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ABSTRACT

A single-mode, nonlinear analysis of the 2XIIB experimental results is under development. A Model-I (infinite geometry, no mirror losses or source beams) analysis is nearly complete. Model II (mirror losses, beams, energy drag) is in progress. Nonlinear analysis of the $2\omega_p$ instability near the quarter-critical point in an inhomogeneous plasma is continuing.

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INTRODUCTION

There is considerable experimental evidence that coherent mode processes dominate in many unstable magnetic confinement experiments. For some reason, a single unstable mode (usually the one with the largest linear growth rate) dominates and suppresses all nearby modes. The result is that one often observes a developed nonlinear situation with one very coherent mode, both in frequency and spatial structure. The primary object of this project, for the past several years, has been to exploit these observations and to attempt analysis of these experiments in terms of self-consistent (i.e. Bogoliubov-like) nonlinear single (or few) mode theory.

This last year has been no exception. Emphasis was on analysis of the DCLC instability in mirror geometries. A final manuscript on fitting the PR-6 observations was sent to Physics of Fluids and has been accepted for publication. Extension of this theory to the finite-beta regime, in order to determine the extent to which one can model the 2XIIB observations, is well under way. Finally, although experimental observations here are less specific, we are studying single-mode nonlinear saturation of the $2\omega_p$ parametric instability in inhomogeneous plasma. Details of these topics follow.

1. Fitting the PR-6 observations with single-mode nonlinear theory.

A detailed manuscript entitled, "Nonlinear saturation of the drift cyclotron loss-cone instability: II. Comparison with PR-6 data" by R.C. Myer and Albert Simon was submitted to Physics of Fluids in January. The paper has now been accepted for publication. A preprint is attached to this report.

2. Fitting 2XIIB observations with single-mode nonlinear theory.

We have been following the general scheme of analysis used in the PR-6 case. Initially, we work with a "Model I" in which we deal with an infinite collisionless plasma,¹ with an equilibrium distribution which has a hot component in a Subtracted-Maxwellians loss-cone distribution. There is also a warm plasma stream, and a cool electron background. In this Model I there are no mirror losses or energy drag terms. We have carried out the linear analysis (which is in complete agreement with the theory of Tang, Pearlstein and Berk²) and determined all details of the stability boundary in the parameter space defined by $n_{\text{warm}}/n_{\text{hot}}$ and $T_{\text{warm}}/T_{\text{hot}}$. We then determined the zero-frequency second order corrections to the distribution functions, and electromagnetic potentials. Finally, saturation amplitudes and a nonlinear frequency shift were determined. This Model I treatment can be considered as a nonlinear extension of the work done in reference 2.

We plan extension to an obvious Model II (inclusion of mirror losses, beam sources, and energy drag terms) and then a Model III which will incorporate finite geometry effects.

This work is being carried out by Robert Ferraro and Albert Simon.

3. Single-mode nonlinear saturation of the $2\omega_p$ instability.

In this study, we have considered the parametric decay of laser light into two dominant plasma waves near the quarter critical point in an inhomogeneous plasma. Using fluid theory for the electrons, fixed ions, and the electrostatic approximation for the plasma waves we obtained the well-known linear result for normal incidence.^{3,4} These results were obtained in an improved fashion, with full details of the eigenfunctions included. We also studied s-polarized oblique evidence.

Carrying on to the nonlinear regime, we determined the zero frequency corrections to the densities, velocities and scalar potentials and the nonlinear saturation amplitude and frequency shift of the waves. Dependence of the saturation on the various mechanisms of pump depletion, electron heating, profile modification and nonlinear frequency shift was determined.

Work is continuing to include ion dynamics, and to make contact with laboratory experiments or computer simulation. This work is carried out primarily by Mr. Thierry DeWandre.

3. Personnel

FACULTY -

Albert Simon (ME and Physics) - Dr. Simon has devoted 20% of his time to this project during the academic year, and full time for one and one-half summer months. He supervised two graduates students, Thierry DeWandre and Robert Ferraro.

GRADUATE RESEARCH ASSISTANTS -

(a) Thierry DeWandre (ME Department) - Mr. DeWandre, who completed his undergraduate and Master's level studies at the Catholic University of Louvain, Belgium, is now beginning his fifth year at Rochester. He is working on the application of nonlinear saturation theories to parametric instabilities in plasma.

(b) Robert D. Ferraro (Physics Department) - Mr. Ferraro, who completed his undergraduate studies at Cornell, is beginning his fourth year of graduate studies. He is working on the extension of the DCLC theory to the finite-beta regime and comparison with 2XIIB.

Other Federal Support. None of the personnel working on this contract have other federal support.

