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**Electron Energy and Mean Forward Range
in Air Resulting From the ETA Factor
Treatment of Scattering in the Continuous
Slowing Down Approximation**

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ELECTRON ENERGY AND MEAN FORWARD RANGE IN AIR RESULTING
FROM THE LTA FACTOR TREATMENT OF SCATTERING IN THE
CONTINUOUS SLOWING DOWN APPROXIMATION

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ABSTRACT

This paper provides a method for making quick estimates of the electron energy and associated speed with propagation distance and of the variation of mean electron energy and associated speed with propagation distance in the beam's forward direction. The continuous slowing down approximation is used with a function modification developed by Logunov to account for scattering. The method is applied to electrons with initial kinetic energy ranging from 5 keV to 3 MeV slowing down in air at 1 atm. Comparisons with Monte Carlo calculations indicate that the method is useful in estimating the mean electron range in the forward direction. The method also provides rough estimates of the variation in mean electron energy and associated speed with propagation distance. However, variations are better predicted by readily available electron transport codes. Results for 1 MeV electrons demonstrate that the electron energy is spread over 5 characteristic depth ranges equal to the extreme range of the electrons rather than to the mean forward range.

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ELECTRON ENERGY AND MEAN FORWARD RANGE IN AIR RESULTING FROM THE ETA FACTOR TREATMENT OF SCATTERING IN THE CONTINUOUS SLOWING DOWN APPROXIMATION

Introduction

There are times when a quick estimate is desired for the penetration depth of an electron beam and for the variation of the mean electron energy with depth in a uniform medium. This paper presents a method that may be used to fill this need. The method is illustrated for electrons stopping in air. The results presented are applicable to stopping materials with atomic number near that appropriate for air (~ 7.2). Externally applied electric and magnetic fields are assumed to be so small that they can be neglected.

Computer codes have been developed to calculate electron transport in a large variety of geometries. As one example, the TIGER code¹ treats one-dimensional problems, in a multiple slab geometry, with a spectrum of incident electron energies. Such codes² sometimes allow ionization and relaxation of electron shells with binding energies greater than 1 keV. They also allow coupled electron-photon transport. The quick estimate suggested here will indicate when such detailed calculations are warranted in certain applications.

Extreme Range and Electron Speed

There are several phenomena which occur when a charged particle slows down in a uniform medium. The main energy-loss mechanism is the transfer of energy (via Coulomb interaction) to the electrons in the medium. The rate of energy loss of a primary particle is described by the Bethe stopping-power formula³

$$\frac{-dE}{dx} = \frac{4\pi N Z e^4}{m c^2 \beta^2} \left\{ - \frac{2 m c^2 \beta^2}{\hbar c \omega (1 - \beta^2)} - \frac{2}{\beta^2} \right\}, \quad (1)$$

where

- N = number of atoms/unit volume
- Z = number of electrons/atom in stopping material
- e = 4.8×10^{10} esu of charge
- ze = charge on moving particle
- v = 3.0×10^{10} cm/s
- β = incident particle speed/c
- m = electron mass
- $\hbar$ = 1.0545×10^{-27} erg-s
- E_{av} = average harmonic binding energy for electrons in the medium, 80.5 eV for air.

Bethe's formula agrees with experimental data for all fast particles with $ze^2 \leq 10$ and with $\beta \leq 5.0$ (where $\beta = v/\sqrt{1-\beta^2}$). At higher energies, a density correction factor applies. This factor is neglected here, limiting initial electron energies to those with $v \leq 5$, i.e., $\beta \leq 0.98$ or energies ≤ 2.5 MeV. Two limits to the applicability of Eq. 1 are: (1) the forces caused by external electric fields are negligible, and (2) $\beta > e^2/\hbar c = 0.00729$. The radiation loss during electron deceleration is negligible for air; the loss must be included for high-Z materials.

When applied to an electron traveling through air, Eq. 1 may be written as⁴

$$-\frac{1}{\beta} \frac{dE}{dx} = \frac{0.1526}{\beta^2} \left[\frac{12700\beta^2}{1-\beta^2} - \beta^2 \right] \frac{\text{MeVcm}^2}{\text{gm}}, \quad (2)$$

where ρ is the air density (gm/cm^3). The electron energy is given by

$$E = T + mc^2,$$

where m is the electron rest mass ($mc^2 = 0.511$ MeV), and T is the electron kinetic energy.

