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WIRE GRID BEAM-SPLITTER FOR
MICROWAVE DIAGNOSTICS ON MTX

Jeffrey A. Lorbeck

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Jeffrey A. Lorbeck
University of Wisconsin - Madison
Electrical and Computer Engineering
Madison, WI 53706

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Abstract

A wire grid beam splitter for use in high-power microwave diagnostics is discussed. The principal application considered here is the Microwave Tokamak Experiment (140 GHz ECH using 35 ns FEL pulses of about 3 GW peak power, single pulse, with future upgrade to 250 GHz, 8 GW, 50 ns, and 5 kHz repetition rate). The theory of wire grids of infinite extent and excited by a uniform plane wave is extended to consider grids of finite extent and excited by fields having a Gaussian distribution. Design criteria based on the numerical simulations for the grid to be used in MTX are included along with other appropriate engineering recommendations.

1. Introduction

It is desired to obtain a measurement of the total power incident upon and reflected from the plasma of the MTX tokamak. A wire grid has been proposed to split off a small fraction of the incident and reflected beam energies and to focus the secondary beams thus produced into a pair of photon drag detectors. These detectors have a rapid response time, are insensitive to mode variation in the input beam, and can be used successfully in high pulse-power applications. It has also been suggested that the change in resistance due to ohmic heating of the individual wires of the grid may be measured to monitor the beam profile.

To avoid damaging the photon drag detectors and losing excessive power at the beam splitter due to reflection of the incident wave, it is required that the reflectivity (ratio of reflected power to incident power) of the grid remain small. However, to ensure viable operation of the detectors, a minimum reflectivity must be realized. Only a small fraction of the beam incident upon the plasma may be reflected. If the wire grid were to redirect too small a fraction of this reflected energy, the accuracy of the detectors may not be guaranteed. Given an FEL beam power of about 3 GW, for the initial single pulse experiments at 140 GHz, it has been determined that a reflectivity of approximately 1-2% should satisfy the above constraints.

It would also be desirable to configure a beam splitter such that the reflectivity was independent of the polarization of the incident beam. This requires a second grid whose wires lie perpendicular to those of the first grid but parallel to the plane of the first grid. Such an arrangement will be referred to as a mesh.

The purpose of the analysis to follow is to determine the wire radii and the spacing between the wires of the mesh, to compute the spatial characteristics of the scattered field and the total reflectivity, and to predict the induced wire currents, the accompanying ohmic heating and subsequent temperature rise of the grid wires.

2. Infinite Wire Grid

The primary applications of wire grid structures have been electromagnetic shielding, artificial antenna ground planes, and semi-transparent mirrors (Larsen, 1962). The literature has thus focused almost exclusively on those applications which require a high degree of grid reflectivity. The theoretical results presented therein are limited to a regime of grid construction characterized by wire spacing much less than the wavelength of the excitation.

A more general treatment of the problem given by Wait (1955, 1978) includes an analysis valid for large wire separation, although it is there assumed that the grid is infinitely large and excited by a uniform plane wave. The author of this report chose to begin with Wait's results to gain a rudimentary insight into the problem before proceeding with a solution more appropriate to the finite grid configuration.

The geometry and coordinate axes of the infinite grid are shown in Fig. 2.1. Under the assumption that the wire radius is small relative to a wavelength, only the z-component of the electric field will excite appreciable currents in the grid wires. In addition, the current density in each wire is symmetrical about the axial center of the wire. Wait finds that the current magnitude I_0 of each wire is given by:

$$I_0 = -A_z d / \{ \cos \theta / 2 \cos \phi + (i \mu_0 \omega d / 2 \pi) \cos^2 \theta \cdot [\ln (d / 2 \pi a) + F] + \cos^2 \theta Z_1 d \} \quad (2.1)$$

Figure 2.1 Planar wire grid with plane wave at arbitrary angle.

where A_z = magnitude of the Z-component of the incident electric field,

d = wire spacing,

a = wire radius,

η = free space wave impedance,

μ_0 = free space magnetic permeability,

Z_i = wire internal impedance, and

F = "correction factor".

For a radius a much less than the separation d , the field may be taken to be uniform around each wire. With metallic wires having small displacement current and at microwave frequencies:

$$Z_i \approx \left(\frac{\mu_w \omega}{2\sigma} \right)^{1/2} \left(\frac{1+i}{2\pi a} \right), \quad (2.2)$$

where μ_w is the wire magnetic permeability and σ is the wire electric conductivity. The "correction factor" F arises from an infinite summation of Hankel functions and is given by:

$$F \left(\frac{d \cos \theta}{\lambda}, \phi \right) = \frac{1}{2} \sum_{m=1}^{\infty} \left\{ \left[(m + d \cos \theta \sin \phi / \lambda)^2 - (d \cos \theta / \lambda)^2 \right]^{-1/2} \right. \\ \left. + \left[(m - d \cos \theta \sin \phi / \lambda)^2 - (d \cos \theta / \lambda)^2 \right]^{-1/2} - 2/m \right\}. \quad (2.3)$$

A power reflection coefficient for the lowest order reflected beam may also be found and is given by

$$r_p = \left| \frac{-1}{1 + 2Z_g/K_0} \right|^2, \quad (2.4)$$

with

$$Z_g/K_0 = i \frac{d}{\lambda} \cos \theta \cos \phi \left[\ln \left(\frac{d}{2\pi a} \right) + F \right] + \cos \phi Z_i d / \eta \cos \theta \quad (2.5)$$

The directions of the reflected beams (there may be only one or several depending on the grid wire spacing and angle of incidence) can be determined from purely geometrical considerations when the angle θ of the incident wave is zero. The angles ψ_n and $\psi_n - \pi$ of the secondary (reflected) waves propagating outwards from the grid are found from (MacFarlane)

$$\sin \psi_n = \sin \phi + \frac{n\lambda}{d} \quad n = 0, \pm 1, \pm 2, \dots \quad (2.6)$$

The angles ψ_n are measured with respect to the grid normal opposite the side of incidence (see Fig. 2.2). To obtain real solutions for ψ_n , the following condition must be satisfied.

$$|\sin \phi + \frac{n\lambda}{d}| \leq 1 \quad (2.7)$$

The number of real solutions is, therefore, finite. The values of n not satisfying the above inequality yield infinitely many complex solutions to Eq. 2.6. These solutions represent waves propagating along the grid surface but decaying exponentially in the direction perpendicular to the grid. As the wavelength relative to the wire spacing increases, more and more of the real side waves become attenuated waves until only a single reflected wave remains. Although not explicitly mentioned in his article, one must note that Wait's reflection coefficient r_p considers only the power in the reflected beam described by $n=0$. However, for large grid wire separation, several reflected beams may exist, and the power in the total reflected field must be considered.

The two principal configurations of grid wire excitation investigated using Wait's theory are given in Figs. 2.3 (a-b). The first shows the electric field lying in the plane of incidence; this case will be denoted by $C_{||\infty}$. The second shows the electric field normal to the plane of incidence ($C_{\perp\infty}$), the ∞ subscript being used to imply an infinitely large grid and uniform plane wave excitation.

Figure 2.2. Coordinate definitions for geometrical derivation of reflected beam angles.

Figure 2.3(a). $C_{\parallel\infty}$. Electric field parallel to the plane of incidence.

Figure 2.3(b). $C_{\perp\infty}$. Electric field normal to the plane of incidence.

Given the above beam-grid geometries, the principal ($n=0$) reflected beam lies in the plane of incidence defined by the incident wave vector and grid normal, and appears at 45° on the opposite side of the normal. The computed mainbeam ($n=0$) reflectivities for both $C_{||\infty}$ and $C_{\perp\infty}$ as a function of λ/d are shown in Fig. 2.4. To the right of the first null in each plot, only the principal reflected beam will propagate. As the grid spacing increases (moving to the left in the figure), additional propagating solutions evolve. $C_{\perp\infty}$ adds one additional solution for each resonance, while due to the symmetry of the grid-beam orientation, $C_{||\infty}$ adds two additional solutions. The higher order lobes actually appear symmetrically on either side of the grid so that two and four lobes are added per resonance to $C_{\perp\infty}$ and $C_{||\infty}$, respectively.

