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FEEDBACK CONTROL OF RESISTIVE INSTABILITIES

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FEEDBACK CONTROL OF RESISTIVE INSTABILITIES

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ABSTRACT

Resistive instabilities are responsible for much of the global behavior and the determination of the possible domains of operation of tokamaks. Their successful control could have definite advantages, even making available new regimes of operation. Elimination of sawtoothing might allow operation with higher currents and more peaked current profiles, with q on axis well below unity. In this work different feedback schemes are explored. Simple analytical derivations of the effects of local heating and current drive feedback are presented. Although control of modes with $m \geq 2$ is fairly straightforward, the control of the $m = 1$ mode is more difficult because of its proximity to ideal instability. The most promising scheme utilizes high energy trapped particles.

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I. INTRODUCTION

The nonlinear evolution of single helicity, large-scale resistive tearing modes in low-beta tokamaks is well understood.¹⁻⁷ The dynamics is conveniently described in terms of the helical flux, ψ , with the magnetic field given by

$$\hat{B} = \hat{r} \psi \times \hat{z} - \frac{nr}{mR} \hat{\theta} + \hat{z}, \quad (1)$$

where we have normalized the field to unity on the toroidal axis; r, θ, z are toroidal coordinates; and m, n define the helicity m/n of the flux ψ . The $m = 0$ component of the helical flux is related to the q profile and the current profile through

$$\psi'_0 = r \left(\frac{1}{q} - \frac{n}{m} \right) \quad (2)$$

and

$$j_0 = -\frac{2}{q} + \frac{rq'}{q^2}. \quad (3)$$

The unit of distance is the minor radius, and primes indicate derivatives with respect to r . In the coordinates used the current density is negative, but we will express final results in terms of the q profile, which is a convenient unambiguous dimensionless quantity.

The MHD equations in cylindrical geometry used here are derived in Ref. 7. Ohm's law results in an equation for the time evolution of ψ . The quasilinear analysis of this equation uses the eigenfunctions of the linear theory, and neglects the effect of higher harmonics on the nonlinear

evolution. These approximations are quite good, and simulations of the nonlinear evolution of the tearing mode using the full resistive magnetohydrodynamic (MHD) equations³⁻⁵ are in close agreement with the analytic treatments,^{1,2,6} up to the point at which modes of different helicity interact, as in the final stages of a major tokamak disruption.

In Sec. II the quasilinear mode development is discussed. Section III treats proposed feedback stabilization mechanisms in light of the quasilinear treatment. Section IV consists of a concluding summary.

II. QUASILINEAR MODE DEVELOPMENT

The dynamical behavior of ψ follows from Faraday's law

$$\frac{\partial \psi}{\partial t} + \vec{v} \cdot \vec{\nabla} \psi = -n(j - j_d) \quad , \quad (4)$$

where $\vec{v}$ is the velocity describing the incompressible fluid flow, $\vec{v} = -\nabla \phi \times \hat{z}$, ϕ the stream functions, and j_d is an externally supplied, driven current. The current j is also related to ψ through Ampere's law

$$j = -\nabla^2 \psi - \frac{2n}{m} \quad . \quad (5)$$

Time evolution of the velocity potential ϕ is given by the momentum equation

$$\rho \frac{d}{dt} \nabla^2 \phi = \hat{z} (\nabla \psi \times \nabla j) \quad . \quad (6)$$

Linear eigenfunctions are obtained by solving these equations for a perturbation of a single helicity

$$\psi = \psi_0(r) + \psi_1(r, t) \cos(m\theta) \quad , \quad (7)$$

$$\phi = \phi_1(r, t) \sin m\theta \quad . \quad (8)$$

We are interested in the domain near the rational surface, but outside the tearing layer, and in magnetic islands which are large compared to the tearing layer, which is the width at which nonlinear effects begin to play a role. The tearing layer width is given by $W_T = \sqrt{2}(\eta r_s / -\psi_0'')^{1/3}$ for the $m = 1$ mode, and $W_T = (\eta r_s / -\psi_0'')^{2/5}$ for $m \geq 2$. In this region the linear eigenfunctions have the form

$$\psi_1(x) = \begin{cases} -bx\theta(-x) & m = 1 \\ \psi_1 & m \geq 2 \end{cases} \quad , \quad (9)$$

$$\phi_1 = \begin{cases} \sqrt{2} b W_T \theta(-x) & m = 1 \\ r_s \gamma \psi_1 / m \psi_0'' & m \geq 2 \end{cases} \quad , \quad (10)$$

where $x = r - r_s$, r_s is the rational surface; $q(r_s) = m/n$. Because of the linear behavior of ψ_1 near the rational surface for the $m = 1$ mode, the 0-point is shifted from this surface by an amount Δ , with $\Delta = b/\psi_0'' < 0$. Within the constant ψ approximation¹ the 0-point does not shift for $m \geq 2$.