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LIST OF PUBLICATIONS (Sept. 1980-July 1981)

A. WORK COMPLETED PREVIOUSLY, BUT PUBLISHED THIS YEAR

<u>Title</u>	<u>Authors</u>	<u>Journal/Status</u>	<u>Percentage DOE Support</u>
Time Behavior of Nonlinear Plasma Oscillations	Thierry DeWandre and Albert Simon	Phys. Fluids <u>23</u> , 1922 (1980)	100%

B. WORK COMPLETED THIS YEAR, OR CONTINUING

Nonlinear Saturation of the Drift Cyclotron Loss-Cone Instability: II. Comparison with PR-6 Data.	R.C. Myer and Albert Simon	Accepted for publication by Phys. Fluids.	100%
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Nonlinear saturation of the drift cyclotron loss-cone instability: II.

Comparison with PR-6 data

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The single-mode time-asymptotic nonlinear saturation of the drift cyclotron loss-cone instability is considered. In this model, there is a uniform magnetic field as well as mirror losses. The linear stability boundary, and the saturation levels of unstable oscillations and floating potential near this boundary, are determined. The theory is then compared with observations on the PR-6 mirror experiment. Good agreement is obtained using a simple energy model for the slow time variation of the equilibrium distribution function.

PACS numbers 52

I. INTRODUCTION

In part I of this paper,¹ we studied the nonlinear saturation of a single dominant mode of the drift cyclotron loss-cone instability. The time-asymptotic small amplitude variation of the mode was determined close to the stability boundary. The model used in (I) was that of a collisionless plasma with no mirror losses or particle sources. It was verified that a stable saturated state occurred, and comparison was made with computer simulations.² In Part II of this paper, we extend the model to include mirror losses and source beams. We then use the calculated saturated states to make comparison with experiments on the PR-6 mirror device.³

II. BASIC EQUATIONS AND EQUILIBRIUM CONFIGURATION

We utilize a model in which the magnetic field is uniform and in the z-direction. The ions are described by a collisional equation of the form

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \frac{e}{M} \nabla \phi \cdot \nabla_{\mathbf{v}} f - \Omega \frac{\partial f}{\partial \sigma} = -\bar{n} \gamma f + S \quad (1)$$

where, insofar as is possible, the notation is the same as in (I). Here the loss term $-\bar{n} \gamma f$ represents the combined effects of electron drag and subsequent loss through the mirrors and loss by radial transport. The electron number density is denoted by n and the bar signifies its radial average. The coefficient γ may depend on the electron temperature but does not depend on n or ϕ . The quantity S represents a possible neutral beam source of ions.

Since the electron gyro-radius is always small compared to wavelengths

and gradients of interest, we may use fluid equations for the electrons. We describe the electrons by the equations

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{u}) = -\lambda n + \int S d^3 v \quad (2)$$

$$n \frac{d\mathbf{u}}{dt} = \frac{ne}{m} (\nabla \phi - \frac{\mathbf{u} \times \mathbf{B}}{c}) - \mathbf{u} \int S d^3 v + \int \mathbf{v} S d^3 v. \quad (3)$$

Eqs. (2) and (3) are readily obtained from the electron equivalent of Eq. (1) by taking the usual velocity moments and neglecting electron pressure. Here $\mathbf{u}$ is the electron fluid velocity and λ represents the loss of electrons due to scattering out of the mirrors. We take λ to be dependent on the radially averaged plasma potential in the manner given by Pastukhov.⁴ Thus

$$\lambda = (\sigma v)_{ee} P(\bar{\phi}) \quad (4)$$

where

$$P(\phi) = \frac{2RT_-}{\pi^{1/2}(2R+1)e\phi \ln(4R+2)} \exp(-e\phi/T_-) \quad (5)$$

Here $(\sigma v)_{ee}$ represents the electron collision rate, T_- is the electron temperature and R is the mirror ratio.

The Poisson equation

$$\nabla^2 \phi = -4\pi e (\int f d^3 v - n) \quad (6)$$

completes the set. We use the electrostatic approximation, which restricts application of this model to low-beta experiments. PR-6 operates at a beta of about 0.1% and can be described by the equations above. However,

2XIIB has a beta of about 20% and would require a finite beta analysis. Work on such a model is now underway. Throughout this paper, we assume no variation of any quantities in the z-direction, and use a slab geometry, identifying the x-direction with the radial direction in mirror machines and y with the azimuthal direction. Thus $\bar{n} = \bar{n}(y)$, $\bar{\phi} = \bar{\phi}(y)$ etc. Of course, implicit in the model is a rapid step-like vanishing of the plasma density and potential at values of $|z| > L$ where $2L$ is of the order of the distance between the actual mirrors.

We choose a cylindrically symmetric, time-independent and quasi-neutral equilibrium. The ion distribution function has the form

$$F^0 = N_0 F_0(v_1^2, v_z^2) \exp[-\epsilon(x + v_y/\Omega)] \quad (7)$$

where N_0 is a constant, while F_0 is normalized to unity and has a loss-cone form, i.e.: $F_0(v_1^2 = 0, v_z^2) = 0$. We shall also assume that $(\epsilon a)^2 \ll 1$ where a is the ion gyroradius, and systematically neglect terms of this order. The ion number density is then,

$$N = \int F^0 d^3v = N_0 \exp(-\epsilon x), \quad (8)$$

and the electron number density is equal to this. The ion source term is such that

$$S = \gamma \bar{N} F^0. \quad (9)$$

The constant equilibrium potential ϕ_0 is determined by vanishing of the net electron loss rate and is the solution of

$$\lambda(\phi_0) \bar{N} N = \int S d^3v = \gamma \bar{N} N \quad \text{or} \quad \lambda(\phi_0) = \gamma. \quad (10)$$

While u_1^0 is not identically zero, its components are proportional to $(\lambda \bar{N}/\Omega_+)(m/M)$ and may be neglected.

