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SYSTEM STUDIES OF COMPACT IGNITION TOKAMAKS

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OAK RIDGE NATIONAL LABORATORY

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ABSTRACT

The new Tokamak Systems Code, used to investigate Compact Ignition Tokamaks (CITs), can simultaneously vary many parameters, satisfy many constraints, and minimize or maximize a figure of merit. It is useful in comparing different CIT design configurations over wide regions of parameter space and determining a desired design point for more detailed physics and engineering analysis, as well as for performing sensitivity studies for physics or engineering issues. Operational windows in major radius (R) and toroidal field (B) space for fixed ignition margin are calculated for the Ignifed and Inconel candidate CITs. The minimum R bounds are predominantly physics limited, and the maximum R portions of the windows are engineering limited. For a modified Kaye-Goldston plasma-energy-confinement scaling, the minimum size is 1.15 m for the Ignifed device and 1.25 m for the Inconel device. With the Ignition Technical Oversight Committee (ITOC) physics guidance of $B^2 a/q$ and $I_p > 10$ MA, the Ignifed and Base-line Inconel devices have a minimum size of 1.2 and 1.25 m and a toroidal field of 11 and 10.4 T, respectively. Sensitivity studies show Ignifed to be more sensitive to coil temperature changes than the Inconel device, whereas the Inconel device is more sensitive to stress perturbations.

SYSTEM STUDIES OF COMPACT IGNITION TOKAMAKS

1. INTRODUCTION

Recently, great effort has been dedicated toward the preconceptual design of a small, high-field tokamak, called the Compact Ignition Tokamak (CIT), for study of fully ignited plasma discharges. The use of small, high-field tokamaks to demonstrate fusion ignition was proposed previously.^{1,2} Because of the relatively low cost of CITs compared with the proposed larger devices (TFCX³ and FED⁴), the idea is being revisited. Several different concepts have been proposed for the CIT, including the Ignitor,⁵ the Ignition Studies Project (ISP),⁶ and the Long-pulse Ignited Test Experiment (LITE).⁷ each of which incorporates different schemes to handle the large stress levels in the coils. Design considerations include temperature rises in the resistive coils, volt-second capability, and plasma parameters to provide a reasonable probability of ignition, in addition to the stress problems. In general, CIT designs deal with tightly coupled constraints and nonlinear trade-offs in physics and engineering.

To delineate the CIT design space systematically, we have developed a new code, the Tokamak Systems Code, to help accommodate the many trade-offs involved. The requirements for engineering (stress levels, coil-temperature-rise limits, etc.) and for physics (plasma beta, ignition margin, etc.) are constructed as a set of constraints. The engineering quantities (coil dimensions, current density, etc.) and the physics quantities (plasma density and temperature) are organized as a set of independent variables. Existing mathematical software⁸ is used to find a combination of variables which satisfies the constraints (a "solution"). An additional optimization feature of the software allows solutions to be obtained which minimize (or maximize) a figure of merit (e.g., the major radius). We have found this setup to be a powerful

tool in analyzing the parameter space of CIT devices, as compared with the previously used system study techniques,⁹ which were limited to varying a couple of design variables simultaneously.

The purpose of this paper is to (1) present example applications of the new code and (2) describe studies which led to the choice of the base-line CIT device. The results, which follow a description of the analysis used in the study, are divided into two sections. In the first part of Sect. 3 (Results), CIT operating windows in major radius (R) and toroidal field (TF) are presented as examples of the code application. In the latter part of Sect. 3, conductor material and TF coil configuration trade-offs are presented which help in choosing the March 1986 CIT preconceptual base-line point.

2. MODEL

The new version of the Tokamak Systems Code consists of a set of subroutines that model the physics and engineering aspects of tokamak designs and a numerical optimization routine that controls all variable iteration. The physics and engineering requirements are constructed as a set of constraints to be satisfied. Many of the modeling algorithms for various tokamak components used here are taken directly from ref. 9. This discussion concentrates on the new engineering and physics models introduced to accommodate CIT studies and on the new code structure with regard to the method of finding solutions. The new plasma physics modeling includes profile averaging, beta and density limits, and a refined volt-second analysis. The engineering modifications include coil stress and temperature calculations, power requirements, and limiter-heat-load bounds. Unless otherwise stated, MKS units are used for all quantities with the exception of keV for plasma temperature.

2.1 OPERATING SCENARIO

Many of the calculations described in this report refer to the CIT operating scenario, which is indicated schematically in Fig. 1. During the first period ($0 \leq t \leq t_1$), the ohmic heating (OH) coil and the

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Fig. 1. Schematic of the CIT coil current and plasma waveforms with time, used in the system studies.

poloidal field (PF) coils begin charging. The duration t_1 is an input and is currently set at 2.7 s. During the second period ($t_1 \leq t \leq t_2$), the TF coils begin ramping up. The total TF coil ramp-up time ($t_3 - t_1$), determined from the CIT design criteria, is 20% longer than the plasma current initiation time ($t_3 - t_2$), which is dictated by a maximum plasma current ramp-up rate of 3 MA/s. Time t_2 is referred to as the "beginning of the pulse." The flattop ($t_3 \leq t \leq t_4$) is taken to be $12 \tau_E$ for all results presented here in accordance with CIT design guidelines, where τ_E is the global plasma energy confinement time and time t_4 is referred to as the "end of the flattop." The $12 \tau_E$ pulse length originally was taken to be 10 energy confinement times, but an additional $2 \tau_E$ were added to allow for time to externally heat the plasma. Finally, the plasma, TF coil, and PF coil current ramp-down duration ($t_4 \leq t \leq t_5$) is determined by a maximum plasma current quench rate of 6 MA/s. The plasma current ramp rates are higher than present experimentally observed rates for discharges with edge $q \approx 2.5$ [i.e., TFTR (ref. 10)] but may be possible with techniques such as simultaneous current and field ramping.

2.2 PLASMA PHYSICS

A primary revision of the Tokamak Systems Code consists in the refinement of the plasma physics modeling subroutines. The plasma physics is based on a global (profile-averaged) steady-state model similar to the model used in CIT physics trades studies.^{11,12} Plasma temperature (T) and density (n) profiles are assumed to be of the form

$$n(T) \propto [1 - (r/a)^2]^\alpha,$$

where a is the plasma minor radius. For all results presented here, $\alpha = 0.5$ is used for density and $\alpha = 1$ for temperature. During profile averaging, integration is performed only in the r direction.

The plasma beta is defined as

$$\beta = 3.2 \times 10^{-16} \frac{(\langle n_i T_i \rangle + \langle n_e T_e \rangle + 2/3 n_\alpha E_\alpha) \times \mu_0}{B^2}, \quad (1)$$

where subscripts i and e refer to ions and electrons, respectively; B is the toroidal magnetic field on axis; and n_α and E_α are the fast-alpha-particle density and average energy per alpha, respectively, solved for as a function of plasma temperature and density. For $T_e \approx 10$ keV, the fast-alpha contribution is typically 10% of the beta value. Also, in this study we assume: $T_i = T_e$; a 5% helium ash contribution to the background ions; and the effective plasma charge number $Z_{eff} = 1.5$ (with an average impurity mass number of 8) to calculate n_e/n_i . The "< >" brackets imply a volume average, and μ_0 is the permeability constant ($= 4\pi \times 10^{-7}$ H/m).

The ignition margin (IM) is determined by using the following plasma power balance:

$$\frac{P_\alpha + P_{OH}}{IM} = P_{tr-i} + P_{tr-e} + P_{rad} \quad (2)$$

where P_α is the volume-averaged alpha particle fusion power (assuming a 50-50 deuterium-tritium fuel mixture); P_{OH} is the volume-averaged ohmic heating power (typically only a few percent of P_α near plasma temperatures of 10 keV); and P_{tr-i} and P_{tr-e} represent the ion and electron transport power loss,

$$P_{tr-i(e)} = \frac{3/2 \langle nT \rangle_{i(e)}}{\tau_{i(e)}} \quad ,$$

where the ion energy confinement time τ_i is taken to be one-third that of Chang-Hinton scaling¹³ in this study and τ_e can be chosen from a variety of scalings, including neo-Alcator, Mirnov, and a modified Kaye-Goldston, and where P_{rad} is the volume-averaged brehmsstrahlung power. The expressions used for confinement scalings are described in more detail in Appendix A. Also the global energy confinement time τ_E is defined as

$$\tau_E = \left(\tau_e^{-1} + \frac{n_i T_i}{n_e T_e} \tau_i^{-1} \right)^{-1} \quad .$$

The poloidal beta equation requires that

$$\beta_p = \beta (B/B_p)^2, \quad (3)$$

where β_p is the poloidal beta and B_p is the poloidal magnetic field,

$$B_p = \frac{I_p \text{ (MA)}}{5a[(1 + \kappa^2)/2]^{1/2}},$$

where κ is plasma elongation and I_p (MA) is plasma current in megamperes. The plasma current I_p is calculated by using a fit to magnetohydrodynamic (MHD) calculations¹⁴ and is a function of the plasma aspect ratio A , β_p , and κ . A quantity associated with the plasma current, related to the plasma safety factor q_* , and used throughout this work is

$$q_* = \frac{5a}{A} \sqrt{\frac{1+\kappa^2}{2}} \frac{B}{I_p \text{ (MA)}}.$$

Two inequality constraints associated with the plasma physics are (1) the restriction that the volume-average plasma density be less than the modified Murakami density,

$$n_e \leq \frac{1.5 \times 10^{20} B}{Rq_*}, \quad (4)$$

where R is the plasma major radius, and (2) the restriction that the plasma beta be less than the Troyon beta limit,

$$\beta \leq \frac{0.03 I_p \text{ (MA)}}{Ba}. \quad (5)$$

The modified density limit used for CIT studies incorporates shape factors (through q_*) which are not in the original limit (ref. 15) and presently not experimentally verified. The coefficient 3.0, used in the Troyon beta limit (ref. 16), is consistent with values derived from experiments (ref. 17).