The islands produced by the perturbation have widths $W = b/(r_s \psi_0'')$ for $m = 1$ and $W = 4(\psi_1 / -\psi_0'')^{1/2}$ for $m \geq 2$, provided $1 \gg W \gg W_T$. To obtain the quasilinear evolution of the mode, the convection term is eliminated by averaging Eq. (1) over a flux surface, giving

$$\left\langle \frac{\partial \psi}{\partial t} \right\rangle = -\langle \eta(j - j_d) \rangle, \quad (11)$$

where $\langle f \rangle = \oint (dl/\nabla\psi) f / \oint (dl/\nabla\psi)$. In the case of $m \geq 2$ the inertia can be neglected in Eq. (6), implying that the current is a flux function, $j = j(\psi)$. In the case of $m = 1$ the inertia cannot be neglected, but the nonlinear behavior can be calculated⁶ without making any assumptions about the form of j .

The analysis is significantly different for $m = 1$ and $m \geq 2$ so we examine the two cases separately.

A. Nonlinear Behavior, $m \geq 2$

The use of the linear eigenfunctions, Eqs. (9,10) reduces the problem to a zero-dimensional determination of a function of time alone. It is common in the literature^{1,6} to perform multiple spatial integrals to determine this single function from Eq. (11). Since the spatial form of the quasilinear solution is fixed, this is completely equivalent to the use of particular flux surfaces which facilitate this evaluation. In any case the quasilinear treatment can only be regarded as approximate.

Evaluated at the 0- and x-points Eq. (11) gives

$$\dot{\psi}_0(r_0) + \dot{\psi}_1(r_0) = -\eta(\vec{r}_0)[j(\vec{r}_0) - j_d(\vec{r}_0)] \quad , \quad (12)$$

$$\dot{\psi}_0(r_x) - \dot{\psi}_1(r_x) = -\eta(\vec{r}_x)[j(\vec{r}_x) - j_d(r_x)] \quad . \quad (13)$$

In this case the current is not negligible, and must be evaluated at the x and 0-points, which lie within the tearing layer. By the constant ψ approximation, the exterior solutions can be used for ψ_1 in this region.

Using the narrowness of the island, $W \ll 1$, Eq. (5) and the form of ψ_1 , Eq. (9), we find $j(r_0) = j_0(r_0) - [\Delta'(W)\psi_1/W]\cos m\theta$ where $\Delta'(W) = [\psi_1'(W/2) - \psi_1'(-W/2)]/\psi_1(0)$, and similarly for $j(r_x)$. This approximation is in error by a factor of $\approx \pi/2$. (See Ref. 2.) Allowing perturbations of η and j_d of the form $\eta = \eta_0(r) + \eta_1 \cos m\theta$, $j_d = j_{d1} \cos m\theta$ we then find, on subtracting Eqs. (12, 13) and using $r_0 \approx r_x \approx r_s$,

$$\dot{\psi}_1 = \frac{\eta(r_s)\Delta'(W)\psi_1}{W} + \eta_0(r_0)j_{d1} - j_0\eta_1 \quad (14)$$

The first term describes the usual Rutherford algebraic growth¹ and saturation² of the mode, and the two additional terms describe the effects of local heating and current drive, and will be discussed later. (Note $j_0 < 0$ in our coordinate system.)

B. Nonlinear $m = 1$ Behavior

The $m = 1$ mode analysis differs because of the inward shift of the 0-point due to ψ_1 being linear in x . Because of the form of ψ_1 the $m = 0$ part contributes significantly through the inertial $v \cdot \nabla \psi$ term. Substitution of the linear eigenfunctions into Eq. (11) leads to an equation for $b(t)$,

$$\langle \dot{\psi}_0(x) \rangle = \dot{b} \langle x \cos \theta \rangle = -\langle \eta(j - j_d) \rangle \quad (15)$$

