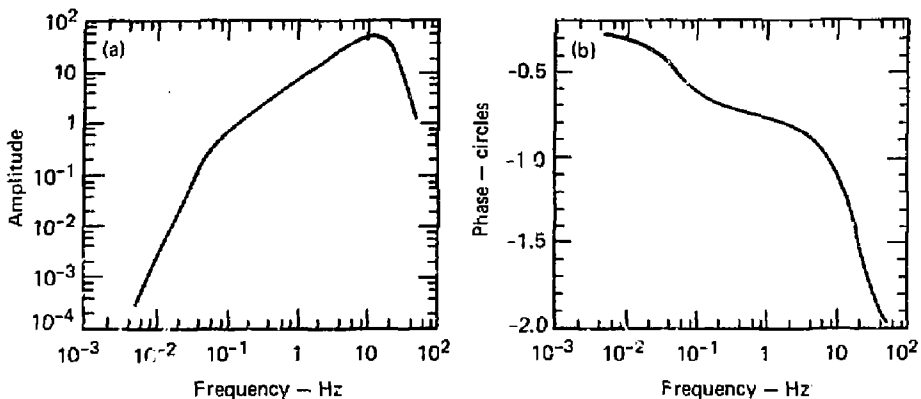


FIG. 4. Amplitude transfer function (a) and phase transfer function (b) for the LLL station at Mina, vertical component.

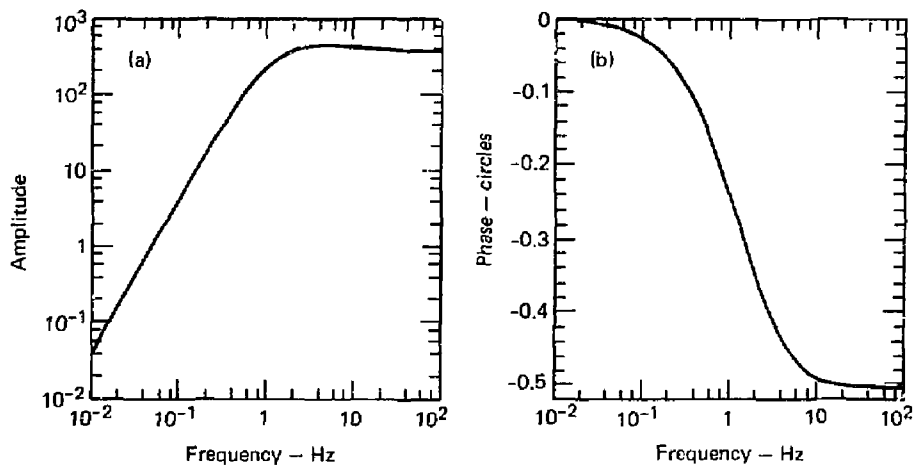


FIG. 5. Amplitude transfer function (a) and phase transfer function (b) for the SRO broadband system.

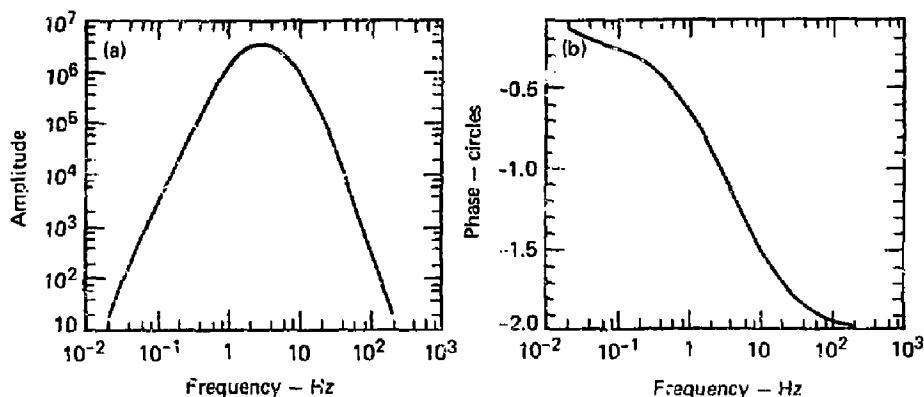


FIG. 6. Amplitude transfer function (a) and phase transfer function (b) for the SRO short-period system.

poles at

$$s = -0.13, -6.02, -8.66, -35.2, -100, -3.93, -0.282$$

$$s = -0.201 \pm 0.241j, -0.134 \pm 0.1j, -0.0251, -0.00942$$

$$s = -0.24 + 0.58j, -0.58 + 0.24j, -0.58 - 0.24j, -0.24 - 0.58j;$$

and a constant multiplier of  $3.94 \times 10^7 \times 0.098 \times 0.0270 \times (0.2\pi)^4$ .