Also

$$E = \gamma mc^2.$$

The electron's β determines (1) the electron's energy, E ; (2) its kinetic energy, T ; and (3) its speed, $v = \beta c$. In the continuous slowing down approximation (CSDA), variation of these quantities with distance traveled may be found through Eq. 2 as the electron slows down in the air. Numerical integration of Eq. 2 gives the distance traveled (i.e., the electron extreme or CSDA Range R) as a function of initial kinetic energy, T , as well as propagation speed as a function of distance traveled for a series of initial energies. With CSDA, the extreme range⁵ is evaluated by:

$$\rho R = \int_0^T -\frac{1}{\beta} \frac{dE}{dx} dx. \quad (3)$$

Refer to Fig. 1. Similar data have been tabulated by Berger and Seltzer⁶ for electron energies from 10 keV to 1 GeV and for a series of materials using a more accurate formula for stopping power. They are also calculated with still greater accuracy by TIGER.¹ At 1 MeV, for example, TIGER indicates the extreme range $R = 0.492 \text{ g/cm}^2$ rather than the 0.446 g/cm^2 value in Fig. 1(a).

Fig. 1 Extreme Range-Energy Curves for Electrons in Air: (a), (b) and (c) Span Differing Intervals of Initial Kinetic Energy.

The kinetic energy is obtained as a function of distance traveled (times c) by

$$T(x) = T(0) + \int_0^x \frac{1}{c} \frac{dE}{dx} dx, \quad (4)$$

the $\alpha(x)$ determined by

$$\alpha = \frac{\sqrt{\alpha^2 + 2\alpha}}{1 + \alpha} \quad (5)$$

(with $\alpha = T/mc^2$) is given in Fig. 2. After a translation along the distance axis to alter the initial particle speed, the 3-M-V curve contains all the other data presented (Fig. 2). However, the multiple curves allow a more convenient estimate for speed after propagation through a variable thickness of air.

Account for Angular Straggling - The ETA Factor

As the electron travels a path distance (dx), the corresponding element of length traversed in the forward direction (dy) is slightly smaller because the electron is scattered several times. The elements of length are assumed to be connected by the eta (ϵ) factor

$$dx = \epsilon dy$$

where

$$\epsilon = \left\langle \frac{1}{\cos \theta} \right\rangle$$

Here we interpret the electron ϵ as being the average electron of a very large number that travel distance, dx. The $\left\langle \frac{1}{\cos \theta} \right\rangle$ then corresponds to the average value of the inverse of the cosine of the scattered angle for the large number of electrons. As the electrons traverse the medium, the scattering angle becomes large and ϵ approaches infinity as θ nears $\pi/2$. The average electron never reverses direction. Values of ϵ are bounded by 1 and infinity. Large values of ϵ prevent significant progress for the average electron in the forward direction.

The mean squared angle $\langle \theta^2 \rangle$ of an initially collimated beam increases with the distance traveled. This variation $\frac{1}{7}$ is approximated by

$$\frac{d\langle \theta^2 \rangle}{dx} = \frac{8-NZ^2c^4}{2m^2c^4} \ln \frac{131\sqrt{\frac{2}{Z}} - 1}{Z^{1/3}} \quad (6)$$

(a)

(b)

(c)

Fig. 2 Electron Speed vs Distance Traveled in Air
for a Series of Initial Electron Kinetic Energies.

The change in $\langle \theta^2 \rangle$ is small in traveling the distance (dx) . Here, following Longmire⁷, $\langle \theta^2 \rangle$ is used to represent the mean square angle of the entire distribution, not just the relatively narrow Gaussian portion. Equation 6 is appropriate only when $\langle \theta^2 \rangle$ is small. Near the end of the particle range, for example, the mean square scattering angle $\langle \theta^2 \rangle$ of Eq. 6 attains the theoretical value of infinity. However, when cast in terms of the τ factor this difficulty does not occur.⁷

For small θ ,

$$\tau = 1 + \frac{\langle \theta^2 \rangle}{2} \quad (8)$$

and

$$\frac{d\tau}{dx} = \frac{1}{2} \frac{d\langle \theta^2 \rangle}{dx} \quad (9)$$

In terms of the forward direction (y), the loss of energy becomes

$$-\frac{1}{\tau} \frac{dE}{dy} = -\frac{1}{c} \frac{dE}{dx} \frac{dx}{dy} = -\frac{1}{c} \tau \frac{dE}{dx} \quad (9)$$

Near the end of the particle range, τ becomes infinite, the mean value of $\cos \theta$ becomes 0, and no more forward progress is possible (for the average electron). An initially collimated beam has $\tau = 1$. Knowing τ allows determination of a forward component of velocity

$$\hat{s}_y = \frac{1}{\tau} \hat{s}_x \quad (10)$$

Because of restrictions on validity of Eqs. 6 and 7 to small $\langle \theta^2 \rangle$, the equations involving τ are valid only for τ near 1. However, following Longmire⁷, we use the τ formulation as an approximation to the behavior at large scattering angles. This approximation can only be justified by comparison with more accurate treatments (such as Monte Carlo). The approximation does exhibit reasonable physical behavior near the end of a particle path where the approximation is expected to be poor. In contrast, Eq. 6 cannot reasonably be extended for use at large values of $\langle \theta^2 \rangle$.