2.3 VOLT-SECOND ANALYSIS

An additional consideration that is a driving force in tokamak design is the criterion that the volt-second requirement, $(V-s)_{req}$, for poloidal flux be less than the volt-second capability, $(V-s)_{cap}$, of the PF coil system:

$$(V-s)_{req} \leq (V-s)_{cap} \quad (6)$$

The V-s requirement includes three components: poloidal flux associated with the full plasma current, resistive losses during startup, and resistive losses during the flattop (or burn) portion of the pulse. The resistive startup V-s requirement is taken to be one-half of the internal flux at full current,

$$(V-s)_{su} = 2.5 \times 10^5 \times I_p \text{ (MA)} \times \mu_0 R \ell_i,$$

where ℓ_i is a normalized plasma internal inductance which depends on the plasma current profile and which is taken from a fit of MHD equilibrium calculations (typically, $1.0 \leq \ell_i \leq 1.5$). The V-s requirement associated with the poloidal flux at full plasma current is

$$(V-s)_I = I_p \text{ (MA)} \times 10^6 \times (\mu_0 R \ell_i / 2 + L_{ext}),$$

where L_{ext} is the external inductance (see ref. 18); the V-s requirement during the burn is

$$(V-s)_b = f_{st} \tau_b \times 10^6 \times I_p \text{ (MA)} R_p,$$

where

$$\tau_b = t_4 - t_3,$$

$$R_p = \text{plasma resistance } (2R\eta/a^2\kappa),$$

$$\eta_{||} = \text{parallel resistivity} = 1.65 \times 10^{-3} Z_{eff} \ln(\Lambda)/T_e^{3/2} \times a_{n.c.},$$

$$\ln(\Lambda) = \text{Coulomb logarithm}$$

$$a_{n.c.} = \text{neoclassical correction factor (ref. 19)}$$

(for parameters typical of CITs, this factor is ~ 2.0),

$$f_{st} = \text{factor (3.0) used to account for sawteeth effects (ref. 20),}$$

The total volt-second requirement is

$$(V-s)_{\text{req}} = (V-s)_{\text{SJ}} + (V-s)_I + (V-s)_E.$$

The volt-second capability is calculated by

$$(V-s)_{\text{cap}} = \sum_i (I_{i-\text{BOP}} M_{i-p} - I_{i-\text{EOF}} M_{i-p}),$$

where

Σ = summation of i over all the PF coils (OH coil, shaping field coils, and vertical field coils),

M_{i-p} = mutual inductance between coil i and the plasma,

$I_{i-\text{BOP}}$ = the current in coil i at BOP,

$I_{i-\text{EOF}}$ = current in coil i at EOF,

BOP = beginning of pulse,

EOF = end of flattop.

The mutual inductance calculations are described in ref. 9. The OH coil current density is either input by the user or treated as a variable, and the other PF coil currents are determined with a scaling routine. This routine scales the end-of-flattop coil currents with I_p , B_p , and the device size from a reference MHD equilibrium calculation; the routine also calculates the coil currents that null the poloidal field at the beginning of the pulse. As the device parameters are iterated in the solutions process, the PF coil locations are determined to avoid physical interference with TF coils and the OH coil. The shaping PF coils on top of the plasma scale with the quantity $(R-a)$, and the outer PF coils scale with the location of the outer TF coil leg. The plasma and PF coils are up-down symmetric for all cases presented here. The $(V-s)_{\text{cap}}$ was required to be 5% larger than the $(V-s)_{\text{req}}$ because of the uncertainties involved in some of the modeling procedures described. Also, no bootstrap current effects (ref. 21) are included here. Although they may reduce the required burn volt-seconds during the flattop, this component of the total required flux is small, so the overall impact of bootstrap current effects is minimal for CITs.

2.4 COIL STRESS

A primary engineering concern in designing CIT devices is the high stress levels in the TF coils and the OH solenoid. In the Tokamak Systems Code, the TF coil stress (σ_{TF}) is required to be less than the allowable stress level (σ_{alw}):

$$\sigma_{TF} \leq \sigma_{alw} \quad (7)$$

Analytic formulas taken from ref. 22 which account for the electromagnetic forces in a current-carrying annulus are used to model the TF coil stress (see Appendix B). Also, the stresses in the OH coil (σ_{OH}) at the beginning of the pulse and at the end of the flattop are limited to the allowable level:

$$\sigma_{OH-BOP} \leq \sigma_{alw} \quad (8)$$

and

$$\sigma_{OH-Eof} \leq \sigma_{alw} \quad (9)$$

The OH stress model²² is also analytic and is described in Appendix B. The allowable stress levels are determined by ref. 23:

$$\begin{aligned} \sigma_{alw} &= \text{minimum } (0.6 \times \text{ultimate stress, } 0.67 \times \text{yield stress}) \\ &\quad \text{for self-constrained coils (LITE-like configuration)} \\ &= \text{minimum } (0.7 \times \text{ultimate stress, } 0.85 \times \text{yield stress}) \\ &\quad \text{for externally constrained coils (Ignitor-like configuration).} \end{aligned}$$

The self-constrained allowables are slightly lower, since there is no external frame available to limit the tension in the inner leg of the TF coil. Figure 2 shows the allowable stress levels vs coil temperature for the copper alloys considered in this study which are incorporated via a table look-up scheme. Also indicated are the stress levels for Inconel, which is used with Cu-102 as a composite material.²⁴ For the composite material the allowable stress is taken to be

$$\sigma_{alw} = \sum_i \sigma_{Ai} \times f_i \quad ,$$

where σ_{Ai} is the allowable stress of material i and f_i is the volume fraction of material i . The end-of-flattop coil temperature is used to

Fig. 2. Allowable stress levels used in CIT systems code studies.

evaluate the allowable stress levels for σ_{OH-EOP} and σ_{TF} and the beginning-of-pulse temperature for σ_{OH-BOP} .

2.5 COIL TEMPERATURE

Another important concern in the design of resistive-coil tokamaks is limiting the coil temperature rise. The CIT devices use an inertial cooling scheme in which the coils are initially cooled to 80 K and no active cooling is provided during the pulse. During the pulse the coil temperature rises because of resistive heating and for TF coils some nuclear heating. The Tokamak Systems Code can limit the end-of-pulse temperature (at t_6) of the TF coils and the OH solenoid through the equations

$$\langle T_{TF} \rangle_{EOP} \leq T_{TF-allow} \quad (10)$$

and

$$\langle T_{OH} \rangle_{EOP} \leq T_{OH-allow} \quad (11)$$

The coil-temperature-rise model in the Tokamak Systems Code uses an average current density and calculates the time dependence of the average coil temperature by using current waveforms of the type displayed in Fig. 1. If Inconel-copper composite coils are used, it is assumed that only the copper carries the current, but both copper and Inconel heat capacities are included in the temperature rise calculation, with a ratio of Inconel to copper heat capacity of 1.13. The end-of-pulse coil temperature, $\langle T \rangle_{EOP}$ is calculated by

$$\langle T \rangle_{EOP} = T_0 + \int_0^{t_5} (dT/dt) dt ,$$

where

T_0 = initial temperature (80 K),

$$\frac{dT}{dt} = \frac{[J(t)/[(1 - VF)(1 - f_{inc})]]^2 \rho + Q_{nuc}}{S \times d [1 + [f_{inc}/(1 - f_{inc})] \times 1.13]} \quad (K/s) ,$$

$J(t)$ = overall current density (A/m^2),

VF = void fraction of coil (10% for TF coils and 15% for the OH coil),

ρ = resistivity ($\Omega\cdot m$) (see Fig. 3),

S = specific heat of copper ($J/kg\cdot K$) (see Fig. 3),

f_{inc} = volume fraction of Inconel,

d = copper density ($8.89 \times 10^3 \text{ kg/m}^3$),

Q_{nuc} = nuclear heating (used only for the TF coil inboard leg).

