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Neutron Electric Dipole Moment Revisited**

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Strong CP Violation and the Neutron Electric Dipole Moment Revisited

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Abstract

The relation between the CP violating operator $\theta F\tilde{F}$ and its effective version L_{CP}^{eff} is reconsidered on the basis of the anomalous Ward-Takahashi (WT) identity. The consistency of the previous phenomenological calculations of the neutron electric dipole moment (NEDM) with the WT identity is critically examined. A consistent evaluation of an $O(N_c^0)$ contribution to NEDM is given and the result is compared with the leading term in the chiral expansion which is $O(N_c^{-1})$.

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I. Introduction

The absence of the $\theta F\tilde{F}$ term is one of the long-standing problems in QCD, which have stimulated the search for the rationale such as the Peccei-Quinn symmetry¹ and its variations (see the review, ref.2). Experimentally, the most stringent bound on θ comes from the measurement of the neutron electric dipole moment (NEDM)³ which should vanish if there is no CP violation. (As for the recent review of the general electric dipole moments, see ref.4.) On the theoretical side, the precise calculation of NEDM is rather difficult since it requires the non-perturbative evaluation of the nucleon matrix element of $F\tilde{F}$. Instead of taking such a matrix element, it has been proposed to replace $F\tilde{F}$ by an effective CP violating operator L_{CP}^{eff} written only by the quark fields,⁵ which makes one possible to apply the current algebra techniques and the effective Lagrangians of QCD at low energies. So far, a number of papers has been published based on this idea. (See the reviews, ref..^{2,6}) One of the problems of these approaches is that it is not clear whether the results and the approximations adopted are consistent with the anomalous Ward-Takahashi (WT) identity. Actually, some of the previous calculations break the constraint from the WT identity as we will show.

The purpose of this paper is to clarify the above issue in view of a general principle and also to point out the existence of a consistent tree-level contribution to NEDM.⁷ First, on the basis of the WT identity, we generalize and develop the argument by Aoki, Gocksch, Manohar and Sharpe⁸ where the danger in the naive use of L_{CP}^{eff} instead of $F\tilde{F}$ is pointed out. The analysis is useful to clarify the problematic points in the previous calculations of NEDM. Although some parts of section 2 overlap the argument of ref.8, we put them in order to make our paper clear and self-contained. Then, in section 3, we re-analyse NEDM based on an effective Lagrangian which is consistent with the WT identity. A contribution which is leading in the large N_c expansion (but not leading in the chiral perturbation) is estimated and compared to the term in the opposite limit (i.e. the term which is leading near the chiral limit but subleading in large N_c). Section 4 is devoted to the summary and concluding remarks.

II. Anomalous WT Identity

We first assume the following anomalous WT identity for QCD without a θ term:

$$\langle (\partial^\mu J_\mu^5(x) - 2m\bar{\psi}i\gamma_5\psi(x) - N_f \frac{g^2}{16\pi^2} F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a(x)) \mathcal{O} + \delta_x \mathcal{O} \rangle = 0, \quad (2.1)$$

where J_μ^5 is the (flavor singlet) axial current, m is the quark mass,⁹ g is the QCD coupling constant, N_f is the number of flavors, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ with $\epsilon_{0123} = 1$, and δ_x is the (infinitesimal) variation under the local chiral rotation at site x , $\mathcal{O}$ is an arbitrary operator and $\langle \cdot \rangle$ denotes the average over all the configuration of quarks and gluons.¹⁰ If one recalls the derivation of the axial anomaly,¹¹ it is easy to see that the above WT identity indeed implies the following two independent equations when $\mathcal{O}$ contains the quark operators:

$$\langle (\partial^\mu J_\mu^5(x) - 2m\bar{\psi}i\gamma_5\psi(x)) \mathcal{O} \rangle_{dis} = N_f \frac{g^2}{16\pi^2} \langle F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a(x) \mathcal{O} \rangle, \quad (2.2)$$

$$\langle (\partial^\mu J_\mu^5(x) - 2m\bar{\psi}i\gamma_5\psi(x)) \mathcal{O} \rangle_{con} = -\langle \delta_x \mathcal{O} \rangle, \quad (2.3)$$

where we have decomposed eq.(2.1) into the disconnected and connected pieces following Aoki, Gocksch, Manohar and Sharpe⁸; $\langle A \mathcal{O} \rangle_{dis}$ denotes the insertion of A into the quark loop and $\langle A \mathcal{O} \rangle_{con}$ denotes the insertion of A into the operator $\mathcal{O}$. The famous triangle diagram which produces the axial anomaly is a former example. In other words, Eq. (2.2) means that, if we connect the fermion and anti-fermion operators in A to create the fermion loop, the sum of $\partial^\mu J_\mu^5$ and $2m\bar{\psi}i\gamma_5\psi$ produces the anomaly at the lowest order (g^2), and the radiative corrections to these diagrams are always cancelled between $\partial^\mu J_\mu^5$ and $2m\bar{\psi}i\gamma_5\psi$ (Adler-Bardeen Theorem¹²). Eq. (2.3) says that the sum of the insertions of $\partial^\mu J_\mu^5$ and $2m\bar{\psi}i\gamma_5\psi$ into the fermion line, which starts from or ends at the operator $\mathcal{O}$, is always equal to the local chiral variation of the operator $\mathcal{O}$.

Now we consider the space-time integration of the anomalous WT identity eqs. (2.2-3). Since there are no massless particles which couple to the axial current J_μ^5 as long as $m \neq 0$, we can safely use the integration by parts:

$$\int d^4x \langle \partial^\mu J_\mu^5(x) \mathcal{O} \rangle_{dis} = \int d^4x \langle \partial^\mu J_\mu^5(x) \mathcal{O} \rangle_{con} = 0. \quad (2.4)$$

Thus we obtain

$$\int d^4x \, 2m \langle \bar{\psi} i \gamma_5 \psi(x) \mathcal{O} \rangle_{dis} = -2N_f \langle Q \mathcal{O} \rangle, \quad (2.5)$$

$$\int d^4x \, 2m \langle \bar{\psi} i \gamma_5 \psi(x) \mathcal{O} \rangle_{con} = \int d^4x \langle \delta_x \mathcal{O} \rangle, \quad (2.6)$$

where the topological charge Q is defined as

$$Q = \frac{g^2}{32\pi^2} \int d^4x F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a(x) \equiv \int d^4x q(x). \quad (2.7)$$

Equation (2.5) implies that the *disconnected* insertion of $\int d^4x \, m \bar{\psi} i \gamma_5 \psi$ is identical to the insertion of $-N_f Q$.

