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UCRL-ID--105439

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Deconvolution Using a Neural Network

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November 15, 1990

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Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

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Deconvolution Using a Neural Network*

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November 15, 1990

Abstract— Viewing one dimensional deconvolution as a matrix inversion problem, we compare a neural network backpropagation matrix inverse with LMS, and pseudo-inverse. This is largely an exercise in understanding how our neural network code works.

I INTRODUCTION

THE NOISE free convolution of two one-dimensional signals is defined as:

$$g(n) = \sum_{m=0}^{M-1} f(m)h(n-m) \quad (1)$$

where $f(m)$, the input signal; $h(k)$, the kernel; and $g(n)$, the result, are defined on

$$\begin{aligned} 0 &\leq m \leq M-1 \\ 0 &\leq k \leq K-1 \\ 0 &\leq n \leq M+K-1 = N \end{aligned}$$

In matrix form:

$$\mathbf{g} = \mathbf{H}\mathbf{f} \quad (2)$$

where $\mathbf{H}$ is the matrix that transforms $\mathbf{f}$ to $\mathbf{g}$:

$$\mathbf{H} = \begin{bmatrix} h(0) & 0 & 0 & \dots & 0 & 0 \\ h(1) & h(0) & 0 & \dots & 0 & 0 \\ h(2) & h(1) & h(0) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & 0 & \vdots \\ h(K-2) & h(K-3) & h(K-4) & \dots & & \\ h(K-1) & h(K-2) & h(K-3) & & h(1) & h(0) \\ 0 & h(K-1) & h(K-2) & & h(2) & h(1) \\ 0 & 0 & h(K-1) & & h(3) & h(2) \\ \vdots & \vdots & & & \vdots & \vdots \\ 0 & 0 & & h(K-1) & h(K-2) & h(K-3) \\ 0 & 0 & & 0 & h(K-1) & h(K-2) \\ 0 & 0 & \dots & 0 & 0 & h(K-1) \end{bmatrix} \quad (3)$$

*Work performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

The problem is to find $\mathbf{A}$ such that

$$\mathbf{f} = \mathbf{A}\mathbf{g} \quad (4)$$

We are also interested in the convolution problem. That is finding $\mathbf{H}$ such that

$$\mathbf{g} = \mathbf{H}\mathbf{f} \quad (5)$$

We compare three methods for determining either $\mathbf{H}$ given $\mathbf{A}$; or $\mathbf{A}$ given $\mathbf{H}$; or $\mathbf{H}$ and $\mathbf{A}$ given an appropriate set of $\mathbf{f}$ s and $\mathbf{g}$ s. The methods are:

1. Pseudo-inverse ;
2. Neural network ;
3. LMS.

II PSEUDO-INVERSE

Given $\mathbf{H}$, $\mathbf{A}$ is determined by:

$$\begin{aligned} \mathbf{H}^T \mathbf{g} &= \mathbf{H}^T \mathbf{H} \mathbf{f} \\ (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{g} &= \mathbf{f} \\ \mathbf{A} \mathbf{g} &= \mathbf{f} \end{aligned}$$

Where

$$\mathbf{A} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \quad (6)$$

Similarly, given $\mathbf{A}$:

$$\mathbf{H} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \quad (7)$$

III NEURAL NETWORK

To determine $\mathbf{H}$, we use a one layer network with M inputs and $M + K - 1$ outputs and train on a set of impulses and their responses. There are $2M$ elements in the set since we have to train on negative as well as the positive impulses. (When trained using only the positive impulses, the network converges but not correctly.)

Once converged, the weights make up $\mathbf{H}$. Because this is a linear problem the biases are all zero.

To determine $\mathbf{A}$, we use a network with $M + K - 1$ inputs and M outputs. We use the same training set as above except the impulse responses are the inputs and impulses are the targets.

Note in both networks we do not use threshold functions.

IV LMS

Given a set of input $\{\mathbf{f}_i\}$ and output $\{\mathbf{g}_i\}$ data determine $\mathbf{H}$.

For the derivation of the LMS solution we temporarily drop the vector index i since we use tensor notation and do not want to confuse the it with the vector element index.