The nuclear heating is approximated as

$$Q_{nuc} = \frac{\alpha \Gamma}{\Delta_{TF}} \{1 - \exp[-(\Delta_{TF}/\lambda)]\},$$

where

Γ = neutron first wall load (W/m^2),

Δ_{TF} = TF coil thickness (m),

λ = neutron mean free path in copper (0.09 m),

$\alpha = 0$ for $0-t_3$ and t_4-t_5

$= 1.0$ for t_3-t_4 (t_k 's are the time periods designed in Fig. 1).

Coil temperatures are calculated for both inboard and outboard TF coil segments (with different current densities for each segment), for the PF coils, and for the OH coil. At present, the same current density is used for the OH coil as for the other PF coils.

The reference allowable average temperature is 373 K for wound coils and 325 K for coils of plate design. The lower allowable average temperature for plate coil designs is used to provide a margin for nonuniform spatial current profiles that give rise to nonuniform heating. More sophisticated one-dimensional temperature rise models²⁵ indicate that, for Ignitor- and LITE-type devices, the peak temperature rises ~20% more than predicted by the average-temperature-rise model for plate-type coils, whereas wound coils have a uniform spatial temperature rise. To limit the peak temperature in wound- and plate-type coils to the same value, we impose the lower limit on the average temperature for the plate-type coils. All CIT designs examined here use plate construction for the TF coils and wound-type coils for the OH coil.

Fig. 3. Electrical properties of copper used in CIT systems code studies.

2.6 POWER REQUIREMENTS

The options to limit the total power (P), MVA, and energy (E) requirements of the device are implemented through the following equations:

$$P_{\max} \leq P_{\text{alw}} , \quad (12)$$

$$MVA_{\max} \leq MVA_{\text{alw}} , \quad (13)$$

$$E_{\text{tot}} \leq E_{\text{alw}} , \quad (14)$$

where $P_{\max}$ is the maximum instantaneous resistive power consumption (including PF coils and TF coils); $MVA_{\max}$ is the maximum power requirement (including resistive and inductive components); E_{tot} represents the total energy requirements including both the resistive losses and the stored magnetic energy requirements at the end of the flattop. These quantities are calculated as described subsequently.

Power and MVA levels are checked at the end of each time period indicated in Fig. 1 for the maximum power and MVA requirements. The resistive power requirement for TF coils is

$$PR_{\text{TF}} (W) = N_{\text{TF}} A_{\text{TF}} L N_{\text{TF}} J_{\text{TF}}(t) \frac{\rho_{\text{TF}}(t)}{(1 - VF_{\text{TF}})(1 - f_{\text{inc-TF}})} ,$$

where

N_{TF} = number of TF coils,

A_{TF} = cross-sectional area of TF coil (m^2)

$$= \pi(R_{i1}^2 - R_{i2}^2)/N_{\text{TF}} ,$$

$L N_{\text{TF}}$ = length of TF coil (m) (ref. 9),

$R_{i1(2)}$ = outer (inner) radius of the inboard TF coil leg.

This calculation is done for both inboard and outboard TF coil legs. The length of the TF coil is calculated assuming an approximately constant tension shaped TF coil (option SHAPTF = 3 in ref. 9). For the PF systems, the resistive power is

$$PR_{\text{PF}} (W) = \sum_i I_i(t) J_i(t) \rho(t) \times \frac{2\pi R_i}{(1 - VF_i)(1 - f_{\text{inc-OH}})} ,$$

where

Σ_i = sum over all PF coils (including OH solenoid),

I_i = current in coil i (A),

J_i = current density in coil i (A/m²),

R_i = radius of coil i (m),

VF_i = void fraction of coil i .

Also, $P_{\max}$ is the maximum of $PR_{TF} + PR_{PF}$ for all times t_k .

The MVA requirement for the TF coil is

$$PI_{TF} \text{ (VA)} = PR_{TF} + L_{TF} N_{TF}^2 A_{TF}^2 J_{TF}(t) \frac{d}{dt} J_{TF}(t),$$

where L_{TF} is inductance of the TF coil (H) which is approximated as a series of filament arcs (see ref. 9). Also, the current density derivative is evaluated at both sides of each time t_k in Fig. 1.

For the PF system, the MVA requirement is

$$PI_{PF} \text{ (VA)} = PR_{PF} + \Sigma_i \Sigma_j L_{ij} I_i \frac{d}{dt} I_j,$$

where

Σ_i = sum over all PF coils (including solenoid),

Σ_j = sum over all PF coils and plasma current,

L_{ij} = mutual inductance of circuits i and j (H) (see ref. 9),

I_j = current in circuit (derivatives evaluated at each side of time t_k , Fig. 1).

The total MVA requirement is the maximum of $PI_{TF} + PI_{PF}$ and is checked at each time t_k .

For the TF coils, the resistive component of the energy requirement is

$$E_{TF} = \int_0^{t_5} N_{TF} A_{TF} L_{TF} \frac{[J_{TF}(t)]^2}{(1 - VF_{TF})(1 - f_{inc-TF})} \rho(t) dt.$$

For the OH solenoid, the resistive component is

$$E_{OH} = \int_0^{t_5} A_{OH} \frac{[J_{sol}(t)]^2}{(1 - VF_{OH})(1 - f_{inc-OH})} \rho(t) dt ,$$

where A_{OH} is the cross-sectional area of the OH coil.

For the other PF coils, the resistive component of the energy requirement is calculated as

$$E_{PF} = \sum_i \sum_j \overline{PR}_{ij} \Delta t_j ,$$

where

$\sum_i$ = sum over PF coils,

$\sum_j$ = sum over time periods j indicated in Fig. 1,

$\overline{PR}_{ij}$ = average (of the sum of the beginning and end of time period j) resistive power as explained previously for PR_{PF} .

The stored energy component is

$$E_{sto} = \sum_i 1/2 L_i I_i^2 ,$$

where i is summed over the PF coils, the OH coil, the TF coils, and the plasma by using the current values at the end of time period t_4 , and the total energy requirement is

$$E_{tot} = E_{sto} + E_{TF} + E_{PF} + E_{OH} .$$

When the Princeton Plasma Physics Laboratory (PPPL) facility limits are considered (i.e., the motor generator set capability), the following values are used:

$$P_{alw} = 760 \text{ MW} ,$$

$$MVA_{alw} = 760 \text{ MVA} ,$$

$$E_{alw} = 3.6 \text{ GJ} .$$

These numbers allow for losses of 20% in bussing, convertors, etc.

2.7 LIMITER HEAT LOAD

Present CIT designs use inertially cooled limiters with no active cooling during the pulse. The heat flux to the limiter is governed in the Tokamak Systems Code through

$$\Gamma_{\text{lim}} \leq \Gamma_{\text{alw}} \quad (15)$$

The maximum heat flux to the limiter, Γ_{lim} , is calculated by assuming that 80% of the alpha power is deposited on the limiter, that the limiter encompasses one-third of the first wall area, and that the peak to average limiter heat flux is 1.6. The allowable limiter heat flux as a function of heating duration ($t_4 - t_5$) is taken from ref. 26 and shown in Fig. 4. This limit is for a 3-cm-thick graphite tile limiter mounted on a 0.5-cm-thick stainless steel substrate with a low gap conductivity ($<10 \text{ W/m}^2\text{-K}$) and corresponds to the heat load and heating duration for which the graphite tile reaches a 2000°C limit.

2.8 CONFIGURATIONAL CONSTRAINTS

In addition to the physics and engineering constraints described previously, we include a radial build relation,

$$\Delta_{\text{bore}} + \Delta_{\text{OH}} + \Delta_{\text{TF}} + \Delta_{\text{FW}} + \Delta_{\text{gap}} + a = R \quad (16)$$

where

Δ_{bore} = thickness of the hole in the OH coil,

Δ_{OH} = thickness of the OH coil,

Δ_{TF} = thickness of the inner TF coil leg,

Δ_{FW} = first wall and limiter thickness,

Δ_{gap} = thickness of all gaps in inboard radial build.

We also include an equation for the toroidal field on axis,

$$B = \frac{\mu_0 J_{\text{TF}} A_{\text{TF}}}{2\pi R} \quad (17)$$

where A_{TF} is the total cross-sectional area of the inner TF coil leg at midplane (see ref. 9).

Fig 4. Allowable limiter heat flux vs heating duration.

2.9 OPTIONAL REQUIREMENTS

Two other optional constraints have been incorporated into the Tokamak Systems Code for CIT systems studies. Both the plasma current I_p and the proposed figure of merit $B^2 a / q_*$ can be required to be greater than or equal to some prescribed input levels (I_{in} and X_{in}):

$$I_p \geq I_{in} , \quad (18)$$

$$\frac{B^2 a}{q_*} \geq X_{in} . \quad (19)$$

These constraints have been added recently in response to the Ignition Technical Oversight Committee (ITOC)²⁷ recommendations. When these equations are used, the ignition margin constraint, Eq. (2), is generally not used.