One can use the above relations to evaluate the effect of small θ on the physical quantities:

$$\begin{aligned} \langle \mathcal{O} \rangle_\theta &\equiv \langle \mathcal{O} \exp(-i\theta Q) \rangle \\ &\simeq \langle \mathcal{O} \rangle - i\theta \langle Q \mathcal{O} \rangle \\ &= \langle \mathcal{O} \rangle + i\theta \frac{m}{N_f} \int d^4x \langle \bar{\psi} i \gamma_5 \psi(x) \mathcal{O} \rangle_{dis} \\ &\simeq \langle \mathcal{O} \exp[i\theta \frac{m}{N_f} \int \bar{\psi} i \gamma_5 \psi] \rangle_{dis} \\ &\simeq \langle \mathcal{O} \exp[im \int \bar{\psi} (e^{i\theta \gamma_5 / N_f} - 1) \psi] \rangle_{dis}. \end{aligned} \quad (2.8)$$

Eq.(2.8) represents the precise meaning of the usual statement that the effect of $-i\theta Q$ can be replaced by the complex fermion mass term. (Note that one can prove this equivalence even if θ is not infinitesimal.) However one should remark that the *connected* insertion of $\int d^4x \, 2m \bar{\psi} i \gamma_5 \psi(x)$ has nothing to do with the θ term as eq.(2.6) tells us; this produces only the chiral variation of the external fermions in the operator $\mathcal{O}$. This chiral rotation cannot produce any physical effect as the equivalence theorem¹³ tells us (we show it explicitly in the last part of this section), which means that *the connected insertions are essentially zero for the physical processes* once the on-shell condition are imposed on the external lines. Note that our argument here using the anomalous WT identity is applicable only to the first order effect of θ . The advantage to assume small θ (which is actually true in the real

world) is that one can treat the θ -term by the first order perturbation theory and does not have to worry about the change of the hadron propagators in the θ -vacuum. For general θ , one has to calculate the two point functions e.g. the θ -dependence of the nucleon propagator to extract the neutron electric dipole moment.⁸

It will be useful here for the phenomenological evaluation of NEDM to comment on the possible source of the violation of the above constraint. If the *connected* insertion of $\int d^4x \, 2m\bar{\psi}i\gamma_5\psi(x)$ is considered and the finite results are obtained for NEDM, one can imagine several possible origins of such a fake result: (i) On the lattice, the violation of the WT identity by the finite lattice spacing may cause the fake result.^{14,8} (ii) If the chiral non-invariant intermediate states were used to calculate NEDM, it would break the equality (2.6) and a non-zero result would be obtained. In fact, there always exist contributions from the other intermediate states which ensure the chiral invariance and tend to cancel the fake contribution. (Note that the identity operator $1 = \sum_n |n\rangle\langle n|$ is chiral invariant.) (iii) If the correct on-shell condition for the nucleon is not imposed (which often occurs when one uses the extended models of the nucleon which does not have an exact covariance), a fake contribution remains since r.h.s. of eq.(2.6) does not vanish.

Here let us briefly see what happens when one takes the chiral limit ($m \rightarrow 0$). One is not allowed to take the limit separately in eq.(2.5) and (2.6) since each matrix element can have a massless pole, while it does not appear in the sum:¹⁵

$$\int d^4x \, 2m\langle\bar{\psi}i\gamma_5\psi(x)\mathcal{O}\rangle = -2N_f\langle Q\mathcal{O}\rangle + \int d^4x \, \langle\delta_x\mathcal{O}\rangle. \quad (2.9)$$

By taking the chiral limit in this formula, one gets

$$2N_f\langle Q\mathcal{O}\rangle = \int d^4x \, \langle\delta_x\mathcal{O}\rangle, \quad (2.10)$$

which is a well-known relation telling us that the effect of the θ term is equivalent to the chiral rotation with the angle $-\theta/2N_f$ in the chiral limit. However, one should remember that it does not imply that there is no effect of θ in the matrix elements. In fact, off-shell matrix elements may be non-zero

for the chiral non-invariant operators, e.g.,

$$\langle \bar{\psi} i \gamma_5 \psi(y) \rangle_\theta \propto \int d^4x \langle \delta_x (\bar{\psi} i \gamma_5 \psi(y)) \rangle = -2 \langle \bar{\psi} \psi(y) \rangle \neq 0. \quad (2.11)$$

Of course, as we mentioned and will show soon, the *on-shell* quantities for $\int d^4x \delta_x \mathcal{O}$ are always zero, which implies that one cannot observe the physical effect of θ in the chiral limit. This gives a constraint on the structures of the low energy effective theory of QCD; the terms induced by the $U_A(1)$ anomaly of QCD should satisfy e.g. (2.10) which is violated in some previous calculation of NEDM (e.g., ref.16). We will return to this point in section 3.

Now we give a brief proof of the vanishing of $\delta \mathcal{O}$ for on-shell physical quantities as we promised.¹⁷ (We have defined $\delta = \int d^4x \delta_x$ as the $U(1)$ global chiral rotation.) Let us consider the following general operator:

$$\mathcal{O} = \prod_{i=1}^l B_i \prod_{j=1}^m F_j \prod_{k=1}^n \bar{F}_k, \quad (2.12)$$

where B_i is a bosonic composite operator made of quarks, which produces a physical meson with mass M_i , and F_j ($\bar{F}_j$) is a fermionic composite operator made of quarks, which produces a physical fermion (anti-fermion) with mass m_j .¹⁸ For example, $B_i = \bar{\psi} \gamma_5 \psi$ or $\bar{\psi} \psi$, $F_j = \psi \psi \psi$ and $\bar{F}_j = \bar{\psi} \bar{\psi} \bar{\psi}$.¹⁹

The on-shell amplitude $S(\mathcal{O})$ is proportional to the pole residue $T(\mathcal{O})$ which is obtained from $\langle \mathcal{O} \rangle$ by cutting the external legs and multiplying the external wave functions with proper on-shell conditions according to the reduction formula.