III. LINEAR THEORY AND STABILITY BOUNDARY

We linearize Eqs. (1), (2), (3) and (6) about the equilibrium, taking Fourier transforms as in (I). The ion equation is:

$$i(s - i\tau + \frac{kv_y}{\Omega}) f_1 + \frac{v_x}{\Omega} \frac{\partial f_1}{\partial x} - \frac{\partial f_1}{\partial \sigma} = i \frac{ck}{B} \frac{\partial F^0}{\partial v_y} \phi_1 - \tau \frac{\bar{n}_1}{\bar{N}} F^0 \quad (11)$$

where we introduce the notation $\tau \equiv \gamma \bar{N}/\Omega = \lambda \bar{N}/\Omega$ and assume $\tau \ll 1$. The local approximation is also assumed. Solving for f_1 by the method of characteristics, and keeping terms to first order in τ , the result is:

$$f_1 = -N \exp(-\epsilon v_y/\Omega) \sum_n \frac{i^n J_n(z) (\frac{e}{M} H_n \phi_1 - \frac{i\tau \bar{n}_1}{\bar{N}} F_0)}{\hat{s} + n} \exp(-i\gamma_n) \quad (12)$$

where we have defined $\hat{s} \equiv s - i\tau$.

The linearized electron equations are:

$$i\omega n_1 + \nabla \cdot (N u_1) = -\lambda \bar{N} n_1 - \lambda \bar{n}_1 N - \lambda' \bar{\phi}_1 \bar{N} N \quad (13)$$

$$i\omega u_1 = \frac{e}{m} (\nabla \phi_1 - \frac{u_1 \times B}{c}) - \lambda \bar{N} u_1 \quad (14)$$

where λ' denotes $d\lambda/d\phi$ evaluated at $\phi = \phi_0$. Neglecting terms in $\tau m/M$ we find

$$u_1 = \frac{-c}{B} (\nabla \phi_1 \times B) \quad (15)$$

$$n_1 = -N(\bar{\phi}_1/s) [(c\epsilon k/\Omega B)(1 + 2i\tau/s) - i\tau(\lambda'/\lambda)] \quad (16)$$

where, as in I, we neglect the x -dependence of ϕ_1 compared to that of N . Eq. (16) also allows us to rewrite the expression for f_1 in a more compact form. We find

$$f_1 = \frac{-e\phi_1 N}{M} \exp(-\epsilon v_y / \Omega) \sum_n \frac{i^n J_n K_n}{\hat{s} + n} \exp(-i\gamma n) \quad (17)$$

where $K_n = H_n + i\tau\theta F_0$, and we have introduced the notation $\theta \equiv k\epsilon/\omega\Omega$. The Bessel function J_n will always be understood to have the argument z .

Upon substitution in Poisson's equation, we obtain the linear dispersion relation. Since we are interested in determining the stability boundaries, we assume ω real. The real and imaginary parts of the "local" dispersion relation are then

$$k^2 = -\omega_p^2 \left(\int \sum_n \frac{H_n J_n^2}{s+n} d^3v - \theta \right) \quad (18)$$

$$0 = \int \sum_n \frac{H_n J_n^2}{(s+n)^2} d^3v + \theta \int \sum_n \frac{F_0 J_n^2}{s+n} d^3v - \frac{2\theta}{s} + \frac{M}{es} \frac{\lambda'}{\lambda} \quad (19)$$

where $\omega_p^2 \equiv 4\pi Ne^2/M$ and we have kept terms to first order in τ only. Note that the collisional terms have virtually dropped out of the dispersion relation, and indeed are represented only by the logarithmic derivative of $\lambda(\phi)$.

The stability boundaries have been determined for the specific F_0 of subtracted Maxwellians,

$$\int F_0(v_1^2, v_z^2) dv_z = \frac{1}{\pi v_h^2 (\eta^2 - 1)} [\exp(-v^2/v_h^2 \eta^2) - \exp(-v^2/v_h^2)]$$

which corresponds to Case III in (I). We did not study the delta function

or broadened delta distributions, since we are interested in comparison with the PR-6 experiments, and wish to consider only reasonably realistic distributions. The detailed dispersion relation for the subtracted Maxwellian is listed in Appendix A.

The procedure for evaluating the boundary curve was to fix the density and wavenumber k and then solve for ω_R and ω_I for a given ϵ . One then varied ϵ until ω_I vanished. This procedure was repeated for various values of k until a minimum ϵ was obtained. This yields one point on the stability boundary. One then repeats the procedure for other values of n .