2.10 SOLUTION METHOD

Equations (1) through (19) constitute the set of constraints satisfied by the Tokamak Systems Code in finding a solution. The set of variables that may be chosen for iteration includes R , A , a , B , β , β_p , I' , T_e , n_e , Δ_{bore} , Δ_{OH} , Δ_{TF} , J_{OH-EOF} , J_{OH-BOP} , J_{TF} , f_{inc-OH} , and f_{inc-TF} . Also, we introduce a dummy variable (which may be iterated) for each of the relations (1)-(19) that is an inequality. For example, the volt-second relation [Eq. (6)] is treated as

$$(V-s)_{req} = f_{V-s} \times (V-s)_{cap} ,$$

where the dummy variable f_{V-s} is bounded by $0 \leq f_{V-s} \leq 1.0$. To find a solution, we use the nonlinear equation optimization routine VMCON.⁸ Input to the code includes defining which of the constraints [Eqs. (1)-(19)] are to be considered and which of the parameters described previously are to be variables. Finally, a figure of merit is specified which can be either minimized or maximized. The code returns the combination of variables which satisfies the constraints and minimizes (or maximizes) the given figure of merit. Also, upper and lower bounds can be introduced for all variables. The most commonly used figure of merit for the CIT studies is minimizing the major radius R since small size is an important consideration in reducing cost. In

general, not all the constraints are included, and often some quantities are held fixed to compare solutions with results from other cases. The numerical methodology used by the Tokamak Systems Code has also been used in the Tandem Mirror systems studies for Minimars.²⁸

3. RESULTS

3.1 CODE APPLICATION, EXAMPLES

The new Tokamak Systems Code is quite flexible and can be used in many ways. The first example we present is a calculation of the CIT design windows for major radius and toroidal field at fixed ignition margins. These calculations provide a good example of the ability of the code to simultaneously push many physics and engineering constraints to their limits in order to maximize (or minimize) a figure of merit. Also, we present the sensitivity of the "ignition margin" results to the available power supply capabilities at a given site (e.g., the Tokamak Fusion Test Reactor facilities). The physics and engineering guidelines used in the studies are summarized in Table 1. The plasma elongation of 1.8 is slightly larger than the experimental experience of the limiter D-III results (ref. 29). The edge q value is within the experimental range for limiter plasmas (refs. 10, 29).

3.1.1 Design Windows

As a first application of the new system code, we examined an Ignifed-type device, described in Table 2. Ignifed was proposed by the Fusion Engineering Design Center (FEDC) and is similar to the Ignitor⁵ concept; the major differences are that the TF coils for Ignifed are bucked and wedged (bucked only for Ignitor) and that the OH solenoid does not require the TF coil bucking pressure for acceptable stress levels (and therefore is referred to as a self-supported OH solenoid). For CIT applications, bucked TF coils refer to the situation in which the TF coils are allowed to move radially inward and press against the OH coil through the action of the centering force. Wedged TF coils means that the coils are restricted from moving radially inward by

Table 1. Physics and engineering guidelines
used in CIT systems code studies

Quantity	Limit	
<u>Physics constraints</u>		
Plasma current I_p , MA	≥ 10	} ITOC
Figure of merit $B^2 a / q_*$, T ² -m	≥ 25	
or		
Ignition margin	$\geq$ Input	
Plasma current ramp-up rate, ^a MA/s	≤ 3	
Plasma current ramp-down rate, ^a MA/s	≤ 6	
Flat-top time τ_{flat}	$12\tau_E$	
Energy confinement time ^b τ_E , s	$2 \times 10^{20} / \langle n \text{ (m}^{-3} \rangle$	
Average density ^a $\langle n \rangle$, m ⁻³	$\leq 1.5 \times 10^{20} B \text{ (T)} / R \text{ (m)} q_*$	
Plasma beta ^a β , %	$\leq 3 \times I_p \text{ (MA)} / a \text{ (m)} B \text{ (T)}$	
Volt-second capability	$\geq 1.05 \times$ volt-second requirement	
Plasma safety factor q_ψ	2.6	
Plasma elongation κ	1.8	
<u>Engineering constraints</u>		
TF coil and OH coil temperature at end of pulse	$\leq 100^\circ\text{C}$	

Table 1 (continued)

Quantity	Limits
<u>Engineering constraints (continued)</u>	
TF coil stress	$\leq \sigma_{alw}$
Tresca stress of average component stresses for wedged TF coils	Minimum (0.6 × ultimate, 0.67 × yield) for self-constrained coils
Tresca stress with the peak radial stress for bucked-and-wedged TF coils	Minimum (0.7 × ultimate, 0.85 × yield) for externally constrained coils
OH coil stress σ_{OH}	$\leq \sigma_{alw}$
Peak stress at the inner radius	(Self-constrained coil)
$\int_0^{EOP} \text{limiter load } (r_{lim}) dt$	Limiter temperature of 2000°C
TF coil (OH coil) packing fraction	0.90 (0.85)
Inboard radial build	
Vacuum vessel and limiter thickness	4 cm
Gap between TF coil and OH coil	1 cm (wedged TF coils) 0.5 cm (bucked-and-wedged TF coils)
Gap between TF coil and vacuum vessel	0.5 cm

^aQuantities are fixed at the stated limit when ITOC physics is used.

^bUsed only with ITOC physics.

Table 2. Ignited and Inconel device configuration descriptions

	Ignited	Inconel CIT
<u>TF system</u>		
Material	Cu-102	Cu-102 and Inconel
Inconel fraction, %	0	<35
Preload, 10^6 kg	≤ 23	≤ 23
Configuration	Bucked and wedged (externally constrained stress allowable)	Wedged (self- constrained stress allowable)
<u>PF system</u>		
OH coil material	Cu-175	Cu-102 and Inconel
OH coil Inconel fraction, %	0	≤ 35
PF coil material	Cu-102	Cu-102
PF coil Inconel fraction, %	0	0
OH coil configuration	Self-supported (self-constrained stress allowable)	Self-supported (self-constrained stress allowable)
<u>Radial build spaces, cm</u>		
First wall thickness	4.0	4.0
Gap between TF coil and OH coil	0.5	1.0
Gap between Tr coil and first wall	0.5	0.5
<u>Plasma</u>		
Elongation	1.8	1.8
q_ψ at edge	2.6	2.6
Flat-top time	$12\tau_E$	$12\tau_E$

pressing tightly against each other in the direction of the toroidal field. Figure 5 shows the operating windows in R (major radius) and B (toroidal field on axis) space for the indicated ignition margin (IM) levels with the modified Kaye-Goldston (K-G) scaling (ref. 30) for electron energy confinement and three times Chang-Hinton scaling for ion energy confinement. We chose the modified Kaye-Goldston scaling, which is a pessimistic scaling, to be conservative in selecting a machine design. We later examined the machine performance, using more optimistic physics scaling assumptions. Each point on the contours satisfies constraints (1) through (17) described in the preceding section. (In the following text, the modified Kaye-Goldston scaling is referred to as Kaye-Goldston scaling.)

The lower portion of the contours is found by solving for the minimum R at the indicated ignition margin level, and the upper portion is found by solving for the maximum R. The maximum ignition margin point (IM = 1.05) is also indicated in Fig. 5. All points on the contours are limited by some of the inequality constraints described in the preceding section. The lower portions of the contours are limited primarily by the plasma beta and density constraints, the right-hand sides of the contours are limited primarily by the MVA and TF coil temperature constraints, and the upper bound of the contour is limited primarily by the MVA constraint. The lower part of the curves can be thought of as primarily "plasma-physics" limited, and the upper right-hand side portions are "engineering" limited. All points are limited by the volt-second constraint. For the engineering-limited region, there is little change in the position of the contours as the ignition margin changes.

Also indicated in Fig. 5 are contours of constant aspect ratio. (Note that a lower bound of $A = 2.5$ was imposed on all runs presented here.) The solutions shown in Fig. 1 have low aspect ratios ($A < 3.0$) due to the favorable scaling of the Kaye-Goldston energy confinement time with plasma current.

Table 3 lists some parameters and the fractions of the limits associated with the inequality constraints for the cases: (1) minimum R and an ignition margin of 1 and (2) maximum ignition margin. As is

Fig. 5. Modified Kaye-Goldston ignition margin contours for an Ignifed-type device, with aspect ratio levels superimposed.