This transfer function is plotted in Fig. 7. An anti-alias filter is included for the long-period but not for the short-period output.

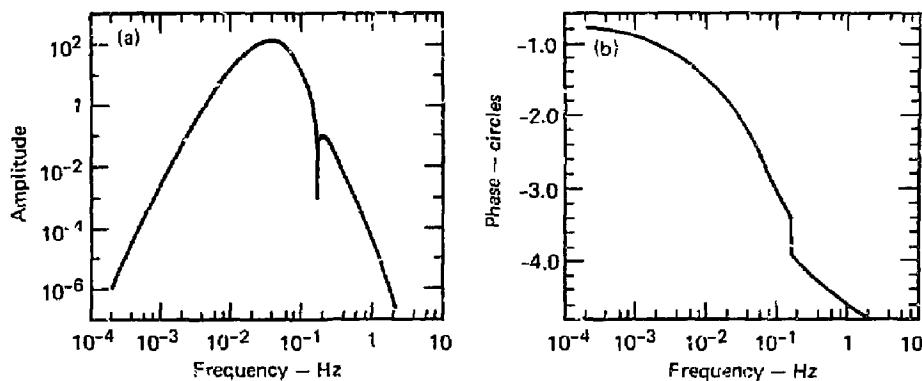


FIG. 7. Amplitude transfer function (a) and phase transfer function (b) for the SRO long-period system.

# EXAMPLES OF THE USE OF THE ROUTINE

As an example of the use of the routine, group delays were computed for each seismic system. The phase spectra for the instruments were obtained from the subroutine. The group delay was then obtained by differentiating the phase spectra with respect to frequency (i.e.,  $t = d\phi/d\omega$ ). These delays ( $t$ ) are illustrated in Figs. 8 through 14. As can be seen in these figures, the group delay can be a significant portion of the total travel time for long-period waves recorded at short source-to-receiver distances.

Another use of the instrument transfer function is to obtain ground displacement from a seismogram. The top trace in Fig. 15 is a seismogram recorded at Mina, Nevada at the time of a nuclear test. The middle trace has been deconvolved to remove the instrument by dividing the fast Fourier transform (FFT) of the top trace by the transfer function, and then taking the inverse FFT of the result. This result may be compared to the bottom trace, which was obtained by integrating the top trace. Since the system response is

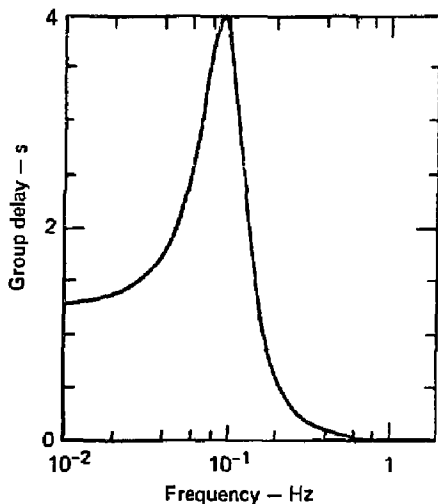


FIG. 8. Group delay for the Wiechert system.

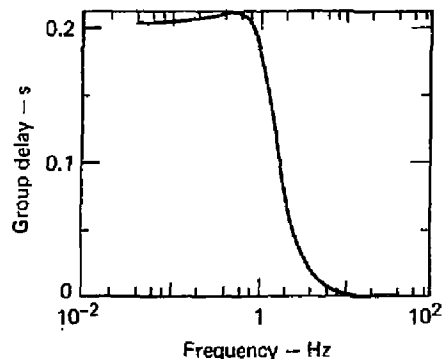


FIG. 9. Group delay for the Wood-Anderson system.

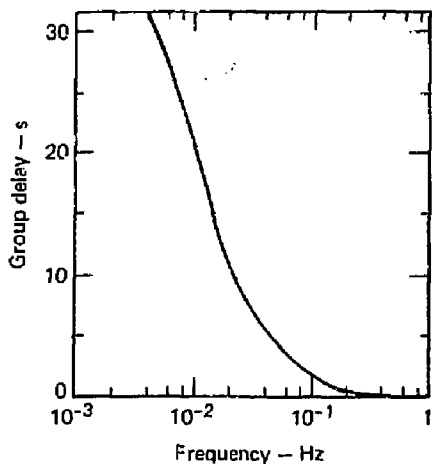


FIG. 10. Group delay for the WSSN-LP system.