The computed current magnitudes shown in Fig. 2.5 assume that reflection in the vertical plane is being employed (see Fig. 2.3(c)). The dominant polarization of the field from the FEL used for MIX is oriented in the horizontal direction. A small cross-polarized field in the vertical direction has also been considered. The electric intensity of the horizontal field component has been chosen somewhat arbitrarily at 1.1×10^7 V/m, and that of the vertical polarization at 2.6×10^5 V/m. The relative values of these fields are given such that the power density of the vertical component is 5% of the power density of the horizontal component. Recall from Eq. 2.1 that the current magnitude scales linearly with the incident electric field intensity. The current and reflectivity for a particular grid vanish at identical nulls given by d/λ satisfying

$$m \pm \frac{d}{\lambda} \cos\theta \sin\phi = \frac{d}{\lambda} \cos\theta \quad , \quad (2.8)$$

as is easily seen from Eqs. 2.3 and 2.5. Except for the factor of $\cos\theta$, Eq. 2.8 simply defines the values of λ/d where for a given m , the inequality of Eq. 2.7 becomes an equality. The factor of $\cos\theta$ is due to the nature of the dependence of the fields scattered from each wire on the angle θ of the incident beam (explicit expressions for which are given in Chapter 3.)

The curves shown in Fig. 2.4 indicate that the resonances for $C_{\perp\infty}$ are much more sharply defined than for $C_{||\infty}$. The design of a grid with a small reflectivity will put one near a position of resonance. The grid relying on

Figure 2.3(c). Vertical and horizontal beam splitting.

REFLECTIVITY

Figure 2.4. Main beam reflectivity versus λ/d .

CURRENT MAGNITUDE (AMPS)

Figure 2.5. Current magnitude versus λ/d . For $C_{||\infty}$, $E_{inc} = 26$ kV/cm while for $C_{\perp\infty}$, $E_{inc} = 110$ kV/cm.

the coupling employed in the case of $C_{||\infty}$ will be less sensitive to deviations in wire spacing and will require a lower density of wires per inch than the $C_{\perp\infty}$ grid.

The infinitely large grid and uniform plane wave excitation model was next used to examine the current magnitude and principal beam reflectivity as a function of wire radius. The orientation of the grid relative to the incident beam in both the θ and ϕ directions was also varied to determine the sensitivity of the grid behavior to grid positioning. The case of the electric field parallel to the plane of incidence, $C_{||\infty}$, will first be discussed.

Figures 2.6 (a-b) show the grid current magnitude as a function of λ/d for several wire radii ranging from 5 to 100 μm (recall that 1 mil = 25.4 μm exactly). Here the electric intensity of the incident field was taken to be 5×10^6 V/cm (so that the component parallel to the grid wires was $(5/\sqrt{2}) \times 10^6$ V/cm). Plotting the current magnitude (normalized for a maximum of one) versus the wire radius for a spacing of $\lambda/d = 1$, one obtains the curve shown in Fig. 2.7. Beyond about 40 μm the current rises linearly with wire radius. Similar computations were performed for the main beam reflectivity (see Fig. 2.8). The reflectivity was also found to vary nearly linearly with wire radius as shown in Fig. 2.9.

The angle of incidence of the input beam was next allowed to vary. Figure 2.10(a) shows plots of the reflectivity for several values of ϕ ranging from 0° to 20° while θ remains fixed at -45° . By allowing the incident beam to strike the grid at greater absolute ϕ , for a given λ/d the beam will "see" a higher wire density which effectively pushes the nulls in the direction of greater actual grid wire density. A similar effect is observed by varying θ from -35° to -55° (and keeping ϕ fixed at 0°), although for an entirely different reason (see Fig. 2.10(b)). Here the varied null position as a function of θ is due to the nature of the scattered fields arising from a traveling wave current induced by the incident plane wave. The traveling wave current radiates as a Hankel function whose argument has a trigonometric dependence on the angle θ of the incident beam (cf. Eq. 3.5). This changes the fields scattered from each wire and alters the coupling among the grid wires.

Figure 2.6. $C_{||\infty}$. Current magnitude versus λ/d for wire radii ranging from 5 to 100 μm , and $E_{\text{inc}} = 50 \text{ kV/cm}$.

Figure 2.7. $C_{||\infty}$. Current magnitude versus wire radius
for $\lambda/d = 1$ and $E_{inc} = 50$ kV/cm.

Figure 2.8. $C_{||\infty}$. Main beam reflectivity versus λ/d for wire radii ranging from 20 to 100 μm .

Figure 2.9. $C_{||\infty}$. Main beam reflectivity versus wire radius for $\lambda/d = 1$.

Figure 2.10(a). $C_{||\infty}$. Main beam reflectivity versus λ/d for ϕ ranging from 0° to 20° and θ constant at -45° .

Figure 2.10(b). $C_{||\infty}$. Main beam reflectivity versus λ/d for θ ranging from -35° to -55° and ϕ constant at 0° .

These results suggest that error in the grid positioning which could cause the incident beam to strike the grid at an angle more nearly aligned with the grid normal than the design specifies may cause undesired reflected beams to emerge. To allow for small grid misalignment, one should operate to the right of the null at the expense of a slightly higher reflectivity.

Computations identical to those described above were also carried out for the second grid configuration, $C_{\perp\infty}$, with an electric intensity of the incident wave given as 2.25×10^7 V/m (see Figs. 2.11 to 2.15). Here it is interesting to note that although the shift in the nulls as a function of the incident angle is less pronounced than for $C_{\parallel\infty}$, because the resonances are more sharply defined for $C_{\perp\infty}$, the overall effect becomes more severe. This may be another reason to choose the coupling implied by $C_{\parallel\infty}$ over that of $C_{\perp\infty}$ if only one grid is to be used (or at least to orient the grids of a mesh such that the dominant polarization relies upon the $C_{\parallel\infty}$ type of field coupling).

The fact that the current and main beam reflectivity vanish at the nulls in the two cases studied above is a direct consequence of the assumptions that the grid is infinitely large and being excited by a uniform plane wave. One may suspect that (the resonances will not be as deep) for a grid of finite dimensions excited by a field of limited extent. Chapter 3 of this report will consider a method for obtaining the individual wire currents and scattered fields for the finite grid configuration.

3. Finite Grid Theory

The title of this chapter is somewhat of a misnomer since the grids to be considered here are composed of a finite number of wires, but each of infinite length (oriented parallel to the z-axis and lying in the x^y -plane (see Fig. 3.1). The incident beam magnitude profile will be allowed to vary as a function of x , although it will remain constant along the length of each wire. A more rigorous analysis that incorporates a two-dimensional beam variation and a grid finite in both directions could, in principle, be performed using a moment method such as finite elements. However, a rough estimate of the number of basis functions required for reasonably accurate results indicates that a matrix inversion involving a 150,000 by 150,000

Figure 2.11. C_{100} . Current magnitude versus λ/d for wire radii ranging from 5 to 100 μm and $E_{inc} = 225$ kV/cm.

Figure 2.12. C_{100} . Current magnitude versus wire radius for $\lambda/d = 2$ and $E_{inc} = 225$ kV/cm.

Figure 2.13. $C_{1\infty}$. Main beam reflectivity versus λ/d for several wire radii ranging from 20 to 100 μm .

Figure 2.14. $C_{1\infty}$. Main beam reflectivity versus wire radius for $\lambda/d = 2$.

REFLECTION COEFFICIENT VS. SEPARATION

Figure 2.15(a). C_{100} . Main beam reflectivity versus λ/d for θ ranging from 0° to 20° and ϕ constant at -45° .

REFLECTION COEFFICIENT VS. SEPARATION

Figure 2.15(b). C_{100} . Main beam reflectivity versus λ/d for ϕ ranging from -35° to -55° and θ constant at 0° .

Figure 3.1. Coordinate axis definitions for the finite grid.

element matrix would be necessary. The size of this task is prohibitively large and a more manageable, though less accurate, algorithm has been sought. It should be noted that finite element type solutions are used quite successfully in other parallel wire structures (e.g., the Yagi-Uda antenna array), although here the wire lengths are on the order of a wavelength and far fewer basis functions are required.

Computer simulations of the incident beam from the FEL at the position of the proposed beam splitter indicate that the beam profile may be approximated by a two-dimensional Gaussian distribution (i.e., the beam varies as a Gaussian in two perpendicular directions). A useful property of this particular profile is that the full two-dimensional variation can be obtained as the product of two one-dimensional Gaussian distributions, each varying in one of two perpendicular directions and constant in the other direction.