Now we will prove that $S(\delta \mathcal{O})=0$ holds provided that the external momenta satisfy the *same* on-shell conditions with those of $S(\mathcal{O})$. From eq. (2.12),

$$\delta \mathcal{O} = \sum_i \delta B_i \mathcal{O}_{B_i} + \sum_j \delta F_j \mathcal{O}_{F_j} + \sum_k \delta \bar{F}_k \mathcal{O}_{\bar{F}_k}, \quad (2.13)$$

where $\mathcal{O}_A$ is defined as $\mathcal{O} = A \mathcal{O}_A$ with $A = B_i, F_j$ or $\bar{F}_k$. The $U_A(1)$ chiral rotation among the fields B_i, F_j and $\bar{F}_k$ can be written using the c-number matrices C, D and D' ; $\delta B_i = \sum_{j \neq i} C_{ij} B_j$, $\delta F_i = \gamma_5 D_{ii} F_i + \sum_{j \neq i} D_{ij} F_j$, $\delta \bar{F}_i = D'_{ii} \bar{F}_i \gamma_5 + \sum_{j \neq i} D'_{ij} \bar{F}_j$. The non-zero off-diagonal matrix elements

of C , D and D' mean the presence of the chiral multiplet. The explicit construction of these operators for the nucleon can be seen in ref.20.

To prove $S(\mathcal{O})=0$, it is enough to prove $S(\delta A \mathcal{O}_A) = 0$ for an arbitrary $A = B_i, F_j$ or $\bar{F}_k$. If the chiral symmetry is not broken, $S(\delta A \mathcal{O}_A)$ vanishes trivially since $\langle \mathcal{O}(A) \rangle = \langle \mathcal{O}(A + \delta A) \rangle$. If the chiral symmetry is (explicitly or spontaneously) broken, the masses of the particles in the chiral multiplet become different, i.e. $M_i \neq M_j$ and $m_i \neq m_j$ for $i \neq j$.²¹ On the other hand, the infinitesimal chiral rotation turns the original operator into the one creating a particle with different mass or into the same particle with extra γ_5 . Therefore it is easy to see that the on-shell condition for the original particle kills the matrix element $S(\delta \mathcal{O})$: e.g.

$$S(\delta B_i \mathcal{O}_{B_i}) \sim \sum_{i \neq j} C_{ij} \frac{p_i^2 - M_i^2}{p_i^2 - M_j^2} T(B_j \mathcal{O}_{B_i}) = 0, \quad (2.14)$$

for the boson operators. (The same thing holds for fermions if one notes $\{\gamma_5, \gamma_\mu\}=0$.)

Thus, $S(\delta A \mathcal{O}_A) = 0$ for all A irrespective of the realization of the chiral symmetry, which completes the proof. As we mentioned before, the assumption of small θ makes us possible to use the on-shell conditions at $\theta = 0$ in the proof.

III. Electric Dipole Moment of Neutron

The measurement of the neutron electric dipole moment (NEDM) gives the most stringent bound on the value of the θ -parameter at present. To extract a bound on θ from experiment, we need to calculate the following matrix element in a reliable way:

$$i \int d^4x \langle n(p') | T[J_\mu(0) L_{CP}(x)] | n(p) \rangle = -D_n \bar{u}(p') \sigma_{\mu\nu} \gamma_5 k^\nu u(p), \quad (3.1)$$

where $L_{CP}(x) = -\theta q(x)$ with $q(x)$ being the topological charge density. (We have already used the first-order perturbation for θ .) Various authors have calculated the l.h.s. of eq.(3.1) using the low energy effective theories of

QCD (see the review, ref.2) or the numerical simulation on the lattice.¹⁴ As we discussed in section 2, however, one should keep in mind that only the disconnected part has to be properly taken into account in the calculation when one uses the effective operator L_{CP}^{eff} in place of $-\theta q$.

In the following, we will give a direct estimate of the l.h.s. of eq.(3.1), i.e. the matrix element of $-\theta Q$ using an effective meson-baryon Lagrangian. We will first calculate D_n in the 2-flavor case to show the essential points. Then we examine the 3-flavor case and show that there exists a tree-level $O(N_c^0)$ contribution to NEDM which has not been addressed so far. Essential constraints from the WT identity discussed in sec.2 are (i) the vanishing of the matrix element in the chiral limit and/or in the absence of the axial anomaly²² and (ii) the independence of the result under the change of variables.

The meson part of the low energy effective action inspired by the large N_c QCD reads²³

$$\begin{aligned} \mathcal{L}_M(U, \theta) = & \frac{f_\pi^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U^\dagger) + \frac{f_\pi^2}{4} \text{Tr}(MU + M^\dagger U^\dagger) \\ & + \frac{i}{2} q(x) \text{Tr}(\log U - \log U^\dagger) + \frac{1}{af_\pi^2} q^2(x) - \theta q(x), \end{aligned} \quad (3.2)$$

where $f_\pi=93\text{MeV}$, M is the explicit chiral symmetry breaking ($M=\text{diag}(m_\pi^2, m_\pi^2)$ for flavor $SU(2)$ and $M=\text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$ for $SU(3)$), a is a constant which controls the strength of anomaly and U is a chiral field defined by

$$U = \exp(i\phi^a \lambda^a / f_\pi), \quad (3.3)$$

with λ^a ($a=0 \sim N_f$) being the flavor matrices normalized as $\text{Tr}(\lambda^a \lambda^b) = 2\delta_{ab}$. $\mathcal{L}_M$ has $U_L(N_f) \otimes U_R(N_f)$ invariance except for the explicit mass terms in the large N_c limit. The breaking of the $U_A(1)$ symmetry due to the anomaly, which occurs in the next leading order of the large N_c expansion, is summarized in the 3rd term in eq.(3.2) representing the mixing of $F\tilde{F}$ with ϕ^0 . In the $N_f = 2$ case ($\phi^{1,2,3}=\pi^{1,2,3}$, $\phi^0=\eta$), the equation of motion at the tree level²⁴ yields the PCU_1C relation²⁵

$$q(x) = af_\pi \eta(x), \quad (3.4)$$

and the parameter a is written as

$$a = \frac{1}{2}(m_\eta^2 - m_\pi^2), \quad (3.5)$$

which is $O(1/N_c)$ and vanishes in the absence of anomaly.