The goal is to minimize:

$$\begin{aligned} \mathcal{E}^2 &= (\mathbf{g} - \mathbf{H}\mathbf{f})^T (\mathbf{g} - \mathbf{H}\mathbf{f}) \\ &= (\mathbf{g}^T - \mathbf{f}^T \mathbf{H}^T) (\mathbf{g} - \mathbf{H}\mathbf{f}) \\ &= \mathbf{g}^T \mathbf{g} - 2\mathbf{g}^T \mathbf{H}\mathbf{f} + \mathbf{f}^T \mathbf{H}^T \mathbf{H}\mathbf{f} \end{aligned}$$

In tensor notation:

$$\mathcal{E}^2 = g_i g_i - 2g_i H_{ij} f_j + f_i H_{ji} H_{jk} f_k \quad (8)$$

Where g_i is the i^{th} component of the vector $\mathbf{g}$ and not the i^{th} vector in the set $\{\mathbf{g}_i\}$.

$$\begin{aligned}
\frac{\partial \mathcal{E}^2}{\partial H_{mn}} &= -2g_i \frac{\partial H_{ij}}{\partial H_{mn}} f_j + f_i \frac{\partial [H_{ji} H_{jk}]}{\partial H_{mn}} f_k \\
&= -2g_i \delta_{im} \delta_{jn} f_j + f_i [\delta_{jm} \delta_{in} H_{jk} + \delta_{jm} \delta_{nk} H_{ji}] f_k \\
&= -2g_m f_n + f_i [\delta_{in} H_{mk} + \delta_{nk} H_{mi}] f_k \\
&= -2g_m f_n + f_n H_{mk} f_k + f_i H_{mi} f_n \\
&= -2g_m f_n + f_n H_{mk} f_k + f_n H_{mi} f_i \\
&= -2g_m f_n + 2f_n H_{mk} f_k
\end{aligned}$$

Back to matrix notation:

$$\frac{\partial \mathcal{E}^2}{\partial \mathbf{H}} = -2\mathbf{g}\mathbf{f}^T + 2\mathbf{H}\mathbf{f}\mathbf{f}^T \quad (9)$$

Summing over the data set, $\{\mathbf{f}_i\}$ and $\{\mathbf{g}_i\}$, and equating to zero:

$$\sum_i \frac{\partial \mathcal{E}_i^2}{\partial \mathbf{H}} = -2 \sum_i \mathbf{g}_i \mathbf{f}_i^T + 2\mathbf{H} \left[\sum_i \mathbf{f}_i \mathbf{f}_i^T \right] = 0 \quad (10)$$

$$\Rightarrow \mathbf{H} = \left[\sum_i \mathbf{g}_i \mathbf{f}_i^T \right] \left[\sum_i \mathbf{f}_i \mathbf{f}_i^T \right]^{-1} \quad (11)$$

In a similar manner we may determine $\mathbf{A}$:

$$\mathbf{A} = \left[\sum_i \mathbf{f}_i \mathbf{g}_i^T \right] \left[\sum_i \mathbf{g}_i \mathbf{g}_i^T \right]^{-1} \quad (12)$$

V RESULTS

For a detailed transcript of the computer results see the appendices. They present a session training the neural network to deconvolve, a session training it to convolve, and a MathematicaTM [1] session used to verify the results.

As our test case we use:

$$\begin{aligned}
M &= 5 \\
K &= 3 \\
N &= 7
\end{aligned}$$

$$h(k) = \{ 1 \quad -1 \quad 1 \quad \} \quad (13)$$

$$\Rightarrow \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

The set of impulses and their responses used in training are:

$$\begin{aligned}
 \mathbf{f}_0^T &= [1 & 0 & 0 & 0 & 0 & 0] \\
 \mathbf{f}_1^T &= [0 & 1 & 0 & 0 & 0 & 0] \\
 \mathbf{f}_2^T &= [0 & 0 & 1 & 0 & 0 & 0] \\
 \mathbf{f}_3^T &= [0 & 0 & 0 & 1 & 0 & 0] \\
 \mathbf{f}_4^T &= [0 & 0 & 0 & 0 & 1 & 0] \\
 \mathbf{f}_5^T &= [-1 & 0 & 0 & 0 & 0 & 0] \\
 \mathbf{f}_6^T &= [0 & -1 & 0 & 0 & 0 & 0] \\
 \mathbf{f}_7^T &= [0 & 0 & -1 & 0 & 0 & 0] \\
 \mathbf{f}_8^T &= [0 & 0 & 0 & -1 & 0 & 0] \\
 \mathbf{f}_9^T &= [0 & 0 & 0 & 0 & 0 & -1]
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 \mathbf{g}_0^T &= [1 & -1 & 1 & 0 & 0 & 0 & 0] \\
 \mathbf{g}_1^T &= [0 & 1 & -1 & 1 & 0 & 0 & 0] \\
 \mathbf{g}_2^T &= [0 & 0 & 1 & -1 & 1 & 0 & 0] \\
 \mathbf{g}_3^T &= [0 & 0 & 0 & 1 & -1 & 1 & 0] \\
 \mathbf{g}_4^T &= [0 & 0 & 1 & 0 & 1 & -1 & 1] \\
 \mathbf{g}_5^T &= [-1 & 1 & -1 & 0 & 0 & 0 & 0] \\
 \mathbf{g}_6^T &= [0 & -1 & 1 & -1 & 0 & 0 & 0] \\
 \mathbf{g}_7^T &= [0 & 0 & -1 & 1 & -1 & 0 & 0] \\
 \mathbf{g}_8^T &= [0 & 0 & 0 & -1 & 1 & -1 & 0] \\
 \mathbf{g}_9^T &= [0 & 0 & 0 & 0 & -1 & 1 & -1]
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 \mathbf{g}_{\text{null}0}^T &= [1 & 1 & 0 & -1 & -1 & 0 & 1] \\
 \mathbf{g}_{\text{null}1}^T &= [0 & -1 & -1 & 0 & 1 & 1 & 0]
 \end{aligned} \tag{17}$$

Using Mathematica™, we used (6) to compute $\mathbf{A}$ from (14). We then verified $\mathbf{A}_{\text{math}} \cdot \mathbf{H} = \mathbf{I}$ and $\mathbf{A}_{\text{math}} \mathbf{g}_i = \mathbf{f}_i$. This is the pseudo-inverse solution from Mathematica™:

$$\mathbf{A}_{\text{math}} = \begin{bmatrix} 3/4 & -1/8 & 1/8 & 1/4 & 1/8 & -1/8 & -1/4 \\ 5/8 & 9/16 & -1/16 & 3/8 & 7/16 & 1/16 & -3/8 \\ 0 & 1/2 & 1/2 & 0 & 1/2 & 1/2 & 0 \\ -3/8 & 1/16 & 7/16 & 3/8 & -1/16 & 9/16 & 5/8 \\ -1/4 & -1/8 & 1/8 & 1/4 & 1/8 & -1/8 & 3/4 \end{bmatrix} \tag{18}$$

When we applied (7) to (18) to get $\mathbf{H}$ back again, we got a Mathematica™ error saying $\mathbf{A}_{\text{math}}^T \mathbf{A}_{\text{math}}$ was singular.

The neural network we trained to convolve yielded (14) exactly. However, the network we trained to deconvolve, found a different $\mathbf{A}$ than the pseudo-inverse method:

$$\mathbf{A}_{\text{nn}} = \begin{bmatrix} 0.4049 & -0.2637 & 0.3314 & 0.5951 & 0.2637 & -0.3314 & -0.5951 \\ 0.5227 & 0.6245 & 0.1018 & 0.4773 & 0.3755 & -0.1018 & -0.4773 \\ 0.1600 & 0.4017 & 0.2417 & -0.1600 & 0.5983 & 0.7583 & 0.1600 \\ -0.1343 & 0.0583 & 0.1926 & 0.1343 & -0.0583 & 0.8074 & 0.8657 \\ -0.2143 & -0.2585 & -0.0442 & 0.2143 & 0.2585 & 0.0442 & 0.7857 \end{bmatrix} \tag{19}$$

We verified $\mathbf{A}_{\text{nn}} \cdot \mathbf{H} = \mathbf{I}$ and $\mathbf{A}_{\text{nn}} \mathbf{g}_i = \mathbf{f}_i$. We entered (19) into Mathematica™ and applied (6) to it but got the singular matrix error.

