THE EFFECT OF NONCONDENSABLE GASES ON
THE PERFORMANCE OF GEOTHERMAL
STEAM POWER SYSTEMS

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The Effect of Noncondensable Gases on the Performance of Geothermal Steam Power Systems

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Abstract

The influence of dissolved carbon dioxide on the thermodynamic performance of geothermal steam systems is analyzed. The system is divided into its main component: the flash tank, the turbine, the condenser and the gas extraction system, and the effect of noncondensables is studied for each. The effect of the noncondensible gas on the output of the whole system is deduced from its effect on the individual components. The analysis of actual systems is preceded by an analysis of an ideal system. The optimum condenser pressure for actual systems is obtained for different gas extraction system efficiencies. Economic considerations, however, are only qualitatively addressed.
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1. Introduction

Geothermal fluids contain dissolved, noncondensable gases in quantities that vary from almost zero to as much as 5 percent by weight. The major component of these dissolved gases is carbon dioxide which constitutes about 80% of the gases, typically. When a compressed geothermal liquid that contains dissolved carbon dioxide is flashed down to a low pressure, practically all the dissolved gas is liberated in the flash tank and appears as a mixture with the flash steam produced. The presence of the noncondensable gas in the mixture affects the performance of a geothermal steam power plant significantly. First, the fraction of mass separated from the geofluid in the gaseous phase increases. Secondly, the specific expansion work in the turbine decreases. Thirdly, and most importantly, the presence of noncondensables has an adverse effect on the performance of a condenser. This last effect is manifested both as a pronounced deterioration in the heat transfer coefficient in the condenser and as an equally pronounced increase in the power expenditure for noncondensable gas extraction. The combination of these effects results in a deterioration of the specific work produced per unit mass of geofluid by a geothermal steam system as the amount of noncondensable gases increases. The decrease of output is not the only economic penalty resulting from the presence of noncondensable gases in the geofluid, the deterioration of the overall heat transfer coefficient in the condenser increases the surface area requirements and raises the capital cost of the condenser. The capital cost of the plant is further increased by the need for expensive gas extraction equipment.

The present study treats the effect of noncondensable gases on the thermodynamic performance of the main components in a geothermal steam (or flash steam) power plant. The study is divided in three distinct stages:
i) The effect of dissolved gases on the fraction of mass liberated as a steam-gas mixture in a flash tank.

ii) The effect of noncondensable carbon dioxide on the specific exergy (available work) of a given mixture of steam and carbon dioxide, including the effect of CO₂ on the turbine expansion work.

iii) The determination of the optimum condenser pressure when an appreciable amount of CO₂ is present in the turbine exhaust steam.

The study addresses only the thermodynamic aspects although economic considerations are discussed in a qualitative manner. A number of assumptions had to be made in order to simplify the analysis. These assumptions, as well as the justification for their adoption in the present analysis, constitute the subject of the next section.

2. Basic Assumptions

Because carbon dioxide constitutes the major fraction (~ 80%) of the noncondensable gases present in typical geothermal fluids, we shall replace the mixture of the noncondensables with a CO₂-equivalent mass fraction. Henceforth we shall be using the terms noncondensables and carbon dioxide (CO₂) synonymously, except where explicitly mentioned otherwise.

We assume a geothermal fluid which is composed of pure water with carbon dioxide dissolved in it. The fluid enters the flash tank as a compressed liquid at a sufficiently high pressure to guarantee that all the carbon dioxide is in solution. Once the pressure is reduced in the flash tank, the pressure and temperatures prevailing will be such that no carbon dioxide will remain in solution. That is, we assume that all the dissolved carbon dioxide will be liberated from the liquid in the flash tank. The assumption is justified on the grounds that the solubility of carbon dioxide
in water at the pressures and temperatures most likely to exist in the flash tank is extremely low. This is indicated in Fig. 1 which depicts the Henry's-law coefficient, $K_H$, for CO$_2$ in NaCl-H$_2$O solutions as a function of temperature and NaCl molality [1, 2]. We notice here that the solubility decreases sharply ($K_H$ increases) at the higher temperatures usually encountered in geothermal systems and that the presence of dissolved sodium chloride (NaCl) in the geothermal fluid further reduces the solubility of carbon dioxide in it [1, 2].