If the plane of incidence of the input beam is parallel to the xy-plane, only the variation of the beam normal to the grid wire axes affects appreciably the grid wire current magnitudes at a particular value of z . For the actual two-dimensional beam, the induced current magnitudes will, of course, vary as a function of z . However, because the scattered fields from each wire propagate radially from the wire axis, only grid currents at a given value of z will couple.

If the plane of incidence is not parallel to the xy-plane, a component of the Poynting vector parallel to the grid wire axes will result. Here, induced wire currents at different positions along the z -axis may couple. In the algorithm to be described below, the input field is allowed to vary as a Gaussian in the direction normal to the wire axes only. This implies that the wire current magnitudes along the length of each wire will be uniform. The field striking the grid wires induces a traveling wave current along the length of each wire that varies sinusoidally with time and position along the wire. In discussions of current magnitude, it is assumed that the coefficient of the sinusoidal variation is being considered and that the phase variation is implied. Hence, the effect of the currents varying as a function of z on the current coupling at different positions along the z -axis has been ignored.

Results to be presented below indicate that the one-dimensional Gaussian beam varying normal to the wire axes induces wire currents that follow the

exact Gaussian distribution from wire to wire for a grid separation far from a resonance. It seems intuitively obvious that by varying the beam as a Gaussian along the z-axis, one will also obtain a Gaussian distribution along the length of each wire. The full two-dimensional Gaussian profile of the input beam will thus be duplicated in the grid wire currents (although discretely in the y-direction and continuously along z). The current coupling at different positions along the z-axis does not ultimately change the Gaussian current distribution normal to the wire axes. Here, varying currents along z may be considered to merely translate the current distribution in the direction parallel to the wire axes. The effects of finite grid size and varying input field magnitude may, therefore, be investigated by allowing a beam varying only as a function of x to strike a grid composed of a finite number of wires, but each of infinite length (as shown in Fig. 3.1).

The method used to obtain the grid currents that will be described below involves the inversion of a much smaller matrix than that required for the finite element solution. It will be shown that the matrix is of size M by M, where M is the total number of grid wires (on the order of 100 for the grids to be considered here). Obviously, since the input field does not vary along the length of the grid wires, the reflected fields will also show no (magnitude) variation as a function of z. One must be certain, therefore, to scan the radiation pattern perpendicular to the wires to observe field structure. The reflected lobes are expected to be Gaussian. The full two-dimensional analysis would also provide for a Gaussian variation along the direction parallel to the wire axes.

The choice of coordinate axes for the grid of finite extent along the dimension of the Gaussian distribution of the incident field is again shown in Fig. 3.2 along with an incident plane wave, $\vec{E}^P$. The grid is composed of $2N + 1$ infinitely long wires with indices assigned as indicated on the figure. The incident plane wave may be written:

$$\begin{aligned}\vec{E}^P(x,y,z) &= \vec{A}(x,y,z) e^{-i\vec{k} \cdot \vec{r}} \\ &= \vec{A}(x,y,z) e^{ik(x\sin\theta\cos\phi + y\sin\theta\sin\phi + z\cos\theta)}\end{aligned}\quad (3.1)$$

Figure 3.2. Finite grid with incident plane wave showing new θ and ϕ definitions.

The z-component is given by

$$E_z^p = A_z(x, y, z) e^{ik(x \sin \theta \cos \phi + y \sin \theta \sin \phi + z \cos \theta)} \quad (3.2)$$

For the one-dimensional Gaussian beam

$$A_z(x, y, z) = E_0 e^{-(x/w)^2} \quad (3.3)$$

where E_0 is the on-axis electric field intensity and w is the 1/e field radius (the beam waist). One may observe that specifying A_z in this manner yields a function that does not satisfy the free-space wave equation. However, it may be shown that for a waist size large relative to the grid wire spacing, the simple expression given in Eq. 3.3 may be used with reasonable accuracy.

Given a single wire coincident with the z-axis and carrying the current $I(z)$ given by

$$I(z) = I_0 e^{ikz \cos \theta} \quad (3.4)$$

the z-component of the electric field produced by this current will be

$$E_z = \frac{I_0 \mu_0 \omega \sin^2 \theta}{4} H_0^{(2)}(k \sqrt{x^2 + y^2} \sin \theta) e^{ikz \cos \theta} \quad (3.5)$$

where

- μ_0 = free space permeability,
- ω = radian frequency,
- k = free space wave number, and
- $H_0^{(2)}(t)$ = Hankel function of the second type of order zero.

This solution assumes that the wire radius is much smaller than the wavelength of the excitation. Only longitudinal wire currents may be supported to any appreciable degree, and just one Hankel function is required to describe the radiated field. In the more general case considering wires of much larger

radii, a series of Hankel functions is necessary. If one now considers a plane wave exciting a traveling wave current on each wire of a grid beam-splitter, the z-component of the scattered field may be written as a sum over the fields due to the individual wire currents:

$$E_z^S = \frac{\mu_0 \omega \sin^2 \theta}{4} e^{ikz \cos \theta} \cdot \sum_{n=-N}^N I_n e^{iknd \sin \theta \cos \phi} H_0^{(2)} [ks \sin \theta \sqrt{(nd-x)^2 + y^2}] \quad (3.6)$$

The additional phase factor arises from the phase difference of the excitation of each wire caused by the beam striking the grid at an angle. All phase terms disappear for normal incidence.

Here one may observe the explicit dependence of the scattered field on the angle θ . The phase of the induced wire traveling wave currents affects the radiation pattern in a manner that has also been exploited in the design of the so-called Beverage antenna. The dependence on $\sin \theta$ through the argument of the Hankel function changes the grid wire coupling as a function of θ which ultimately determines the positions of current resonances.

Since the induced currents are confined to the z-direction, the scattered field can be derived from an electric Hertz vector which has only a z-component, π_z^S (Panoisky and Phillips). The scattered field is then given by

$$\vec{E}^S = k^2 \pi_z^S \vec{z} + \text{grad} \frac{\partial \pi_z^S}{\partial z} \quad (3.7)$$

where $\vec{z}$ is the unit vector in the z-direction. However, since

$$\frac{\partial}{\partial z} = ik \cos \theta \quad , \quad (3.8)$$

the field components may be written

$$E_x^S = ik \cos \theta \frac{\partial \pi_z^S}{\partial x}, \quad (3.9a)$$

$$E_y^S = ik \cos \theta \frac{\partial \pi_z^S}{\partial y}, \text{ and} \quad (3.9b)$$

$$E_z^S = k^2 \sin^2 \theta \pi_z^S \quad (3.9c)$$

The total electric field is then

$$\vec{E} = \vec{E}^P + \vec{E}^S \quad (3.10)$$

The complex magnitudes I_n of the currents along the wires are calculated by imposing the boundary condition that the tangential fields are continuous at the surface of the wires,

$$E_z^P + E_z^S = -I(z) \cdot Z_i \quad (3.11)$$

If the wire radius, in addition to being much smaller than a wavelength, is also much less than the grid wire separation, the incident field may be taken to be uniform around each wire. Z_i then becomes the wire internal impedance of Eq. 2.2. Written out explicitly, the boundary condition for the m th wire becomes

$$\begin{aligned} E_0 e^{-(md/w)^2} e^{ikmd \sin \theta \cos \phi} + \frac{v_0 \omega \sin^2 \theta}{4} e^{ikz \cos \theta} \\ \cdot \left\{ \sum_{\substack{n=-N \\ n \neq m}}^N I_n e^{iknd \sin \theta \cos \phi} \cdot H_0^{(2)} [k \sin \theta |m-n|d] \right. \\ \left. + I_m e^{ikmd \sin \theta \cos \phi} \cdot H_0^{(2)} (ka \sin \theta) \right\} \\ = -I_m e^{ikmd \sin \theta \cos \phi} \cdot Z_i \quad (3.12) \end{aligned}$$

Doing this for each wire leads to a set of $2N+1$ equations in the $2N+1$ unknown complex current magnitudes I_n . This linear system may then be solved using familiar matrix inversion techniques. The coefficient matrix may be cast into a form that is Toeplitzian to allow for more efficient matrix evaluation. Having solved for the currents one may then calculate the scattered fields using the series in Eq. 3.6 and Eqs. 3.9 (a and b).

