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DETERMINATION OF COHESIVE AND NORMAL STRESSES AND
SIMULATION OF FLUIDIZATION USING KINETIC THEORY

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LIST OF SYMBOLS

a	Velocity of propagation
c	Instantaneous particle velocity
C	Peculiar particle velocity = $c - v$
C_d	Drag coefficient
D	Diffusion coefficient
d_p	Particle diameter
e	Restitution coefficient
F	External force per unit mass acting on the particle
g	Gravitational constant
g_0	Radial distribution function,
G	Solids modulus = $(\partial P_s / \partial \epsilon_s)$
I	Unit tensor
m	Mass of particle
M	Molecular weight
P	Pressure
q	Flux of fluctuating energy
$\bar{S}$	Rate of shear tensor
t	Time
v	velocity

GREEK LETTERS

β	Two-phase drag coefficient
ϵ, ϵ_s	Gas and solid volume fraction
Θ	Granular temperature = $\frac{1}{3} \langle C^2 \rangle$

κ	Conductivity of granular temperature
μ	Shear viscosity
ξ	Bulk viscosity
ρ	Density, bulk or particle = $\epsilon_s \rho_s$
τ	Stress
ψ	Single particle quantity, like mass, momentum or energy

ABSTRACT

The general objective of this study is focussed on the solid stresses involved in gas - solid flow. These stresses are generally included in the momentum conservation equations, essentially for stability and to prevent particles from collapsing to unreasonably low values of gas volume fraction. The first half of this work undertakes the measurement of the stresses in various powders by direct means, while the second part uses a newly developed kinetic theory constitutive equation for stress to predict the flow and also the solids viscosity in a CFB.

The cohesive or tensile stress found to exist in some classes of powders is measured using a Cohetester, based on which a cohesive force model is derived, which is sensitive to the characteristic properties of the powder material. The normal stress is measured using a Consolidometer, and the powder solids modulus is obtained as a function of the volume fraction. The solids modulus is seen to vary with particle size and particle type, with the smaller size particles being more compressible.

The simulation of flow in the CFB using Gidaspow's (1991) extension of Ding's (1990) kinetic theory model to dilute phase flow, predicts realistic values of solids viscosity that are comparable to viscosities obtained experimentally by Miller (1991). However, to obtain a match between the two the value of the restitution coefficient has to be close to unity. The flow behavior showed periodic oscillations of flow (turbulence) as seen in a real system.

CHAPTER I

INTRODUCTION

The hydrodynamic approach to fluidization was first introduced by Davidson in 1961, when he analyzed single bubble motion in a fluidized bed using two continuity equations and an expression for relative velocity in terms of Darcy's law for flow in porous media. Modelling of fluidization has come a long way since then, and Gidaspow's (1986) review of the hydrodynamics of fluidization presented a viable hydrodynamic model that predicted the variations of void fraction, pressure, gas and solids velocities as a function of time and position. This approach is based on the principles of conservation of mass, momentum and energy, derived from the early work of Jackson (1963). Computer codes have been developed based on this approach and have been demonstrated to be adaptable to a variety of problems (Syamlal, 1985). Although the continuity and momentum equations used are generally accepted by the majority of two-phase flow investigators, to overcome the ill-posedness discussed in detail by Lyczkowski et al (1978), a normal component of solids stress associated with particle-particle interaction must be included in the model. This term not only provides stability, but also keeps the particles from collapsing to unreasonably high solid volume fractions.

The interparticle shear stress, from one dimensional statics and the Coulumb failure condition (Gidaspow, (1991) chapter 4) is found to be a sum of a function of the normal stress and the cohesive or tensile stress. Most of the recent fluidization models generally include an expression for the normal stress, since most of the commonly fluidized particles are non-cohesive, such as the particles classified under Geldhart's group 'B'. While working with Geldhart's group 'A' or 'C' particles however, it is necessary to also account for the cohesive force.

Ding (1989) included an empirical cohesive force model in his fluidization model for fine 'A' particles to obtain a match between the predicted and experimentally measured porosities. In this study, the cohesive forces of powders are measured directly using a cohetester, which provides us with information regarding the cohesive behavior of powders. An attempt has been made to derive a cohesive force model, based on these measurements.

The normal component of the solids stress plays a role similar to pressure (Gidaspow, (1991) chapter 4). The interparticle stress term was first introduced to fluidization by Rietema and Musters (1973) who measured the solids modulus in a fluidized bed experiment. Gidaspow and Ettahadieh (1983) used a fitted model of Rietema and Musters (1973) data in their fluidization model, which was later also used by Tsuo and Gidaspow (1990) to calculate the solids viscosity in the simulation of a circulating fluidized bed. Bouillard et al (1989) derived a generalized model for the solids modulus based on Orr's (1966) theory of powder compaction, in order to interpret the physical significance of the solid pressure. While most of the predictions of the solids modulus available so far are from empirical correlations or indirect measurements, chapter three of this study discusses a direct means of measuring this stress by using a consolidometer. The solids modulus is obtained as a function of the volume fraction, and the dependence of this stress on various factors such as particle size and type are also studied in detail.

Many features of fluidized beds such as bubble shapes and sizes are predicted by inviscid models (Gidaspow, 1986), which are based on the classical Davidson model where the solids are assumed to be in potential flow, and that the dominant method of energy dissipation is the drag between the particles and fluid. This does not hold good when the flow is dense or when the particles are recirculated rapidly, as in the bubbling bed or circulating fluidized bed. In such cases the additional dissipation due to particle - particle collisions must be considered. The theory and

experimental work of granular flow solids started by Bagnold (1954) and continued by Savage(1984), Lun et al (1984) and Ding and Gidaspow (1990) forms a rational basis for including granular stresses into a fluidization model. Based on the kinetic theory of gases and concepts derived from turbulence theories, Ding and Gidaspow (1990) developed a constitutive model for solids viscosity, while Tsuo and Gidaspow's (1990) model calculates the solids viscosity from an empirical expression of the solids modulus. Ding and Gidaspow's (1990) kinetic theory model was able to successfully predict flow in a bubbling bed, this application involves dense phase flow. For flow in a circulating fluidized, which is relatively dilute they used a turbulence model (Deardoff, 1971) to account for dilute phase particle interactions.

Gidaspow's (1991) model however overcomes this limitation by an extended kinetic theory model which includes the dilute limit. Chapter four uses this model to simulate fluidization in a CFB to predict realistic values of the solids viscosity. The predicted flow patterns are qualitatively compared with those observed experimentally. The solid viscosity predicted by the model is comparable to the measurements made by Miller (1990),but is however found to be extremely sensitive to the restitution coefficient used in the computation. The solids viscosity appears to go through a minimum value with increasing solid volume fraction. The predicted higher viscosities, at low solids volume fractions correspond to large granular temperatures. This code is also able to predict oscillations of flow in the riser relative to periodic recirculation/discharge of solids from the standpipe to the riser, and a comparison with the earlier model used by Ding indicates that the present model is quite competent.

CHAPTER II

MEASUREMENT OF COHESIVE FORCES IN FINE POWDERS

2.1 Introduction

In the simulation of fluidization of fine particles, Gidaspow and Ding (1988), found that by taking into account the cohesive forces found to exist between the particles, a good agreement was obtained between the measured and computed time averaged porosities in the bed. Their fluidization model included an empirical cohesive force model. The empirical model used however, provided only a rough estimate of the force exerted by the particles, and does not account for the dependence of the force on the charge, particle size or particle type. The Cohetester used here is a device by which the cohesive force can be directly measured.

According to Geldart's classification of powders, the class 'C' particles comprise of cohesive powders, which are generally difficult to fluidize. The cohesiveness of these powders is primarily because of the strong interparticle forces which may be due to very small particle size, electrostatic charges, wet sticky particle surfaces, soft solids or irregular particle shapes. Difficulties in effectively fluidizing these powders arise because the cohesive forces are greater than what the fluid can exert on the particles, thus causing the powders to lift as a plug or causing agglomeration or channeling. It is, therefore, important to estimate the cohesiveness while working with class 'C' particles.

Finely-ground oil shale particles can be classified under the group of 'C' particles, and their cohesiveness can be attributed mainly due to the electrostatic forces (Gupta, 1990) acting between the positively charged kerogen, the carbonaceous constituent of shale, and the negatively charged mineral matter, the non-carbonaceous constituent.