The resulting stability boundaries differ so little from the corresponding curves (3a, 3b, 3c and 3d) in Figure 2 of (I) that we do not replot them here. The only change is in the critical value of ω associated with any given point on the curve. Here the change is generally less than 10%. The critical k remains virtually unchanged.

IV. NONLINEAR EXPANSION AND MODIFIED DENSITY AND POTENTIAL

As in (I), we study the time-asymptotic behavior near the stability boundary by increasing ϵ from its critical values ϵ_c to $\epsilon_c(1+\Delta)$. The single mode expansion is made as in (I) and the amplitude Γ and nonlinear frequency shift $\delta\omega^2$ are introduced as before.

The zero-frequency equation of order Δ for the ions is now:

$$\begin{aligned} \frac{1}{\Omega} \vec{v} \cdot \nabla f_0^2 - \frac{\partial f_0^2}{\partial \sigma} - \frac{c}{B} \nabla \phi_0^2 \cdot \nabla_v F^0 = \frac{c}{B} (\nabla \phi^1 \cdot \nabla_v f_1^* + \text{c.c.}) \\ - \tau [f_0^2 + \frac{1}{N} (\overline{n_1 f_1^*} + \text{c.c.}) + \frac{\overline{n_0^2}}{N} F^0]. \end{aligned} \quad (20)$$

The corresponding electron equations are:

$$\begin{aligned} \nabla \cdot (\vec{u}^1 n^1 * + \text{c.c.}) + \nabla \cdot (N \vec{u}_0^2) = -\lambda \overline{N} n_0^2 - \lambda \overline{n_0^2} N - \lambda (\overline{n^1 n^1 *} + \text{c.c.}) \\ - \lambda' (\overline{\phi^1 n^1 *} + \text{c.c.}) N - \lambda^1 \overline{\phi^1} n^1 * \overline{N} - \lambda' \phi_0^2 \overline{N} N \\ - \lambda'' \overline{\phi^1} \phi^1 * \overline{N} N \end{aligned} \quad (21)$$

where λ'' denotes $d^2 \lambda / d\phi^2$ evaluated at $\phi = \phi_0$ and

$$\begin{aligned} N [(\vec{u}^1 \cdot \nabla) \vec{u}^1 * + \text{c.c.}] = \frac{e}{m} [n^1 * (\nabla \phi^1 - \frac{\vec{u}^1 \times \vec{B}}{c}) + \text{c.c.}] \\ + N \frac{e}{m} (\nabla \phi_0^2 - \frac{\vec{u}_0^2 \times \vec{B}}{c}) - \vec{u}_0^2 \int S d^3 v. \end{aligned} \quad (22)$$

Finally, the second order Poisson equation is:

$$\frac{d^2 \phi_0^2}{dx^2} = -4\pi e \left(\int f_0^2 d^3 v - n_0^2 \right). \quad (23)$$

It is simple to solve Eq. (22) for $\vec{u}_0^2$, recalling that $\int S d^3 v = \gamma \overline{N} N$. Neglecting terms of order $(\tau m / M)$, we obtain:

$$N \vec{u}_0^2 = - \left\{ [n^1 * (\nabla \phi^1 - \frac{\vec{u}^1 \times \vec{B}}{c}) + \text{c.c.}] + N \nabla \phi_0^2 \right\} \times \frac{\vec{b} c}{B}. \quad (24)$$

If we recall that the zero frequency components are independent of y , we obtain:

$$(N \vec{u}_0^2) = -\frac{c}{B} [\nabla n^1 * \times \nabla \phi^1 \cdot \vec{b} + \text{c.c.}] - \nabla \cdot [n^1 * \vec{u}^1 + \text{c.c.}].$$

Substituting this in Eq. (21), we have:

$$\begin{aligned} \lambda(\overline{Nn}_0^2 + \overline{Nn}_0^2) &= \left[\frac{c}{B} \nabla n^{1*} \times \nabla \phi^1 \cdot \underline{b} - \lambda \overline{n^1} n^{1*} \right. \\ &\quad \left. - \lambda' \overline{\phi^1} (\overline{n^1}^* N + n^{1*} \overline{N}) + \text{c.c.} \right] - \lambda' \overline{\phi_0^2} \overline{NN} - \lambda'' \overline{\phi^1} \overline{\phi^1}^* \overline{NN} \end{aligned}$$

we may write this in the form

$$\lambda(\overline{Nn}_0^2 + \overline{Nn}_0^2) = T.$$

Since $n^1 = \Gamma N \phi_1 \beta$, where β is a complex constant, we may also write

$$\begin{aligned} T &= |\Gamma|^2 \left[\left(\frac{2ck\epsilon}{B} N \beta_I + 2\lambda \overline{NN} \beta \beta^* + 4\lambda' \overline{NN} \beta_R \right) |\phi_1|^2 \right. \\ &\quad \left. + (\lambda' \overline{\phi_0^2} + \lambda'' \overline{\phi_1} \phi_1^*) \overline{NN} \right]. \end{aligned}$$