Before more detailed field calculations are performed, the two grid wire configurations to be explored will first be described. These correspond exactly to the cases $C_{||\infty}$ and $C_{\perp\infty}$ of the infinite wire grids but with a

minor coordinate axis transformation (see Figs. 3.3 (a and b)). The direction of the Gaussian field variation is also indicated for both $C_{\perp f}$ and $C_{\parallel f}$ (the f for finite). The angle definitions were chosen to correspond more naturally with common usage of θ , the polar angle measured from the z-axis, and ϕ , the azimuthal angle measured from the xz-plane and starting with positive values of x.

Field Expressions

Using Eq. 3.6 and the fact that $\phi = \pi/2$ and $\theta = 3\pi/4$, the scattered electric field for $C_{\parallel f}$ becomes

$$E_z^S = \frac{\mu_0 \omega}{8} e^{-ikz/\sqrt{2}} \cdot \sum_{n=-N}^N I_n \cdot H_0^{(2)}(k\rho_n/\sqrt{2}) \quad (3.13)$$

where

$$\rho_n = \sqrt{(nd - x)^2 + y^2}$$

The boundary condition at the m th wire may now be written

$$\left\{ e^{-(md/w)^2} + \frac{\mu_0 \omega}{8} \sum_{\substack{n=-N \\ n \neq M}}^N I_n \cdot H_0^{(2)}(kd|m-n|/\sqrt{2}) + I_m \cdot H_0^{(2)}(ka/\sqrt{2}) \right\} = -I_m \cdot Z_i \quad (3.14)$$

In terms of the electric Hertz vector, all three electric field components are given as

$$E_x^S = (-ik/\sqrt{2}) \partial \pi_z^S / \partial x \quad (3.15a)$$

$$E_y^S = (-ik/\sqrt{2}) \partial \pi_z^S / \partial y \quad (3.15b)$$

$$E_z^S = k^2 \pi_z^S / 2 \quad (3.15c)$$

Since

$$\pi_z^S = 2E_z^S/k^2, \text{ one finds that}$$

$$E_z^S = \frac{\mu_0 \omega}{4k^2} e^{-ikz/\sqrt{2}} \cdot \sum_{n=-N}^N I_n \cdot H_0^{(2)}(k\rho_n/\sqrt{2}) \quad (3.16)$$

The following derivative is required

$$\frac{\partial H_0^{(2)}(u)}{\partial u} = -H_1^{(2)}(u) \frac{\partial u}{\partial w} \quad (3.17)$$

Performing the differentiations indicated and after a bit of simplification, one may obtain

$$E_y^S = \frac{\mu_0 \omega}{8} e^{-ikz/\sqrt{2}} \cdot \sum_{n=-N}^N \left(\frac{y}{\rho_n}\right) I_n \cdot H_1^{(2)}(k\rho_n/\sqrt{2}) \quad (3.18a)$$

$$E_x^S = \frac{\mu_0 \omega}{8} e^{-ikz/\sqrt{2}} \cdot \sum_{n=-N}^N \left(\frac{x - nd}{\rho_n}\right) I_n \cdot H_1^{(2)}(k\rho_n/\sqrt{2}) \quad (3.18b)$$

The scattered field is the sum of E_x^S , E_y^S , and E_z^S : $\vec{E}^S = E_x^S \hat{x} + E_y^S \hat{y} + E_z^S \hat{z}$. From simple antenna theory, one may recall that in a spherical coordinate system only the field components E_θ and E_ϕ will remain in the far-field. The following unit vector transformations are required

$$\hat{x} = \hat{r} \sin \theta' \cos \phi' + \hat{\theta} \cos \theta' \cos \phi' - \hat{\phi} \sin \phi' \quad (3.20a)$$

$$\hat{y} = \hat{r} \sin \theta' \sin \phi' + \hat{\theta} \cos \theta' \sin \phi' + \hat{\phi} \cos \phi' \quad (3.20b)$$

and

$$\hat{z} = \hat{r} \cos \theta' - \hat{\theta} \sin \theta' \quad (3.20c)$$

where $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ are the spherical coordinate unit vectors and ϕ' and θ' are the azimuthal and polar angles in the observation coordinate system (not to be confused with ϕ and θ associated with the incident field).

Substituting Eqs. 3.20(a-c) into the expression for the total field, one obtains after some simplification

$$E_\theta = (E_x^S \cos \phi' + E_y^S \sin \phi') \cos \theta' - E_z^S \sin \theta' \quad (3.21a)$$

Figure 3.3(a). $C_{||t}$. Electric field parallel to the plane of incidence.

Figure 3.3(b). $C_{\perp t}$. Electric field normal to the plane of incidence.

and

$$E_{\phi} = E_x^S (-\sin\phi') + E_y^S \cos\phi' \quad (3.21b)$$

The results for C_{1f} are quite a bit simpler. Since $\theta = \pi/2$ and $\phi = 3\pi/4$, one finds from Eq. 3.6 that

$$E_z^S = \frac{\mu_0 \omega}{4} \cdot \sum_{n=-N}^N I_n e^{-inkd/\sqrt{2}} \cdot H_0^{(2)}(k\rho_n) \quad (3.22)$$

The boundary condition at the m th wire becomes

$$E_0 e^{-(md/w)^2} e^{-imkd/\sqrt{2}} + \frac{\mu_0 \omega}{4} \left\{ \sum_{\substack{n=-N \\ n \neq m}}^N I_n e^{-inkd/\sqrt{2}} \cdot H_0^{(2)}(kd|m-n|) + I_m e^{-imkd/\sqrt{2}} \cdot H_0^{(2)}(ka) \right\} = -I_m e^{-imkd/\sqrt{2}} \cdot Z_1 \quad (3.23)$$

Using Eqs. 3.9(a-c) and $\theta = \pi/2$, one may find that $E_x^S = 0$ and $E_y^S = 0$. From Eqs. 3.21(a-b) one may then observe that

$$E_{\theta} = -E_z^S \sin\phi' \quad , \quad \text{and} \quad (3.24)$$

$$E_{\phi} = 0 \quad .$$

4. Finite Grid Currents and Scattered Fields

The current evaluation and scattered field computations were initially performed on a Hewlett-Packard personal computer. The number of grid wires required was chosen such that the grid dimension in the direction of the Gaussian profile variation was about three times the waist size. (If the grid is placed at an angle to the beam path, it is the projection of the grid onto a plane normal to the beam path that is taken to be three times the waist size.) The first computations assumed that reflection in the vertical plane was to be employed. Given the appropriate grid orientations and waist sizes expected, the HP pc had insufficient memory to handle all values of wire separation required. However, the HP-BASIC code was used to verify the solution algorithms and yielded valuable insight into the grid behavior, especially in a region of resonance.

Before presenting the computed results, the determination of field intensity given an incident beam power will first be discussed. In calculations of wire absorption due to ohmic heating (figures for which will be given in the next section), it is assumed that the length of the wires is equal to three times the beam waist in the direction parallel to the axes of the wires (the z-axis). This waist is not, in general, the same size as the waist used in calculating the field exciting the grid. In fact, recall that for current and scattered field computations, the grid wires are assumed to be infinitely long.

Given that the beam has a Gaussian profile along x and is constant along z, the power in the beam per unit length in the z-direction may be obtained as follows. The power density per unit length is given by

$$S_d(x) = S_0 e^{-2(x/w)^2} \quad , \quad (4.1)$$

where S_0 is the on-axis power density. The power per unit length P_Δ may then be found by integration over x (recall that the beam is taken to be a plane wave),

$$P_\Delta = \int_{-3w/2}^{3w/2} S_d(x) dx \quad . \quad (4.2)$$

Allowing the limits of integration to extend to plus and minus infinity introduces only small error but greatly simplifies the analysis. The integral then becomes

$$\begin{aligned} P_\Delta &= \int_{-\infty}^{\infty} S_0 e^{-2x^2/w^2} dx \\ &= 2 \int_0^{\infty} S_0 e^{-2x^2/w^2} dx \quad . \end{aligned} \quad (4.3)$$

Recall that the error function, $\text{erf}(x)$, is given by

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad , \quad (4.4)$$

so that P_A becomes

$$P_A = \sqrt{\pi}/2 w S_0 \operatorname{erf}(w) = \sqrt{\pi}/2 w S_0 \quad (4.5)$$

The power in the beam of finite extent is found by multiplying P_A by the length of a grid wire projected onto a plane normal to the beam path. This procedure may also be executed in reverse order to obtain the power density (and ultimately the electric field intensity) given a total beam power and grid size.