Another assumption we adopted is to treat the H$_2$O-CO$_2$ system in the vapor phase as an ideal gas mixture. This assumption is justified in view of the relatively low pressures encountered in geothermal steam turbines and condensers. The water vapor present in a CO$_2$-H$_2$O mixture coexisting with a liquid phase is in equilibrium with the liquid, i.e. the vapor exists in the mixture at a partial pressure equal to the equilibrium saturation pressure of H$_2$O.

The preceding assumptions are supplemented by a number of other assumptions regarding the turbine expansion efficiency and its variation with the dryness fraction of the expanding mixture, etc. These additional assumptions will be introduced in the text as needed for the analysis.

3. Thermodynamic Analysis

A geothermal steam power plant can be divided into the following subsystems:

---

For the range of conditions encountered in geothermal applications, the Henry's-law coefficient, $K_H$, is equal to the partial pressure of CO$_2$ in the vapor phase divided by the mole fraction of CO$_2$ in the liquid phase. The latter can be taken as a measure of solubility.
1) Flash tank (or separator)
2) Turbine
3) Condenser
4) Gas extractor.

In this report we shall consider only single-flash or direct steam systems. Double-flash or more sophisticated systems will not be considered here although the method of analysis adopted in this report can be straightforwardly applied to more complex systems.

3.1 Flash Process

3.1.1 Vapor augmentation

We assume that compressed geothermal water containing dissolved carbon dioxide is flashed isenthalpically in a flash tank in order to generate a vapor working fluid that would be suitable for expansion in a conventional steam turbine. The dissolved gas will be completely liberated and will exist in a homogeneous mixture with the flashed steam. The latter is in equilibrium with the water phase at the flash temperature. At a given flash temperature, the presence of carbon dioxide in the mixture augments the fraction of fluid to be admitted to the turbine.

The specific enthalpy of the liquid before flashing is given by

\[ h_{g}^{(l)} (T_{g}, P_{g}) = f_{g} h_{CO_{2}}^{(g)} (T_{g}, P_{g}) + \left[ 1 - f_{g} \right] h_{H_{2}O}^{(g)} (T_{g}, P_{g}) \]  

in which the suffix "g" refers to the geothermal liquid and \( f \) is the mass fraction of dissolved \( CO_{2} \). Equation (1) is valid under the assumption that the solution is an ideal one. The specific enthalpy of a mixture of water vapor and carbon dioxide at the flash temperature, \( T_{1} \), is calculable from
\[ h^{(1)}_{\text{mix}} = f \cdot h_{\text{CO}_2}^{(1)}(T_1) + (1-f) \cdot h''_{\text{H}_2\text{O}}(T_1) , \]  

in which \( f \) is the mass fraction of \( \text{CO}_2 \) in the ideal mixture. The steam in this mixture is in equilibrium with the water at \( T_1 \). Since the "products" of the flashing process are a liquid phase made of pure water and a vapor-gas mixture in equilibrium with it, conservation of mass necessitates that

\[ f = x_1 f , \]  

and that

\[ [1-f] = [1-x_1] + x_1 [1-f] , \]

where \( x_1 \) is the fraction of mass liberated as steam and carbon dioxide. The fraction \( x_1 \) can be obtained by applying the first law of thermodynamics to the isenthalpic flash process:

\[ h_{\text{g}} = [1-x_1] \cdot h'_{\text{H}_2\text{O}}(T_1) + x_1 \cdot h^{(1)}_{\text{mix}} \]  

Utilizing eqs. (3a) and (3b) we obtain

\[ x_1(f_{\text{g}}) = f_{\text{g}} + \left\{ f_{\text{g}} \left[ h(T_{\text{g}}, P_{\text{g}}) - h(T_1) \right]_{\text{CO}_2} + 
\right. \\
\left. 
[1-f_{\text{g}}] \left[ h^{(\ell)}(T_{\text{g}}, P_{\text{g}}) - h'(T_1) \right]_{\text{H}_2\text{O}} \right\}/L(T_1) , \]

#To avoid confusion, parentheses will be reserved for function arguments except when they appear in a superscript.
in which \( L(T_1) \) is the latent heat of water at \( T_1 \). If the geothermal water was free of \( \text{CO}_2 \) then

\[
x_1^* = x_1(0) = \left[ h^L(T_g, P_g) - h^l(T_1) \right] / L(T_1).
\]  

(6)