Table 3. Parameters of Ignifed and Inconel with minimum radius, ignition margin = 1, and maximum ignition margin

Parameter	Ignifed		Inconel	
	Max. IM	Min. R, IM = 1	Max. IM	Min. R, IM = 0.95
<u>Principal device dimensions</u>				
R, m	1.40	1.24	1.43	1.29
B, T	10.1	10.2	9.29	10.2
A	2.75	2.66	2.72	2.79
I _p , MA	10.6	10.2	10.2	9.48
K-G IM ^a	1.05	1.0	0.965	0.95
NA IM ^a	1.26	1.17	1.12	1.09
MI IM ^a	1.34	1.27	1.18	1.15
B ² a/q _x , T ² -m	27.7	26.2	24.3	25.4
τ _b , s	4.20	3.68	4.38	3.74
<u>Radial build</u>				
Δ _{bore} , m	0.332	0.263	0.417	0.355
Δ _{OH} , m	0.324	0.284	0.251	0.236
Δ _{TF} , m	0.176	0.173	0.172	0.178
a, m	0.508	0.467	0.524	0.462
<u>Current densities</u>				
J _{OH-BOP} , kA/cm ²	6.27	7.26	6.15	6.75
J _{OH-EOF} , kA/cm ²	2.50	5.37	2.83	4.45
J _{TF} , kA/cm ²	8.46	9.05	7.96	8.47
<u>Fraction of Limits</u>				
σ _{OH-BOP}	1.0	1.0	1.0	1.0
σ _{OH-EOF}	0.06	0.39	0.06	0.23
σ _{TF}	0.81	0.72	1.0	1.0
T _{OH}	0.51	0.92	0.39	0.52
T _{TF}	1.0	1.0	1.0	1.0
V-s	1.0	1.0	1.0	1.0
β	1.0	1.0	1.0	1.0
n _e	1.0	0.97	0.98	0.94
r _{lim}	0.90	0.88	0.70	0.73
MVA	1.0	1.0	1.0	1.0
Energy	1.0	1.0	0.85	0.78
Power	0.68	0.72	0.62	0.73

^aK-G IM, NA IM, and MI IM represent Kaye-Goldston, neo-Alcator, and Mirnov ignition margin, respectively.

frequently seen in CIT optimization runs, both of these cases are hitting several limits. A common characteristic for the points in Fig. 1 is that the OH coil is not pushed to its stress or temperature limit at the end of flattop, because of the strong influence of the MVA limit, which tends to distribute the OH coil and TF coil peak MVA requirements to different times. Since the TF coil MVA peak is at time t_3 (see Fig. 1), the OH coil MVA contribution at t_3 is lowered by reducing the OH current density at the end of the flattop relative to the beginning of the pulse.

Also indicated in Table 3 is the figure of merit, B^2a/q_* , for both cases. This figure of merit was proposed by the ITOC physics panel²⁷ and is suggested to be ≥ 25 , along with the criterion that the plasma current be ≥ 10 MA. Figure 6 shows B^2a/q_* contours and plasma current levels superimposed on the Kaye-Goldston ignition margin levels, which indicate that the minimum-radius Kaye-Goldston ignition point and some points with Kaye-Goldston ignition margins of <1 satisfy both of these criteria. Figure 7 shows the maximum ignition margins obtainable by using neo-Alcator and Mirnov scaling for the electron energy confinement time (three times Chang-Hinton scaling is used for the ions in both these cases). Here we keep the device (i.e., radial build) fixed at the Kaye-Goldston values and allow the flexible quantities, such as coil current densities and plasma density and temperature, to vary. These additional ignition margin curves are the maximum achievable with a device that is optimized for Kaye-Goldston scaling if the energy confinement is neo-Alcator or Mirnov. Since the confinement time is longer for the latter scalings compared with those for Kaye-Goldston scaling, the burn time tends to be longer and the coil current densities and field levels are somewhat lower than the corresponding Kaye-Goldston values in order to satisfy the coil temperature constraints. The attainable ignition margins with neo-Alcator and Mirnov scalings are about 20 to 30% larger than the Kaye-Goldston ignition margins. At the minimum R and the Kaye-Goldston ignition margin = 1 point, the peak ignition margin for neo-Alcator scaling is 1.17, the toroidal field is 8.9 T, and the burn time is 5.7 s. The corresponding values for Mirnov scaling are: ignition margin = 1.27, $B = 8.0$ T, and burn time = 7.9 s.

Fig. 6. B^2a/q_* and plasma current levels superimposed on the Kaye-Goldston ignition margin contours for Ignited.

Fig. 7. Maximum Mirnov and neo-Alcator ignition margins superimposed on the Kaye-Goldston ignition margin contours.

The general behavior of the neo-Alcator and the Mirnov ignition margin contours is qualitatively similar to the Kaye-Goldston contours; the strong influence of the "engineering limits" in the upper right-hand region can be seen even for the neo-Alcator and the Mirnov contours.

The configuration for a CIT candidate device that uses an Inconel composite material for the coils (called the "Inconel device" here) is described in Table 2. This device uses only wedging in the TF coils to handle the centering forces to avoid complications inherent with bucked-and-wedged TF coil design. With wedging only, the stress levels in the TF coils are somewhat higher than those in bucked-and-wedged TF coils; therefore, a composite material made of Inconel sandwiches bonded to Cu-102 plates is proposed for use in this device because of the strengthening properties of the Inconel.

Figure 8 shows the 0.95 and 0.925 Kaye-Goldston ignition margin contours for the Inconel device with the aspect ratio levels superimposed, again satisfying constraints (1) through (17). The maximum Kaye-Goldston ignition margin for this device, for the same conditions used in the previous Ignifed example, is only 0.965. The behavior of the contours for this device is like that of the Ignifed device contours except that the upper and lower right-hand parts of the curves are not so closely grouped. The reason for this is that the upper and right-hand side of the Inconel ignition margin contours are both physics and engineering limited. Table 3 indicates the Inconel device parameters for the maximum Kaye-Goldston ignition margin and the minimum-radius Kaye-Goldston ignition margin of 0.95. Again we see that these cases simultaneously meet several of the limits imposed by the inequality constraints. As with the preceding Ignifed examples, the Inconel cases do not push the OH coil stress or temperature limits at the end of the flattop, because of the strong influence of the MVA limit.

Figure 9 shows the B_a^2/q_* and I_p contours for the Inconel device. In this case there is no region with

$$\frac{B_a^2}{q_*} \geq 25 \text{ T}^2\text{-m} \quad \text{and} \quad I_p \geq 10 \text{ MA} .$$

Fig. 8. Modified Kaye-Goldston ignition margin contours for an Inconel-type CIT device, with aspect ratio levels superimposed.

Fig. 9. B^2a/q_* and plasma current levels superimposed on the Kaye-Goldston ignition margin contours for the Inconel device.

Figure 10 shows the maximum neo-Alcator and Mirnov ignition margins for the Inconel device. These alternate ignition margin contours are found in the same manner described for the Ignifed device and have characteristic values of 1.1-1.3. Their behavior is quite different from the behavior for the Ignifed example, however. This difference is due to the additional effect of changing material properties (i.e., changing Inconel fractions). As the Kaye-Goldston contours move to lower B, higher R regions, the Inconel fraction drops. This reduces the severity of the coil temperature rise, which can have a strong impact on the alternative ignition margin cases. We stress that the neo-Alcator and Mirnov ignition margin contours shown here are maximized using fixed radial builds that were determined from the Kaye-Goldston ignition margin cases.

The neo-Alcator and Mirnov ignition margin contours look quite different when optimized with the same degree of flexibility as the Kaye-Goldston contours used in Figs. 5 and 8, as indicated by the neo-Alcator 1.2 ignition margin contour for the Inconel device shown in Fig. 11. The ignition margin region of ≥ 1.2 in R vs B space is much larger than the corresponding region in Fig. 10, and the aspect ratio variation ($2.5 \leq A \leq 4.1$) over the 1.2 ignition margin contour is also larger than observed in Fig. 5. The maximum neo-Alcator ignition margin point (1.43) is also indicated in Fig. 11.

3.1.2 Power Supply Sensitivity

The Tokamak Systems Code can also be used to investigate sensitivities of the results to various assumptions and modeling procedures. For example, in Fig. 12, we indicate the sensitivity of the Ignifed device performance to the permissible power supply capabilities (MVA and energy). Here, we show the maximum Kaye-Goldston ignition margin vs normalized power supply capabilities (normalized to the present PPPL motor generator set power supply capabilities deliverable to the coils, i.e., 760 MVA and 3.6 GJ). There is a 10% increase in the maximum ignition margin with the first 20% increase in power supply and a further 5% increase in ignition margin with each additional 20%

Fig. 10. Maximum Mirnov and neo-Alcator ignition margins superimposed on the Kaye-Goldston ignition margin contours for the Inconel device.

Fig. 11. Neo-Alcator 1.2 ignition margin contour for the Inconel-type device in R vs B space.