For the computations performed here, the vertical waist is taken as 1.2 cm and the horizontal waist as 2.34 cm. Since vertical reflection is being employed, the grids are tilted back 45° with respect to the input beam path which causes the reflected beam principal lobes to be directed normal to the incident wave vector. The grid dimensions are thus found to be

$$\text{Height} = (1.2 \text{ cm})(3)/\cos 45^\circ$$

$$\approx 31 \text{ cm, and}$$

$$\text{Width} = (2.34 \text{ cm})(3)$$

$$\approx 7 \text{ cm.}$$

The power in the horizontal polarization is 3 GW, while due to possible cross-polarization, the vertical polarization is given as 5% of 3 GW or 150 MW. Using Eq. 4.5, the electric intensities of the horizontal and vertical polarizations were found to be 2.25×10^7 V/m and 5.0×10^6 V/m, respectively.

Turning first to C_{eff} (which implies coupling to the vertical polarization), the current and scattered field for $\lambda/d = 1$ are shown in Figs. 4.1 (a and b). Note from Fig. 2.4 that for this wire separation, the grid is well within the single reflected lobe regime (i.e., to the right of the first null). Figure 4.1(a) is actually two plots superimposed: the first is the magnitude of the current as a function of the wire index, and the second is the incident field at the position of each wire normalized so that the maxima of the two plots coincide. (The current plot should actually be represented with discrete points showing the current at the individual wires and not as a continuous curve. The current magnitude along each wire

CURRENT MAGNITUDE

Figure 4.1(a). C_{llf} . Current magnitude versus wire index for $\lambda/d = 1$ and $E_{inc} = 50$ kV/cm.

E_THETA FIELD MAGNITUDE

Figure 4.1(b). C_{llf} . E_{θ} versus ϕ for $\lambda/d = 1$ ($\theta = 45^\circ$).

is, of course, taken to be constant since the field is uniform in the z-direction.) Far from a resonance, the current magnitudes follow the exact Gaussian variation of the incident beam.

In Fig. 4.1(b) the normalized scattered field is plotted as a function of the angle ϕ while keeping θ constant at 45° . For this configuration the principal reflected lobe always lies along the ray given by $\theta = 90^\circ$ and $\theta = 45^\circ$. Simulated results indicate that the coupling to E_ϕ is very weak and so the field plots for this polarization have not been included.

For a separation of $\lambda/d = 0.73$, the infinite grid would not have yet reached the first resonance. Figures 4.2 (a and b) show the simulated results for the finite grid at this grid wire separation. The current magnitude has deviated significantly from the Gaussian distribution near the outer edges of the grid. In addition, two higher order lobes have fully begun to make the transition from evanescent field along the grid surface to propagating solution. These lobes represent power lost from the incident beam and raise the grid reflectivity significantly.

At $\lambda/d = 0.5$ the grid is between the first and second resonance and two higher order lobes have lifted completely from the grid surface (see Figs. 4.3(a and b)). For applications such as the beam splitter considered here, the existence of higher order lobes can be a problem for at least two reasons. The first is that in order to avoid possible interference effects at the photon drag detectors, microwave absorber must be placed in the Tokamak heating port to eliminate the power of the undesired lobes. The second is that the higher order lobes represent a substantial energy loss from the incident beam and greatly reduce the grid transmissivity. Although scattered fields have only been plotted for ϕ ranging from 0° to 180° , an identical radiation pattern is obtained by sweeping ϕ from 360° down to 180° . Plots of the dramatic rise in the total reflectivity beyond the first null will be presented in the next chapter.

Results for the case of $C_{\perp f}$ (which implies coupling to the horizontal polarization) are similar to those just discussed. Because the symmetry inherent in the configuration is different from $C_{\parallel f}$, only one additional higher order solution for ϕ between 0° and 180° is added as one sweeps through a resonance by reducing λ/d . Here again the identical radiation pattern is obtained for ϕ ranging from 360° down to 180° . For this grid orientation the principal reflected beam lies at $\phi = 45^\circ$ and $\phi = 90^\circ$.

CURRENT MAGNITUDE

Figure 4.2(a). C_{llf} . Current magnitude versus wire index for $\lambda/d = 0.73$ and $E_{inc} = 50$ kV/cm.

E_THETA FIELD MAGNITUDE

Figure 4.2(b). C_{llf} . E_{θ} versus ϕ for $\lambda/d = 0.73$ ($\theta = 45^\circ$).

CURRENT MAGNITUDE

Figure 4.3(a). C_{llf} . Current magnitude versus wire index for $\lambda/d = 0.5$ and $E_{inc} = 50$ kV/cm.

E_THETA FIELD MAGNITUDE

Figure 4.3(b). C_{llf} . E_θ versus ϕ for $\lambda/d = 0.5$ ($\theta = 45^\circ$).

Figures 4.4 (a and b) show the simulated results for $\lambda/d = 1$. From Fig. 2.4 one may observe that this separation lies between the first and second nulls which suggests that a single higher order lobe will be present (as is clearly shown at $\phi \approx 107^\circ$).

The plots obtained for $\lambda/d = 1/3$ will be used to verify the geometrical derivation of lobe positions. This case (shown in Figs. 4.5 (a and b)) also reveals two other important features of large grid wire separation. The first is that an appreciable amount of the reflected energy has obviously been diverted away from the principal lobe. The second feature is that one of the additional beams may be reflected in a direction 180° to that of the principal lobe where it will be absorbed by the opposite photon drag detector.

Recall from Chapter 2 that the positions of the reflected lobes (when the plane of incidence is the xy-plane) are given by Eq. 2.6 repeated here in a slightly modified form to accommodate a new coordinate system

$$\sin \psi_n = -\cos \phi + \frac{n\lambda}{d} \quad , \quad n = 0, \pm 1, \pm 2 \quad , \quad \dots$$

One seeks real values of ψ_n satisfying

$$\begin{aligned} \psi_n &= \sin^{-1} \left(-\cos 135^\circ + \frac{n}{3} \right) \\ &= \sin^{-1} \left(\frac{\sqrt{2}}{2} + \frac{n}{3} \right) \quad , \end{aligned} \tag{4.6}$$

with the sign convention as shown in Fig. 4.6. For $n = 0$,

$$\psi_0 = \sin^{-1} \left(\frac{\sqrt{2}}{2} \right) = 45.0^\circ \text{ and } 135.0^\circ \quad ,$$

which corresponds to $\phi = 315.0^\circ$ and 45.0° ; respectively. The solution for $\phi = 45.0^\circ$ is the principal reflected solution, whereas $\phi = 135.0^\circ$ represents the transmitted field. For $n = -1$,

$$\psi_{-1} = \sin^{-1} \left(\frac{\sqrt{2}}{2} - \frac{1}{3} \right) = 22.0^\circ \text{ and } 158.0^\circ \quad ,$$

or

$$\phi_{-1} = 292.0^\circ \text{ and } 68.0^\circ \quad .$$

CURRENT MAGNITUDE

Figure 4.4(a). C_{1f} . Current magnitude versus wire index for $\lambda/d = 1$ and $E_{inc} = 225$ kV/cm.

FIELD MAGNITUDE

Figure 4.4(b). C_{1f} . E_{θ} versus ϕ for $\lambda/d = 1$
($\theta = 90^\circ$).

CURRENT MAGNITUDE

Figure 4.5(a). C_{lf} . Current magnitude versus wire index for $\lambda/d = 1/3$ and $E_{inc} = 225$ kV/cm.

FIELD MAGNITUDE

Figure 4.5(b). C_{lf} . E_{θ} versus ϕ for $\lambda/d = 1/3$ ($\theta = 90^\circ$).

Figure 4.6. New coordinate definitions for geometrical derivation of reflected beam angles.

Real solutions are also possible for $n=-2, -3, -4$, and -5 with the results tabulated in Table 4.1. The reflected lobes at $\phi = 45.0^\circ, 68.0^\circ, 87.7^\circ, 107.0^\circ, 128.8^\circ$, and 163.7° are easily seen in Fig. 4.5(b).