Before considering the baryon sector, we will examine an implication of the anomalous WT identity in this effective action. By changing the variables in the flavor singlet meson such that $\eta \rightarrow \eta - f_\pi \theta / 2$, we can eliminate $-\theta q(x)$ in (3.2) and get

$$\begin{aligned} \mathcal{L}_M(U e^{-i\theta\lambda^0/2}, 0) = & \frac{f_\pi^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U^\dagger) + \frac{f_\pi^2}{4} \text{Tr}(M U e^{-i\theta\lambda^0/2} + M^\dagger e^{i\theta\lambda^0/2} U^\dagger) \\ & + \frac{i}{2} q(x) \text{Tr}(\log U - \log U^\dagger) + \frac{1}{af_\pi^2} q^2(x), \end{aligned} \quad (3.6)$$

where θ appears only in the explicit symmetry breaking term corresponding to the operator $m\bar{\psi}e^{-i\theta\gamma_5/2}\psi$ in the quark level. Both effective actions $\mathcal{L}_M(U e^{-i\theta\lambda^0/2}, 0)$ and $\mathcal{L}_M(U, \theta)$ should give the same result since the change of variables does not affect the physical quantities.¹³ Therefore we can conclude immediately that the CP violation in the physical quantities should vanish in the absence of anomaly ($a = 0$) and/or in the chiral limit ($m_\pi^2 = 0$).

Let us demonstrate this explicitly in the case of the $\eta \rightarrow 2\pi$ decay. From (3.2) and (3.4), the matrix element for this process is given by the zero momentum insertion of $-\theta q(x) = -\theta f_\pi a \eta(x)$,

$$-i\theta f_\pi a \int d^4x \langle \eta | \eta(x) | \pi^+ \pi^- \rangle, \quad (3.7)$$

at the tree level, while from (3.6), we obtain

$$i\theta \frac{f_\pi}{2} m_\pi^2 \times \int d^4x [\langle \eta | \eta(x) | \pi^+ \pi^- \rangle - \frac{1}{f_\pi^2} \langle \eta | \eta \pi^+ \pi^-(x) | \pi^+ \pi^- \rangle]. \quad (3.8)$$

Since at the tree level

$$\int d^4x \langle \eta | \eta | \pi^+ \pi^- \rangle = \frac{m_\pi^2}{f_\pi^2 m_\eta^2} \quad \text{and} \quad \int d^4x \langle \eta | \eta \pi^+ \pi^- | \pi^+ \pi^- \rangle = 1,$$

both (3.7) and (3.8) give the same result:

$$i\theta \frac{a}{f_\pi} \frac{m_\pi^2}{m_\eta^2}, \quad (3.9)$$

which has the correct property mentioned above. If one takes the approximation $m_\pi/m_\eta \ll 1$, which may be regarded as the large anomaly limit ($a \rightarrow \infty$) or the chiral perturbation for m_π (but not for m_η), (3.9) reduces to $i\theta m_\pi^2/2f_\pi$ being consistent with the result in ref.26. Therefore, the formula (3.9) has a correct behavior both for $a \rightarrow 0$ and $a \rightarrow \infty$.

If there exists a small isospin breaking term,

$$\frac{f_\pi^2}{4} \delta m^2 \text{Tr}[\lambda^3(U + U^\dagger)],$$

it causes the mixing between η and the neutral pion (π^0). Then the matrix element is modified as

$$-i\theta f_\pi a \times \int d^4x [\cos \phi \langle \eta | \eta | \pi^+ \pi^- \rangle - \sin \phi \langle \eta | \pi^0 | \pi^+ \pi^- \rangle], \quad (3.10)$$

where

$$\int d^4x \langle \eta | \eta | \pi^+ \pi^- \rangle = \frac{(m_\eta^2 - m_\pi^2)2 \sin^2 \phi + m_\pi^2(1 + 2 \cos^2 \phi) + \delta m^2 \sin 2\phi}{3f_\pi^2 m_\eta^2},$$

and

$$\int d^4x \langle \eta | \pi^0 | \pi^+ \pi^- \rangle = \frac{(m_\eta^2 - m_\pi^2) \sin 2\phi - m_\pi^2 \sin 2\phi + \delta m^2 \cos 2\phi}{3f_\pi^2 m_{\pi^0}^2}.$$

Here m_π is the charged pion mass and the mixing angle ϕ satisfies $\tan 2\phi = \delta m^2/a$ with $m_\eta^2 = m_\pi^2 + a + \sqrt{a^2 + (\delta m^2)^2}$ and $m_{\pi^0}^2 = m_\pi^2 + a - \sqrt{a^2 + (\delta m^2)^2}$. (Therefore the quantity a is expressed in terms of the meson masses: $a = (m_\eta^2 + m_{\pi^0}^2 - 2m_\pi^2)/2$.) It is also easy to check that both (3.2) and (3.6) give the same result.

The baryon sector in $N_f=2$ is written in terms of the baryon field transforming as a fundamental representation of $SU(2)$,

$$\mathcal{L}_B(B, U) = \bar{B}(i\gamma \cdot \partial - \alpha U_5^\dagger)B - \gamma \bar{B} M B + \bar{B} \mu \sigma_{\mu\nu} U_5^\dagger B (\partial^\nu A^\mu). \quad (3.11)$$

Here the first term is manifestly chiral invariant since $U_5 = \exp(i\gamma_5 \phi^a \lambda^a / f_\pi)$ with $\lambda^a = (1, \vec{\lambda})$; the second term breaks $U(2)$ in the same way with the meson sector and the last term denotes the phenomenological (anomalous) magnetic moment of the baryon with μ being a 2×2 matrix and A_μ being the photon field. A few comments are in order. First, we have constructed this effective action to make $\mathcal{L}_B$ $U_A(1)$ invariant (see $U_5^\dagger$ in the last term. It can be just $U_5^\dagger = \exp(-i\gamma_5 \phi^0 \lambda^0 / f_\pi)$, but the difference does not matter in the tree-level calculation of NEDM.). This is because the electromagnetism does not break $U_A(1)$ although it breaks $SU(2)$. (In principle, one can add terms which break $U_A(1)$ but vanish in the chiral limit. Here we have neglected these terms to make the argument simple. For the realistic $SU_f(3)$ case, see the later discussion.) Secondly, we did not introduce the usual vector coupling $A_\mu [\bar{B} \gamma_\mu (\lambda_3 + 1) B] / 2$ since it does not contribute to NEDM at the tree level because of its vector nature.