Fig. 12. Minimum radius for Kaye-Goldston ignition and maximum Kaye-Goldston ignition margin vs normalized power supply capability.

increase in power supply. It should be noted that the size of the maximum ignition margin devices increases with increased power supply. For example, the device with 1.6 times the PPPL power supply and a Kaye-Goldston ignition margin of 1.25 has a major radius of 1.81 m.

Figure 12 shows the sensitivity of the minimum major radius for a Kaye-Goldston ignition margin of 1 to the allowable power supply. Increasing the power supply above 1.36 times the reference value offers no further decrease in the major radius, because above this point other constraints (stress, temperature, and plasma beta) are the dominant limitations. Figure 12 also indicates the sensitivity of the same parameters for the Inconel device. The maximum Kaye-Goldston ignition margin for the Inconel device is about 10% lower than that for Ignifed at the reference allowable power supply, but the difference decreases to about 4% at 1.5 times the reference allowable power supply. With the Inconel device, obtaining a Kaye-Goldston ignition margin of 1 requires a power supply at least 5% greater than the reference value. The minimum size for an ignition margin of 1 decreases sharply above this value until 1.18 times the reference power supply is reached, at which point other constraints dominate.

Table 4 lists some device parameters and fractions of the limits reached for the minimum-size devices for different allowable power supplies. With additional power supply capabilities, different limits are approached. For Inconel with a larger power supply, the OH coil current density is larger at the end of the flat-top than at the beginning of the pulse, as opposed to the devices with lower power supply limits. In this case, the OH coil current density is adjusted so that the stress limit is met at both the end of the flat-top and the beginning of the pulse.

3.2 CIT DESIGN CHOICE AND SENSITIVITIES

In this section, trade studies are presented, which help in defining the March 1986 preconceptual base-line CIT device. The motivation behind these studies is to find the minimum-cost device for a fixed performance. We present results for minimum major radius rather than

Table 4. Parameters for minimum major radius Kaye-Goldston ignited devices with different allowable power supply capabilities

Parameter	Ignifed		Inconel	
<u>Principal device dimensions</u>				
Normalized power supply	1.0	1.36	1.06	1.18
Major radius R, m	1.24	1.12	1.40	1.22
Toroidal field B, T	10.2	12.2	9.67	10.9
Aspect ratio A	2.66	2.93	2.74	2.80
Plasma current I_p , MA	10.2	8.85	10.3	9.58
$B^2 a / q_*$, T ² -m	26.2	29.9	25.7	27.7
τ_b , s	3.68	2.94	4.25	3.44
<u>Radial build</u>				
Δ_{bore} , m	0.263	0.202	0.393	0.281
Δ_{OH} , m	0.284	0.299	0.252	0.243
Δ_{TF} , m	0.173	0.181	0.182	0.199
a, m	0.467	0.382	0.512	0.435
<u>Coil current densities</u>				
J_{OH-BOP} , kA/cm ²	7.26	7.32	6.23	6.79
J_{OH-EOF} , kA/cm ²	5.37	6.51	3.49	7.53
J_{TF} , kA/cm ²	9.05	10.0	7.89	8.36
<u>Fraction of limit</u>				
σ_{OH-BOP}	1.0	1.0	1.0	1.0
σ_{OH-EOF}	0.39	0.63	0.14	1.0
σ_{TF}	0.72	1.0	1.0	1.0
T_{OH}	0.92	1.0	0.47	1.0
T_{TF}	1.0	1.0	1.0	0.96
V-s	1.0	1.0	1.0	1.0
β	1.0	1.0	1.0	1.0
n_e	0.97	0.92	0.99	0.95
Γ_{lim}	0.88	0.97	0.77	0.86
MVA	1.0	1.0	1.0	1.0
Energy	1.0	0.85	0.83	0.76

minimum cost, but the cost generally scales with size. Previous system code results (ref. 31), as well as earlier tokamak studies (ref. 32), have shown this trend.

3.2.1 Conductor Material and TF Configuration

Another example of the Tokamak Systems Code's use for examining engineering trade-offs is shown in Fig. 13, which gives the minimum major radius of CIT vs applied vertical preload for three material choices for TF coils. The 23×10^6 kg preload limit used previously is not an absolute upper bound but is representative of the level above which there is a large impact on the design. Each material choice is done with wedged-only TF coils and with bucked-and-wedged TF coils. For this trade-off study, we used the ITOC physics guidance that $I_p \geq 10$ MA and $B^2 a/q_* \geq 25$. Because the size is being minimized and the edge q is being held fixed, both I_p and $B^2 a/q_*$ generally end up at their lower bounds in the final solutions. Achieving higher values of I_p and/or $B^2 a/q_*$ corresponds to larger sizes. Since no specific requirement exists for an ignition margin with any particular scaling law, the energy confinement time (used in the TF coil flattop time calculation) is calculated by assuming $n\tau_{ig} = 2 \times 10^{20}$ and using the density at the Murakami density. The plasma temperature is calculated by using the beta limit in Eq. (5), which results in temperatures of ~ 10 keV for all cases here. Also, for these runs the power supply limitations of Eqs. (12) through (14) are not incorporated. In all cases, Cu-175 is used for the OH coil material, except for the Inconel machine, which uses the Inconel composite for both the TF coil and the OH coil.

With the exception of the Cu-175 device, size decreases as the applied preload increases, up to the point at which all the Vertical Separating Force (VSF) in the coils is canceled (indicated by the * on the plot). The sharp increase in size as the preload decreases from 1×10^6 to 0 kg is somewhat arbitrary and is due to the decrease in the assumed allowable stress in a coil going from an externally constrained coil (with preload) to a self-constrained coil (no preload). For the Cu-102 devices, there is a large difference in the

Fig. 13. Minimum major radius vs applied vertical preload for different alloy choices and TF coil configurations.

minimum size between the wedged-only and the bucked-and-wedged TF coils. The minimum size is 1.54 m (at 34×10^6 kg) for wedged-only coils and 1.21 m (at 28×10^6 kg) for bucked-and-wedged coils. Above 16×10^6 kg for bucked-and-wedged Cu-102, the TF coil is at the stress and temperature limit, and further increases in preload have only a small effect on the device size; i.e., increasing the preload from 16×10^6 to 28×10^6 kg (where the VSF is canceled) reduces the major radius only about 0.015 m.

The Inconel devices are less sensitive than the Cu-102 devices to changes in preload and to the bucked-and-wedged or wedged-only choice of TF coil configuration. This is due in part to the flexibility of the Inconel device, which permits "tailoring" of the material properties to best meet the constraints. As the preload level increases, the TF coil stress level is lowered, and the Inconel volume fraction of the TF coil drops, providing a lower strength, lower resistivity composite.

The minimum-size Cu-175 devices are insensitive to preload levels or to changing from wedged-only to bucked-and-wedged TF coils. For Cu-175, the minimum size is limited primarily by the temperature constraints, and changes that affect the TF coil stress have only a minimal effect. Above 23×10^6 kg of preload, reducing the major radius for any of the cases shown in Fig. 13 provides little benefit.

The points corresponding to the LITE, Ignifed, and Base-line Inconel candidate CIT devices are shown in Fig. 13. LITE has wedged-only TF coils, uses the high-strength Cu-175 alloy for the OH and TF coils, and has no preload. Two criteria considered in choosing the present CIT base-line device were: (1) wedged-only TF coils should be used to avoid problems introduced when the TF and OH coils are in contact with each other; and (2) the major radius R should be ≤ 1.2 -1.25 m. Figure 13 indicates that the Inconel composite material with a preload of $\geq 23 \times 10^6$ kg is required to meet these criteria. Figure 14 shows the progression of the device points in R vs B space moving from LITE to Ignifed. The Base-line Inconel toroidal field is between that for LITE and that for Ignifed, but its major radius is closer to that of Ignifed. This figure also indicates that some changes that have little

Fig. 14. Progression of CITS in R vs B space going from LITE to Ignifed. (1) LITE reference point, (2) includes preload, (3) switches to Inconel composite coils (Inconel Base-line point), (4) uses a bucked-and-wedged TF coil configuration, (5) uses Cu-102 TF coils and Cu-175 OH coil, and (6) restricts the preload to 23×10^6 kg (Ignifed).

effect on the minimum size (for instance, going from Inconel point 4 to Ignifed point 6) can affect the magnitude of B.

3.2.2 CIT Sensitivity Studies

Figure 15 shows the sensitivity of the minimum size of the Ignifed and Inconel reference points to the allowable coil temperature (normalized to the reference allowable of 373 K). These cases also use the $B^2a/q_* > 25$ and $I_p \geq 10$ MA physics constraints. The Inconel device has a 2.5-cm change in major radius per 10% variation in the allowable coil temperature near the reference value; the Ignifed device has a 4-cm change in major radius per 10% allowable temperature variation. The reduced sensitivity of the Inconel device to temperature changes is due to the flexibility in changing the Inconel fraction in the coils. The TF coil Inconel fraction drops from 31% at the reference allowable temperature to 22% at a 30% reduction in allowable temperature. Both designs become more sensitive to changes in allowable temperatures as the temperature limit decreases. Permitting higher preloads ($>23 \times 10^6$ kg) in either device or higher Inconel fractions ($>35\%$) for the Inconel device has a negligible impact on the results shown in Fig. 15.