We applied (11) in Mathematica™ to the data set, $\{\mathbf{f}_i\}$ and $\{\mathbf{g}_i\}$, we presented to the neural network. It computed $\mathbf{H}$ exactly. However (12) resulted in a singular matrix error when Mathematica™ tried to invert $\sum_i \mathbf{g}_i \mathbf{g}_i^T$. This is because half of the $\{\mathbf{g}_i\}$ are linearly dependent.

The $\{\mathbf{f}_i\}$ span an M -space. The $\{\mathbf{g}_i\}$ span an M -dimensional subspace of an N -dimensional space. Recall $M < N$.

H maps an M -space to an N -space. Since we have $M (=5)$ linearly independent $\{f_i\}$, we can use (11) to find H . However in finding the mapping, A , from a 7-space to a 5-space, we have only 5 independent $\{g_i\}$, thus (12) will not work. It needs more information. The neural network somehow "guesses" at the additional information in order to find the inverse.

The solution the neural network finds is not the minimum norm solution. Equation (18) is the minimum norm solution since the pseudo-inverse is the LMS solution. The solution, A_{nn} , the neural network finds can be written as:

$$A_{nn} = A_{\parallel} + A_{\perp} \quad (20)$$

where $A_{\parallel} (= A_{math})$ is parallel to the 5-subspace spanned by $\{g_i\}$ and $A_{\perp}$ is perpendicular to it. The minimum norm solution is $A_{\parallel}$. By subtracting (18) from (19), $A_{nn} - A_{math}$, we obtained an $A_{\perp}$. We verified in MathematicaTM that $A_{\perp} \cdot g_i = 0$.

We used MathematicaTM to find the null-space vectors of A_{math} , equation (17), and included them with the set of vectors used in training the neural network to deconvolve. The resulting weights were (18) exactly.

Note $A_{\parallel}$ is the best solution for noise elimination since the noise would span the full 7-space.

VI CONCLUSIONS

A feed forward, backpropagation trained, one layer neural network with no thresholding functions can be trained to convolve and deconvolve noise free data.

When given identical vectors set, the neural network was able to find the convolution matrix whereas the LMS method was not.

VII ACKNOWLEDGMENTS

The author wishes to acknowledge the help and encouragement of J. P. Fitch and D. T. Gavel.

REFERENCES

- [1] Stephen Wolfram, *Mathematica, A System for Doing Mathematics by Computer*, Addison-Wesley Publishing Company, NY 1988.

A DECONVOLVING NETWORK

This is the shell script used in training the network to deconvolve. The case presented includes the two null-space vectors, thus the A found is that of the LMS solution.

First the training code, `learn`, is called, followed by `apply`, the testing code and then `disect` which "disects" the network to examine the weights.

For a detailed description on code operation contact the author.

```
#!/bin/sh

Train=dtrain0-null
Test=dtest

Range=1.0e12
Err=1.e-10

if( test -z "$1" ) then Method=1 ; else Method=$1 ; fi

set 'wc -l $Train $Test'
Elem1=$1
Elemt=$3
```

```

echo time \
learn -ve $Err -m $Method -F 2 deconvolve decon$Method $Eleml < $Train
time learn -ve $Err -m $Method -F 2 deconvolve decon$Method $Eleml < $Train

echo
echo apply -v -a "%8.2f" decon$Method $Elemt < $Test
apply -v -a "%8.2f" decon$Method $Elemt < $Test

echo
echo dissect -a "%10.5f" deconvolve decon$Method << TheEnd
dissect -a "%10.5f" deconvolve decon$Method << TheEnd
1 0
1 1
1 2
1 3
1 4
TheEnd