Let us evaluate the matrix element (3.1) in two different ways which will clarify how the general discussion in section 2 is realized and also what is wrong in some of the previous calculations (in particular the ref.16). Here we use (3.2) as an effective action for mesons. If we expand $\mathcal{L}_B$ up to the first order of the meson fields, we get

$$\begin{aligned} \mathcal{L}_B^I = & \bar{B}(i\gamma \cdot \partial - m_N)B + \bar{B}\mu\sigma_{\mu\nu}B(\partial^\nu A^\mu) \\ & + g_s \bar{B}i\gamma_5 \lambda^a B \phi^a - \bar{B}\mu\sigma_{\mu\nu}i\gamma_5 \lambda^a B(\phi^a / f_\pi)(\partial^\nu A^\mu), \end{aligned} \quad (3.12)$$

where $g_s = \alpha / f_\pi$ and $m_N = \alpha + \gamma m_\pi^2$ are the pion-nucleon coupling constant and the physical nucleon mass respectively. Another form of $\mathcal{L}_B$ is obtained by defining the chiral invariant nucleon $N = \sqrt{U_5^\dagger} B$;

$$\begin{aligned} \mathcal{L}_B^{II} = & \bar{N}(i\gamma \cdot \partial - m_N)N + \bar{N}(\sqrt{U_5^\dagger} i\gamma_\mu \partial^\mu \sqrt{U_5})N \\ & - g_w \bar{N} \lambda^a i\gamma_5 N \phi^a + \bar{N}\mu\sigma_{\mu\nu}N(\partial^\nu A^\mu), \end{aligned} \quad (3.13)$$

where $g_w = \gamma m_\pi^2 / f_\pi$. One should note that this change of variables does not alter the final result.¹³ If one uses (3.12) (case I), we have a pseudoscalar (ps) nucleon-meson coupling and an additional contact photon-meson-baryon coupling. On the other hand, in (3.13) (case II), we have a ps-coupling from the explicit symmetry breaking and a pseudovector (pv) coupling from the invariant term.

Our CP-violating operator $L_{CP} = -\theta q$ is written by the gluon field carrying zero momentum. Therefore the pv-coupling does not contribute at the tree level and only two diagrams shown in Fig.1 are relevant in case II. The explicit evaluation of them at the tree level gives

$$D_n = \theta(a f_\pi) \frac{1}{m_\eta^2} \frac{1}{m_N} g_w \mu_n. \quad (3.14)$$

The meaning of each term in D_n is self-evident. $a f_\pi$ just denotes the coupling strength of $q(x)$ to the physical η meson. Rewriting it by the physical quantities, we obtain

$$D_n = \theta \frac{m_\eta^2 - m_\pi^2}{2m_N^2} \frac{m_\pi^2}{m_\eta^2} \tilde{\gamma} \mu_n, \quad (3.15)$$

where we have defined a dimensionless number $\tilde{\gamma} = \gamma m_N$ characterizing the strength of the explicit symmetry breaking. (3.15) shows that D_n vanishes in the absence of anomaly ($m_\eta \rightarrow m_\pi$) and/or in the chiral limit ($m_\pi \rightarrow 0$), which is consistent with our constraint. The former aspect ensures that we took into account only the disconnected piece correctly and the latter is related to the fact $\langle \delta \mathcal{O} \rangle|_{on-shell} = 0$. One should note that if one adopts the first order of the chiral perturbation for m_π or takes $a \rightarrow \infty$, a disappears from the formula; $D_n \rightarrow \theta(m_\pi^2/2m_N^2) \tilde{\gamma} \mu_n$.

Let us now examine the large N_c behavior of our D_n in (3.15). m_N and μ_n are known to be $O(N_c)$ quantities as the quark model tells us.²⁷ γ is also $O(N_c)$, since it is proportional to the $\pi - N$ sigma term. Therefore, together with the fact $m_\eta^2 - m_\pi^2 \sim O(N_c^{-1})$, we conclude that our D_n is $O(N_c^0)$ quantity. (Here one should not confuse the artificial limit $a \rightarrow 0$ with the large N_c limit.)

If we perform the same calculation using $\mathcal{L}_B^I$, we have three diagrams at the tree level (fig.2). The sum of the first two gives

$$D_n^{12} = -\theta(a f_\pi) \frac{1}{m_\eta^2} \frac{1}{m_N} g_s \mu_n = -\theta a \frac{1}{m_\eta^2} \frac{c}{f_\pi m_N} \mu_n, \quad (3.16)$$

which does not vanish in the chiral limit. In other words, *if one were not to make $\mathcal{L}_B$ $U_A(1)$ invariant, it gives a wrong result.* The remedy comes from

the third term related to the $U_A(1)$ invariant magnetic moment which gives

$$D_n^3 = \theta a \frac{1}{m_\eta^2} \mu_n. \quad (3.17)$$

This term cancels a part of D_n^{12} and recovers the desired result, eq.(3.15).

Since it becomes now clear how to do the consistent evaluation of NEDM by the above exercise, let us give a diagnosis of the previous calculations. They are classified in two categories: (i) The direct evaluation of $-\theta Q$ using the effective Lagrangians with an explicit gluon field^{28,29} or with the instanton induced vertex,³⁰ and (ii) calculations using $L_{CP}^{eff} \sim \theta m \bar{\psi} i \gamma_5 \psi$ in place of $-\theta Q$.^{5,16,31-35}

In the former approach, there arises no contradiction to the general principle since only the disconnected part is treated from the beginning. In ref.28, the photoproduction of η' off the nucleon is adopted to estimate $D_n \sim \langle n | J_\mu F \tilde{F} | n \rangle$. In ref.29, the pion loop contribution, which is a dominant term in the chiral limit because of the factor $\log(m_N^2/m_\pi^2)$, is adopted following ref.32. In ref.30, the instanton-induced 't Hooft type vertex³⁶ is used to calculate NEDM in $N_f = 2$ case.