Glass beads with particle sizes of $40\text{ }\mu\text{m}$, are classified under Geldhart's group

of 'A' particles, which are only slightly cohesive. A characteristic of these powders are that the minimum bubbling velocity is greater than the minimum fluidization velocity. Ding and Gidaspow (1989) via their fluidization model for fine particles showed that even the small cohesive behavior of the glass beads played an important role in the characterization of the fluidized bed. It is therefore necessary to account for the cohesive forces between the particles.

In this study measurements of the cohesive forces of various shales and glass beads were made using a Cohetester. The cohesiveness of the different shales were compared, as well as the difference between the cohesiveness of type A and type C particles.

2.2 Instrument

The Cohetester used for the measurements, was manufactured by the Hosokawa Micron Corporation of Japan. It is an instrument especially designed to measure the cohesive forces or agglomeration tendencies of powders. Figure 1 shows a schematic diagram of the Cohetester. It essentially consists of 3 units:

1. The main unit, which comprises of 3 sections, viz. motor section for pulling the cell, a suspended cell section and the housing, in which the motor and cell are enclosed. The suspended cell section consists of 2 half cells (suspended by copper plates), one of which is fixed. The other movable half cell is connected to a strain gauge, which is also coupled to the motor. The output of the strain gauge (which is obtained on the Y axis of the recorder) is calibrated to be proportional to the applied load. Displacement (obtained on the X axis) is measured by means of a linear voltage differential transformer (LVDT).
2. The operational amplifier consists of a strain amplifier, a displacement meter, and a signal processor. It is used to amplify and/or process signals from sensors

in the main unit, which detects tensile stress and displacement of the powder layer, and to output the signals to the X-Y recorder. The signal processor indicates the actual load by subtracting the blank load required to pull the cells apart.

3. The X-Y recorder, is used to obtain a relation between the tensile stress and displacement of the powder layer. It records displacement on the X-axis and tensile strength on the Y- axis.

2.3 Test Procedure

Prior to actual measurement, a blank test must be conducted to tune the amplifier and other instrument settings and to check for any warping of plates, or the levelling of the instrument, etc. The blank test is carried out by switching on the motor and pulling the half cells apart, with no load.

The actual test is then conducted. The cell and its contact surfaces are thoroughly cleaned and the cell is firmly housed using the securing knob. The cell extension tube is placed on the cell and a known weight of sample is filled in. The shale is compacted using the compaction weight and additional weights. The weights are kept on long enough to ensure the formation of a stable layer, after which the weights as well as the extension tube are removed. The motor is then switched on, the recorder pen is set down, and the cell is gently released by releasing the securing knob.

The motor pulls the cell apart, until the compacted sample layer is broken. The amplifier detects the force required to break the layer, which the recorder records as a peak. The value corresponding to this peak is proportional to the tensile strength of the powder. This tensile strength represents the cohesive force of the powder.

Several tests were conducted to validate the measurements made. These tests

Figure 1. Schematic Diagram of Split Cell Type Coheter

included testing the cohesiveness of wheat flour as a function of volume fraction, the data for which had already been supplied to us by the Hosokawa Micron Corporation (HMC). A comparison of our data with the data supplied by HMC for the wheat flour is shown in Figure 2 and shows a similar behavior, although we do find a small variation which can be attributed to the experimental conditions being different, such as surface moisture, chemical nature of the flour, and particle density. The samples were also tested for consistency in the values of cohesiveness.

2.4 Results and Discussion

The first sample tested was a raw shale (bag fines), which had a kerogen content of approximately 13 percent. The cohesive force was observed to increase with an increase in the solids volume fraction as can be seen from Figure 3, which is plot of tensile stress as a function of solids volume fraction. The increase in the cohesiveness with volume fraction is expected, since the particles are now in closer contact with each other, and the electrostatic forces acting between the positively charged kerogen and negatively charged mineral matter are also much stronger.

The second sample tested was a 'beneficiated shale', which had a higher kerogen content (about 28 percent). This sample exhibited a similar trend of the cohesive force increasing with an increase in the solids volume fraction as shown in Figure 3. The cohesive force for the beneficiated shale is however much higher than that for the raw shale, as can be seen in the figure, which compares the cohesiveness of both the shale samples as a function of volume fraction. The higher cohesiveness can be attributed to the higher kerogen (carbon) content, which results in a higher charge.

The test on glass beads ($40\text{ }\mu\text{m}$) also showed an increase in cohesiveness with solids volume fraction as can be seen in figure 4. The cohesive forces exerted by the glass beads was however much lower than that of the shales, and the behavior of the curve is also quite different, due to an inherent difference in the cohesiveness,

Figure 2. A Comparison of our Data with the Data Supplied by HMC for the Wheat Flour

since glass beads are not highly charged particles.

2.5 Cohesive Force Model

An earlier empirical model for cohesive forces between particles was developed by Ding (1989) for class 'A' particles, which are slightly cohesive. While working on an empirical model for fluidization of fine particles, inclusion of an empirical cohesive force model, gave a much better agreement between computed and experimentally determined porosities. The model developed by Ding (1989) was based on the assumption that the cohesive force per unit area is a function of volume fraction alone. Using an empirical fit to correlate data supplied by Hosakawa Micron corporation, Japan, it was estimated that the cohesive forces varied with porosity as

$$\tau_c = 10^{-a\epsilon + b} \quad (2.1)$$

Where a and b are powder characteristics. For glass beads of $40\mu\text{m}$, the characteristic parameters used by Ding (1989) were $a = 10.6$ and $b = 7.0$.

A comparison of the cohesive force predicted by the existing empirical model in Equation 1.1 and the experimentally determined cohesiveness of the glass beads can be seen in figure 5 . Although the predicted values are of the same order of magnitude as that measured experimentally, the model does not predict a similar behavior, which is due to the values of the powder characteristic used.

A correlation similar to Ding's model can be obtained from the experimental measurements of the cohesiveness of the shale and glass beads. The results indicate that the cohesive force is an exponential function of porosity. A generalized expression can be written as,

$$\tau_c = b \exp(a\epsilon_s) \quad (2.2)$$

Figure 3. A Comparison of Cohesive Forces of Raw Shale and Beneficiated Shale

Figure 4. Cohesive Forces of Glass Beads ($40\ \mu\text{m}$) as a Function of Volume Fraction

Figure 5. A Comparison of Cohesive Force of Glass Beads ($40\mu\text{m}$) with the Predictions of Ding's Model

where a and b are the powder characteristic parameters which depend on properties of the particles, such as the size, density and charge. In figures 6, 7 and 8 the cohesive force model expressed in the form of equation 1.2 has been compared with the experimental data, with values of the powder characteristic parameters as shown below.

$a = 10.6$ and $b = 0.015$ for the raw shale sample.

$a = 9.18$ and $b = 0.125$ for the beneficiated shale sample.

$a = 3.81$ and $b = 0.212$ for the glass beads ($40\ \mu\text{m}$) sample.

These characteristic parameters are obtained from plots of the experimental data. The values of the powder characteristics indicate that while the two shale samples have parameters of the same order, the glass beads belonging to Geldhart's group of 'A' particles has a lower characteristic parameter. The relatively small difference between the characteristics of the two shales can be attributed to a difference in charge of the particles, since the beneficiated shale sample carries a higher charge because of its higher kerogen content. The difference in the characteristics of the glass beads from the group 'C' particles is probably due to a difference in the nature of the cohesiveness. While the 'C' particles are cohesive primarily because of electrostatic charges (Gupta, 1990), the glass beads or 'A' particles (which usually do not possess any charge) are cohesive only when the particle sizes are very small, which result in small and almost negligible charge. The glass beads are also much less cohesive than the shales.

2.6 Conclusions

The determination of the cohesive forces in fine powders provides a useful estimate of the order of magnitude of the interparticle forces. The cohesive force model

Figure 6. A Comparison of the Cohesive Force Model with the Raw Shale Data

Figure 7. A Comparison of the Cohesive Force Model with the Beneficiated Shale Data

Figure 8. A Comparison of the Cohesive Force Model with the Glass Beads Data

obtained by correlating the experimental results provides a useful relationship between the cohesive forces and the porosity. Such a relationship is necessary in the computation of fluidization or flow behavior of cohesive 'C' particles or even fine particles belonging to class 'A', for accurate prediction. The powder characteristics account for the difference in the type of particles, and indicate the nature of the cohesiveness of the different powders.