It is clear that $\overline{T} = \overline{TN}/N$. Hence $\lambda \overline{Nn}_0^2 = T/2$ and then

$$\begin{aligned} \overline{n_0^2} &= - \frac{|\Gamma \phi_1|^2}{\lambda \overline{N}} \left[\frac{ck\epsilon}{B} I_m \left(\frac{n_1}{\phi_1} \right) + \lambda \text{Re} \left(\frac{\overline{n_1} n_1^*}{\phi_1^2} \right) + 2\lambda' \overline{N} \text{Re} \left(\frac{n_1}{\phi_1} \right) \right. \\ &\quad \left. + \frac{\lambda' \overline{NN}}{2|\Gamma \phi_1|^2} \overline{\phi_0^2} + \frac{\lambda''}{2} \overline{NN} \right]. \end{aligned}$$

Using Eq. (16), we obtain the following reduced expression,

$$\frac{\overline{n_0^2}}{N} = |\Gamma \phi_1|^2 \left[\frac{e\theta}{Ms} \left(\frac{\lambda'}{\lambda} + \frac{e\phi}{Ms} \right) - \frac{\lambda''}{2} \right] - \frac{\lambda'}{2\lambda} \overline{\phi_0^2}. \quad (25)$$

Note that this expression, which is correct to order τ , reduces to that given in Eq. (43) of (I) for the special case where λ is a constant.

Note also that no indeterminacy occurs. This is due to the existence of loss terms in our basic equations.

The ion equation may be written as

$$\begin{aligned} \tau f_0^2 + \frac{v_x}{\Omega} \frac{\partial f_0^2}{\partial x} - \frac{\partial f_0^2}{\partial \sigma} &= \frac{c}{B} \frac{d\phi_0^2}{dx} \frac{\partial F^0}{\partial v_x} \\ &+ \left(\frac{c}{B} \nabla \phi^1 \cdot \nabla_v f^1* - \frac{\tau}{N} \bar{n}^1 f^1* + \text{c.c.} \right) - \frac{\tau n_0^2 F^0}{N}. \end{aligned}$$

Comparing this with Eq. (29) of (I), we see that the principal change is the replacement of $-2\theta/\Omega$ with τ and the addition of two extra terms on the right hand side. Solving by the method of characteristics, one obtains the solution for f_0^2 as the sum of four terms, designated as I_1 , I_2 , I_3 and I_4 . Following the same Taylor expansion as in (I), the first term becomes:

$$\begin{aligned} I_1 &= - \frac{2cN}{B} \frac{\partial F_0}{\partial v_1^2} \left\{ \frac{d\phi_0^2}{dx} (vy - \tau v_x) \right. \\ &\quad \left. + \frac{v_1^2}{2\Omega} \frac{d^2 \phi_0^2}{dx^2} [(1 - \frac{1}{2} \cos 2\sigma) - \tau \frac{3}{8} \sin 2\sigma] \right\} \exp(-\epsilon v_y / \Omega). \end{aligned} \quad (26)$$

This is identical to Eq. (33) of (I) in the limit where $\tau \rightarrow 0$. The second term, I_2 has the integral form given in Eq. (34) of (I), with the obvious replacement of $-2\theta/\Omega$ by τ in the definition of I_0 . The integrated result is not identical due to the extra terms in f_1 . The result is

$$\begin{aligned} I_2 &= - \frac{2c^2 k}{B^2 \hat{a}} |\Gamma \phi_1|^2 \text{Im} \sum_n [Q_n J_n (\epsilon \hat{a} - n \frac{\partial}{\partial z}) L_n \\ &\quad + (Q_n J_n)' i^{n+1} \exp(-iY_n) + s(Q_n J_n)' L_n \\ &\quad - (J_n K_n)' L_n] \exp[-\epsilon(x + v_y / \Omega)]. \end{aligned}$$

where the prime denotes a derivative with respect to z , $\hat{a} \equiv v_{\perp}/\Omega$, and

$$L_n \equiv (-1)^n \sum_m \frac{iJ_m(m+n+i\tau)}{m(m+n)^2 + \tau^2} \exp[-i(m+n)(\sigma + \frac{\pi}{2})]$$

$$Q_n \equiv (\hat{s}+n)K_n/(s+n)^2.$$

The quantity I_3 is,

$$I_3 = \frac{\tau}{N} |\Gamma\phi_1|^2 \int_0^\sigma \left(\frac{\bar{n}_1}{\phi_1} \frac{f_1^*}{\phi_1} + \text{c.c.} \right) I_0 d\sigma'$$

and the resulting integral is

$$I_3 = - \frac{2e^2\theta N}{M^2} |\Gamma\phi_1|^2 \exp(-\epsilon v_y/\Omega) \sum_n \frac{H_n J_n^2}{s+n} \quad (28)$$

to first order in τ . Finally

$$I_4 = \frac{\tau}{N} \int_0^\sigma \bar{n}_0^2 F^0 I_0 d\sigma' = - \bar{n}_0^2 (N/\bar{N}) F_0 \exp(-\epsilon v_y/\Omega). \quad (29)$$