Therefore eq. (5) can be rewritten as

\[
x_1(f_g) = [1 - f_g] x_1^* + f_g \left\{ 1 + \left[ h(T_g, P_g) - h(T_1) \right] \right\}_{\text{CO}_2}/L(T_1).
\]  

(7)

We define the vapor augmentation factor, \( \alpha \), by

\[
\alpha(f_g) = x_1(f_g)/x_1^*.
\]  

(8)

The augmentation factor \( \alpha \) has been computed on the assumption that carbon dioxide behaves as a perfect gas at \( T_g \) and \( P_g \), i.e.

\[
h_{\text{CO}_2}(T_g, P_g) - h_{\text{CO}_2}(T_1) = c_p [T_g - T_1].
\]  

(9)

The results are displayed in Fig. 2, which depicts the augmentation factor as a function of the flash temperature \( T_1 \) for a geothermal liquid at \( T_g = 200^\circ\text{C} \) containing 0.5%, 1% and 5% dissolved \( \text{CO}_2 \), respectively. It can be seen that the mass flow rate to the turbine will be augmented by about 10% when a 200\(^\circ\text{C}\) geothermal water, containing 1% by mass dissolved \( \text{CO}_2 \), is flashed down to 150\(^\circ\text{C}\). Figure 3 shows the variation of the mass fraction of \( \text{CO}_2 \) in the vapor phase, \( f \), with the flash temperature, \( T_1 \), for various concentrations of \( \text{CO}_2 \) in the geothermal water.
3.1.2 Available work (exergy)

The flash process results in entropy production and thus leads to work dissipation. It is, therefore, important to evaluate the available work (exergy) of the steam-carbon dioxide mixture resulting from the flash process. For an ideal mixture of gases at state (1) we write

\[ p(1) = p_{CO_2}(1) + p_{H_2O}(1). \]  

(10)

The partial pressure of \( H_2O \) at state (1) is equal to the vapor pressure at \( T_1 \), i.e.

\[ p_{H_2O}^{(1)} = p_v(T_1). \]  

(11)

The partial pressure of \( CO_2 \) is related to that of \( H_2O \) through the mole fraction, \( \nu \), which depends in turn on the mass fraction, \( f \). That is

\[ \nu_1 = \left( \frac{f/M_{CO_2}}{f/M_{CO_2} + [1-f]/M_{H_2O}} \right). \]  

(12)

or,

\[ \nu_1 = \mu f/\left( \mu f + 1-f \right). \]  

(12')

Here \( M \) is the molecular mass and \( \mu \) is the ratio \( M_{H_2O}/M_{CO_2} = 0.4092 \). The partial pressure of \( CO_2 \) is thus given by

\[ p_{CO_2}^{(1)} = \nu_1 p_v(T_1)/[1-\nu_1]. \]  

(13)
The specific enthalpy of an ideal mixture of steam and carbon dioxide is given by eq. (2). The specific entropy of this ideal mixture is given by

\[ s_{mix}^{(1)} = \int s_{CO_2}^{(1)}(T, P^{(1)}_{CO_2}) + (1-f)s_{H_2O}^{(1)}(T) \] \hspace{1cm} (14)

The entropy of CO\textsubscript{2} in the ideal-gas limit can be written as [3]

\[ s_{CO_2}^{(1)}(T^*, P^*) + \int_{T^*}^{T_1} c_p \, dT/T - R_{CO_2} \ln \left( \frac{P_{CO_2}^{(1)}}{P^*} \right) \], \hspace{1cm} (15)

in which the stars refer to a reference state. Since changes of entropy, and not entropy itself, is what interests us here, we can choose \( s^* = 0 \). Furthermore, for a thermally perfect gas, \( c_p \) is independent of temperature and eq. (15) reduces to

\[ s_{CO_2}^{(1)} = c_p \ln \left( \frac{T_1}{T^*} \right) - R_{CO_2} \ln \left( \frac{P_{CO_2}^{(1)}}{P^*} \right) \]. \hspace{1cm} (16)

The calculation of the exergy of the mixture requires the definition of a dead state. A natural choice for the problem at hand is an equilibrium mixture of CO\textsubscript{2} and H\textsubscript{2}O at the ambient pressure, \( P_0 \), and the ambient temperature \( T_0 \). This mixture will consist of a liquid phase made essentially of pure water and a gaseous mixture of carbon dioxide and saturated steam in equilibrium with the liquid phase (water). Thus

\[ P^{(0)}_{H_2O} = P_v(T_0) \], \hspace{1cm} (17)

and
\[ \nu_0 = 1 - P_v \left( T_0 \right) / P_o \]  \hspace{1cm} (18)