CHAPTER III

CONSOLIDATION IN POWDERS

3.1 Introduction

When any powder is subject to an increase in pressure or load, a readjustment of the powder volume occurs, which corresponds to a reduction in void ratio. Consolidation is generally defined as time-dependent settlement.

In most fluid flow problems, the momentum equations must include solid stress terms, that are functions of porosity, pressure, and the displacement tensors of solids velocity, gas velocity and relative velocity, particularly when the flow is dense. This interparticle pressure is a result of either collision of particles with each other, or due to elastic or inelastic contact with each other and generally acts as a repulsive pressure at high solids fraction. Molerus (1975) interpreted this phenomenon as a direct result of the Hertz contact pressure, which is a function of the particle Young's elasticity modulus. A compressive force must be applied to bring these particles together and thereby change the volume fraction causing a displacement. In view of the existing theory and measurements and in the absence of the effect of particle-particle collisions, the constitutive equation for the normal component of the stress (Gidaspow, 1986) is

$$\sigma = \sigma(\epsilon_s) \quad (3.1)$$

The derivative of the stress is the modulus of elasticity, or the particle-to-particle interaction coefficient

$$G = \frac{d\sigma}{d\epsilon_s} \quad (3.2)$$

and

$$a = \sqrt{G/\rho_s} \quad (3.3)$$

is the velocity of propagation through the powder.

Measurements of the solids modulus has been reported previously by Rietema and Musters (1973) in a tilted fluidized bed experiment, and their was fitted by Gidaspow and Ettehadieh (1983), by equations such as

$$G = 10^{-8.76\epsilon + 5.43} \quad (3.4)$$

This expression was also used by Tsuo and Gidaspow (1990) in their fluidization model to estimate the solids viscosity. The solids modulus becomes of numerical significance only when the void fractions go below the minimum fluidization void fraction. Keeping the coulombic stress component in perspective a satisfactory expression for the mechanism of powder compaction is the experimental observation that a plot of the logarithm of the consolidation pressure versus the particle volume fraction yields a straight line for metallic and non-metallic powders undergoing compaction. (Orr, 1966). Bouillard et al (1989), used Orr's simple theory of powder compaction to derive a generalized expression for G as

$$G = G_o \exp(-c(\epsilon - \epsilon^*)) \quad (3.5)$$

Where 'c', the compaction modulus, is the slope of $\ln(G)$ versus ϵ with a typical value of 600. ϵ^* is the compaction porosity and is typically around 0.376. Bouillard et al (1989) converted all the existing expressions to the generalized form above which predict a negligible solids modulus when the flow is dilute.

All of the existing expressions for the solids modulus are either empirically fitted or derived from fluidization experiments. A direct measurement of the normal stress would provide a better understanding of the influence of various factors on the solids modulus, particularly since some powders may posses a charge when the particles are very fine. In this study, the solids modulus, G, has been experimentally determined

as a function of porosity using a consolidometer and compared with the previously existing models.

3.2 Equipment

The consolidometer used for the experiments is typically used for measuring the consolidation of soil, which is an important civil engineering problem. Figure 9 shows the experimental setup used, which consisted of a consolidation cell and a loading device. A dial gage was used to measure the displacement at selected elapsed intervals of time with various loads.

The consolidation cell is essentially used for compacting the sample. It is made up of a brass base containing a porous stone at the center, on which is placed a brass ring of diameter 6.36 cm and height, 2.5 cm. The ring is clamped on to the base by means of an outer brass ring with 6 bolts. The sample is usually filled into the inner brass ring and trimmed. An Aluminum plate of diameter 6.2 cm is placed on the sample instead of the porous stone which is used for most consolidation experiments. A brass load block is placed on top of the plate, and a ball seated on it comes in contact with the loading device. The loading device used in this study was a levered load hanger, so that high pressures could be applied.

3.3 Test procedure

The consolidation cell was thoroughly cleaned and weighed initially and then again after filling in the sample. The sample was carefully trimmed, the aluminum plate and load block were placed on top, and the consolidation cell was then placed under the loading device. The loading device was carefully adjusted using a seating load. The dial gage was placed on the loading bar, as shown in figure 9. and the dial was corrected to read an initial zero displacement.

Figure 9. Schematic Representation of the Consolidometer

The load was placed on the load hanger, and a stop watch was started to record the maximum displacement,(after about 2 hours). The load was increased in steps and the displacement was recorded as a function of the pressure applied.

After the maximum load had been applied, the procedure was reversed and a plot of the displacement versus the unloading pressure was obtained, to observe the effect of hysteresis and permanent deformation on the sample.

3.4 Results

Since the normal stresses in particles can be related to material properties of the powders, the consolidation behavior of various powders were studied. The consolidation test was tried out on fine as well as on coarse grained particles to observe the effect of particle size. In the case of coarse grained particles, consolidation is almost instantaneous and can be considered to be simply a compression of the powder, while for fine grained and cohesive particles, the diffusion of air is much slower and process takes a much longer time. A comparison of the solids modulus of glass beads of particle sizes 40 microns and 500 microns showed an increase in the Hertz contact pressure and therefore on the solids modulus, due to the larger surface area of contact in the smaller particles. The other materials tested were shale (10 microns), which is classified under Geldhart's cohesive 'c' particles, and atomic aluminum (6 microns).

3.4.1 Glass Beads - 500 μm . The consolidation of the glass beads is shown by the plot of the loading pressure versus solids volume fraction is shown in figure 10. The solids modulus calculated from the slopes of the loading pressure curves in figure 10, is plotted as a function of porosity in figure 11. A hysteresis effect can also be observed in figure 10, which indicates a permanent or plastic deformation of the sample. This effect is accompanied by the breaking of the glass beads at pressures between 1000 to 2000 kPa.

Figure 12 shows a comparison of the experimentally determined values of G with existing models. The model fitted to Rietema and Musters (1973) data is quite inappropriate, since the porosity range (0.62) of their data was much higher than in this study. Gidaspow and Syamlal's (1986) model on the other hand is appropriate only at a compaction porosity near minimum fluidization. Bouillard's (1989) model has a high compaction modulus and a low compaction porosity, and predicts that the solids modulus becomes negligible above a porosity of 0.375. Extrapolation of the experimental plot indicates that G becomes negligible only at a porosity of 0.6.

A plot of the velocity of propagation through the powder as a function is shown in figure 13. The experimental data indicates that the velocities are comparable to that through solids, as would be expected at such a high compaction.

3.4.2 Glass Beads - 40 μm . To study the effect of particle size on the solids modulus, the consolidation of glass beads of 40 μm was measured and compared with the 500 μm glass beads. The plot of solids volume fraction versus pressure, (figure 14) shows a similar consolidation and hysteresis behavior. The comparison in figure 15, indicates the difference in the compaction porosities and that the smaller particle size is more compressible. Figure 16, which is a comparison of the solids modulus of the two glass bead samples indicates that the 40 μm glass beads have higher values of the solids modulus.

3.4.3 Shale - 10 μm . As mentioned earlier shale belongs to the class of 'c' particles which are difficult to fluidize and handle. It is therefore important to account for and to estimate the cohesive forces as well as the normal stresses involved while working with these particles.

The pressure - volume fraction plot for shale is shown in figure 17, and the solids modulus is plotted in figure 18. The consolidation behavior of shale is not very different from that of the 40 μm glass beads. It is however noticeably more compressible, than the 40 μm glass beads, although its solids modulus is not as high

Figure 10. Consolidation of Glass Beads of Particle Size = 500 μm

Figure 11. Solids Modulus of Glass Beads ($500 \mu\text{m}$) as a Function of Porosity

Figure 12. Comparison between Experimentally Determined and Model Predictions of Solid Modulus of Glass Beads of Particle Size = $500\ \mu\text{m}$

Figure 13. Velocity of Propagation through Glass Beads of Particle Size = 500 μm

Figure 14. Consolidation of Glass Beads of Particle Size = 40 μm

Figure 15. Comparison between the Consolidation Behavior of Glass Beads of Particle Sizes 500 μm and 40 μm

Figure 16. Comparison between the Solids Modulus of Glass Beads of Particle Sizes 500 μm and 40 μm

at low porosities. The hysteresis effect on consolidation of the shale is also larger than the 40 μm glass beads as can be seen from figure 17, which is due to the cohesiveness of the particles caused by the charge present in it.