In terms of the distribution (9), Eqs. 3 and 4 become

$$R_N = \int_0^T T \cdot \frac{1}{T} \frac{dT}{dy} dy \quad (12)$$

and

$$T(x) = T(0) \cdot \int_0^x \frac{1}{T} \frac{dT}{dx} dx \quad (13)$$

The $T(x)$ is the kinetic energy remaining after traveling forward distance x . If $T = 0$, then, enters into evaluation of dE/dy is determined from:

$$\frac{dn}{dy} = - \frac{d\tau}{dx} \quad (15)$$

where from Eqs. 6 and 8,

$$\frac{d\tau}{dx} = \frac{4\pi N Z^2 e^4}{\sqrt{2} m v^4} \ln \left[\frac{131 \sqrt{v^2}}{Z^{1/3}} + 1 \right] \quad (16)$$

For propagation in air⁸, this becomes

$$\frac{1}{\sqrt{2}} \frac{d\tau}{dx} = \frac{2.156}{\sqrt{2} E^2} \ln \left[67.8 \sqrt{E^2} + 1 \right] \quad (17)$$

In this approximation the 3 and 5 vary with kinetic energy but are unique functions of x . For application with Eq. 11,

$$\frac{d\tau}{dE} = \frac{1}{\sqrt{2}} \frac{d\tau}{dx} \frac{1}{\left(\frac{1}{\sqrt{2}} \frac{dE}{dx} \right)} \quad (18)$$

This τ -factor treatment yields Figs. 3 through 5.

The mean forward range-energy relation⁹ for electrons may also be estimated from

$$R_e = \frac{0.312 T^2}{\rho(0.3 + T)} \quad (19)$$

where R_e is the mean forward range in cm, T is the electron kinetic energy in MeV, and ρ is the air density in g/cm³. This relation is indicated by the dashed curve in Fig. 3.

(a)

(b)

(c)

Fig. 3 Electron Mean Forward Range in Air vs Initial Kinetic Energy.

(a)

(b)

(c)

Fig. 4 Electron Mean Forward Speed vs Forward Distance Traveled in Air for a Series of Initial Kinetic Energies.

(a)

(b)

(c)

Fig. 5 Electron Mean Kinetic Energy (MeV) vs Forward Distance Traveled in Air for a Series of Initial Kinetic Energies.

Comparison With More Detailed Calculations for 1-MeV Electrons

As a test of the accuracy of the continuous slowing down approximation presented here, as an illustration of the variations from the mean range that are expected, and as an indication of the information required for more detailed calculations such as occur in electron transport codes, the TIGER code¹ is used to model a 1-MeV electron incident upon 400 cm² of air.

The air is represented in Table I. The initial electron energy and air density are inserted as 1-MeV and 1.2929×10^{-3} g/cm³, respectively. The air thickness corresponds to 0.51714 g/cm² of air. Four thousand electron histories generated an estimate of the distribution function describing the energy deposited (per incident electron) as a function of forward range traversed. This estimate was used to form Fig. 6 giving the average electron energy as a function of the forward distance traveled by the average electron. Figure 6 indicates that the energy from a 1-MeV electron is spread over 0.50 g/cm² in air rather than over the mean forward range near 0.28 g/cm² indicated in Fig. 5(a). The 0.50 g/cm² range is more than the 0.446 g/cm² extreme range resulting from Eq. 3 and shown in Fig. 1.

Fig. 6
Average Electron Energy Remaining
at a Given Forward Distance.

TABLE I
Air Model

Material	Mole Fraction (%)	Molecular Weight	Weight in 1 "Mole" of air	Weight Fraction
N ₂	78.09	28.016	21.878	0.7556
O ₂	20.95	32.000	6.074	0.2316
Ar	0.93	39.944	0.371	0.0128
Total	99.97		28.953	1.0000

From the data we estimated that the air model in Table 1 has an effective $\bar{W}_{\text{eff}} = 26.8 \text{ eV}_e$, $\bar{W}_{\text{eff}} = 26.8 \text{ eV}_e$ for $\bar{W}_{\text{eff}} = 26.8 \text{ eV}_e$ evaluating Eq. 1.