We now substitute $f_0^2 = I_1 + I_2 + I_3 + I_4$ into Poisson's equation to obtain a differential equation for ϕ_0^2 . The azimuthal integrals are carried out quite readily. The integral of I_1 takes exactly the form given by Eq. (38) of (I). However,

$$\int I_2 d^3v = \frac{2e^2\theta Ns}{M^2} |\Gamma\phi_1|^2 \int \sum_n \frac{J_n}{s+n} \cdot$$

$$\left(\frac{H_n J_n}{s+n} + \theta J_n F_0 + \epsilon \hat{a} J_n' H_n \right) d^3v$$

while

$$\int I_3 d^3v = - \frac{2e^2 \theta N}{M^2} |\Gamma \phi_1|^2 \int \sum_n \frac{H_n J_n^2}{s+n} d^3v$$

and

$$\int I_4 d^3v = - \overline{n_0^2} N / \overline{N}.$$

Poisson's equation takes the form

$$\frac{d}{dx} \left\{ \left[1 + \left(\frac{\omega_p}{\Omega} \right)^2 \right] \frac{d\phi_0^2}{dx} \right\} = 4\pi N e (\tilde{A} + \frac{\lambda'}{\lambda} \overline{\phi_0^2}) \quad (30)$$

where

$$\begin{aligned} \tilde{A} \equiv & |\Gamma \phi_1|^2 \left(\frac{4\theta e}{M} \frac{\lambda'}{\lambda} - \frac{\lambda''}{\lambda} - \frac{2\theta k_e^2 e^2}{M^2 \omega_{po}^2} \right. \\ & \left. - \frac{2\theta s e^2}{M^2} \int \sum_M \frac{H_n \epsilon \hat{a}_n J_n J_n'}{s+n} d^3v \right). \end{aligned} \quad (31)$$

Here $\omega_0^2 \equiv 4\pi N_0 e^2 / M$. Note that the last term in $\tilde{A}$ is identical to the right hand side of Eq. (40) in (I).

Integrating Eq. (30) once, using the vanishing of $d\phi_0^2/dx$ at the origin, we have

$$\left[1 + \left(\frac{\omega_p}{\Omega} \right)^2 \right] \frac{d\phi_0^2}{dx} = \frac{4\pi N_0 e}{\epsilon} \left(\tilde{A} + \frac{\lambda'}{\lambda} \overline{\phi_0^2} \right) [1 - \exp(-\epsilon x)].$$

Integrating again, assuming ϕ_0^2 vanishes at some boundary $x = R$, we obtain

$$\phi_0^2 = - \frac{4\pi N_0 e}{\epsilon^2} \left(\tilde{A} + \frac{\lambda'}{\lambda} \overline{\phi_0^2} \right) U(x)$$

where

$$U(x) \equiv \int_x^R \frac{1 - \exp(-\epsilon x)}{1 + (\omega_p/\Omega)^2} \epsilon dx$$

is a positive shape function. With the further definition,

$$\tilde{B} = - \frac{4\pi N_0 e}{\epsilon^2} \frac{\lambda'}{\lambda},$$

we obtain

$$\phi_0^2 = - \frac{4\pi N_0 e \tilde{A}}{\epsilon^2} \frac{U(x)}{1 + \tilde{B}U}. \quad (32)$$

It should be noted that λ'/λ is generally negative and approximately equal to $-(e/T_-)$, as deduced from the Pastukhov loss rate, Eq. (5). Thus $\tilde{B}$ is positive and $\approx (\lambda_D)^{-2}$ where λ_D is the electron Debye length. Since this Debye length is usually much smaller than the scale length in mirror experiments, we see that $\tilde{B}$ is much larger than unity. To a good approximation, one may then write:

$$\phi_0^2 \approx - \frac{4\pi N_0 e \tilde{A}}{\tilde{B}\epsilon^2} \frac{U(x)}{U}$$

or $\phi_0^2 \approx \tilde{A}(\lambda/\lambda')$ (33)

which is the result one would have obtained from Eq. (30) by neglecting the left side.

In any event, ϕ_0^2 has the opposite sign to $\tilde{A}$. The third term in $\tilde{A}$, which incorporates the Model I radial transport, has already been shown in (I), to be positive. The remaining mirror terms are all negative, and are generally dominated by the λ'' term which has the approximate value

$-(e/T_-)^2$. This is easily seen to be larger than the radial term by $(T_+/T_-)^2(ka)^2$. Hence, the induced potential is positive, and

$$\overline{\phi_0^2} \approx |\Gamma\phi_1|^2 \frac{e}{T_-}. \quad (34)$$

From Eq. (41) of (I), one may easily estimate ϕ_0^2 in the model used in (I). It is

$$(\phi_0^2)_I \approx -|\Gamma\phi_1|^2 \frac{e}{T_+}. \quad (35)$$

Hence we see that the sign of ϕ_0^2 is reversed in going from Model I to our new case, and its magnitude is increased by T_+/T_- . This is an appreciable factor in PR-6⁽³⁾ (≥ 15). Thus the general statement made at the end of Sect. IV of (I) is verified, although the increase in magnitude is T_+/T_- rather than $(\epsilon a)^{-2}$ as stated there.