3.4.4 Atomic Aluminum - 6 μm . The study of consolidation of atomic aluminum is of interest to us for problems such as when it is mixed with other materials and subjected to very high pressures. A typical consolidation plot is shown in figure 19. The consolidation behavior seen here, shows an initial sharp increase in solids volume fraction with applied pressure, which probably indicates that the material reaches its yield limit, after which the modulus of the material as well as the particles effect the compaction of the sample. The solids modulus plot shown in figure 20, shows a higher sensitivity to the volume fraction than the other samples measured, which is possibly due to the metallic properties of the powder. Comparison of this data with predictions of the empirical models, however is rather poor as can be seen in figure 21.

3.5 Conclusions

The consolidation measurements above, provide valuable estimates of the normal interparticle solids stress, which enables us to estimate the solids viscosity at different porosities. The values of the solids modulus obtained are comparable to the modulus of elasticity of the solids, and the velocity of propagation through the powder, at low porosities is of the same order of magnitude as that through solids.

The results indicate that the solids modulus cannot be accurately predicted by the existing generalized model as can be seen in figure 12 and 21. The solids modulus is characteristic of the material properties of the powder, and figure 22, which is a comparison between the measured stresses of all the powders tested, also shows that the compaction porosities of the samples are quite different from each other. The effect of particle size on the solids modulus can be seen in figure 16.

Figure 17. Consolidation Behavior of Shale Particles (10 μm)

Figure 18. Solids Modulus of Shale Particles ($40 \mu\text{m}$)

Figure 19. Consolidation of Atomic Aluminum of Particle Size = 6 μm

Figure 20. Solids Modulus of Atomic Aluminum of Particle Size = 6 μm

Figure 21. Comparison between Model Predicted and Experimentally Determined Solids Modulus of Atomic Aluminum of Particle Size = 6 μm

which implies that the smaller particle size and larger surface area of contact results in a greater interparticle Hertz contact pressure and therefore higher modulus. The type of particles also influence the modulus. The cohesive shale particles have a high solids modulus even when the flow is relatively dilute.

Figure 22. Comparison Between the Solids Modulus of Various Powders

CHAPTER IV

SIMULATION OF FLOW THROUGH A CFB

4.1 Introduction

Circulating fluidized bed combusters have become quite popular because of the advantages of higher energy utilization and clean combustion. The design and modelling of a CFB combustor however, is complicated and the hydrodynamics of the solids is not well understood. The CFB does not operate in a true steady state, and the observed transient behavior referred to as "turbulence" has been modelled using mixing length or other more sophisticated turbulence models derived from single phase flow as in computer codes developed by Spalding and co-workers, such PHOENICS or FLUENT.

Our approach essentially utilizes a 2 dimensional multiphase computer code for the solutions of the generalized Navier Stokes equations for multiphase particulate flow. The solids viscosity term which arises due to the random collisions of particles is needed to account for the energy dissipation between particles. In Tsuo and Gidaspow's (1990) study a constant solid viscosity coefficient was used, while Ding and Gidaspow's (1990) used the dense phase kinetic theory approach. The kinetic theory approach calculates the solids viscosity from the fluctuating energy equation, which is derived from the Boltzman equation. Ding's (1990) approach works well for the case when the solid concentration is high. When the flow is dilute however, it is necessary to additionally account for the effect of gas phase turbulence on particle collision. In the CFB the flow is predominantly in the dilute phase, and for this system Ding (1990) added a turbulent viscosity model for the gas phase, which utilizes a simple subgrid scale model (SGS), first used by Deardoff (1971) to estimate turbulent viscosity for channel flow. While this model predicts the flow behavior, it computes very low values of the solids viscosity.

In this investigation we have used the more recent kinetic theory model, which was developed by Gidaspow (1991). This model directly extends the kinetic theory approach to the dilute solid limit, without using an additional turbulence model. This theory predicts a flow behavior quite similar to that of Ding's model with turbulence, while the predicted values of solid viscosity are much higher than those predicted by Ding and are comparable to experimentally determined viscosities, when the restitution coefficient is close to unity.

4.2 Hydrodynamic Model

The physical principles used in our code are the conservation of mass, momentum and fluctuating energy (which is related to the granular temperature) for each phase, which are listed below along with the constitutive equations for the gas and solid phase. These equations are derived from the Boltzman equation for particle velocity distribution (Gidaspow,1990). The equation for granular temperature (Θ) is given by conservation of the fluctuating energy. The constitutive equation for solids stress can be interpreted in terms of the dynamic solids pressure ($\mathbb{P}_s$), the solids bulk viscosity (ξ_s) and the solids shear (μ_s) viscosity, which are given as functions of the granular temperature (Θ) in the next section

In order to calculate the solids stress we used Gidaspow's (1991) extension of the dense phase kinetic theory model (Ding and Gidaspow 1990) which includes the dilute limit. The shear viscosity term used here is the sum of the dense phase viscosity term $\epsilon_k \mu_s$, used by Ding (1990) and a dilute phase viscosity ($\mu_{s,dil}$) term. When the flow is dilute as in this case, the contribution to the viscosity by the dense phase is negligible and it is appropriate to use the dilute phase viscosity, as shown by Gidaspow (1991). This takes care of the effect of gas phase turbulence on the particle interactions when the flow is dilute, and no extraneous model for the gas phase turbulence is required.

4.3 Governing Equations

CONTINUITY EQUATION FOR PHASE ($k = g, s$)

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k) + \nabla \cdot (\epsilon_k \rho_k v_k) = \dot{m}_k \quad (4.1)$$

$$\sum_k \epsilon_k = 1 \quad (4.2)$$

MOMENTUM EQUATION FOR PHASE ($k = g, s; l = g, s$)

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k v_k) + \nabla \cdot (\epsilon_k \rho_k v_k v_k) = -\epsilon_k \nabla P_g + \epsilon_k \rho_k g + \nabla \cdot \bar{\bar{\tau}}_k + \beta(v_l - v_k) + \dot{m}_k v_k \quad (4.3)$$

CONSTITUTIVE EQUATION FOR STRESS

$$\bar{\bar{\tau}}_k = [-P_k + \epsilon_k \xi_k \nabla \cdot v_k] \bar{\bar{I}} + 2\mu_s \bar{\bar{S}}_k \quad (4.4)$$

where

$$\bar{\bar{S}}_k = \frac{1}{2} [\nabla v_k + (\nabla v_k)^T] - \frac{1}{3} \nabla \cdot v_k \bar{\bar{I}} \quad (4.5)$$

GAS PHASE STRESS

$$\bar{\bar{\tau}}_g = 2\epsilon \mu_g \bar{\bar{S}}_g \quad (4.6)$$

SOLID PHASE STRESS (Kinetic Theory Model - Gidaspow, 1991)

Solids Phase Pressure

$$P_s = \rho_p \epsilon_s \Theta [1 + 2(1 + e)g_o \epsilon_s] \quad (4.7)$$

Solids Phase Bulk Viscosity

$$\xi_s = \frac{4}{3} \epsilon_s \rho_p d_p g_o (1 + e) \left(\frac{\Theta}{\pi} \right)^{\frac{1}{2}} \quad (4.8)$$

Solids Phase Shear Viscosity

$$\mu_s = \frac{2\mu_{s_{dil}}}{(1 + e)g_o} \left[1 + \frac{4}{5} (1 + e) g_o \epsilon_s \right]^2 + \frac{4}{5} \epsilon_s^2 \rho_p d_p g_o (1 + e) \left(\frac{\Theta}{\pi} \right)^{\frac{1}{2}} \quad (4.9)$$

Solids Phase Dilute Viscosity

$$\mu_{s_{dil}} = \frac{5\sqrt{\pi}}{96} \rho_p d_p \Theta^{\frac{1}{2}} \quad (4.10)$$

Radial Distribution Function

$$g_o = \frac{3}{5} \left[1 - \left(\frac{\epsilon_s}{\epsilon_{s_{max}}} \right)^{\frac{1}{3}} \right]^{-1} \quad (4.11)$$

FLUCTUATING ENERGY ($\frac{3}{2}\Theta = \frac{1}{2} \langle C^2 \rangle$) EQUATION

$$\frac{3}{2} \left[\frac{\partial}{\partial t} (\epsilon_s \rho_s \Theta) + \nabla \cdot (\epsilon_s \rho_s v_s \Theta) \right] = \bar{\bar{\tau}}_s : \nabla v_s - \nabla \cdot q - \gamma \quad (4.12)$$