The mass fraction, \( f_o \), of CO\(_2\) in the gaseous mixture is related to the mole fraction, \( \nu_o \), by

\[ f_o = \nu_o / \{ \nu_o + [1-\nu_o] \mu \} \]  \hspace{1cm} (19)

Conservation of mass necessitates that the dryness fraction, \( x \), of the H\(_2\)O component be related to \( f \) and \( f_o \) by

\[ x = f \left[ 1-f_o \right] / f_o \left[ 1-f \right] \]  \hspace{1cm} (20)

The enthalpy and entropy of the system at the dead state can be determined from

\[ h^{(0)} = f h_{CO_2} \left( T_0 \right) + x \left[ 1-f \right] h''_{H_2O} \left( T_0 \right) + \left[ 1-x \right] \left[ 1-f \right] h'_{H_2O} \left( T_0 \right) ; \]  \hspace{1cm} (21)

\[ s^{(0)} = f s_{CO_2} \left( T_0, P_{CO_2}^{(0)} \right) + x \left[ 1-f \right] s''_{H_2O} \left( T_0 \right) + \left[ 1-x \right] \left[ 1-f \right] s'_{H_2O} \left( T_0 \right) . \]  \hspace{1cm} (22)

The available work is expressed by

\[ \mathcal{W}^0 = h^{(1)} - h^{(0)} - T_0 \left[ s^{(1)} - s^{(0)} \right] \]  \hspace{1cm} (23)

Upon substituting for the various quantities we obtain
\[ W^o = f \left[ c_p (T_1 - T_o) + \left[ 1 - f \right] \left( h''_{H_2O} (T_1) - h''_{H_2O} (T_o) - x L_o \right) \right] - \]

\[ T_o \left\{ f \left[ c_p \ln \left( \frac{T_1}{T_o} \right) - R \ln \left( \frac{P_{CO_2}}{P_{CO_2}} \right) \right] + \right. \]

\[ \left. (1 - f) \left( s''_{H_2O} (T_1) - s''_{H_2O} (T_o) - x L_o / T_o \right) \right\} \]

in which \( L_o \) is the latent heat of \( H_2O \) at \( T_o \).

Figure 4 shows the normalized specific ideal (available) work for the \( H_2O-CO_2 \) system at various values of \( T_1 \). The work is normalized with respect to its value at \( f = 0 \) (uncontaminated steam). The dead state temperature, \( T_o \), is taken as 298.15K (25°C) and the dead state pressure, \( P_o \), is taken as 0.1 MPa (~1 atm). A flow diagram of the ideal system is also shown. It comprises an isentropic expander (turbine), an isothermal condenser operating at \( T_o \) and an isothermal gas extractor (compressor) and liquid pump, both operating at \( T_o \). The entropy production in the system is zero. It can be seen that the dependence of the normalized specific available work on the initial temperature, \( T_1 \), is very weak and that the effect of \( CO_2 \) is such that the normalized work drops by about 1% for each 1% mass increment of \( CO_2 \).

In order to refer the available work obtained above to a unit mass of geothermal fluid instead of a unit mass of the steam-carbon dioxide mixture, we multiply the normalized ideal work by the vapor augmentation factor, \( \alpha \). Therefore we define the augmented normalized ideal work, \( \psi^o \), as

\[ \psi^o (f_g) = \alpha (f_g) \frac{W^o (f)}{W^o (0)} \]
in which \( f \) and \( f_g \) are related through eqs. (3a) and (7). Figure 5 shows the dependence of \( \psi^0 \) on \( T_1 \) for \( T_g = 200^\circ C \) and \( f_g = 0.01 \). The mass fraction \( f(T_1) \) under the same conditions is also shown. On the basis of these calculations we note that the augmentation of the vapor fraction by carbon dioxide offsets, to some degree, the loss in the specific available work resulting from the presence of the noncondensable gas. The drop in the augmented normalized ideal work in the case cited amounts to no more than 3.3% when the flash temperature is as low as 100°C. It is also interesting to note that for flash temperatures that are higher than about 177°C the augmentation effect more than compensates for the specific work loss.