The histogram of the data for primary electron charge deposition generated by TIGER is shown in Fig. 6. The following expectation value for range is

$$\bar{R} = \frac{\sum_{i=1}^N i \Delta R_i P_{i-1/2}(R)}{\sum_{i=1}^N P_{i-1/2}(R)} \quad (18)$$

where ΔR_i is the bin width of the i^{th} bin in the histogram. This prescription and the data in Fig. 6 give

$$\bar{R} = 0.270 \text{ g/cm}^2.$$

The proper factor treatment gives

$$\bar{R} = 0.286 \text{ g/cm}^2.$$

The range energy formula from Langmuir⁷ (Eq. 17) gives

$$\bar{R} = 0.240 \text{ g/cm}^2.$$

This result suggests that the proper range in Fig. 3 lies between the two given curves.

Fig. 7 The Distribution Function of Forward Range for 1-MeV Electrons in Air.

The π -factor⁷ was first proposed as a method for determining the self-consistent peak EMP. Figure 1a and 1b show the results of the π -factor treatment for the 1000 and 5000 electron distributions. The right-hand plots represent rough estimates of the peak EMP obtained with the 1000 and 5000 electron data. The 5000 results match the 0.50 g cm^{-2} value of the 1000 electron data, but the 1000 values are off by a factor of 2 before falling to the peak EMP. A factor of 2 is also the ratio of the extrapolated values in HGLB to the present estimates in HGLB. This is a consequence of the π -factor treatment.

Lange and⁷ proposed the π -factor treatment as a reasonably approximate but useful convenience, that is, sufficiently accurate for several types of EMP calculations. The method is especially convenient for treating the self-consistent case where the EMP distribution is affected by the fields they create. Comparisons with Monte Carlo calculations imply that using complete distributions of Compton electrons instead of average values is a worthwhile effect on the peak EMP.¹⁰ The main difficulties with the π -factor treatment occur a few tens of nanoseconds after the peak EMP. Here the method has provided a quick estimate of the magnitude of mean electron energy and associated speed with propagation distance in the forward direction.

APPENDIX A

Formulation of Equation 2

In Eq. 2, the hw term is taken as 30.5 eV as proposed by Livingston *et al.* (1964).²⁴ Thus, the logarithmic term $2\pi e^2 Z^2 hw$ becomes $12700 \cdot 10^{-16} \text{J} \cdot \text{m} \cdot \text{kg}^{-1} \cdot \text{s}^{-2} \cdot \text{C}^{-2}$, the factor 10^{-16} is hw in J $\cdot$ s for additional material.

TABLE A.1

Mean Ionization Potentials

Material	hw , eV	Material	$Z/A \cdot 10^{-4}$
H	15.6	Cu	0.0047
Li	34.0	Ag	0.0035
Be	60.4	Fe	0.0048
C	76.4	W	0.0040
Al	150.0	Pb	0.0039
Pb	241.0	U	0.0039

The hw values are approximately equal to 10% for other materials. For example, the hw term enters in a logarithm, causing only small alterations in the result. N/V gives the density of electrons/cm³ in air while N = number of atoms and volume. Air density is 1.29 g/cm³, the atomic weight, or $N = 1 \times 6.023 \times 10^{23}/M$ where M is the air density (see Table for "atomic weight for air" ($M = 14.493$ grams).

$$\frac{4\pi N Z e^4 \gamma^2}{mc^2} = \frac{4\pi Z e^4 \gamma^2}{mc^2} \cdot \frac{6.023 \times 10^{23}}{M} \cdot 1 \times 0.1526 \text{ MeV} \cdot \text{cm}^2/\text{g}, \quad \text{Eq. 1}$$

Since Z/M for all materials ≈ 6.5 , this coefficient varies only slightly as the stopping medium is varied. In Eq. 2, note that for $\gamma^2 = 7.83 \times 10^{-5}$ the in term quickly becomes negative, indicating a nonphysical gain in energy with distance traveled. The σ_{eff} evaluated by Eq. 1 has a maximum for γ^2 near 2.15×10^{-4} . When γ^2 is less than 2.15×10^{-4} , the stopping power is evaluated at $\gamma^2 = 2.15 \times 10^{-4}$ in the calculations presented here.

APPENDIX B

Formulation of Equation 15

With $Z = 7.22$ for air, the $\ln$ term becomes $\ln 67.8\sqrt{z}^{-1} = 1.6$. Since $N = z \times 6.023 \times 10^{23} \times N_A$, the coefficient becomes $2.156 \times 10^{-2} z^{-4}$. As $z \rightarrow 0$, the $z+1$, in the argument of the $\ln$ term, in Eq. 15, approaches 67.84. Thus $z = 1/67.8 = 0.01475$ gives the nonphysical result that $d\epsilon/dx$ is a maximum at $\ln 67.8 \pm 1/z^4$ occurs near $z = 0.019$. In evaluating Eq. 15 for $z = 0.019$, the $z+1$ is altered to 0.019 to prevent nonphysical values of $d\epsilon/dx$. The results do not apply to $z < 0.019$.

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