Collisional Energy Dissipation γ

$$\gamma = 3(1 - e^2) \epsilon_s^2 \rho_s g_o \Theta \left(\frac{4}{d_p} \left(\frac{\Theta}{\pi} \right)^{\frac{1}{2}} - \nabla \cdot v_s \right) \quad (4.13)$$

Flux of Fluctuating Energy q

$$q = -\kappa \nabla \Theta \quad (4.14)$$

Conductivity of Fluctuating Energy

$$\kappa = \frac{2}{(1 + e)g_o} \left[1 + \frac{6}{5} (1 + e) g_o \epsilon_s \right]^2 \kappa_{dil} + 2\epsilon_s^2 \rho_p d_p g_o (1 + e) \left(\frac{\Theta}{\pi} \right)^{\frac{1}{2}} \quad (4.15)$$

Dilute Phase ("Eddy Type") Granular Conductivity

$$\kappa_{dil} = \frac{75}{384} \sqrt{\pi} \rho_p d_p \Theta^{\frac{1}{2}} \quad (4.16)$$

GAS - SOLID DRAG COEFFICIENTS

For $\epsilon_s < 0.8$ (based on Ergun equation)

$$\beta = 150 \frac{\epsilon_s^2 \mu_g}{\epsilon d_p^2} + 1.75 \frac{\rho_g \epsilon_s |v_g - v_s|}{d_p} \quad (4.17)$$

For $\epsilon_s > 0.8$ (based on empirical correlation)

$$\beta = \frac{3}{4} C_d \frac{\epsilon \epsilon_s \rho_g |v_g - v_s|}{d_p} \epsilon^{-2.65} \quad (4.18)$$

where

$$C_d = \frac{24}{Re_p} \left[1 + 0.15 Re_p^{0.687} \right], \text{ for } Re_p < 1000 \quad (4.19)$$

$$C_d = 0.44, \text{ for } Re_p > 1000 \quad (4.20)$$

$$Re_p = \frac{\epsilon \rho_g |v_g - v_s| d_p}{\mu_g} \quad (4.21)$$

4.4 Initial and Boundary Conditions

The governing equations shown in the previous section require appropriate boundary conditions for the gas phase pressure, gas and solids velocities and for the granular temperature. The initial bed height and porosity are prescribed along with the inlet gas velocity and pressure. At the outlet, the mass flux is assumed to be continuous.

At the impenetrable solid wall, a no-slip boundary condition is prescribed for the gas phase velocities in the two directions which are both zero. The no-slip boundary condition cannot however, always be applied to the solids motion, since the particle diameter is usually larger than the length scale of surface roughness of

the rigid wall. The boundary conditions used here are given by Ding and Gidaspow (1990), with the slip parameter determined by

$$\lambda_p = \frac{d_p}{\epsilon_s^{\frac{1}{3}}} \quad (4.22)$$

The boundary condition for the granular temperature is given by assuming the energy flux at the wall to be zero.

4.5 Numerical Scheme

The governing equation along with the constitutive equations are solved for the pressure, gas and particle velocities, porosity and granular temperature, which are differenced based on the IMF (implicit multifield) solution method (Harlow and Amsden, 1975), which is an extension of the ICE (implicit continuum Eulerian) technique. This method differences the continuity equation as well as the implicit gas pressure gradient and the partial implicit coupling between the phases in the momentum equation, fully implicitly. The momentum and granular temperature equations are explicitly differenced, except for the energy production and dissipation terms of the granular temperature equation, which are differenced implicitly. The difference equations of the continuity, momentum and granular temperature are written as

$$\frac{(\epsilon_k \rho_k)^{n+1} - (\epsilon_k \rho_k)^n}{\delta t} + \nabla \cdot (\epsilon_k \rho_k \vec{v}_k)^{n+1} = 0 \quad (4.23)$$

$$\frac{(\epsilon_k \rho_k \vec{v}_k)^{n+1} - (\epsilon_k \rho_k \vec{v}_k)^n}{\delta t} + \nabla \cdot (\epsilon_k \rho_k \vec{v}_k \vec{v}_k)^n = -(\epsilon_k \nabla P_g)^{n+1} + \epsilon_k \rho_k g^n + \nabla \cdot \bar{\bar{\tau}}_k^n + \beta^n (\vec{v}_l - \vec{v}_k)^{n+1} \quad (4.24)$$

$$\frac{3}{2} \left[\frac{(\epsilon_s \rho_p \Theta)^{n+1} - (\epsilon_s \rho_p \Theta)^n}{\delta t} + \nabla \cdot (\epsilon_s \rho_p \vec{v} \Theta)^n \right] = \bar{\bar{\tau}}_s^{n+1} : \nabla v_s^{n+1} - \nabla \cdot q^n - \gamma^{n+1} \quad (4.25)$$

The computations are carried out using donor cell differencing in a staggered mesh of finite difference cells. In a staggered difference cell, all scalar variables are defined in the cell center and the vector variables are defined at the boundaries.

The equations are written in the form found in the K-FIX computer code (Rivard and Torrey, 1977) for gas - liquid flow, as developed by Ettehadieh (1982) and Syamlal (1985) for gas - solid fluidization. The program used in this study is based on Syamlal's (1985) computer code which was modified by using variable grid size and by adding the kinetic theory model (Ding, 1990). The use of non-uniform grid saves storage space and computation time. In addition, we have used fine grids for regions where the physical variables change sharply and coarse grids when they change smoothly. For accuracy the ratio between two adjacent increments of grid size should be less than two. In this computation a fine grid is used near the walls of the riser, and gradually increased to a maximum cell size of 30 cm at the riser center. The time step is chosen according to the Courant stability criteria, which is given as

$$\partial t < \frac{\min (\partial r_i, \partial z_j)}{\max (\text{velocity})} \quad (4.26)$$

Ding's (1990) version of the code has been modified for this computation by including the extended kinetic theory model for dilute flow (Gidaspow, 1991). The changes in the code involved a modification of the kinetic theory model in the relevant subroutines, which are listed in the appendix. The fluctuating energy equation is solved for the granular temperature by assuming that the granular temperature is zero when the energy production equals the energy dissipation.

4.6 Results and Discussions

Simulation of two phase flow in the Pyropower CFB shown in figure 23, was carried out using the simplified geometry shown in figure 24. The bed is assumed

to be isothermal and running at room temperature. The inlet conditions are shown in Table 1. The top of the standpipe is open to the atmosphere, but protected by a screen to prevent particles from leaving the system. The non-uniform finite difference grid used for the computation is shown in figure 25. The time step used was 5×10^{-4} seconds.

Table 1. Inlet Conditions

Diameter of particles	=	150 μm
Density of particles	=	1714 Kg/m^3
Terminal velocity of particles	=	1.1 m/s
Gas flow rate at bottom of standpipe (Minimum fluidization velocity)	=	2.5 cm/s
Air inlet (riser) velocity	=	2 m/s
Initial bed height	=	4.5 m
Initial porosity	=	0.42

4.6.1 Flow Behavior. The variables solved for were pressure, gas and solid velocities in the radial and axial directions, volume fraction, viscosity and granular temperature. Figures 26, 27 and 28 show the particle distributions in the CFB from 1 to 12 seconds, every second. The plots indicate that it takes approximately 4 seconds for the particles to reach the top of the riser for an inlet gas velocity of 2 m/s. At 5 seconds, the flow in the riser is considered to be developed, and the particle distribution plot shows a dilute core, which is consistent with the observations made in Miller's CFB (1991). The distributions however, are not steady and fluctuate periodically with time, as with the discharge from the standpipe to the riser. A closer look at figure 27 and 28, shows the dilute region in the lower section of the riser, at the 7th second travels upward during the 8th and 9th second, and at the 10 th second, starts again from the bottom.

Figure 23. The PYROFLOW Circulating Fluidized Bed Combustor

Figure 24. Simplified Full-Scale PYROFLOW CFB with Dimensions and Flow Conditions (Gidaspow, Ding and Jayaswal, 1990).