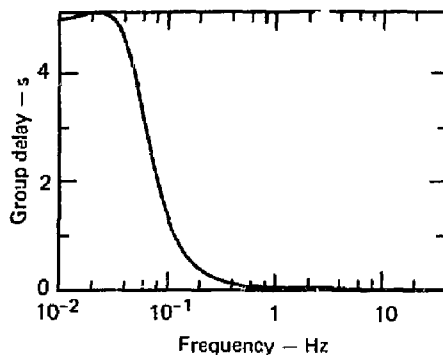


FIG. 11. Group delay for the LLL Mina vertical system.

known to be flat in velocity, it is not surprising that there appears to be little difference between the deconvolved and the integrated signals except in the surface waves. Here, the period of the predominant signal is fairly long compared to the body waves, and a phase shift of a fraction of a circle may result in a group delay of several seconds.

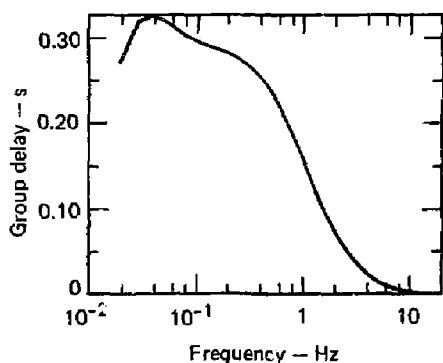


FIG. 12. Group delay for the SRO-BB system.

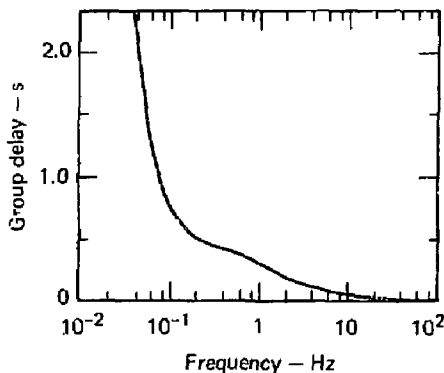


FIG. 13. Group delay for the SRO-SP system.

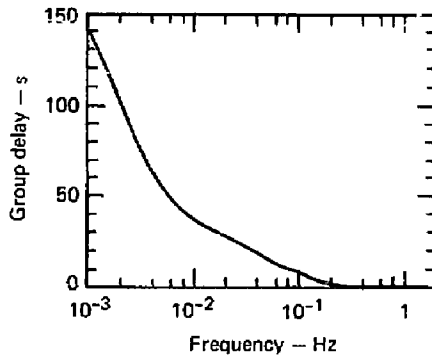


FIG. 14. Group delay for the SRO-LP system.

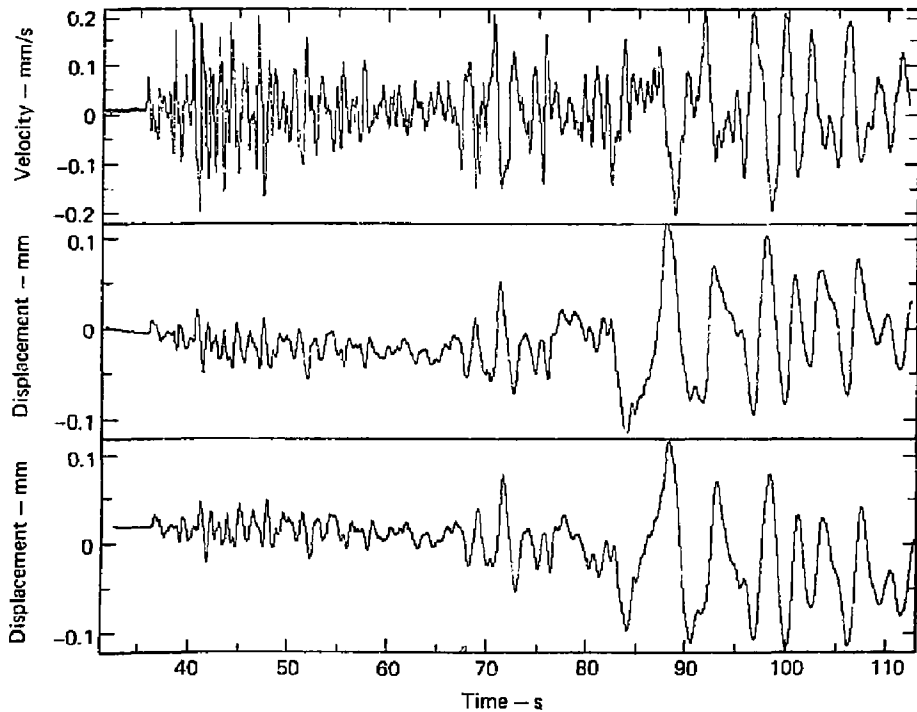


FIG. 15. Top trace: seismogram of the Carpetbag event, recorded at Mina. Middle trace: ground displacement calculated by deconvolving the seismogram by the instrument transfer function. Bottom trace: integrated seismogram.

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