V. ADJOINT FUNCTION AND SATURATION CONDITION

The third order equations, for frequency ω , take the form:

$$\begin{aligned} 0_f(f^3, \phi^3, \overline{n^3}) = & -i \frac{\delta\omega^2}{\Omega} f^1 + \frac{c}{B} (\nabla\phi_0^2 \cdot \nabla_v f^1 + \nabla\phi^1 \cdot \nabla_v f_0^2) \\ & - \frac{ikc\epsilon}{B\Omega} \phi^1 F^0 \Delta - \frac{\tau}{N} (\overline{n}_0^2 f^1 + \overline{n}^1 f_0^2) \end{aligned} \quad (36)$$

$$\begin{aligned} 0_n(n^{3 \rightarrow 3}, \phi^3) = & -i\delta\omega^2 \overline{n}^1 + \epsilon \Delta N \overline{u}^1 \cdot \vec{i}_x - \nabla \cdot (n^{1 \rightarrow 2} \overline{u}_0 + n_0^{2 \rightarrow 1}) \\ & - \lambda (\overline{n}^1 n_0^2 + \overline{n}_0^2 \overline{n}^1) - \lambda' [\phi^1 (\overline{N} n_0^2 + \overline{n}_0^2 N) \\ & + \phi_0^2 (\overline{N} n^1 + \overline{n}^1 N) + \phi^1 (\overline{n}^1 n^{1*} + \text{c.c.}) + \phi^{1*} \overline{n}^1 n^1] \end{aligned}$$

$$\begin{aligned}
& - \lambda'' [\phi^1 \phi^1 * (\bar{N}n^1 + \bar{n}^1 N) + \frac{1}{2} \phi^1 \phi^1 (\bar{N}n^1 * + \bar{n}^1 * N) \\
& + \phi_0^2 \phi^1 \bar{N}N] - \frac{1}{2} \lambda''' \phi^1 \phi^1 \phi^1 * \bar{N}N
\end{aligned} \tag{37}$$

$$\begin{aligned}
N\vec{O}_u(\phi^3, \vec{u}^3, n^3) &= -i\delta\omega^2 \vec{u}^1 N - i\omega(n^1 \vec{u}_0^2 + n_0^2 \vec{u}^1) \\
&- N[(\vec{u}^1 \cdot \nabla) \vec{u}_0^2 + (\vec{u}_0^2 \cdot \nabla) \vec{u}^1] - n^1 [(\vec{u} \cdot \nabla) \vec{u}^1 * \\
&+ (\vec{u}^1 * \cdot \nabla) \vec{u}^1] - n^1 * (\vec{u}^1 \cdot \nabla) \vec{u}^1 + n^1 \frac{e}{m} \nabla \phi_0^2 \\
&+ n_0^2 \frac{e}{m} \nabla \phi^1 - n^1 \frac{e}{mc} \vec{u}_0^2 \times \vec{B} - n_0^2 \frac{e}{mc} \vec{u}^1 \times \vec{B}
\end{aligned} \tag{38}$$

where O_f , O_n and $\vec{O}_u$ represent the collected forms of the linear equations given in Eqs. (11), (13) and (14), but with f_1 , n_1 , ϕ_1 , replaced by f^3 , n^3 , ϕ^3 . In addition, there is the third order Poisson equation,

$$\nabla^2 \phi^3 + 4\pi e (\int f^3 d^3v - n^3) = 0. \tag{39}$$

As in (I), we may write these set of equations in the matrix form

$$\vec{O} \cdot \vec{\psi} = \vec{D} \tag{40}$$

where now

$$\vec{\psi} = [f^3, n^3, u_x^3, u_y^3, \phi^3] \tag{41}$$

$$\vec{D} = [d_f, d_n, d_{ux}, d_{uy}, 0] \tag{42}$$

$$\vec{0} = \begin{bmatrix} \alpha_1 & \alpha_2 & 0 & 0 & \alpha_5 \\ 0 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \\ 0 & 0 & N\gamma_3 & N\gamma_4 & N\gamma_5 \\ 0 & 0 & N\delta_3 & N\delta_4 & N\delta_5 \\ \epsilon_1 & \epsilon_2 & 0 & 0 & \epsilon_5 \end{bmatrix} . \quad (43)$$