3.2 Expansion Process

In this section we consider the adiabatic expansion of a mixture of saturated steam and carbon dioxide, initially at \( T_1 \), down to a specified back pressure, \( P_2 \). In the ideal case, the mixture will follow an isentropic expansion line. The specific enthalpy and entropy of the mixture at state (1) are given by eqs. (2) and (14), respectively. At state (2) a mixture of \( CO_2 \), \( H_2O \) vapor and \( H_2O \) liquid will exist at a temperature \( T_2 \) lower than the steam saturation temperature corresponding to \( P_2 \). The temperature \( T_2 \) is determined by an iterative procedure. Assuming that the temperature \( T_2 \) has been found, the determination of the properties at state (2) proceeds as described for the dead state (0) in the previous section. The correct value of \( T_2 \) is that which satisfies the constraint

\[
\frac{s^{(2)}}{s^{mix}} = \frac{1}{s^{mix}}. \tag{25}
\]

In an iterative procedure, a trial value of \( T_2 \) is selected and the entropy at state (2), corresponding to this trial value, is calculated and compared
with the initial value of the specific entropy, \( s_{\text{mix}}^{(1)} \). The value \( T_2 \) is then incremented until condition (25) is satisfied within an acceptable tolerance. However, it was found that, owing to the nature of the equations, unacceptable numerical uncertainties arise if we obtain the steam properties by interpolation in the steam tables. This is particularly true as the mass fraction of \( \text{CO}_2 \) tends to zero because eq. (20) becomes indeterminate. For this reason we employed the Clausius-Clapeyron equation for the determination of the saturated steam properties at exhaust conditions. The Clausius-Clapeyron equation was integrated after substituting a linear expression for the latent heat, \( L(T) \), and a perfect gas law for the saturated vapor. Explicitly:

\[
L(T)/[\text{kJ/kg}] = 3156.2 - 2.394 [T/K] ,
\]

(26)

and

\[
v'' = R_{\text{H}_2\text{O}} T/P_v >> v' .
\]

(27)

These assumptions lead to the following expression for the vapor pressure in the temperature range \( 298.15 < T < 333.15 \text{K} \):

\[
\ln [P_\text{v}/0.003169 \text{MPa}] = [1 - 298.15K/T] - 5.183 \ln [T/298.15] .
\]

(28)

Furthermore, the enthalpy and entropy of the saturated liquid are evaluated on the assumption that the liquid behaves as an incompressible fluid with a constant specific heat. Thus

\[
h'/[\text{kJ/kg}] = 4.187 [T - 273.15]/K ,
\]

(29)

and
\[ s'/[kJ/kg K] = 4.187 \ln [T/273.15K], \quad (30) \]

in which we assumed that the datum for enthalpy and entropy is located at \( T = 273.15K \).

The normalized isentropic expansion work, \( W_e^o(f)/W_e^o(0) \), for various initial temperatures and a fixed back pressure of 0.012349 MPa (about 3.7" Hg) is shown as a function of the mass fraction of \( CO_2, f \), in Fig. 6. It is seen that the dependence of the normalized expansion work on the initial temperature is weak and that, approximately, a loss of 0.5% occurs for each increment of 1% in the mass fraction of \( CO_2 \). We also note that the presence of \( CO_2 \) in the expanding steam results in lowering the exhaust temperature, \( T_2 \), by as much as 4°C below the saturation value corresponding to the selected \( P_2 \) (50°C).

The presence of a noncondensable gas in the exhaust mixture alters its dryness fraction. The latter is defined here by

\[ x(f) = f + x [1-f], \quad (31) \]

in which \( x \) is the true quality of the \( H_2O \) component. Since the expansion efficiency in a real expansion depends, among other factors, on the percentage of the moisture present in the exhaust steam, as manifested by the familiar Baumann rule [4], it is expected that the actual value of the normalized work, \( W_e(f)/W_e(0) \), will depend also on the dryness fraction. Therefore, we write

\[ W_e(f)/W_e(0) = \xi W_e^o(f)/W_e^o(0), \quad (32) \]
in which \( \xi \) is the ratio of the isentropic expansion efficiencies with and without \( \text{CO}_2 \), i.e.

\[
\xi = \frac{\eta(f)}{\eta(0)}.
\]  

(33)

Adopting the Baumann rule for the evaluation of the moisture loss, we obtain

\[
\xi = \frac{x(f)}{x(0)}.
\]  

(34)

Figure 7 illustrates the dependence of \( \xi \) on the mass fraction of \( \text{CO}_2 \) for various initial temperatures. The effect of this modifier on the normalized work is represented by the dashed line in Fig. 6. It can be noticed that the effect of \( \text{CO}_2 \) is such that the effective dryness fraction of the exhaust mixture is somewhat higher. This results in attenuating the rate at which the expansion work declines with increasing \( \text{CO}_2 \) mass fraction.