Figure 16 shows the sensitivity of the Ignifed and the Inconel devices to changes in the allowable stress (normalized to the reference allowable levels). The Ignifed device is not sensitive to reductions in the allowable stress down to 75% of the reference allowable levels, below which there is a strong increase in size. For allowable stress levels above this critical value, the Ignifed minimum-size devices have B^2a/q_* values greater than 25; and, as the allowable stress level decreases, reductions in the B^2a/q_* value moderate the increase in the minimum major radius. At the 75% normalized stress point, both TF coil stress and B^2a/q_* are at their limits, and further decreases in the allowable stress require large increases in size to satisfy both of these constraints.

Increasing the preload limit can alleviate this problem for Ignifed, although the effect of preload increases at higher allowable stress

Fig. 15. Minimum major radius vs maximum coil temperature (normalized to 373 K) for the Inconel and Ignifed devices.

Fig. 16. Minimum major radius vs allowable stress (normalized to the reference allowables) for the Ignifed- and Inconel-type CITs.

levels is small. The Inconel device is more strongly influenced by changes in the allowable stress than Ignifed. In this case, the B^2a/q_* value is at 25 for all allowable stress levels; as the allowable stress decreases, the Inconel fraction in the coils increases, reducing the impact on the size increase. Removing the preload and Inconel fraction limits has a strong impact on the Inconel device size at the reduced allowable stress levels. For the 1.35-m point at 70% of the reference allowable stress, the preload is 28×10^6 kg, and the TF and OH coil Inconel fractions are 46 and 39%, respectively.

The sensitivity of three candidate CIT designs (Ignifed, Inconel, and LITE) to the B^2a/q_* level is shown in Fig. 17. All cases here also satisfy the 10-MA plasma current requirement. Each of the devices exhibits a flat region of minimum size at low B^2a/q_* levels which is predominantly coil temperature limited. At higher B^2a/q_* levels the curves are either stress limited or wall-load [e.g., Eq. (15)] limited, the wall-load limit being independent of the particular design configuration.

Ignifed is wall-load limited above $B^2a/q_* = 28$. Releasing this limit offers little further benefit in reducing the size before the stress limit is met. If larger preload levels are also allowed for Ignifed, substantial reductions in the minimum size at large B^2a/q_* values are possible. For example, at $B^2a/q_* = 35$, increasing the preload from 23×10^6 to 37×10^6 kg reduces the size from 1.71 m to 1.22 m (at which point the wall-load limit is 26% over the allowable reference value). In view of the strong impact on minimum size in this case, reevaluating the design to accommodate the higher wall-load and preload levels would be worthwhile if the larger B^2a/q_* values are necessary.

The Inconel device hits the stress limit before the wall-load limit as B^2a/q_* increases, and the transition from the temperature-limited region to the stress-limited region is smoother than the Ignifed transition because of the changing Inconel fraction in the coils. Above $B^2a/q_* = 30$, the wall-load limit dominates the Inconel design. As with the Ignifed example, substantial reduction in size at high B^2a/q_* values is possible if the wall-load, preload, and Inconel fraction limits are

Fig. 17. Minimum size vs $B^2 a / q_*$ for LITE, Inconel, and Ignifed CITs.

released. At $B^2a/q_* = 35$, the Inconel device can be reduced from 1.55 m to 1.29 m if 1.22 times the reference wall-load level, 40×10^6 kg of preload, and 40% coil Inconel fractions are possible. The LITE case is stress-dominated between $B^2a/q_* = 25$ and $B^2a/q_* = 33.5$, above which the wall load dominates.

The slope of the stress-dominated portion of the curves in Fig. 17 is related to the strength of the materials used in each device. LITE is the least sensitive of the three devices in the stress-dominated region, but its temperature bound restricts the major radius to >1.45 m, even at low B^2a/q_* levels. On the other hand, the Ignifed device is the most stress sensitive at high B^2a/q_* levels, but increasing the preload (which reduces the stress in the TF coils) can greatly alleviate this sensitivity.

3.2.3 CIT Design Point

The new Tokamak Systems Code has been instrumental in scoping the CIT design space and providing initial design points for which more detailed analysis can be performed. The March 1986 CIT base-line design point was arrived at with the Tokamak Systems Code by using the guidelines summarized in Table 1 (with the ITOC options) and searching for the minimum-size device with wedged-only TF coils. With these conditions the Inconel-copper composite material for the coils provides the smallest device, as seen in Fig. 13. Using other coil materials results in minimum major radii at least 20 cm larger; thus the coil material choice is quite clear. Also, preload levels of $\sim 25 \times 10^6$ kg are necessary to realize the minimum size. The parameters associated with the March 1986 CIT base-line design point are listed in Table 5.

4. SUMMARY

We have developed a new Tokamak Systems Code and used it to investigate Compact Ignition Tokamaks. The new code can simultaneously vary many parameters, satisfy many constraints, and minimize or maximize a figure of merit. It is a useful tool in comparing different CIT design configurations over wide regions of parameter space and

Table 5. CIT base-line design point parameters

Parameter	Value
<u>Principal device parameters</u>	
Major radius, m	1.25
Toroidal field, T	10.4
Aspect ratio	2.72
Plasma current, MA	10
B^2a/q_* , T ² -m	25
Flat-top time, s	3.82
Fusion power, MW	530
Kaye-Goldston-IM	1.0
Mirnov-IM	2.2
Neo-Alcator-IM	1.9
<u>Radial build</u>	
Δ_{bore} , m	0.282
Δ_{OH} , m	0.258
Δ_{TF} , m	0.189
a, m	0.460
<u>Current densities</u>	
$J_{\text{OH-BOP}}$, kA/cm ²	6.52
$J_{\text{OH-EOF}}$, kA/cm ²	7.19
J_{TF} , kA/cm ²	8.42
<u>Fraction of limits</u>	
$J_{\text{OH-BOP}}$	1
$J_{\text{OH-EOF}}$	1
J_{TF}	1
T_{OH}	1
T_{TF}	1
V-S	1
Wall load	0.88
I_p	1
B^2a/q_*	1

determining a desired design point for more detailed physics and engineering analysis. Also, the systems code is useful for performing sensitivity studies for either physics or engineering issues.

Operational windows in R and B space for fixed ignition margin were calculated for the Ignifed and Inconel candidate CITs. The minimum R bounds are predominantly physics limited and the maximum R portions of the windows are engineering limited. For Kaye-Goldston plasma-energy-confinement scaling, the Ignifed device has a minimum size of 1.15 m, and the Inconel-type device has a minimum size of 1.25 m.

With the ITOC physics guidance of $B^2a/q_* > 25$ and $I_p > 10$ MA, the Ignifed-type CIT has a minimum size of 1.2 m and a toroidal field of 11 T, and the base-line Inconel device has a minimum size of 1.25 m and a field of 10.4 T. Sensitivity studies show Ignifed to be more sensitive to coil temperature changes than the Inconel device, whereas the Inconel device is more sensitive to stress perturbations. Also, when larger B^2a/q_* values are required, modifying the design to permit larger limiter heat loads and larger TF coil preload levels can greatly reduce the minimum size of both types of devices to nearly the minimum values.

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APPENDIX A

The details of the plasma-energy-confinement time scalings are presented here. For ions, the Chang-Hinton¹¹ energy confinement is

$$\tau_i (s) = \frac{3}{16} \frac{a^2}{\chi} ,$$

where

$$\chi (m^2/s) = 6.5 \times 10^{-22} \frac{C_1 Z_{eff} A^{1.5} n_e q_x^2}{\sqrt{\tau_i} B^2} ,$$

$$C_1 = (2/3 + 1.88 \sqrt{\epsilon'} - 1.54 \epsilon') (1 + 3/2 \epsilon'^{-2}) ,$$

$$\epsilon' = 1/(2A) .$$

The three options for electron energy confinement are

$$\text{Neo-Alcator: } \tau_{NA} (s) = 7 \times 10^{-22} n_e a R^2 q_x ,$$

$$\text{Mirnov: } \tau_{MI} (s) = 0.15 a \sqrt{k} I_p (MA) ,$$

$$\text{Modified Kaye-Goldston: } \tau_{KG} (s) = \left(\frac{1}{\tau_{KG}^{0.2}} + \frac{1}{\tau_{NA}^{0.2}} \right)^{-\frac{1}{2}} ,$$

where

$$\begin{aligned} \tau'_{KG} (s) = & 2.77 \times 10^{-8} \kappa^{0.28} I_p^{1.24} (kA) n_e^{0.26} (10^{19} m^{-3}) R^{1.65} (cm) \\ & \times 3^{0.25} / [B^{0.09} (T) a^{0.49} (cm) P_\alpha^{0.58} (MW)] . \end{aligned}$$

Here the Kaye-Goldston confinement scaling is modified with the $3^{0.25}$ factor to correct for the presence of tritium due to isotope mass effects (ref. 33).