```

This is a typescript of the execution of the above shell script.

```

roadrunner[101] decon
time learn -ve 1.e-10 -m 1 -F 2 deconvolve decon1 12 < dtrain0-null

```

```

Network file: deconvolve
Weight file: decon1
Dimension of input: 1
  Dimension 0: 7
Inputs: 7
Hidden layers: 0
Outputs: 5
No. of layers: 2
Layers: 7 5
No. of weights: 40

```

```

Optimization method: Polak - Ribiere (1)
Maximum linesearch evaluations: 20
Convergence parameter sigma: 0.1
Maximum restarts: 5
Maximum system error: 1e-10
Maximum number of function evaluations: 5000
Random number seed: 1
Neuron output function: Linear - USE WITH CARE (2)

```

Total number of input patterns: 12

```

-Inputs-
1 -1 1 0 0 0 0
0 1 -1 1 0 0 0
0 0 1 -1 1 0 0
0 0 0 1 -1 1 0
0 0 0 0 1 -1 1
-1 1 -1 0 0 0 0

```



```

0 -1 1 -1 0 0 0
0 0 -1 1 -1 0 0
0 0 0 -1 1 -1 0
0 0 0 0 -1 1 -1
1 1 0 -1 -1 0 1
0 -1 -1 0 1 1 0

```

-Targets-

```

1 0 0 0 0
0 1 0 0 0
0 0 1 0 0
0 0 0 1 0
0 0 0 0 1
-1 0 0 0 0
0 -1 0 0 0
0 0 -1 0 0
0 0 0 -1 0
0 0 0 0 -1
0 0 0 0 0
0 0 0 0 0

```

Error at start: 0.91579, gnorm = 1.08015

Iteration	1, error =	0.277851, gnorm =	0.375408
Iteration	2, error =	0.13769, gnorm =	0.273439
Iteration	3, error =	0.0555376, gnorm =	0.178131
Iteration	4, error =	0.0246831, gnorm =	0.109281
Iteration	5, error =	0.0060042, gnorm =	0.0578176
Iteration	6, error =	0.0015934, gnorm =	0.0214331
Iteration	7, error =	0.000140781, gnorm =	0.00510232
Iteration	8, error =	4.20221e-28, gnorm =	3.22214e-14

Total number of iterations is 8
 Total number of function evaluations is 18
 Normalized system error is 4.20221e-28
 Norm of the gradient is 3.22214e-14
 Maximum pattern error is 9.08972e-28

```

real      0.3
user      0.0
sys       0.0

```

apply -v -a %8.2f decon1 6 < dtest

Network file: deconvolve
 Weight file: decon1
 Dimension of input: 1
 Dimension 0: 7
 Inputs: 7
 Hidden layers: 0
 Outputs: 5
 No. of layers: 2

Layers: 7 5
No. of weights: 40

-Inputs-

0 1 -1 0 1 -1 0
1 0 1 1 1 0 1
0 1 1 2 3 -1 4
0 1 1 0 1 1 0
1 -2 3 -3 3 -2 1
5 -10 15 -15 15 -10 5

-Outputs-

1)	0.00	1.00	0.00	-1.00	0.00
2)	1.00	1.00	1.00	1.00	1.00
3)	0.00	1.00	2.00	3.00	4.00
4)	0.00	1.00	2.00	1.00	0.00
5)	1.00	-1.00	1.00	-1.00	1.00
6)	5.00	-5.00	5.00	-5.00	5.00

disect -a %10.5f deconvolve decon1 << TheEnd

Network file: deconvolve

Weight file: decon1

Dimension of input: 1

Dimension 0: 7

Inputs: 7

Hidden layers: 0

Outputs: 5

No. of layers: 2

Layers: 7 5

No. of weights: 40

Enter dissection layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 7

Layer: 1

Neuron: 0

Bias: 1.89735e-16

0.75000 -0.12500 0.12500 0.25000 0.12500 -0.12500 -0.25000

Layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 7

Layer: 1

Neuron: 1

Bias: 5.88234e-15

0.62500 0.56250 -0.06250 0.37500 0.43750 0.06250 -0.37500

Layer & neuron (~D to quit):
 Dimension of sample: 1
 Dimension 0: 7
 Layer: 1
 Neuron: 2
 Bias: 8.44377e-15