Now evaluate $\dot{\psi}_0$ quasilinearly, using the linear eigenfunctions. From Eq. (4) we have for the $m = 0$ component

$$\dot{\psi}_0 = \frac{-b^2 W_T}{\sqrt{2} r_s} - \eta_0 j_0 \quad (16)$$

The first term on the rhs is an inertial contribution, arising from $\mathbf{v} \cdot \nabla \psi$, and scales as $\eta^{1/3}$. Note that a similar evaluation of the $\mathbf{v} \cdot \nabla \psi$ term for modes with $m \geq 2$ gives zero within the constant ψ approximation. From the linear eigenfunctions the current j is independent of η and the current terms are thus negligible. Then we have, in the exterior region

$$\frac{b^2 W_T}{\sqrt{2} r_s} + \dot{b} \langle x \cos \theta \rangle = \langle \eta_1 j_o \cos \theta \rangle - \langle \eta_o j_{d1} \cos \theta \rangle \quad (17)$$

Just outside the island the flux surfaces can be approximated as shifted circles, with the shift at $\theta = 0$ twice that of the 0-point. Thus in this region $x = \Delta(1 + \cos \theta)$ on a flux surface. We then find $\langle x \cos \theta \rangle = \Delta/2 = b/2\psi_o''$ giving in the absence of local heating or current drive $\dot{b} = -\sqrt{2}\psi_o'' W_T b/r_s$, or exponential growth at the linear growth rate. This is what is observed in the simulations,^{3,4} that is, the mode continues at approximately the linear growth rate far into the nonlinear regime. Thus the inertia continues to play a role in the evolution of the $m = 1$ mode in the nonlinear regime, with plasma flow entering into the force balance equation. This means that the plasma does not evolve through a succession of MHD equilibria, as it does for the $m = 2$ mode. This fact is of relevance to recent calculations of ideal stability of $m = 1$ magnetic island states.^{8,9} It indicates that the relevant state for considerations of ideal stability is a growing $m = 1$ mode, not an ideal equilibrium, which if it exists, appears to possess singular current sheets along the separatrix.

III. FEEDBACK STABILIZATION

The equations for the nonlinear evolution of resistive modes, Eq. (14) for $m \geq 2$ and Eq. (17) for $m = 1$, make obvious two means of stabilization, and

provide estimates of the necessary amplitudes. The two cases are significantly different so we again discuss them individually.

A. Stabilization of $m \geq 2$ Modes

From Eq. (14) we find that stabilization can be achieved using current drive of magnitude

$$\frac{j_{d1}}{j_0} = - \frac{\Delta' \psi_1}{W_{J0}} = \frac{\Delta' W}{16} \frac{1}{(2q/\tau q') - 1} \quad (18)$$

Stabilization is thus achieved typically by a local increase of the current density of a few percent at the island 0-point. The necessary modification of the resistivity for stabilization is of the same form,

$$\frac{\eta_1}{\eta_0} = - \frac{\Delta' W}{16} \frac{1}{(2q/\tau q') - 1} \quad (19)$$

but of the opposite sign. Stabilization is thus achieved by a decrease in η at the 0-point, or local heating, again of a few percent.

Theoretical estimates¹⁰ and numerical simulation¹¹ of $m = 2$ mode stabilization have already been reported, and similar numerical experiments have been carried out by the present authors. The results confirm Eq. (19). The possibility of the use of local heating to control the $m = 2$ mode, and thus major disruptions, rests primarily on the efficacy of local heating in a narrow radial band in phase with a rotating $m = 2$ perturbation.¹² Assuming a resistivity of the form $\eta \sim T^{-3/2}$ we find the required local heating to be given by

$$\frac{\delta T}{T} = \frac{\Delta' W}{24} \frac{1}{(2q/\tau q') - 1}.$$

The power balance for the temperature in the island is

$$-\int \kappa_1 \nabla T \cdot ds = \int P dv, \quad (20)$$

where P is the net power deposited in the island (ohmic and auxiliary heating, radiation losses, etc.). For a thin island $\nabla T = -2(\delta T/W)$. Using the fact that the island volume to surface area is $(dv/da) = W$, we find

$$P = \frac{k_1 T \Delta^2}{12 W (2q/\tau q') - 1}. \quad (21)$$

Note also that if heat loss is greater from the island than from the surrounding plasma, due to radiation for example, Eq. (16) provides a destabilizing effect.¹³