3.3 Heat Rejection and Gas Extraction Processes

A schematic representation of the condenser and gas extraction system is shown in Fig. 8 which also shows the temperature distribution for a condenser operating with a large fraction of noncondensables. Unlike the ideal system shown in Fig. 4, condensation does not proceed isothermally. Furthermore, practical gas extraction systems (ejectors, pumps and compressors) do not operate isothermally; their operation is closer to adiabatic than to isothermal and their adiabatic efficiency ranges from as low as 10% (ejectors) to 80% or higher (compressors). Isothermal compression can be approached only by frequent compressor intercooling. 

*Practical considerations limit the number of intercooling stages to two or three.*
As the turbine back pressure, $P_2$, is lowered, the temperature in the condenser approaches that of the available cooling medium, $T_o$. This decreases the temperature difference in the condenser and increases the heat transfer surface area required. Moreover, a lower condenser temperature reduces the subcooling, $\Delta T_s$, of the steam-gas mixture which results in a marked increase in the gas extractor work expenditure. This increase should be balanced against the corresponding increase in turbine output effected by the reduction in the back pressure, $P_2$. In other words, an excessively high back pressure decrease both the turbine work output and the gas extractor work input and vice versa. The net output of the plant, which is the difference between the turbine output and the gas extractor input, may exhibit a maximum at an intermediate value of the back pressure. Naturally, the economic optimum may differ from the thermodynamic optimum because an extremely low back pressure, which may be indicated on thermodynamic grounds in some cases, results in a larger volume flow rate through the exhaust end of the turbine and into the condenser. The higher capital costs of the turbine and the condenser in this case may shift the balance in favor of a higher back pressure.

To investigate the effect of noncondensables on the optimum value of the back pressure in a geothermal steam power plant, we introduced the following additional assumptions:

(a) pressure losses in the condenser are negligible.

(b) noncondensables are withdrawn from the condenser at a temperature $T_3 = 35^\circ C$, irrespective of the back pressure and of the amount of noncondensables present.

(c) the gas extractor is a single-stage adiabatic compressor whose efficiency is independent of the pressure ratio.

(d) the isentropic efficiency of the turbine is independent of the back pressure.
3.3.1 The condenser

For any selected value of the back pressure, $P_2$, the turbine work per unit mass at state (1) and the exhaust temperature, $T_2$, can be calculated according to the procedure described in the previous section.

We need to compute the composition and the quantity of the mixture entering the gas extraction system at state (3). The total pressure at state (3), $P^{(3)}$, is equal to the total pressure at state (2), $P^{(2)} = P_2$, according to assumption (a) above. The partial pressure of $H_2O$ at state (3) is given by

$$P_{H_2O}^{(3)} = P_y(T_3),$$

therefore

$$P^{(3)} = P_2 - P_{H_2O}^{(3)}.$$  \hspace{1cm} (36)

The mole fraction of $CO_2$ at state (3) is given by

$$v_3 = \frac{P^{(3)}_{CO_2}}{P_2} = \frac{\mu f_3}{1 + f_3 [\mu - 1]}.$$  \hspace{1cm} (37)

which leads, together with eq. (36), to

$$f_3 = v_3/[\mu - [\mu - 1] v_3],$$

$$= \frac{[1 - P_y(T_3)/P_2]/\mu - [\mu - 1] [1 - P_y(T_3)/P_2]}{[\mu - [\mu - 1] [1 - P_y(T_3)/P_2]}.$$  \hspace{1cm} (38)

The quantity of gas to be extracted can be determined from the mass conservation equation for carbon dioxide. The gas extraction system will have to
cope with a mass of CO$_2$-H$_2$O mixture

$$m = \frac{f}{f_3},$$

per unit mass of mixture at state (1).