0.00000 0.50000 0.50000 0.00000 0.50000 0.50000 0.00000

Layer & neuron (~D to quit):
 Dimension of sample: 1
 Dimension 0: 7
 Layer: 1
 Neuron: 3
 Bias: 3.40288e-15

-0.37500 0.06250 0.43750 0.37500 -0.06250 0.56250 0.62500

Layer & neuron (~D to quit):
 Dimension of sample: 1
 Dimension 0: 7
 Layer: 1
 Neuron: 4
 Bias: -1.0031e-15

-0.25000 -0.12500 0.12500 0.25000 0.12500 -0.12500 0.75000

Layer & neuron (~D to quit):
 roadrunner[102]

B CONVOLVING NETWORK

This is the shell script used in training a network to convolve. It is identical in form to the deconvolution shell script.

```
#!/bin/sh
```

```
Train=ctrain0
Test=ctest
Range=1e12
Err=1.e-10
```

```
if( test -z "$1" ) then Method=1 ; else Method=$1 ; fi
```

```
set 'wc -l $Train $Test'
Elem1=$1
Elem2=$3
```

```
echo time learn -ve $Err -m $Method -F2 convolve con$Method $Elem1 \< $Train
time learn -ve $Err -m $Method -F2 convolve con$Method $Elem1 < $Train
```

```
echo
echo apply -v -a "%8.2f" con$Method $Elem2 \< $Test
apply -v -a "%8.2f" con$Method $Elem2 < $Test
```

```

echo
echo disect -a "%9.4f" convolve con$Method \<\< TheEnd
disect -a "%9.4f" convolve con$Method << TheEnd
1 0
1 1
1 2
1 3
1 4
1 5
1 6
TheEnd

```

This is a typescript of the execution of the above shell script:

```

roadrunner[101] con
time learn -ve 1.e-10 -m 1 -F2 convolve con1 10 < ctrain0

```

```

Network file: convolve
Weight file: con1
Dimension of input: 1
  Dimension 0: 5
Inputs: 5
Hidden layers: 0
Outputs: 7
No. of layers: 2
Layers: 5 7
No. of weights: 42

```

```

Optimization method: Polak - Ribiere (1)
Maximum linesearch evaluations: 20
Convergence parameter sigma: 0.1
Maximum restarts: 5
Maximum system error: 1e-10
Maximum number of function evaluations: 5000
Random number seed: 1
Neuron output function: Linear - USE WITH CARE (2)

```

```

Total number of input patterns: 10

```

```

-Inputs-
1 0 0 0 0
0 1 0 0 0
0 0 1 0 0
0 0 0 1 0
0 0 0 0 1
-1 0 0 0 0
0 -1 0 0 0
0 0 -1 0 0
0 0 0 -1 0
0 0 0 0 -1
-Targets-
1 -1 1 0 0 0 0

```

```

0 1 -1 1 0 0 0
0 0 1 -1 1 0 0
0 0 0 1 -1 1 0
0 0 0 0 1 -1 1
-1 1 -1 0 0 0 0
0 -1 1 -1 0 0 0
0 0 -1 1 -1 0 0
0 0 0 -1 1 -1 0
0 0 0 0 -1 1 -1

```

Error at start: 2.45993, gnorn = 1.27743

Iteration 1, error = 1.09327, gnorn = 0.855856

Iteration 2, error = 1.10771e-31, gnorn = 4.23388e-16

Total number of iterations is 2

Total number of function evaluations is 6

Normalized system error is 1.10771e-31

Norm of the gradient is 4.23388e-16

Maximum pattern error is 1.72274e-31

```

real      0.2
user      0.0
sys       0.0