TABLE 4.1

n	ψ_n (deg.)	ϕ_n (deg.)
0	45.0	315.0
	135.0	45.0
-1	22.0	292.0
	158.0	68.0
-2	2.3	272.3
	177.7	87.7
-3	-17.0	253.0
	-163.0	107.0
-4	-38.8	231.2
	-141.2	128.8
-5	-73.7	196.3
	-106.3	163.7

By integrating the power in all of the reflected lobes, one may obtain a measure of the grid total reflectivity. These calculations, along with estimates of ohmic heating, will be discussed in Chapter 5.

5. Finite Grid Total Reflectivity and Ohmic Heating

The plots to be presented in this chapter were generated by FORTRAN source code run on the NMFEC computer system. The algorithms used are nearly identical to those implemented on the HP pc in HP-BASIC. One difference can be found in the Hankel function (and its derivatives) evaluation subroutines. The BASIC code employed an ascending series solution for small arguments or an asymptotic form for large arguments. The FORTRAN code relied upon polynomial approximations (Abramowitz and Stegun). The latter algorithm is significantly faster for small arguments with negligible difference in accuracy.

The power dissipated in the grid wires due to ohmic heating was calculated as follows. At the frequency considered for these simulations (140 GHz), one may safely assume that the currents are confined to about a skin depth of the wire cross-section. In that case the ohmic power loss associated with wire m is given by

$$P_{Lm} = 1/2 |I_m|^2 R_L, \quad (5.1)$$

where

I_m = peak complex current of the m th wire ,

and

R_L = skin effect resistance of the wire .

The total ohmic power loss is the sum of the P_{Lm} over all the $2N+1$ wires of the grid. Given a wire of length L , the skin effect resistance may be written

$$\begin{aligned} R_L &= R_e \left(|Z_i| \cdot L \right) \\ &= \left(\frac{\mu_w \omega}{2\sigma} \right)^{1/2} \frac{1}{2\pi a} \cdot L \\ &= \frac{1}{2\pi \sigma \delta} \cdot L, \end{aligned} \quad (5.2)$$

where δ is the wire skin depth.

Using a simple one-dimensional slab model for the grid wire, one may obtain rough estimates for the diffusion time associated with the transport of heat away from the skin depth. Recall that the diffusion time τ is given by

$$\tau \approx \frac{\delta^2}{4D}, \quad (5.3)$$

with D the wire thermal diffusivity and δ the diffusion depth. Say that one is using 25 μm radius Tungsten wire. The room temperature skin depth is given by (all units MKS)

$$\delta = \left(\frac{2}{\mu_w \sigma \omega} \right)^{1/2}$$

$$= \left[\frac{2}{(4\pi \cdot 10^{-7})(5.51 \cdot 10^{-8})^{-1}(2\pi \cdot 140 \cdot 10^9)} \right]^{1/2}$$

$$= 0.32 \mu\text{m}$$

For $\delta < \delta < \text{radius}$, the diffusion times are ($D = 0.78 \text{ cm}^2/\text{s}$ for Tungsten)

$$\tau_{\min} \approx \frac{(0.32 \cdot 10^{-4})^2}{4 \cdot 0.78} \approx 3 \cdot 10^{-10} \text{ sec}$$

and

$$\tau_{\max} \approx \frac{(25.0 \cdot 10^{-4})^2}{4 \cdot 0.78} \approx 2 \cdot 10^{-6} \text{ sec}$$

For a pulsed power repetition rate of 5 kHz, the time between pulses is

$$1/(5 \cdot 10^3) = 2 \cdot 10^{-4} \text{ sec}$$

which is two orders of magnitude greater than the maximum diffusion time. Hence, although the ohmic losses will initially appear to be confined to within about a skin depth, between pulses the heat will have sufficient time to diffuse throughout the wire. To calculate the temperature rise of the grid wire, therefore, the mass of the entire wire cross-section will be used. Since the center wire is in a position of greatest field intensity, its temperature change ΔI will be taken as an upper bound on wire heating. The temperature rise of the center wire is given by

$$\Delta I = \frac{P_{L0} \Delta t}{W_m C_p} \quad (5.4)$$

where

Δt = pulse duration,
 W_m = wire mass,
 C_p = wire specific heat, and
 P_{Lo} = ohmic power loss of the center wire.

The simulated results to follow assume that reflection in the horizontal plane is now being employed so that the dominant polarization relies on the coupling implied by C_{Hf} . In addition, more recent data on the beam characteristics had become available and both magnitude and phase information on the beam profile have been incorporated. The new waist sizes in the vertical and horizontal polarizations are taken as 8.9 cm and 4.0 cm, respectively. Each phase characteristic was approximated by a parabola with reasonably good fit.

The projection of the grid onto a plane perpendicular to the incident wave vector is again taken to be three times the waist sizes in both directions. The grid wires with horizontal alignment are thus 17 cm long with a grid height of 27 cm, while the grid wires with vertical alignment are 27 cm long with a grid width of 17 cm. In both cases, the total number of wires is obviously dependent on the wire separation.

The power densities of the vertical and horizontal polarizations were calculated from Eq. 4.5 assuming that the total power in the horizontal component of the beam is 3 GW and that in the vertical component is 150 MW (5% cross-polarization). The calculation also assumed the grid sizes given above and that the side-to-side beam splitting shown in Fig. 2.3 (c) is now being employed. The on-axis electric field intensity of the horizontal polarization may then be found to be 1.54×10^7 V/m. For the C_{Hf} type coupling, this implies that the field strength parallel to the wire is 1.09×10^7 V/m, a factor of $\cos 45^\circ (= \sqrt{2})$ less. The field strength of the vertical polarization may similarly be found to be 3.44×10^6 V/m. However, due to a small programming error, a value of 4.1×10^6 has been used in the simulations for the C_{Hf} field coupling.

The results of the numerical analysis for C_{lf} (coupling to the vertical polarization) are plotted in Figs. 5.1(a-f). In all the figures provided, a particular grid characteristic is shown plotted as a function of λ/d in the neighborhood of the first resonance occurring at about $\lambda/d = 1.71$. From the results presented earlier in this report, one should suspect that beyond the first resonance the total power reflected due to the presence of multiple lobes will be too great to yield a beam splitter having a small reflectivity. Hence, only beam characteristics near the first null will be of interest. The validity of this assertion will be seen shortly.

In Fig. 5.1(a) are plotted the main beam reflectivities of both the finite and the infinite grids for 25, 50, and 75 μm radius Tungsten wires (labeled 1, 2, and 3, respectively). The main beam reflectivity vanishes at the null for the infinite grid but gets only as low as $\approx 4\text{-}5\%$ for the finite grid. As one moves away from the null the reflectivities of the two grids agree quite well. The deviation from ideal behavior at the position of the infinite grid resonance is probably due more to the Gaussian variation of the incident field than to the finite dimensions of the grid, although both do contribute to the null filling. As the beam waist and grid size are made larger, one may observe that the nulls become deeper, verifying the expected asymptotic behavior.

In Fig. 5.1(b) are plotted both the main beam reflectivity and total reflectivity of the finite grid. Well before the first resonance (at the far right in the figure), the two curves coincide. Here only a single reflected beam, the $n=0$ principal order solution, is observed. As one approaches the null, the next higher order lobe begins to make the transformation from evanescent wave along the surface of the grid to a fully propagating reflected lobe. If the grid were of infinite extent and excited by a uniform plane wave, the transition from single to multiple reflected lobes would occur precisely at the first resonance. Because the real grid and input beam are finite, however, the required current configuration for vanishing total grid currents cannot be supported, and additional beams begin to emerge somewhat before the infinite grid resonance. One may also observe the dramatic increase in total grid reflectivity beyond the first resonance. In an actual

Figure 5.1(a). C_{1f} . Main beam reflectivity versus λ/d .

Figure 5.1(b). C_{1f} . Total reflectivity versus λ/d .

Figure 5.1(c). C_{1f} . Center wire current magnitude versus λ/d for $E_{inc} = 41$ kV/cm.

Figure 5.1(d). Center wire temperature rise versus λ/d .

Figure 5.1(e). C_{Lf} . Grid ohmic heating loss versus λ/d for $E_{inc} = 41$ kV/cm.