### 3.3.2 The gas extraction system

The isentropic compression of a saturated CO$_2$-H$_2$O mixture will proceed from a point of 100% relative humidity to a point of lower relative humidity. In other words the water vapor will become progressively superheated as the pressure is raised isentropically. Consequently the composition of the mixture in an adiabatic compressor will remain constant and the mixture can be treated as a pseudo-single gas whose specific heat is given by

$$\bar{c}_p = f_3 \bar{c}_p (CO_2) + (1-f_3) \bar{c}_p (H_2O)(\gamma).$$

In a similar manner we define a gas constant, $\bar{R}$ for the mixture by

$$\bar{R} = f_3 R_{CO_2} + (1-f_3) R_{H_2O}.$$

The isentropic exponent for the mixture is given by

$$\bar{\gamma} = \frac{\bar{c}_p}{[\bar{c}_p - \bar{R}]}.$$

and the specific compression work from $P_2$ to $P_0$ is expressed as

$$w_c = \bar{c}_p T_3 \left(r^{(\bar{\gamma}-1)/\bar{\gamma}-1}\right)/n_c.$$
in which \( r \) is the pressure ratio \( P_0/P_2 \) and \( \eta_c \) is the compression efficiency. The work expended in compression per unit mass of mixture at the turbine inlet [state (1)] is calculable from

\[
W_c = m w_c = m c_p T_4 \left( r \frac{(\gamma - 1)}{\gamma} - 1 \right) / \eta_c .
\] (44)

Therefore, the net work output for a plant comprising a gas extraction system is

\[
W_n = W_e - W_c .
\] (45)

Here \( W_e \) designates the specific turbine expansion work which can be evaluated as described in section 3.2.

Figures 9 and 10 show the variation of the net work with the back (condenser) pressure, \( P_2 \), for various values of CO\(_2\) mass fraction. Figure 9 corresponds to a turbine admission temperature, \( T_1 \), of 150°C whereas Fig. 10 is for \( T_1 = 200°C \). In both cases the turbine expansion efficiency was taken as 0.75 and the compressor efficiency as 0.7. It can be seen that the net work increases very steeply with the back pressure close to the point of zero subcooling. The net work soon reaches a maximum then declines less steeply towards the output of a noncondensing turbine (\( P_2 = 1 \) atm). The decline becomes milder as the concentration of noncondensables increases until the maximum occurs at \( P_2 = 1 \) atm. This circumstance occurs at lower values of CO\(_2\) mass fraction if the efficiency of the gas extraction system is lower than the value assumed in the present analysis.

The variation of the normalized maximum value of the net work, \( W_n^* (f)/W_n^* (0) \), with the initial mass fraction of CO\(_2\) is shown in Fig. 11. As would be expected, the maximum work decreases with increasing CO\(_2\) concentration.
Figure 12 shows the variation of the optimum back pressure, $P_2^*$, with the mass fraction of CO$_2$ for $\eta_c = 0.7$, 0.5 and 0.2. The latter case roughly corresponds to the use of an ejector for gas extraction.

The efficiency of the turbine, $\eta_e$, depends on the moisture content of the expanding mixture. To study the effect of the variation in turbine efficiency associated with the variation in the dryness of the exhaust steam with $P_2$ we repeated the calculations for $T_1 = 200^\circ$C, $f = 0.2$ and $\eta_c = 0.7$, assuming that the expansion efficiency changes with the exhaust dryness fraction according to the Baumann rule [4]. The results are shown plotted in Fig. 13 which also depicts the variation of the expansion efficiency with the back pressure. The solid line represents the results obtained previously for $\eta_e = 0.75$. Figure 13 demonstrates that the work declines even more mildly past the optimum point when the effect of the variation in turbine efficiency is taken into account.

The optimum back-pressure in an actual installation will be dictated by economics rather than by thermodynamics. The thermodynamic optimum, however, represents the lower limit of the optimum back pressure. The higher cost of a turbine/condenser/gas extractor system designed for operation at a low back pressure may more than offset the thermodynamic gain. Consequently, the economic optimum will, most likely, occur at values of the back pressure that are somewhat higher than those indicated in Figs. 9 and 10, particularly in view of the argument given in the preceding paragraph.