B. Stabilization of the $m = 1$ Mode

From Eq. (17) the local heating or current drive necessary for stabilization of the $m = 1$ mode must be localized within the island with amplitude given approximately by

$$\frac{\eta_1}{\eta_0} \approx - \frac{\sqrt{2} b^2 w_T}{r_s j_0 \eta_0} \quad (22)$$

or by j_1/j_0 being of the same magnitude, but of the opposite sign. Simulation results are shown in Fig. 1, where the maximum amplitude $A_{\parallel} = \psi_{\max}$ is plotted as a function of time, obtained using the nonlinear resistive MHD code HIB.¹⁴ A peaked current profile was used, $q = q_0[1 + (r/r_0)^2]$, with $q_0 = 0.7$ and $r_0 = 0.7$. The resistivity was taken to be $\eta_0 j_0 = 10^{-4}$, giving $w_T = 6 \times 10^{-2}$. The eigenfunction was found to have the same form during its full

evolution, and the maximum amplitude was related to its slope through $\psi_{lmax} = br_s/3$. The various curves in Fig. 1 show the evolution of ψ_{max} with different values of $\eta_1/\eta_0 = \delta$, and the feedback was initiated at $t = 75$, measured in the units of the Alfvén time τ_A . The perturbation δ was taken to be a Gaussian in r , centered at $r = r_s$, and the results were insensitive to the width. Substituting these expressions into Eq. (19) we find a theoretical prediction of stabilization for $\delta = -10^{-2}$ in reasonable agreement with the simulations. These simulations are, of course, highly simplified in that they assume a given $m = 1$ perturbation of η or j_d . Realistic treatments must address also the localized deposition and the maintenance of the temperature or current profile against its outward diffusion.¹² Although this result would appear to make stabilization of the sawtooth instability a possibility, these results are for $\eta = 10^{-4}$, and the scaling of Eq. (19) as $\eta^{-2/3}$ makes it not feasible for realistic tokamak temperatures. In Fig. 2 is plotted a numerical verification of this scaling. The critical stabilizing amplitude, δ is plotted versus $S = \tau_R/\tau_A \sim \eta^{-1}$. Present-day tokamaks operate at temperatures giving values of S of 10^6 , 10^7 , so stabilization in this manner is clearly impossible unless at high temperatures there exists some stabilizing effect on the nonlinear behavior of the mode due to physical effects not included in this analysis.

C. Stabilization Using Ideal Effects

The scaling of the current drive and local heating necessary to stabilize the $m = 1$ resistive kink as $\eta^{-2/3}$ is due to the proximity of this mode to ideal instability.¹⁵ If the ideal internal kink mode is made sufficiently stable, then the linear growth rate reduces from $\gamma \sim \eta^{1/3}$ to $\gamma \sim \eta^{3/5}$, and the nonlinear behavior of the mode assumes the character of the tearing mode, with

$m \geq 2$. Ideal stabilizing forces would thus make the $m = 1$ mode amenable to feedback stabilization with local heating or current drive. Some stabilization exists due to toroidal curvature.¹⁶ The stabilizing contribution to δW from toroidal curvature and plasma pressure is

$$\delta W_T = 3v\Delta q \left[\frac{13}{48(v+4)} - \beta_p^2 \int_{r_0}^{r_2} \frac{dr}{r_0} \left(\frac{r}{r_0} \right)^{v-5} \right] \quad (20)$$

for a q profile of the form $1 - q(r) = \Delta q [1 - (r/r_0)^v]$, $q(r_0) = 1$, $q(r_2) = 2$,

$$\beta_p = \frac{2}{B^2} \int_0^{r_0} \left(\frac{r}{r_0} \right)^2 \left(-\frac{dp}{dr} \right) dr, \quad ,$$

and $\delta W \sim (a/R)^2 \delta W_T$. Unfortunately, to have a significant effect δW must be of magnitude $\delta W \gg \eta^{-1/3}$, or $\delta W \geq 10^{-2}$ for present-day tokamaks. This is difficult to achieve since δW is of the form $\delta W \sim \Delta q \cdot O[(a/R)^2]$ and Δq is assumed small. The expression for δW due to triangularity of the plasma cross section is similar,^{17,18} so significant ideal stabilization by this means is very difficult.

Simulations have been carried out using bean-shaped plasma cross sections,¹⁹ as used on PBX. Although the ideal mode was found to be stable, due to the triangularity of the flux surfaces, for reasonable values of η the resistive mode showed no sign of stabilization, continuing exponential growth with $\gamma \sim \eta^{1/3}$ well into the nonlinear regime. In addition simulations have been carried out using a strongly triangular plasma cross section, obtained by a direct modification of the $n = 0$, $m = 3$ amplitude at the plasma edge. These simulations confirmed the result that it is impossible to obtain a significant reduction of the resistive growth for reasonable temperatures using plasma profile shaping.