In approach (ii), one has to make sure that the direct insertion defined in section 2 is properly eliminated. For example, the direct insertion of L_{CP}^{eff} to the quark field gives a finite result on the lattice¹⁴ which is an artifact due to the violation of the WT identity by the finite lattice spacing.⁸

In the bag model, the non-zero result of NEDM is obtained by the insertion of L_{CP}^{eff} into the *valence* quarks in the lowest order of the bagged perturbation theory.^{5,31} If one assumes that the valence quark in the bag model is the same with the current quark in QCD Lagrangian, the contribution to the NEDM in the above calculation should vanish. Hence, the claim of the non-zero result should be a fake either due to the use of the chiral non-invariant intermediate states or the inappropriate on-shell condition for nucleon. If the valence quark is much more complicated object than the current quark (which is more plausible), there is no systematic way to single out only the disconnected pieces to calculate the matrix element of L_{CP}^{eff} . Therefore, the the result of the bag model is not reliable also in this case because of the uncontrollable contamination of the connected piece.

In ref.16 , it is claimed that a rather large $|D_n|$ is obtained at the tree level. However, the result has serious problems. First, the contamination from the direct insertion is unavoidable since the matrix element $\langle 0 | \bar{q}i\gamma_5 q | \eta' \rangle$, which has both a connected and a disconnected piece, is evaluated (note that η' has both quark and gluon contents due to the axial anomaly.). Second, $U_A(1)$ non-invariant magnetic moment of neutron is introduced, which gives an erroneous result being non-vanishing in the absence of anomaly as we already remarked in eq.(3.16). Once one introduces the $U_A(1)$ invariant magnetic moment in their approach and uses the $\eta' - N$ coupling from the quark model, NEDM vanishes due to the contact photon- η' -nucleon coupling.³⁷

In ref.32, the current algebra and the chiral perturbation is used to evaluate the CP-violating pion-nucleon coupling $\bar{g}_{\pi N}$ induced by L_{CP}^{eff} , which gives rise to NEDM from the one pion loop. At the first look, the result does not seem to vanish when $m_{\eta'} \rightarrow m_\pi$. However, if one treats the CP violating pion-nucleon coupling $\bar{g}_{\pi N}$ more carefully, one finds that the factor $a/(1+a\text{Tr}M^{-1})$ should be multiplied when a is finite. The factor reduces to 1 in the limit $m_\pi/m_{\eta'} \ll 1$ but vanishes in the limit $a \rightarrow 0$. (See the derivation of D_n in ref.29.) Therefore, as far as one has keep in mind this correction (numerically this is a minor correction though), the result is all right. Only a remaining question, although it has nothing to do with the consistency to the WT identity, is whether this leading term near the chiral limit is really a main term in the real world where $m_\pi = 140\text{MeV}$, which was properly addressed by Cea and Nardulli.¹⁶ We will come to this point later. $\bar{g}_{\pi N}$ is also calculated by using the chiral quark model,³³ the chiral bag model^{34,31} and the Skyrme model,³⁵ which are essentially the variant of ref.32.

It is instructive to consider more about the relation between (i) and (ii) within our effective Lagrangian approach. In approach (i) we have to use the following total action:

$$\mathcal{L}_M(U, \theta) + \mathcal{L}_B(B, U), \quad (3.18)$$

while in approach (ii) there are several choices for $\mathcal{L}_B$ with the mesonic part given by (3.6). The most naive choice is

$$\mathcal{L}_M(Ue^{-i\theta\lambda^0/2}, 0) + \mathcal{L}_B(B, Ue^{-i\theta\lambda^0/2}), \quad (3.19)$$

where

$$\begin{aligned}\mathcal{L}_B(B, Ue^{-i\theta\lambda^0/2}) &= \bar{B}(i\gamma \cdot \partial - \alpha U_5^\dagger e^{i\theta\gamma_5\lambda^0/2})B \\ &\quad - \gamma \bar{B}MB + \bar{B}\mu\sigma_{\mu\nu}U_5^\dagger e^{i\theta\gamma_5\lambda^0/2}B(\partial^\nu A^\mu).\end{aligned}\quad (3.20)$$

At the quark level, this choice is rather strange since the quarks in the baryon are unchanged while the quarks in the meson are transformed under the $U_A(1)$ rotation. Furthermore, the CP violating effect appears in several places; the complex baryon mass, the CP violating baryon-meson interaction, the explicit electric dipole moment of the baryon and so on. Each term does not have a definite physical meaning; only the sum of all effects has physical relevance. A more reasonable and simpler choice is

$$\begin{aligned}\mathcal{L}_B(e^{-i\theta\gamma_5\lambda^0/4}B, Ue^{-i\theta\lambda^0/2}) &= \bar{B}(i\gamma \cdot \partial - \alpha U_5^\dagger)B \\ &\quad - \gamma \bar{B}Me^{-i\theta\gamma_5\lambda^0/2}B + \bar{B}\mu\sigma_{\mu\nu}U_5^\dagger B(\partial^\nu A^\mu),\end{aligned}\quad (3.21)$$

where the CP violating term appears only in the explicit symmetry breaking term. Another choice adopted in ref.29 is a little complicated:

$$\mathcal{L}_M(Ue^{-ib\theta\lambda^0}, \theta) + \mathcal{L}_B(e^{ic\theta\gamma_5\lambda^0/4}B, Ue^{-ib\theta\lambda^0}) \quad (3.22)$$

where $b = a/m_\eta^2 = (m_\eta^2 - m_\pi^2)/(2m_\eta^2)$ and $c = -2b\alpha/m_N$. This is chosen so that $\theta\eta^0$ term is absent in $\mathcal{L}_M$ and the baryon mass is γ_5 free. (Here we consider the $N_f = 2$ case for simplicity, though $N_f = 3$ in ref.29,32.) In this choice of variables, the CP violating coupling $\bar{B}\lambda^a B\pi^a$ appears from the original CP preserving interaction $\bar{B}\lambda^a B\pi^a\eta$ by the replacement $\eta \rightarrow \eta - (2b + c)\theta f_\pi/2$ and this factor becomes $\gamma m_\pi^2 f_\pi/(2m_N)$ ($b = 1/2$) in the large a limit. This CP violating interaction in this limit was used to calculate the NEDM.^{29,32}