Figure 5.1(f). C_{Lf} . Ratio of ohmic heating power to input beam power versus λ/d .

grid design requiring low total reflectivity, one will clearly choose to construct a grid such that the separation of the wires places it before the first resonance.

The absolute current magnitudes for both the finite and infinite grids are shown in Fig. 5.1(c). The finite grid current plotted is that which occurs along the center wire at the position of maximum field intensity (recall that the current amplitude is constant along each wire). This maximum field intensity, 154 kV/cm, was used as the electric field intensity of the uniform plane wave exciting the infinite grid. Here one may observe the excellent agreement between the two solutions when the separation is not near the resonance. At the position of the infinite grid resonance, the currents vanish (a finer separation of plotted values would be necessary to see this), while the null of the finite grid has been filled somewhat and displaced slightly from that of the infinite grid.

The current of the center wire is used to calculate the maximum temperature rise due to ohmic heating of the grid wires (cf. Eq. 5.4). For the range of wire sizes used, it has been shown in Chapter 4 that the current I is proportional to the wire radius a , or $I \propto a$. The skin effect resistance goes as $1/a$ so that the ohmic power loss of the center wire, P_{Lo} , is proportional to a . The temperature rise ΔT varies as the power P_{Lo} divided by the wire mass W_m . Since $W_m \propto a^2$, one may see immediately that $\Delta T \propto 1/a$. This result seems to be borne out, at least approximately, as shown in Fig. 5.1(d). Since this analysis assumes that the grid current is constant along the length of the wire, the computed temperature rise is clearly an upperbound on the grid heating.

The total ohmic power loss of the complete grid is a sum over all the grid wires of the loss incurred in each (see Fig. 5.1(e)). Expressed as a ratio of the total input power (Fig. 5.1(f)), one finds that the ohmic power loss accounts for at most a few tenths of a percent of the power in the input beam.

Probably the single most important result of the simulations for C_{lf} is the relatively high total grid reflectivity, even at the position of the null. In order to allow for grid misalignment and variation in wire spacing, one would probably choose to operate near $\lambda/d = 1.8$. One may see from Fig. 5.1(b) that for the wire radii used, the reflectivity ranges from about

12 to 22%, which is too large. Given these values of reflectivity, the beam reflected off the grid would have to be further attenuated to provide for polarization insensitivity of a beam splitting mesh. Here it has been assumed that a grid oriented along the horizontal polarization of the input beam can be found to have a much smaller reflectivity. Since the horizontal grid couples to the dominant polarization, this criterion must be met to ensure a high grid transmissivity.

The results of the numerical analysis for the coupling by $C_{\parallel f}$ indicate that such a grid may be found. Once again, this grid couples to the horizontal polarization such that the principal order reflected beam is directed at an angle of 90° with respect to the incident wave vector and is made to lie in the horizontal plane (i.e., side-to-side beam splitting). Here the input power is taken to be 3 GW with the simulated results shown in Fig. 5.2(a-f). The behavior of this grid as a function of wire spacing is qualitatively very similar to that demonstrated for $C_{\perp f}$. However, it has already been noted in Chapter 2 that the nulls for $C_{\perp \infty}$ are much more sharply defined than for those of $C_{\parallel \infty}$. Hence, in the neighborhood of a resonance one should expect a lower reflectivity for $C_{\parallel f}$ over that of $C_{\perp f}$.

The main beam reflectivities plotted in Fig. 5.2(a) indicate that this is indeed the case. Here again, both the results for $C_{\parallel f}$ and $C_{\parallel \infty}$ have been shown. The reflectivity for $C_{\parallel \infty}$ vanishes at the resonance, whereas the reflectivity for $C_{\parallel f}$ becomes quite small but then exhibits a slight peaking. Although a detailed analysis of the precise causes of the peaking has not been undertaken, the subsidiary maxima observed here and in Figs. 5.2(b-f) have been shown to slowly disappear as the grid size and input beam waist are made larger, demonstrating the appropriate asymptotic behavior.

From Fig. 5.2(b) it is clear that the total reflectivity of this grid makes it the natural choice to couple to the dominant polarization. Well before the first resonance, say $\lambda/d = 0.76$, the total grid reflectivities for the 25, 50, and 75 μm radius wires are 0.9, 1.3, and 1.6%, respectively. This represents a full order of magnitude reduction in reflectivity over that found for $C_{\perp f}$.

The excellent agreement between grid currents for $C_{\parallel \infty}$ and $C_{\parallel f}$ far from the resonance is again clear from Fig. 5.2(c). The null for $C_{\parallel f}$ is

Figure 5.2(a). C_{llf} . Main beam reflectivity versus λ/d .

Figure 5.2(b). C_{llf} . Total reflectivity versus λ/d .

Figure 5.2(c). C_{11f} . Center wire current magnitude versus λ/d for $E_{inc} = 154 \text{ kV/cm}$.

Figure 5.2(d). C_{11f} . Center wire temperature rise versus λ/d .

Figure 5.2(e). C_{llf} . Grid ohmic heating loss versus λ/d for $E_{inc} = 154$ kV/cm.

Figure 5.2(f). C_{llf} . Ratio of ohmic heating power to input beam power versus λ/d .

shifted slightly from the resonance of $C_{||\infty}$ and is not as deep, both effects already noted from $C_{\perp f}$. The temperature rise of the center wire, total ohmic power loss, and the ratio of ohmic power loss to incident power are shown in Figs. 5.2(d-f).

For $\lambda/d = 0.76$ and the smallest wire radius simulated ($25 \mu\text{m}$), Fig. 5.2(d) indicates that the maximum temperature rise of the center wire will be about 2.6°C . A simple calculation will show that if the ohmic power were absorbed entirely within the skin depth (which is approximately true before the heat has had time to diffuse), the temperature rise will be about 120°C at the wire surface. The change in the dc resistance of a Tungsten wire for a 3°C rise is only about 1.5%. If one were able to make a dc measurement of the resistance before the heat had diffused throughout the wire cross-section, a change of about 2.5% could be observed. Here the higher resistivity of the skin depth causes the current to flow through the slightly narrowed interior of the wire. However, it has already been shown that the diffusion time lies somewhere between 10^{-10} and 10^{-6} second so that a dc measurement of this type would be quite difficult. For multiple pulses much of the difficulty is evidently eliminated.

6. Design Recommendations and Engineering Considerations

Comparing grid reflectivity characteristics with respect to viability in a beam-splitting application on MTX, it has been shown that the grid orientation that results in coupling to the input electric field parallel to the plane of incidence ($C_{||f}$) is more desirable than that which couples to the input electric field normal to the plane of incidence ($C_{\perp f}$). To be more precise, the $C_{||f}$ reflectivity is less sensitive to wire spacing error and grid misalignment, and given identical grid wire radii, is an order of magnitude lower than that of $C_{\perp f}$ at the minima of the single reflected-lobe regime. Placing a wire grid in the ECH port of the Tokamak such that the dominant polarization of the FEL beam couples to the grid with its electric field parallel to the plane of incidence results in the principal reflected lobe being directed in a horizontal plane (i.e., side-to-side reflection).

One might also consider the placement of a second grid relying on the $C_{\perp f}$ type of coupling to measure the power in the vertical polarization.

However, if the reflectivities of the two grids which couple to orthogonal polarizations are not identical, then without a precise knowledge of the power expected in each polarization, one cannot determine the total power incident upon the beam splitter. Since for the same grid wire radii the $C_{\perp f}$ reflectivity is an order of magnitude greater than the $C_{\parallel f}$ reflectivity, some sort of additional attenuation would be required to realize equal reflectivities. One could use an additional wire grid to couple to the vertical polarization of the beam reflected off of the primary, vertically-oriented grid. However, given the uncertainty in grid alignment and wire spacing, it may be quite difficult to obtain a specific value of reflectivity for the multiply-reflected beam.

One may also attempt to use much smaller grid wires to reduce the $C_{\perp f}$ reflectivity to match that of $C_{\parallel f}$. It is rather questionable whether wires of the size required (10 μm or less in diameter) could withstand the shock wave accompanying the incident pulses or the subsequent thermal cycling. It is for these reasons that the author of this report recommends that a single grid using the $C_{\parallel f}$ grid orientation be chosen for the beam splitter on MIX.