```

apply -v -a %8.2f con1 6 < ctest

Network file: convolve

Weight file: con1

Dimension of input: 1

Dimension 0: 5

Inputs: 5

Hidden layers: 0

Outputs: 7

No. of layers: 2

Layers: 5 7

No. of weights: 42

-Inputs-

```

0 1 0 -1 0
1 1 1 1 1
0 1 2 3 4
0 1 2 1 0
1 -1 1 -1 1
5 -5 5 -5 5

```

-Outputs-

```

1)  0.00  1.00 -1.00  0.00  1.00 -1.00  0.00
2)  1.00  0.00  1.00  1.00  1.00  0.00  1.00

```

3)	0.00	1.00	1.00	2.00	3.00	-1.00	4.00
4)	0.00	1.00	1.00	0.00	1.00	1.00	0.00
5)	1.00	-2.00	3.00	-3.00	3.00	-2.00	1.00
6)	5.00	-10.00	15.00	-15.00	15.00	-10.00	5.00

disect -a %9.4f convolve con1 << TheEnd

Network file: convolve

Weight file: con1

Dimension of input: 1

Dimension 0: 5

Inputs: 5

Hidden layers: 0

Outputs: 7

No. of layers: 2

Layers: 5 7

No. of weights: 42

Enter dissection layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 5

Layer: 1

Neuron: 0

Bias: -2.22045e-16

1.0000 0.0000 0.0000 0.0000 0.0000

Layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 5

Layer: 1

Neuron: 1

Bias: -6.41848e-17

-1.0000 1.0000 0.0000 0.0000 0.0000

Layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 5

Layer: 1

Neuron: 2

Bias: -1.66533e-16

1.0000 -1.0000 1.0000 0.0000 0.0000

Layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 5

Layer: 1

Neuron: 3

Bias: 2.77556e-16

0.0000 1.0000 -1.0000 1.0000 0.0000

Layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 5

Layer: 1

Neuron: 4

Bias: -5.55112e-17

0.0000 0.0000 1.0000 -1.0000 1.0000

Layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 5

Layer: 1

Neuron: 5

Bias: 1.11022e-16

0.0000 0.0000 0.0000 1.0000 -1.0000

Layer & neuron (~D to quit):

Dimension of sample: 1

Dimension 0: 5

Layer: 1

Neuron: 6

Bias: 0

0.0000 0.0000 0.0000 0.0000 1.0000

Layer & neuron (~D to quit):

roadrunner[102]

C MATHEMATICA RESULTS

This is the Mathematica™ script used in computing the various results:

```
(*
H      : The convolution kernel ;
Amath  : The pseudo-inverse solution ;
Ann    : The inverse solution obtained from the weights of
         the trained neural network ;
Aperp  : The perpendicular part of Ann ;
Alms   : The LMS solution for A ;
Hlms   : The LMS solution for H ;
f[n]   : Positive & negative impulses ;
g[n]   : Responses to positive & negative impulses ;
gnull0 , gnull1 : The null space vectors of Amath.
*)

H/: H := {{1, 0, 0, 0, 0}, {-1, 1, 0, 0, 0}, {1, -1, 1, 0, 0},
          {0, 1, -1, 1, 0}, {0, 0, 1, -1, 1}, {0, 0, 0, 1, -1}, {0, 0, 0, 0, 1}}

Amath/: Amath := Inverse[ Transpose[H] . H ] . Transpose[H]
```

```

Ann/: Ann := {
{0.4049, -0.2637, 0.3314, 0.5951, 0.2637, -0.3314, -0.5951},
{0.5227, 0.6245, 0.1018, 0.4773, 0.3755, -0.1018, -0.4773},
{0.1600, 0.4017, 0.2417, -0.1600, 0.5983, 0.7583, 0.1600},
{-0.1343, 0.0583, 0.1926, 0.1343, -0.0583, 0.8074, 0.8657},
{-0.2143, -0.2585, -0.0442, 0.2143, 0.2585, 0.0442, 0.7857}}

Aperp/: Aperp := Ann - Amath

f/: f[n_] := If[ n<=4, Table[If[i == n, 1, 0], {i, 0, 4}],
Table[If[i == n, -1, 0], {i, 5, 9}]]

g/: g[n_] := H.f[n]

gnull0/: gnull0 := NullSpace[Amath][[1]]

gnull1/: gnull1 := NullSpace[Amath][[2]]

fgt/: fgt[n_] := Outer[Times, f[n], g[n]]

gft/: gft[n_] := Outer[Times, g[n], f[n]]

ggt/: ggt[n_] := Outer[Times, g[n], g[n]]

fft/: fft[n_] := Outer[Times, f[n], f[n]]

ggd/: ggd[n_] := g[n].g[n]

ffd/: ffd[n_] := f[n].f[n]

Alms/: Alms := Sum[fgt[n], {n, 0, 11}].Inverse[Sum[ggt[n], {n, 0, 11}]]

Hlms/: Hlms := Sum[gft[n], {n, 0, 11}].Inverse[Sum[fft[n], {n, 0, 11}]]