We come to the point to evaluate a contribution to NEDM which survives even in the large N_c limit or equivalently in the tree level of hadron fields in the realistic $N_f=3$ case. As we have already demonstrated in the $N_f = 2$ case, the $U_A(1)$ invariant magnetic moment is an essential ingredient. Eq.(3.2) is our meson Lagrangian with

$$\begin{aligned}q(x) &= \sqrt{\frac{3}{2}}af_\pi\eta^0, \quad a = \frac{1}{3}(m_\eta^2 + m_\eta^2 - 2m_K^2), \\ \text{and} \quad M &= \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2),\end{aligned}\quad (3.23)$$

where we have neglected the small isospin breaking. We introduce the baryon field by $B = \frac{1}{2}(1 + \gamma_5)B_R + \frac{1}{2}(1 - \gamma_5)B_L$ where B_L and B_R transform as $(3, 3^*)$ and $(3^*, 3)$ representation of the $U_L(3) \otimes U_R(3)$ flavor respectively. Then $\mathcal{L}_B$ is written as

$$\begin{aligned}\mathcal{L}_B = & \text{Tr}[\bar{B}i\gamma \cdot \partial B] - \alpha \text{Tr}[\bar{B}_L U B_R U + \bar{B}_R U^\dagger B_L U^\dagger] \\ & + \delta \text{Tr}[\bar{B}_L U B_R M + \bar{B}_R U^\dagger B_L M] \\ & + \gamma \text{Tr}[\bar{B}_L M B_R U + \bar{B}_R M B_L U^\dagger] \\ & + \epsilon \text{Tr}[MU + MU^\dagger] \text{Tr}[\bar{B}_L U B_R U + \bar{B}_R U^\dagger B_L U^\dagger] \\ & + (\text{magnetic moment}).\end{aligned}\quad (3.24)$$

The δ , γ and ϵ terms denote the $U(3)$ breaking. The last term denotes the magnetic moment of the octet baryons, the precise form of which is irrelevant here except that it is $U_A(1)$ invariant. The parameters δ and γ are determined by the baryon mass splitting, while α and ϵ can be determined^{28,29} by the nucleon mass and the $\pi - N$ sigma term. Note that our α , δ and γ corresponds to $f_\pi/\sqrt{2}$ times α , δ and γ in ref.29. A more general form of the $U_L(3) \otimes U_R(3)$ invariant part of the effective Lagrangian has D and F terms for the pv-coupling.³⁸ In our case, however, such details are irrelevant since pv-couplings do not contribute. On the other hand, in the approaches calculating the one loop diagram, they are important to get the observed $g_{\pi N}$ at the tree level i.e., the Goldberger-Treiman relation $g_{\pi N} = (F + D)m_N/f_\pi$.

To evaluate NEDM, it is convenient to define the chiral invariant nucleon field by

$$B_L = \sqrt{U} N_L \sqrt{U} \quad \text{and} \quad B_R = \sqrt{U^\dagger} N_R \sqrt{U^\dagger}, \quad (3.25)$$

Using this, expanding U up to the first order of the meson fields and retaining only the part relevant for the neutron (n), we get

$$\begin{aligned}\mathcal{L}_B \rightarrow & \bar{n}(i\gamma \cdot \partial - m_N)n + (\text{pv} - \text{coupling}) \\ & - (\eta' g_{\eta'} + \eta g_\eta) \bar{n} i \gamma_5 n + \mu_n \bar{n} \sigma_{\mu\nu} n (\partial^\nu A^\mu),\end{aligned}\quad (3.26)$$

with

$$\begin{aligned}g_{\eta'} &= \frac{1}{f_\pi} [\sqrt{2}\delta(2m_K^2 - m_\pi^2) \cos(\phi + \phi_0) + \gamma m_\pi^2 \sin(\phi + \phi_0)], \\ g_\eta &= \frac{1}{f_\pi} [-\sqrt{2}\delta(2m_K^2 - m_\pi^2) \sin(\phi + \phi_0) + \gamma m_\pi^2 \cos(\phi + \phi_0)].\end{aligned}\quad (3.27)$$

Here $m_N = \alpha - 2\epsilon(2m_K^2 + m_\pi^2) - \delta(2m_K^2 - m_\pi^2) - \gamma m_\pi^2$, μ_n is the neutron magnetic moment and we have used a relation between the flavor eigenstates (η^0 and η^8) and the mass eigenstates (η and η');

$$\begin{aligned}\eta^0 &= \eta' \cos \phi - \eta \sin \phi, \\ \eta^8 &= \eta \cos \phi + \eta' \sin \phi,\end{aligned}\quad (3.28)$$

with the empirical mixing angle³⁹ $\phi = -11^\circ \sim -20^\circ$ and $\phi_0 = 54.73^\circ$ ($\tan \phi_0 = \sqrt{2}$). Owing to the fact that we introduce the magnetic moment as an $U_A(1)$ invariant form, the contact photon- η^0 -neutron coupling does not arise in the choice of the variables here (as in case II in our previous example).