Figure 25. The Non-Uniform Finite Difference Grid used for Computation

Figures 29, 30 and 31 are plots of the solid velocity vectors in the CFB from 1 to 12 seconds. Since the particles take approximately 4 seconds to reach the top of the riser and exit to the standpipe, the velocity vectors in the riser section in figure 29 are all in the upward direction. After reaching the top of the riser however, the particles start sliding down along the wall. The downflow is predominantly along the right wall of the riser where the exit and inlet are, which leads to highly asymmetric radial profiles, which are more pronounced at the low inlet gas velocity.

Figures 32, 33 and 34 are radial time averaged (over the period from 5 to 10 seconds) profiles of the solids velocity, mass flux, and volume fraction respectively. The radial solids volume fraction profile (figure 34) at a riser height of 10 m, shows that the left half of the riser is relatively dilute, compared to the right half, particularly in the region adjacent to the wall. The denser region along the right wall is obviously due to the location of the exit.

The radial solids velocity and mass flux profiles (figures 32 and 33) also show that the asymmetry of radial profiles is pronounced due to the low inlet gas flow rate. At a higher inlet gas velocity of 3 m/s the profile is somewhat parabolic, with the maximum closer to the center of the riser, as one would expect for developed flow, although the downward flux (represented by negative values) is still greater along the right wall of the riser, because of the location of the exits.

Figures 35 and 36 are the instantaneous radial profiles of the radial and axial components of solids velocity at a riser height of 10 m, from 5 to 10 seconds. Although the radial (u component) velocity is quite negligible compared to the axial (v component) velocity, figure 35, shows high velocities in the left section of the riser at 5 seconds. This corresponds to a vortex like formation which can be seen in the 5th second of the particle velocity vector plot in figure 30. This vortex like formation can also be seen at 4 seconds in figure 29, at a position much lower in the riser height.

Figure 36 shows instantaneous asymmetric radial profiles of the axial solids velocity with downflow along the right wall of the riser. The downflow along the right wall however, seems to fluctuate with time and at 9 seconds the velocity adjacent to the wall is almost zero. This fluctuation is captured in figure 37, which is a plot of the axial solids velocity at the location adjacent to the right wall of the riser, at 10 m, as a function of time. It shows that the downflow starts after 4 seconds, i.e, after the particles reach the top of the riser. It is interesting to see that between 8 and 9 seconds, and again at 13 seconds the velocity is almost zero, indicating a somewhat periodic turbulent slug like behavior which has been observed in Miller's CFB (1991) experiment. The fluctuation is probably dependent on the discharge of solids from the standpipe to the riser.

Figure 38 and 39 are plots of the solids discharge velocity and solids flux from the standpipe to the riser as a function of time. The negative velocities and fluxes indicate that the particles from the standpipe move down into the riser (discharge) and the positive values indicate an upward flow. The solids are first recirculated after 6 seconds, prior to which, the particles actually move up the standpipe inlet due to initial effects. After 7 seconds the discharge rates are of the order of 1 m/s which is consistent with the measured data of solids discharge from hoppers (Gidaspow et al.,1986). The discharge rate is somewhat periodic, approximately every 4 - 5 seconds, because of the time required for recirculation.

A contour plot of the pressure drop along the riser is shown in figure 40. The pressure decreases along the riser height as would be expected. In the radial direction however, there is an increase in pressure along the right wall of the riser, which again is caused by the asymmetry in the location of the exits.

4.6.2 Comparison of the Present Model with Ding's Model. The results of the simulation of the CFB using the new kinetic theory model, described above in figures 32 to 39 also show a comparison of the results obtained by using

Figure 26. Particle Distribution (1 to 4 seconds)

Figure 27. Particle Distribution (5 to 8 seconds)

Figure 28. Particle Distribution (9 to 12 seconds)

Figure 29. Particle Velocity (1 to 4 seconds)

Figure 30. Particle Velocity (5 to 8 seconds)

Figure 31. Particle Velocity (9 to 12 seconds)

Figure 32. Time Averaged (6 to 10 seconds) Radial Solids Velocity Profile at 10 m

Figure 33. Time Averaged (6 to 10 seconds) Radial Mass Flux Profile at 10 m

Figure 34. Time Averaged (6 to 10 seconds) Radial Solids Volume Fraction Profile at 10 m

Figure 35. Instantaneous Radial Particle (U Component) Velocity Profiles at a Riser Height of 10 m

Figure 36. Instantaneous Radial Particle (V component) Velocity Profiles at a Riser Height of 10 m

Figure 37. Solids Velocity at the Right Wall of the Riser at a Height of 10 m

Figure 38. Solids Discharge Rate from the Standpipe to the Riser as a Function of Time

Figure 39. Solids Mass Flux from the Standpipe to the Riser as a Function of Time

Figure 40. Pressure Contours at 8 seconds

Figure : Area Averaged Axial Solids Volume Fraction along the Riser Height

Figure 41. Area Averaged Axial Solids Volume Fraction along the Riser Height

the earlier dense phase kinetic theory model developed by Ding, which included a turbulence model to account for the effect of gas phase turbulence in dilute flows.

Figure 41 is a plot of the solids volume fraction, averaged over the cross section area, along the riser height which indicates that the flow is developed only after a riser height of 500 cm. A comparison of the profiles obtained from the present model and that used by Ding's show that although the computed values are not the same, the trends are similar and both profiles show developed flow only after a riser height of 7 m. A comparison of the radial solids volume fraction profile (figure 34) shows that both models predict a denser region on the right wall of the riser. Both models also predict similar asymmetric radial mass flux profiles (figure 33) and the standpipe discharge velocities (figure 38) show similar fluctuations, with an approximately steady discharge rate of 1 m/s.

The comparisons made above lead us to the conclusion that the new model predicts flow behavior similar to that of Ding's and satisfactorily accounts for the dilute limit without the need of an extraneous model for gas phase turbulence. The main difference between the two models is in predicting the solids viscosity. The kinetic theory model of Gidaspow (1991) gives more realistic predictions of solids viscosity as evidenced by experimental data.

4.6.3 Computation of Solids Viscosity. Figures 42, 43 and 44 are plots showing the solid viscosity contours in the CFB from 5, to 10 seconds. The range of solid viscosity predicted by the present model is between 10^{-1} to 10^{-3} Poise, which is more realistic than the values computed by the earlier model used by Ding (10^{-6} Poise). The predicted solids viscosity is quite sensitive to the restitution coefficient, as can be seen in figure 45, which shows radial profiles of the solids viscosity at a height of 10 m with varying values of restitution coefficient. As the restitution coefficient approaches unity the model predicts higher viscosities, which are comparable to the viscosities measured by experimentally by Miller (of the

order of 10^{-2} Poise). Miller's (1991) viscosities however, provides only a qualitative comparison of the solids viscosity since his system, as well as the operating conditions are different from the pyropower CFB.

The sensitivity to the restitution coefficient is not well understood, and actual measurement of the restitution coefficient would provide us with a better explanation. However, it is clear that the restitution coefficient depends on the particle size and, since the particles used by Miller (1991) in his CFB are smaller than the particles used in this computation, the restitution coefficient has to be used as a fitting parameter in order to obtain a match between the viscosities calculated from experimental measurements and from our code.

The radial solids viscosity profiles shown in figure 45, also indicate that the solids viscosity is almost constant in core of the riser, similar to Miller's (1991) radial solid viscosity profiles. This can be explained by the fact that since volume fraction in the core of the riser is constant, the mean free path is constant, and the granular temperature which is also a constant, results in a constant solids viscosities in the core of the riser.

Figures 46, 47 and 48 are contours plots of the granular temperature in the CFB from 5 to 10 seconds. The high temperature regions seen in these plots correspond to the high viscosity regions in the viscosity contour plots since viscosity is calculated from the granular temperature.. The high granular temperature and viscosity regions are observed to start at a lower portion of the riser and rise up at a later time, with a repeating pattern approximately every 2 seconds, as can be seen in the 8, 9 and 10 second contours in figure 43 and 44 and in figure 47 and 48. The high viscosity/granular temperature regions are also seen to correspond to relatively dilute regions in the particle distribution plots and turbulent vortex like regions in the velocity vector plots. The granular temperature computed by the present model is very high when the flow is dilute and when the velocity is high, which occurs due

to larger oscillations of particles. Figure 49 and 50 show the particle distribution, the viscosity contours, and the granular temperature contours at 4 seconds and 8 seconds respectively. These plots clearly mark out the high viscosity and high granular temperature regions correspond to the dilute ($\epsilon_s \approx 0.02$) region. A rough generalized plot of the solids viscosity as a function of solids volume fraction (figure 51) points out a minimum value of viscosity around a volume fraction of 0.09, and a further decrease in the volume fraction results in a sharp increase of the viscosity. This is similar to predictions made by Lun et al (1984) for simple shear flow, which according to his explanation was due to the kinetic stress contribution becoming dominant. According to Gidaspow (1991), under dilute conditions the mean free path for particle oscillations is larger, which results in large granular temperatures and hence high solids viscosities.