```

This is a typescript showing the Mathematica™ results:

```
In[1]:= << decon.m
```

```
In[2]:= Amath
```

```

Out[2]= {{-, -(-), -, -, -, -(-), -(-)}, {-, --, -(--), -, --, --, -(-)},
4 8 8 4 8 8 4 8 16 16 8 16 16 8

```

```

> {0, -, -, 0, -, -, 0}, {-(-), --, --, -, -(--), --, -},
2 2 2 2 8 16 16 8 16 16 8

```

```

> {-(-), -(-), -, -, -, -(-), -}
4 8 8 4 8 8 4

```

```
In[3]:= Alms
```



```
Inverse::sing: Matrix {{2, -2, 2, 0, 0, 0, 0}, {-2, 4, -4, 2, 0, 0, 0},
  {2, -4, 6, -4, 2, 0, 0}, {0, 2, -4, 6, -4, 2, 0},
  {0, 0, 2, -4, 6, -4, 2}, {0, 0, 0, 2, -4, 4, -2},
  {0, 0, 0, 0, 2, -2, 2}}
is singular.
```

```
Out[3]= {{2, -2, 2, 0, 0, 0, 0}, {0, 2, -2, 2, 0, 0, 0},
> {0, 0, 2, -2, 2, 0, 0}, {0, 0, 0, 2, -2, 2, 0},
> {0, 0, 0, 0, 2, -2, 2}}\
> . Inverse[{{2, -2, 2, 0, 0, 0, 0}, {-2, 4, -4, 2, 0, 0, 0},
> {2, -4, 6, -4, 2, 0, 0}, {0, 2, -4, 6, -4, 2, 0},
> {0, 0, 2, -4, 6, -4, 2}, {0, 0, 0, 2, -4, 4, -2},
> {0, 0, 0, 0, 2, -2, 2}}]
```

```
In[4]:= Hlms
```

```
Out[4]= {{1, 0, 0, 0, 0}, {-1, 1, 0, 0, 0}, {1, -1, 1, 0, 0},
> {0, 1, -1, 1, 0}, {0, 0, 1, -1, 1}, {0, 0, 0, 1, -1}, {0, 0, 0, 0, 1}}
```

```
In[5]:= gnull10
```

```
Out[5]= {1, 1, 0, -1, -1, 0, 1}
```

```
In[6]:= gnull11
```

```
Out[6]= {0, -1, -1, 0, 1, 1, 0}
```

```
In[7]:= Amath . H
```

```
Out[7]= {{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0},
> {0, 0, 0, 0, 1}}
```

```
In[8]:= Ann . H
```

```
Out[8]= {{1., 0., 0., 0., 0.}, {0., 1., 0., 0., 0.},
```

```
> {0., 0., 1., -1.11022 10-16, 8.32667 10-17},
```

```
> {0., 0., -6.93889 10-18, 1., 0.},
```

```
> {1.38778 10-17, 0., 0., -1.38778 10-17, 1.}}
```

```
In[9]:= Amath . g[2]
```

```
Out[9]= {0, 0, 1, 0, 0}
```

```
In[10]:= Ann . g[2]
```

```
Out[10]= {0., 0., 1., -6.93889 10-18, 0.}
```

```
In[11]:= Quit
```

-END-

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13

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91