In cylindrical geometry a positive pressure gradient is stabilizing, the modification of the ideal growth rate being given by

$$\gamma = \frac{-\pi}{r_s (r_s q')^2} \int_0^{r_s} g \, dr \quad (21)$$

with

$$g \approx 4r^3(q - 1) + 2r^2 \frac{dp}{dr}.$$

In Fig. 3 is shown a numerical verification of the stabilizing effect of such a pressure profile for both the ideal and resistive modes, obtained with the code HIB.¹⁴ For sufficiently strong negative pressure the resistive mode assumes a Rutherford algebraic growth in the nonlinear stages and is thus amenable to feedback stabilization. The pressure profile used was of the form $p = p_0(1 - r^2)^2$ with the units of p_0 given by $\epsilon^2 B^2$, and $\epsilon = 1/5$. The ideal mode is stabilized for $p_0 \approx -0.1$. The resistive mode growth rate at $p = 0$ is proportional to $\eta^{1/3}$, and thus the necessary stabilizing δW is also proportional to $\eta^{1/3}$. The eigenfunctions obtained at $p_0 = 0$ (B) and $p_0 = -1$ (A) are quite different. The flow pattern at point B is approximately uniform displacement inside $q = 1$, whereas at point A the flow is concentrated in the vicinity of the $q = 1$ surface, much like the structure of a mode with $m \geq 2$. The mode at point A exhibits Rutherford algebraic growth, rather than exponential growth, and is easily stabilized.

Positive pressure gradients are not practical, but this calculation exhibits the essential change in structure of the $m = 1$ mode, provided stabilizing δW of the order of $S^{-1/3}$ can be introduced. One way of introducing an ideal stabilizing effect is to inject high energy, barely

trapped particles. Bounce averages $\oint (dl/v_{||})$ accentuate contributions from the good curvature region, and the resulting pressure contribution is stabilizing. An estimate of the necessary hot, trapped particle poloidal beta necessary for ideal stabilization for $1 - q_0 \sim O(1)$ is $\beta_{ph} \gg \epsilon \beta_{pc}^2$ with β_{pc} the core plasma poloidal beta. For stabilization of the resistive mode one has

$$\epsilon \beta_{ph} \geq S^{-1/3} \quad . \quad (22)$$

with the ordering $1 - q_0 < \epsilon$ the stabilization of the resistive mode requires

$$\epsilon^2 \beta_{ph} \beta_{pc} \geq S^{-1/3} \quad . \quad (23)$$

These estimates are based on a formalism introduced to treat the fishbone instability,²⁰ and will be reported in a future publication. The small magnitude of β_{ph} necessary for stabilization makes this method, in contrast to plasma cross-sectional shaping, seem feasible.

IV. CONCLUSION

Localized current drive and heating are quite promising means of controlling resistive modes with $m \geq 2$, and in particular for controlling the major tokamak disruption by limiting the size of the $m = 2$ mode. Elimination of the sawtooth would provide access to a new regime of tokamak operation, with higher beta and more peaked current profiles, which would tend to be stable also to the $m = 2$ mode. Quasilinear analysis of the nonlinear behavior gives expressions for necessary localized heating or current drive which are in agreement with simulations using resistive MHD codes. The control of the

$m = 1$ mode is much more difficult than the $m = 2$ mode, due to its proximity to ideal instability, and its continued exponential nonlinear growth, scaling as $\eta^{1/3}$. This results in the scaling of the current drive or local heating necessary to stabilize the mode as $\eta^{2/3}$, and makes control impossible without ideal stabilization. Ideal stabilization through cross-sectional shaping appears impossible, because although the mode can be made ideally stable, the contribution to δW from the shaping is so small that the mode does not depart from its $\eta^{1/3}$ nonlinear exponential growth for reasonable plasma temperatures. A possible scheme, which is presently being explored, is the use of high energy, barely trapped particles to produce pressure localized in the good curvature region, making the mode sufficiently ideally stable that the nonlinear growth of the resistive mode becomes algebraic, proportional to η .

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FIGURE CAPTIONS

FIG. 1. The nonlinear development of the $m = 1$ amplitude in the presence of feedback stabilization.

FIG. 2. The scaling of the critical value of δ to achieve stabilization with $S \sim 1/\eta$.

FIG. 3. Ideal and resistive growth rates in cylindrical geometry in the presence of plasma pressure. A positive pressure gradient stabilizes the ideal mode and reduces the resistive mode to algebraic growth.

Fig. 1

Fig. 2

Fig. 3

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