4.7 Conclusions

The results discussed above show that our computer code with the modified kinetic theory model for the stress term can predict realistic values of solid viscosity for a complete range of concentration. The flow behavior is quite similar to that obtained by using Ding's model, which implies that the new model accounts for the effect of gas phase turbulence on particle collisions reasonably well without the need of an extraneous model for turbulence.

The code is also able to predict flow oscillations and periodic fluctuations that are observed in real systems.

Figure 43. Solids Viscosity (Poise) Contours at 7 and 8 seconds

Figure 44. Solids Viscosity (Poise) Contours at 9 and 10 seconds

Figure 45. Effect of Restitution Coefficient on Time Averaged (6 to 10 seconds)
Radial Solids Viscosity Profiles at 7.5 m

Figure 46. Granular Temperature (cm^2/s) Contours at 5 and 6 seconds

Figure 47. Granular Temperature (cm^2/s) Contours at 7 and 8 seconds

Figure 48. Granular Temperature (cm^2/s) Contours at 9 and 10 seconds

Figure 50. Particle Distribution, Viscosity Granular Temperature and Solids viscosity at Time = 4 seconds

Figure 49. Particle Distribution, Viscosity, Granular Temperature and Solids viscosity at Time = 8 seconds

Figure 51. Solids Viscosity as a Function of Volume Fraction

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PROGRAM IFAP
C
C*****
C      IFAP v.3 = ISOTHERMAL FLOW ANALYSIS PROGRAM 2-D      *
C      .....*
C      Developed at Illinois Institute of Technology, Chicago, IL 60616 *
C      by JIANMIN DING *
C*****
C      Modifications .....by RUKMINI BEZBARUANI      MAR 18,1991
C      To use the corrected expressions for MUS and CAPA (KINETIC THEORY)
C      IN SUBR VISCS .... BULK MUS AND P_K TERMS - XIMU
C      SHEAR MUS (NEW)
C      IN SUBR KENGYN .... BX2,BZ2,BX1,BZ1 TERMS - CAPA INSTEAD OF C      MUS*EPS
C      IN SUBR KENMUS .... B0,B1,B3,B4,B5,BEQ2,XIMU,CAPA,MUS-ADDN OF C      DEL TERM
C      IN SUBR TAPEWR,TAPERD .... READ,WRITE XIMU,CAPA
C      ALSO STORE CAPA,XIMU IN DCOM.LIQ
C      .....
C      SUBR KENMUS ..... BEQ2 = (2/3 MU - XIMU) - 2 MU      MAR 26,1991
C      ..... DELETE 3 BETA TERM IN FLUC EN EQ      APR 02,1991
C*****
C
C===== > MAIN PROGRAM
C===== > VISCS
      SUBROUTINE VISCS
      INCLUDE 'dcom.liq'
C .....:
C Calculate solid phase stresses      :
C .....:
      DO 10 J=2,JB1
      DO 10 I=2,IB1
        IJ=I+(J-1)*IB2
        IF(FL(IJ).NE.1) GOTO 10
        CALL SUBSC
        DRP=(DR(I)+DR(I+1))
        DRM=(DR(I)+DR(I-1))
        DZP=(DZ(J)+DZ(J+1))
        DZM=(DZ(J)+DZ(J-1))
        TXX2=2.*MUS(K,IJ)/DR(I+1)*(UK(K,IPJ)-UK(K,IJ))
        TXX1=2.*MUS(K,IJ)/DR(I)*(UK(K,IJ)-UK(K,IMJ))
        TXY2=(UK(K,IPJ)-UK(K,IJ))/DZP*2.+(VK(K,IPJ)-VK(K,IJ))/DRP*2.
        TXY1=(UK(K,IJ)-UK(K,IMJ))/DZM*2.+(VK(K,IPJ)-VK(K,IMJ))/
        & DRP*2.
        TYY2=2.0*MUS(K,IJ)*(VK(K,IPJ)-VK(K,IJ))/DZ(J+1)
        TYY1=2.0*MUS(K,IJ)*(VK(K,IJ)-VK(K,IMJ))/DZ(J)
        TXY2=TXY2
        TXY1=(UK(K,IMJ)-UK(K,IMJ))/DZP*2.+(VK(K,IJ)-VK(K,IMJ))/DRM*2.
        B1=2.5*SQRT(EKI(K,IJ))*1.7725/DK(K)

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PXX2=-RLK(K,IJR)*EKI(K,IJR)-XIMU(K,IJR)/1.6667*B1
TXX2=TXX2+(XIMU(K,IJR)-0.6667*MUS(K,IJR))*
& ((RB(I+1)*UK(K,IPJ)-RB(I)
& *UK(K,IJ))/R(I+1)/DR(I+1)+(VK(K,IPJ)-VK(K,IPJM))/DZ(J))
B1=2.5*SQRT(EKI(K,IJ))*1.7725/DK(K)
PXYC=-RLK(K,IJ)*EKI(K,IJ)-XIMU(K,IJ)*B1/1.6667
TXYC=(XIMU(K,IJ)-0.6667*MUS(K,IJ))*
& ((RB(I)*UK(K,IJ)-RB(I-1)
& *UK(K,IMJ))/R(I)/DR(I)+(VK(K,IJ)-VK(K,IJM))/DZ(J))
TXX1=TXX1+TXYC
IF(ITC.EQ.1) THEN
GMUM=(DR(I)*MUS(K,IJR)*RLK(K,IJR)
& +DR(I+1)*MUS(K,IJ)*RLK(K,IJ))/DRP/RL(K)
T0=GMUM*UK(K,IJ)/RB(I)
C *** ADD ***
UKM2=0.5*(RB(I+1)*UK(K,IPJ)+RB(I)*UK(K,IJ))
UKM1=0.5*(RB(I)*UK(K,IJ)+RB(I-1)*UK(K,IMJ))
VKM2=(DR(I)*VK(K,IPJ)+DR(I+1)*VK(K,IJ))/DRP
VKM1=(DR(I)*VK(K,IPJM)+DR(I+1)*VK(K,IJM))/DRP
T0=T0+GMUM*((UKM2
1 -UKM1)*2./DRP/RB(I)+(VKM2-VKM1)/DZ(J))
ENDIF
B1=2.5*SQRT(EKI(K,IJT))*1.7725/DK(K)
PYY2=-RLK(K,IJT)*EKI(K,IJT)-B1*XIMU(K,IJT)/1.6667
TYY2=TYY2+(XIMU(K,IJT)-0.6667*MUS(K,IJT))
& *((RB(I)*UK(K,IJP)-RB(I-1)
1 *UK(K,IMJP))/R(I)/DR(I)+(VK(K,IJP)-VK(K,IJ))/DZ(J+1))
TYY1=TYY1+TXYC
IF(ICOH.EQ.1) THEN
TAUC1=10.**(-10.6*EP(IJ)+8.6)
TAUC2=10.**(-10.6*EP(IJR)+8.6)
TXX2=TXX2+TAUC2
TXX1=TXX1+TAUC1
TAUC2=10.**(-10.6*EP(IJT)+8.6)
TYY2=TYY2+TAUC2
TYY1=TYY1+TAUC1
ENDIF
VMU21=(DZ(J)*MUS(K,IJT)+DZ(J+1)*MUS(K,IJ))/DZP
VMU22=(DZ(J)*MUS(K,IJTR)+DZ(J+1)*MUS(K,IJR))/DZP
VMU2=(DR(I+1)*VMU21+DR(I)*VMU22)/DRP
VMU11=(DZ(J-1)*MUS(K,IJ)+DZ(J)*MUS(K,IJB))/DZM
VMU12=(DZ(J-1)*MUS(K,IJR)+DZ(J)*MUS(K,IJB))/DZM
VMU1=(DR(I+1)*VMU11+DR(I)*VMU12)/DRP
PVISX(K,IJ)=(PXX2-PXYC)/DRP*2.+(TXX2*R(I+1)
1 -TXX1*R(I))/DRP*2./RB(I)+(VMU2*TXX2-VMU1*TXX1)/DZ(J)-T0/RB(I)
VMU11=(DZ(J)*MUS(K,IJTL)+DZ(J+1)*MUS(K,IJL))/DZP
VMU1=(DR(I)*VMU11+DR(I-1)*VMU21)/DRM