Here several comments are in order. (i) The term proportional to ϵ in (3.24) is irrelevant in our calculation since it does not produce Yukawa coupling. (ii) The $U_A(1)$ breaking due to anomaly is solely governed by the $F\tilde{F} - \eta^0$ mixing in $\mathcal{L}_M$, since we have used the effective action motivated by the large N_c QCD. Therefore, the ps-coupling of η^0 with neutron in eq.(3.26) takes the value in the large N_c limit. (iii) As we already mentioned in the $N_f=2$ case, one can in principle add terms which break $U_A(1)$ invariance in the magnetic moment by the quark masses such as $\text{Tr}(\bar{B}\sigma_{\mu\nu}MB)\partial^\mu A^\nu$. The strength of such terms can be determined by the observed magnetic moment of the octet baryons. We have found that the effect of the terms is $\sim 10\%$ of the contribution from δ and γ terms in eq.(3.24). The detailed explanation of this effect will be given in the separate publication.⁴⁰

Using PCU_1C , the problem is reduced to evaluating the matrix element

$$-i\theta(af_\pi) \int d^4x \langle n(p') | T[\eta'(x) \cos \phi - \eta(x) \sin \phi] J_\mu(0) | n(p) \rangle, \quad (3.29)$$

which is given by the diagrams similar to those in Fig.1. After a straightforward calculation, we get

$$D_n = \theta \frac{m_{\eta'}^2 + m_\eta^2 - 2m_K^2}{3} \sqrt{\frac{3}{2}} \left(\frac{f_\pi}{m_N} \right) \left(\frac{g_{\eta'}}{m_{\eta'}^2} \cos \phi - \frac{g_\eta}{m_\eta^2} \sin \phi \right) \mu_n, \quad (3.30)$$

which is proportional to “ a ” and vanishes in the chiral limit since $g_{\eta'(\eta)}$ is proportional to the current masses. Note that the absolute value of the numbers in eq.(3.31) will be reduced by $\sim 10\%$ if one takes into account the

magnetic moment with explicit $U_A(1)$ breaking due to M as we mentioned. Furthermore, note that our D_n is $O(N_c^0)$ contribution while D_n due to the pion loop is $O(1/N_c)$. The latter can be easily seen if one remembers that the pion-nucleon ps-coupling $g_{\pi N}$ is $O(N_c^{3/2})$ and the parity violating pion-nucleon coupling $\bar{g}_{\pi N}$ is $O(N_c^{-3/2})$ if one considers the multiplicative factor $a/(1 + a\text{Tr}M^{-1})$ which we mentioned before. Taking the experimental numbers for the hadron masses, we get an estimate for the tree level contribution;

$$\begin{aligned} D_n &= -7.2 \cdot 10^{-16} \theta e \cdot \text{cm} \quad \text{for } \phi = -11^\circ, \\ &= -3.9 \cdot 10^{-16} \theta e \cdot \text{cm} \quad \text{for } \phi = -20^\circ, \end{aligned} \quad (3.31)$$

which is different in sign⁴¹ and comparable in magnitude with the one-loop result $(+3.6 \times 10^{-16} \theta)$ in ref.29. This result tells that one can neglect neither of the following two contributions; one is a leading term in the large N_c limit having a structure $N_c^0 \cdot m$ like our D_n and another is the leading term in the chiral expansion having the structure $N_c^{-1} \cdot m \ln m$.

Here we should mention briefly the same kind of calculation for the CP violating condensation parameter or the topological susceptibility K which is defined by

$$\theta K = \langle -q(x) \rangle_\theta \simeq \theta \int d^4 y \langle T q(x) q(y) \rangle. \quad (3.32)$$

If only the disconnected part is properly taken into account in the calculation, K has to be proportional to both a and the current quark mass. The evaluation using the current algebra and the chiral perturbation, however, gives only the latter dependence, as can be seen in ref.26. Although K is an off-shell quantity, (3.2) and (3.6) give the same result, since q is a purely gluonic operator being chiral invariant:

$$K = \frac{f_\pi^2}{2} a \left[1 - 2a \left(\frac{\cos^2 \phi}{m_\eta^2} + \frac{\sin^2 \phi}{m_{\pi^0}^2} \right) \right], \quad (3.33)$$

in $N_f = 2$. Here we took into account the isospin violation. In the large “ a ” or small current quark mass limit, (3.34) reduces to the result of ref.26,

$$K = \frac{f_\pi^2}{4} m_\pi^2. \quad (3.34)$$

As can be seen in all the above examples, the current algebra with chiral perturbation is always missing the factor “ a ”. The reason is easy to understand: Schematically every tree amplitude is proportional to the combination

$$N_f \frac{a}{m_S^2}, \quad (3.35)$$

in the flavor symmetric case. Here m_S is the mass of the flavor singlet pseudoscalar meson. Once one adopts the lowest order of the chiral perturbation, it reduces to unity. This tells us that we can improve the result of the current algebra with chiral perturbation by simply multiplying $N_f a/m_S^2$ which is $O(1/N_c)$.

IV. Conclusion

In this paper, we have extensively examined the consistency of the calculation of the low energy matrix element of $\theta F \tilde{F}$.

The anomalous WT identity tells us that the insertion of $\theta F \tilde{F}$ in the hadronic process is related to the *disconnected* insertion of L_{CP}^{eff} while the connected insertion of L_{CP}^{eff} is merely the change of variables $\delta\mathcal{O}$ which vanishes for the on-shell amplitude.

This simple observation gives a constraint on the use of the effective theory to calculate NEDM and also makes it possible to diagnose the previous calculations. In fact, one cannot construct the effective theory without $U_A(1)$ invariance in the chiral limit. That also implies all the CP violating matrix elements have to be proportional to the current quark masses and the strength of the anomaly. We pointed out, in section 3, the problematic points of the previous calculations of NEDM. Then we gave a new calculation of NEDM which is consistent with the general constraints above and does not vanish in the large N_c limit. The explicit inclusion of the $U_A(1)$ invariant magnetic moment of the neutron is the essential ingredient of the calculation.

Finally we should mention that it is desirable to calculate NEDM from the first principle. One possibility is a lattice QCD calculation, although

there are several problems to be overcome as the recent analysis shows.⁸ Another interesting problem related to the operator $F\tilde{F}$ is the spin content of the nucleon⁴³ where the nucleon matrix element of $F\tilde{F}$ plays a crucial role.⁴⁴ Extensive studies using the effective Lagrangians and the lattice QCD consistent with the WT identity are called for on this problem.

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Figure Caption

Fig. 1 : The tree-level contributions to NEDM in the effective Lagrangian in case II. There is no contact photon-meson-nucleon coupling in this case.

Fig. 2 : The tree-level contributions to NEDM in the effective Lagrangian in case I. There is a contact photon-meson-nucleon coupling from the $U_A(1)$ invariant magnetic moment of the neutron.

Fig.1

Fig.2

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