Chapter 7 will describe efforts to investigate a second grid that could be used to monitor the beam profile but will not be used as a beam splitter per se.

To allow for grid misalignment and wire spacing error, a separation of $\lambda/d_1 \approx 0.76$ for the $C_{\parallel f}$ grid should be chosen (see Fig. 5.2(b)). This places one safely in the operating regime characterized by a single reflected beam. Higher order lobes begin to emerge at $\lambda/d_2 \approx 0.73$. In terms of real lengths at 140 GHz (where $\lambda = 2143 \mu\text{m}$)

$$d_1 = \lambda/0.76 = 2820 \mu\text{m} \quad ,$$

and

$$d_2 = \lambda/0.73 = 2935 \mu\text{m} \quad ,$$

so that with perfect grid alignment, a constant spacing error of over 100 μm could be tolerated. It is believed that a random error in grid wire separation of no greater than $\pm 25 \mu\text{m}$ can be achieved. Imposing this random error on the spacing d in the numerical analysis yields the plot shown in

Figure 6.1. C_{eff} . Total reflectivity versus λ/d . Here a random error of within $\pm 25 \mu\text{m}$ in the individual wire positions has been imposed. Compare with Fig. 5.2(b).

Fig. 6.1. Because $25 \mu\text{m}$ is less than one percent of a wavelength and also less than one percent of the grid separation for λ/d ranging from 0.60 to 0.80, very little deviation from the results for exact grid spacing is expected as is clearly seen by comparing Figs. 6.1 and 5.2(b).

Several mechanical considerations of the grid wires will now be addressed. The following handbook data for Tungsten will be used in the calculations to follow (High-Temperature Materials, Number 3)

Tensile strength	$1.5 \cdot 10^5 \text{ psi}$
Young's modulus, Y	$59 \cdot 10^6 \text{ psi}$
Coefficient of linear expansion, α	$4.58 \cdot 10^{-6}/^\circ\text{C}$

The maximum tension allowable on each wire, T_m , is given as the product of the tensile strength and the wire cross-sectional area, A . If the wires are wound on the grid with a force of $T_m/2$, the amount of stretching for a wire of length l will be

$$\Delta s = \frac{(T_m/2)l}{YA} \quad (6.1)$$

An upper bound on the expansion of the wire due to ohmic heating may be obtained by using the temperature rise ΔT associated with the heat being absorbed entirely within a skin depth (or about $0.32 \mu\text{m}$ at 140 GHz). The expansion is then found from

$$\Delta l = \alpha l \Delta T \quad , \quad (6.2)$$

where both here and in Eq. 6.1, l is 17 cm. Table 7.1 indicates the calculated results for T_m , Δs , and Δl given wire radii of 25, 50, and $75 \mu\text{m}$.

Note that the stretching Δs is independent of the wire radius because the wire tension has arbitrarily been set at $T_m/2$, which is proportional to A . From Eq. 6.1 one may see that Δs no longer depends on area. One may also note that even for a very liberal estimate of the temperature rise of the wire, the expansion due to thermal heating is significantly less than the

increase in length due to stretching under tension. Hence, the input wires are not expected to exhibit appreciable bowing following an input pulse. The larger radius wire should remain quite taut even after multiple pulses.

TABLE 7.1

Radius (μm)	Area, A (mil^2)	ΔT ($^{\circ}\text{C}$)	T_m (lbs)	Δs (μm)	Δl (μm)
25	π	100	0.5	220	79
50	4π	18	1.9	220	14
75	9π	8	4.2	220	6

7. Normal Electron Field Coupling

Heretofore, it has been assumed that the reflectivity of the grid is dominated by that component of the incident field parallel to the grid wires. In fact, assuming incidence normal to the plane of the grid, published results (Marcuvitz, 1948) indicate that for very small wire-grid spacing, the reflectivity of the electric field component parallel to the grid wire axes relative to the component normal to the grid wire axes is still over fifty times greater even for wire radii of $200\ \mu\text{m}$ (and at a frequency of 140 GHz). For smaller grid wires, the disparity grows even larger.

The absorption of power due to ohmic heating by each component of the input field does not, however, exhibit exactly the same behavior as a function of wire radius. For wire radii of $200\ \mu\text{m}$, the absorption due to each field component will be nearly equal (assuming equal input power in both the parallel and normal components). It may be possible to configure a grid such that the heating due to the normal component dominates that of the parallel component at a position of minimum reflectivity for the parallel component. Indeed, if one were able to align the grid wires such that the much stronger horizontal polarization of the beam from the FEL were normal to the grid wire axes, the absorption could be made quite large without strong reflection. By monitoring the grid wire resistance, a beam profile could then be obtained in the horizontal direction.

The complete solution for the normal electric field coupling was found to lie outside the scope of the analysis that has been described in this report. However, a brief sketch of a possible algorithm to determine the grid currents and scattered fields will be given below. It will be shown that the solution here is not a straightforward extension of the method used to solve for parallel electric field coupling. One must be careful not to confuse the notions of an electric field parallel to a wire axis with that of an electric field parallel to the plane of incidence. (A similar comment applies to the normal electric field.) The analysis in earlier chapters always assumed that the electric field that coupled to the wire was parallel to the wire axis.

Given a plane wave striking a cylinder at normal incidence with the electric field normal to the axis of the cylinder, the magnetic field will be aligned parallel to the axis of the cylinder. The scattered magnetic field may then be written (Panofsky and Phillips)

$$H_z^S = H_{z0} \sum_{m=-\infty}^{\infty} a_m H_m^{(1)}(k\rho) e^{im\phi}, \quad (7.1)$$

where

H_{z0} = magnitude of the incident magnetic intensity field,

$H_m^{(1)}$ = Hankel function of the first kind of order m ,

k = free space wave number,

ρ = length from the z -axis in a cylindrical coordinate system,

ϕ = azimuthal angle around the z -axis,

a_m = coefficients of the Hankel functions.

For a perfect conductor, the a_m are found from

$$a_m = -i^m \frac{J_m'(ka)}{H_m^{(1)'}(ka)}, \quad (7.2)$$

where a is the cylinder radius. For small ka , only the $m = 0$ and $m = \pm 1$ terms are important. Equation 7.1 then becomes

$$H_z^S = -H_{z0} \frac{\pi}{4} (ka)^2 \left[2H_1^{(1)}(k\rho) \cos \phi - iH_0^{(1)}(k\rho) \right] . \quad (7.3)$$

from which one may immediately see a fundamental difference between the normal and the parallel electric field coupling. It has been shown that the scattered electric field given an incident electric field parallel to the wire axis is cylindrically symmetrical about the wire. The expression given in Eq. 7.3 clearly indicates a dependence of the scattered field on the azimuthal angle ϕ . Since for a perfect conductor the surface current density is equal to the vector cross-product of the conductor outward unit normal vector and the tangential component of the magnetic intensity field at the surface of the conductor, the surface current density for the wire will also show a dependence on the azimuthal angle ϕ . The matrix inversion technique of Chapter 3 assumed that the current magnitude along the wire (ignoring for now a possible phase term) could be described by a single complex number. For the normal electric field coupling, the current magnitude at a particular value of z must be written as a function of ϕ .

It is possible that a finite element solution for the grid coupling given a normal electric field at normal incidence may be found. If the grid wires are assumed to be infinitely long, the scattered fields will be directed normal to the grid wire axes and only currents at a particular value of z will couple. The current density along the circumference of the wires at a given value of z may be subdivided into small elements. These small elements may then be allowed to couple to one another so that an approximate solution to the current density as a function of ϕ may be obtained.

B. Conclusion

A wire grid beam-splitter for use in microwave diagnostics on the Microwave Tokamak Experiment (MTX) has been described. The theoretical analysis considered a grid of finite dimensions excited by an incident field with a Gaussian distribution. It has been shown that the most favorable orientation of the grid to couple to the dominant (horizontal) polarization of the beam from the FEL requires that the electric field lie in the plane of incidence defined by the input beam wave vector and the grid normal. Grid design criteria that allow for grid misalignment, wire spacing error, wire stretching under tension, and wire expansion during heating have been included.

It has been assumed that the reflectivity is dominated by the component of the electric field parallel to the grid wire axes. A brief outline of a possible solution for the reflectivity due to the electric field component normal to the wire axes has also been given.

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