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      PVISZ(K,IJ)=(RB(I)*VMU2*TX Y2-RB(I-1)*VMU1*TX Y1)
      1 /R(I)/DR(I)+(PYY2-PXYC)/DZP*2.
      & +(TTY2-TYY1)/DZP*2.
10  CONTINUE
      RETURN
      END
C=====>
      SUBROUTINE KENGYN
      INCLUDE 'decom.liq'
C:.....:
C  Calculate the previous time step values in energy equation :
C:.....:
      DO 500 K=1,NSOLID
      DO 200 J=2,JB1
      DO 200 I=2,IB1
        IJ=I+(J-1)*IB2
        IF(FL(IJ).NE.1)GOTO 200
        CALL SUBSC
        F11=UK(K,IJ)*EKI(K,IJ)*RLK(K,IJ)
        IF(UK(K,IJ).LT.0.0)F11=UK(K,IJ)*EKI(K,IJR)*RLK(K,IJR)
        F12=UK(K,IMJ)*EKI(K,IJL)*RLK(K,IJL)
        IF(UK(K,IMJ).LT.0.0)F12=UK(K,IMJ)*EKI(K,IJ)*RLK(K,IJ)
        F1=(F11*RB(I)-F12*RB(I-1))/DR(I)/R(I)
        F21=VK(K,IJ)*EKI(K,IJ)*RLK(K,IJ)
        IF(VK(K,IJ).LT.0.0)F21=VK(K,IJ)*EKI(K,IJT)*RLK(K,IJT)
        F22=VK(K,IJM)*EKI(K,IJB)*RLK(K,IJB)
        IF(VK(K,IJM).LT.0.0)F22=VK(K,IJM)*EKI(K,IJ)*RLK(K,IJ)
        F2=(F21-F22)/DZ(J)
        DZP=DZ(J)+DZ(J+1)
        DZM=DZ(J)+DZ(J-1)
        DRP=DR(I)+DR(I+1)
        DRM=DR(I)+DR(I-1)
        BX2=(DR(I)*CAPA(K,IJR)+DR(I+1)*CAPA(K,IJ))/DRP
        BZ2=(DZ(J)*CAPA(K,IJT)+DZ(J+1)*CAPA(K,IJ))/DZP
        BX1=(DR(I)*CAPA(K,IJL)+DR(I-1)*CAPA(K,IJ))/DRM
        BZ1=(DZ(J)*CAPA(K,IJB)+DZ(J-1)*CAPA(K,IJ))/DZM
        D11=(RB(I)*BX2*(EKI(K,IJR)-EKI(K,IJ))/DRP*2.-
        & RB(I-1)*BX1*(EKI(K,IJ)-
        & EKI(K,IJL))/DRM*2.)/DR(I)/R(I)
        D12=(BZ2*(EKI(K,IJT)-EKI(K,IJ))/DZP*2.-BZ1*(EKI(K,IJ)-
        & EKI(K,IJB))/DZM*2.)/DZ(J)
        DDT=D11+D12
        REK(K,IJ)=1.5*EKI(K,IJ)*RLK(K,IJ)-DT*(1.5*(F1+F2)-DDT)
200  CONTINUE
500  CONTINUE
      RETURN
      END

```

```

C=====>
      SUBROUTINE KENMUS
      INCLUDE 'decom.liq'
C.....:
C  Compute collisional kinetic energy and then solid viscosity :
C.....:
      DO 500 K=1,NSOLID
      DO 100 J=2,JB1
      DO 100 I=2,IB1
        IJ=I+(J-1)*IB2
        IF(FL(IJ).NE.1) GOTO 100
        CALL SUBSC
        DRP=DR(I)+DR(I+1)
        DRM=DR(I)+DR(I-1)
        DZP=DZ(J)+DZ(J+1)
        DZM=DZ(J)+DZ(J-1)
        D11=(UK(K,IJ)-UK(K,IMJ))/DR(I)
        D22=(VK(K,IJ)-VK(K,IMJ))/DZ(J)
        IF(ITC.EQ.1)D33=(UK(K,IJ)+UK(K,IMJ))/R(I)
        D1C=(RB(I)*UK(K,IJ)-RB(I-1)*UK(K,IMJ))/DR(I)/R(I)
        & +(VK(K,IJ)-VK(K,IMJ))/DZ(J)
        U3=0.5*(RB(I)*UK(K,IJP)+RB(I-1)*UK(K,IMJP))/R(I)
        U2=0.5*(RB(I)*UK(K,IJ)+RB(I-1)*UK(K,IMJ))/R(I)
        U1=0.5*(RB(I)*UK(K,IJM)+RB(I-1)*UK(K,IMJM))/R(I)
        V3=0.5*(VK(K,IJP)+VK(K,IPJM))
        V2=0.5*(VK(K,IJ)+VK(K,IMJ))
        V1=0.5*(VK(K,IMJ)+VK(K,IMJM))
        UM2=U2+(U3-U2)*DZ(J)/DZP
        UM1=U1+(U2-U1)*DZ(J-1)/DZM
        VM2=V2+(V3-V2)*DR(I)/DRP
        VM1=V1+(V2-V1)*DR(I-1)/DRM
        D12=0.5*((UM2-UM1)/DZ(J)+(VM2-VM1)/DR(I))
C  IF(ISWIR.EQ.1) THEN
C  W2=(DR(I)*WK(K,IJR)+DR(I+1)*WK(K,IJ))/DRP
C  W1=(DR(I)*WK(K,IJL)+DR(I-1)*WK(K,IJ))/DRM
C  D13=(W2-W1)/DR(I)-WG(IJ)/R(I)
C  W2=(DZ(J)*WK(K,IJT)+DZ(J+1)*WK(K,IJ))/DZP
C  W1=(DZ(J)*WK(K,IJB)+DZ(J-1)*WK(K,IJ))/DZM
C  D23=(W2-W1)/DZ(J)
C  ENDIF
      EPS=RLK(K,IJ)/RL(K)
      IF(EPS.LT.EPSMX0)THEN
        GEPS0=1.0/(1.-(EPS/EPSMX0)**(1./3.))
      ELSE
        GEPS0=2000.
      ENDIF
      B0=(1.+CRES)*GEPS0

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      B1=RLK(K,IJ)*B0
      B2=B1*DK(K)/1.7725
      B3=0.1846*RL(K)*DK(K)/B0*(1.+0.8*B1/RL(K))**2.
      B4=EPS*B2*0.8
      B5=0.6923*RL(K)*DK(K)/B0*(1.+1.2*B1/RL(K))**2.
      DISS=3.*(1.-CRES**2.)*RLK(K,IJ)**2./RL(K)*GEPS0
      & *(2.2568*SQRT(EKI(K,IJ))/DK(K)-D1C)
C    DISS=DISS+3.*KPGV(K,IJ)
      AEQ2=1.5*RLK(K,IJ)+DT*(2.*B1*EPS+RLK(K,IJ))*D1C+DT*DISS
      BEQ2=DT*((0.6667*(B3+B4)-1.6667*B4)*D1C*D1C-2.*(B3+B4)*
      & (D11*D11+D22*D22+D33*D33+
      & 2.*D12*D12+2.*D13*D13+2.*D23*D23))
      CEQ2=-REK(K,IJ)
      IF(EPS.LE.RLIM) GOTO 999
      DELTA=BEQ2**2-4.*AEQ2*CEQ2
      IF(DELTA.LT.0.) DELTA=0.
      EK1(K,IJ)=((-BEQ2+SQRT(DELTA))/2./AEQ2)**2.
C    EK1(K,IJ)=(CEQ2/BEQ2)**2.
      GOTO 11
999  EK1(K,IJ)=0.
11   MUS(K,IJ)=(B3+B4)*SQRT(EK1(K,IJ))
      XIMU(K,IJ)=1.6667*B4*SQRT(EK1(K,IJ))
      CAPA(K,IJ)=(B5+2.5*B4)*SQRT(EK1(K,IJ))
100  CONTINUE
500  CONTINUE
      RETURN
      END

```

APPENDIX

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