APPENDIX B

The details of the TF coil and OH solenoid stress analysis, taken from ref. 16, are presented here. The TF coil stress is

$$\sigma_{TF} = \sqrt{\sigma_{TR}^2 + 4\sigma_{SH}^2} ,$$

where

σ_{SH} = shear stress (assumed to be 30 MPa for CIT devices),

σ_{TR} = Tresca stress,

= maximum of

$$(|\bar{\sigma}'_{\theta} - \bar{\sigma}'_v|, |\bar{\sigma}'_{\theta} - \bar{\sigma}'_r|, |\bar{\sigma}'_v + \bar{\sigma}'_r|) ,$$

$\bar{\sigma}'_r$ = average radial stress

$$= \frac{\int_{r_i}^{r_o} r \sigma_r dr}{\int_{r_i}^{r_o} r dr \times [1 - VF \times (r_i + r_o)/(2r_i)]} ,$$

σ'_{θ} = average hoop stress

$$= \frac{\int_{r_i}^{r_o} \sigma_r dr}{\int_{r_i}^{r_o} dr (1 - VF)} ,$$

$\bar{\sigma}'_v$ = average vertical stress.

Here $r_{i(o)}$ is the inner (outer) radius of the inboard TF leg. For the case of bucked-and-wedged TF coils, the peak stress at the inner surface σ_{r_i} is used in place of σ_{TR} if the former quantity is larger, where

$$\sigma_{r_i} = \frac{\sigma_r(r_i)}{1 - VF \times (r_i + r_o)/(2r_i)} + 2/3 \bar{\sigma}'_v .$$

The $2/3$ coefficient for $\bar{\sigma}_v$ in this expression is taken from comparisons with finite element analysis, which indicate that the vertical separating stress at r_i is $-2/3$ the average value. The radial and hoop stress components are calculated by

$$\sigma_r(r) = A_0(A_1 r^2 + A_2 \ln(r) + A_3 + \frac{A_4}{r^2}) ,$$

and

$$\sigma_\theta(r) = B_0(B_1 r^2 + A_2 \ln(r) + B_2 - \frac{A_4}{r^2}) ,$$

where

$$A_0 = f_r \frac{\mu_0 J_{TF}^2}{4} ,$$

$$A_1 = \frac{3 + \nu}{4} ,$$

$$A_2 = -r_i^2(1 + \nu) ,$$

$$A_3 = C_1(1 + \nu) - r_i^2 ,$$

$$A_4 = -C_2(1 - \nu) ,$$

$$B_0 = f_\theta \frac{\mu_0 J_{TF}^2}{4} ,$$

$$B_1 = \frac{3\nu + 1}{4} ,$$

$$B_2 = C_1(1 + \nu) - \nu r_i^2 ,$$

$$C_1 = \frac{1 - \frac{3+\nu}{4} \left(\frac{r_o}{r_i}\right)^2 + (1+\nu) \ln(r_o) - \frac{1-\nu}{4} \left(\frac{r_i}{r_o}\right)^2 - \frac{C_3}{C_4} (1-\nu) \ln(r_i) \left(\frac{r_i}{r_o}\right)^2}{\frac{1+\nu}{r_i^2} - \frac{C_3}{C_4} \frac{1-\nu}{r_o^2}} ,$$

$$C_2 = \frac{\frac{C_3}{C_4} \left[r_i^2 - \frac{3+v}{4} r_o^2 + (1+v) r_i^2 \ln \left(\frac{r_o}{r_i} \right) \right] - \frac{1+v}{4} r_i^2}{\frac{1+v}{r_i^2} - \frac{C_3}{C_4} \frac{1-v}{r_o^2}}$$

$$C_3 = \frac{E_2}{E_1} - 1$$

$$C_4 = \frac{E_2}{E_1} + 1$$

v = Poisson ratio (0.34),

E_1 = Young's modulus in the OH solenoid,

E_2 = Young's modulus in the TF coil.

For designs with bucking and wedging for the TF coils, $E_2/E_1 = 1$, and with wedging only $E_2/E_1 = 10^6$ is used. The calibration coefficients f_r and f_θ are determined from comparisons with finite element analysis and are set to 0.77 and 0.88, respectively, for all cases presented here (ref. 34).

The vertical stress is calculated as

$$\bar{\sigma}_v = f_v \times 0.25 \frac{RB J_{TF}}{(1 - VF)} \ln \left(\frac{r_{out}}{r_{in}} \right) - \frac{F_{pre}}{A_{TF} N_{TF}}$$

where

$r_{out(in)}$ = the outboard (inboard) TF coil leg radius,

$f_v = 0.92$ for all cases,

F_{pre} = the externally applied vertical compression preload force,

A_{TF} = the cross-sectional area of a TF coil,

N_{TF} = the number of TF coils.

The stress in the OH solenoid is calculated by using the peak stress level for wound coils,

$$\sigma_{OH} = \sigma'_\theta(r_{i-OH}) + \bar{\sigma}'_v$$

and using an average stress for plate coils,

$$\sigma_{OH} = (\bar{\sigma}'_{\theta} - \bar{\sigma}'_r) + \sigma'_v ,$$

where the prime indicates that the filling factor $1/(1 - VF)$ is included in the stress values.

The hoop stress in the solenoid is

$$\sigma_{\theta} = f_{\theta} D_0 \left(D_1 r^2 + D_2 r + \frac{D_3}{r^2} + D_4 \right) ,$$

where

$$D_0 = \mu_0 J^2 r_o^2 ,$$

ν = Poisson ratio,

$$D_1 = \frac{1 + 3\nu}{8r_{o-OH}^2} ,$$

$$D_2 = \frac{-(1 + 2\nu)}{3r_{o-OH}} ,$$

$$D_3 = r_{i-OH}^2 \frac{(7 + 5\nu) - 8(2 + \nu)p + 3(3 + \nu)p^2}{24(1 - p^2)} ,$$

$$D_4 = \frac{(7 + 5\nu) - 8(2 + \nu)p^3 + 3(3 + \nu)p^4}{24(1 - p^2)} ,$$

$$p = \frac{\text{OH inner radius } (r_{i-OH})}{\text{OH outer radius } (r_{o-OH})} ,$$

f_{θ} = calibration coefficient (0.8) determined from comparisons with finite element analysis.

When used for plate designs, the stress is averaged as

$$\bar{\sigma}_{\theta} = \frac{\int_{r_{i-OH}}^{r_{o-OH}} \sigma_{\theta} r \, dr}{\int_{r_{i-OH}}^{r_{o-OH}} r \, dr}.$$

The radial component (used only for the plate designs) is

$$\begin{aligned} \bar{\sigma}_r = D_0 \times & \left[D_5 (r_{o-OH}^4 - r_{i-OH}^4) + D_6 (r_{o-OH}^3 - r_{i-OH}^3) \right. \\ & \left. + D_7 \ln \frac{r_{o-OH}}{r_{i-OH}} + D_8 (r_{o-OH}^2 - r_{i-OH}^2) \right], \end{aligned}$$

where

$$D_5 = \frac{3 + \nu}{32 r_{o-OH}^2},$$

$$D_6 = \frac{-(2 + \nu)}{9 r_{o-OH}},$$

$$D_7 = -r_{i-OH}^2 \frac{(7 + 5\nu) - 3(1 + \nu)p}{2^4 (1 + p)},$$

$$D_8 = \frac{(7 + 5\nu) - 8(2 + \nu)p^3 + 3(3 + \nu)p^4}{48(1 - p^2)}.$$

The axial stress ($\bar{\sigma}_v$) is calculated by

$$\bar{\sigma}_v = f_{OH-v} \times \frac{\mu_0 J_{OH}^2}{3} r_{o-OH}^2 \frac{(1-p^3)}{(1+p)},$$

where f_{OH-v} is the OH solenoid vertical stress component calibration coefficient (0.11) and is found from comparisons with finite element code analysis.

When the OH coil stress is evaluated at the beginning of the pulse, the preceding equations are used as indicated; but, when the stress is evaluated at the end of the flattop, the effect of the field introduced by the plasma current is included by reducing σ_r and σ_{θ} by the ratio of

the field produced by the plasma (modeled as a solenoid of height $2a$) over the peak OH coil field, i.e., by

$$f_I = 1 - \frac{10^6 I_p \text{ (MA)}}{2a \kappa J_{\text{OH-EOF}} (r_{\text{O-OH}} - r_{\text{i-OH}})} .$$

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