4. **Conclusions**

The influence of dissolved carbon dioxide on the performance of various components of geothermal steam power plants has been analyzed in a simple,
thermodynamically consistent manner. The presence of CO$_2$ in the brine reduces the amount of work that can be extracted per unit mass of this brine in a single-flash energy conversion system. The same holds true for the output work of a steam turbine expanding CO$_2$-contaminated steam down to a specified back pressure. In this case the normalized specific expansion work decreases by almost 0.5% for each additional 1% increase in the mass fraction of CO$_2$. When the effects of carbon dioxide on the condenser and the gas extractor are taken into account, the optimum net work output of a geothermal steam plant drops even more sharply. The presence of 10% CO$_2$ in the geothermal steam, which is not unusual at many geothermal sites, results in as much as a 25% decrease in the net work output compared to a clean steam system. The work losses associated with the same concentration of noncondensables in an actual system may even be higher because the design economic optimum may not coincide with the thermodynamic optimum.

A more refined thermodynamic analysis, which takes into account the effect of dissolved salts (NaCl) and which allows for intercooling in the gas extraction system and admits more realistic models for the efficiency of expansion and compression of a multicomponent gas mixture, is left for a future report.

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References


Captions to Figures

Fig. 1 Variation of the Henry's-law coefficient with temperature for CO$_2$ in NaCl solutions.

Fig. 2 Dependence of the vapor augmentation factor on flash temperature.

Fig. 3 Mass fraction of CO$_2$ in the vapor phase as a function of flash temperature.

Fig. 4 Dependence of the normalized specific ideal work on the mass fraction of CO$_2$.

Fig. 5 Augmented normalized ideal work as a function of flash temperature.

Fig. 6 Dependence of the normalized turbine expansion work on the initial temperature.

---------- isentropic ------------ with moisture loss

Fig. 7 The effect of CO$_2$ on the wetness factor.

Fig. 8 Schematic representation of the condenser and gas extraction system.

Fig. 9 The variation of net work output with back pressure for $T_1 = 150^\circ$C.

Fig. 10 The variation of net work output with back pressure for $T_1 = 200^\circ$C.

Fig. 11 The dependence of the normalized maximum net work output on the mass fraction of CO$_2$.

Fig. 12 The optimum back pressure as a function of the mass fraction of CO$_2$.

Fig. 13 The effect of variable expansion efficiency on the net work output.
Fig. 1
Fig. 2

VAPOR AUGMENTATION FACTOR, $\alpha$

$T_g = 200^\circ C$

$\phi_g = 0.05$

0.01

0.005

FLASH TEMPERATURE, $T_1 / ^\circ C$

120 140 160 180 200
Fig. 3
Fig. 4
Fig. 5
Fig. 6

- Standardized Work, $W(f)/W(0)$
- Mass Fraction of CO$_2$, $f$
- $P_1, T_1, f$ to $W$
- $P_2 = 0.012349$ MPa
- $T_1/°C$: 200, 125
- Ideal and Actual lines

$P_2$, $S_2 = S_1$
Fig. 7
CO₂ + H₂O (V)

Wc

ADIABATIC COMpressor

CO₂ + H₂O (V)

(From TURBINE)

CO₂ + H₂O (V + L)

P₂

T₂

CONDENSER

BAFFLE

HOT WELL

H₂O

CONDENSATE

CW IN

OUT

CO₂ + H₂O (V)

SUBCOOLING, ΔTₜₚ

T₃ = 35°C

T₂(P₂,f)

CO₂ + H₂O

COOLING WATER

Fig. 8
Fig. 9

NET WORK, $W_n = W_e - W_c$, kJ/kg

BACK PRESSURE, $P_2$/MPa

$T_g = 150^\circ$C

$\eta_e = 0.75$

$\eta_c = 0.70$
Fig. 10
Normalized maximum work, $W_n^*(f)/W_n^*(0)$

$T_3 = 35^\circ C$, $\eta_e = 0.75$, $\eta_c = 0.7$

$T_1 = 200^\circ C$

$150^\circ C$

Mass fraction of $CO_2$, $f$

Fig. 11
\[ \eta_c = 0.75, \quad T_1 = 150^\circ C \\
\eta_c = 0.75, \quad T_1 = 200^\circ C \]

Fig. 12

- Mass fraction of CO$_2$, $f$
- Optimum back pressure, $p_2^*$ / MPa

- $0.056$ MPa ($T_2 = T_3 = 35^\circ C$)
- $\eta_c = 0.2$ (Ejector)
- $\eta_c = 0.5$
- $\eta_c = 0.7$
Fig. 13

T₁ = 200°C, f = 0.2, ηₑ = 0.7

- ηₑ = CONSTANT = 0.75
- VARIABLE ηₑ