Here d_f , d_n , $\vec{d}_u$ represent the right hand sides of Eqs. (36), (37) and (38), while

$$\alpha_1 = i(s - i\tau + kv_y/\Omega) + \frac{v_x}{\Omega} \frac{\partial}{\partial x} - \frac{\partial}{\partial \sigma}$$

$$\alpha_2 = \frac{\tau F^0}{N} L$$

$$\alpha_5 = -i \frac{ck}{B} \frac{\partial F^0}{\partial vy}$$

$$\beta_2 = i\omega + \lambda \bar{N} + \lambda NL$$

$$\beta_3 = \frac{\partial}{\partial x}(N$$

$$\beta_4 = ikN$$

$$\beta_5 = \lambda \bar{N}NL$$

$$\gamma_3 = i\omega + \lambda \bar{N}$$

$$\gamma_4 = \frac{eB}{mc}$$

$$\gamma_5 = -\frac{e}{m} \frac{d}{dx}$$

$$\delta_3 = -\frac{eB}{mc}$$

$$\delta_4 = i\omega + \lambda\bar{N}$$

$$\delta_5 = -ik \frac{e}{m}$$

$$\epsilon_1 = 4\pi e \int d^3v$$

$$\epsilon_2 = -4\pi e$$

$$\epsilon_5 = d^2/dx^2 - k^2$$

where L is the radial averaging operator, i.e.: $Ln = \bar{n}$. We now introduce the adjoint $\vec{\chi} \equiv [\chi_1, \chi_2, \chi_3, \chi_4, \chi_5]$ defined as in (I), and satisfying Eqs. (52) and (53) of I. The appropriate boundary conditions are easily shown to be the single-valuedness of χ , the vanishing of χ_5 at the outer boundary and of $\partial\chi_5/\partial x$ at the origin. Then Eq. (53) of (I) takes the form

$$[i(s + \frac{kv_y}{\Omega}) + \tau - \frac{v_x}{\Omega} \frac{\partial}{\partial x} + \frac{\partial}{\partial \sigma}] \chi_1 + 4\pi e \int \chi_5 d^3v = 0 \quad (44)$$

$$\frac{\tau}{N} \int \bar{F}^0 \chi_1 d^3v + (i\omega + \lambda\bar{N}) \chi_2 + \lambda\bar{N} \chi_2 - 4\pi e \int \chi_5 d^3v = 0 \quad (45)$$

$$\frac{\partial \chi_2}{\partial x} - (i\omega + \lambda\bar{N}) \chi_3 + \frac{eB}{mc} \chi_4 = 0 \quad (46)$$

$$ik\chi_2 + \frac{eB}{mc} \chi_3 + (i\omega + \lambda\bar{N}) \chi_4 = 0 \quad (47)$$

$$\begin{aligned} & -\frac{ick}{B} \int \frac{\partial F^0}{\partial v_y} \chi_1 d^3v + \lambda' \bar{N} \chi_2 + \frac{e}{m} \frac{d}{dx} (N\chi_3) - ikN \frac{e}{m} \chi_4 \\ & + \left(\frac{d^2}{dx^2} - k^2 \right) \int \chi_5 d^3v = 0. \end{aligned} \quad (48)$$

Eq. (44) is identical to Eq. (55a) of (I) (with obvious replacement of s by $s - i\tau$) and thus, using Eq. (56) of (I), we have

$$\chi_1 = i4\pi e \tilde{\chi}_5 \sum_n \frac{(1)^{-n} J_n}{\hat{s} + n} \exp(i\gamma_n) \quad (49)$$

where we define $\tilde{\chi}_5 \equiv \int \chi_5 d^3v$. It is not difficult to solve the remaining equations to obtain:

$$\chi_2 = \frac{4\pi e \tilde{\chi}_5}{\Omega(is + 2\tau)} (1 - i\tau \int \sum_n \frac{J_n^2 F_0}{\hat{s} + n} d^3v) \quad (50)$$

$$\chi_3 = \frac{M}{im\hat{s}} \chi_4 \quad (51)$$

$$\chi_4 = \frac{m^2 ck\hat{s}}{eBM} \chi_2. \quad (52)$$

Substituting these in Eq. (48), one obtains an identity, using the linear dispersion relation. This verifies the existence of the adjoint solution.

The nonsecularity condition is given by Eq. (54) of (I). It is a simple matter to verify that the terms $\chi_3 d_{ux}$ and $\chi_4 du_y$ are smaller than the terms in $\int \chi_1 d_f d^3v$ and $\chi_2 d_n$ by m/M and may be neglected. The resulting saturation condition is now:

$$\iint \chi_1 d_f d^3v dx + \int \chi_3 d_n dx = 0, \quad (53)$$

which takes the form:

$$\int \Gamma \phi_1 \left(A \frac{\delta \omega^2}{\omega} + C |\Gamma \phi_1|^2 + E \Delta \right) dx = 0. \quad (54)$$

The coefficients may be evaluated using the expression for $\chi_1, f_1, n_1,$

u_1 , f_0^2 , n_0^2 and $\vec{u}_0$. The results are:

$$A = s \int \sum_n \frac{K_n J_n^2}{(\hat{s}+n)^2} d^3v - \theta - i\tau (2\theta + \frac{\lambda'M}{\lambda e}), \quad (55)$$

$$E = \theta[1 - sR + i\tau (2/s - R)], \quad (56)$$

where $R \equiv \int \frac{F_0 J_n^2}{\hat{s}+n} d^3v,$

and

$$\begin{aligned} \left(\frac{M}{e}\right)^2 C = & -i\tau\rho_0 \int \sum_n \frac{K_n J_n^2}{(\hat{s}+n)^2} d^3v \\ & + \left(\frac{k}{\Omega}\right)^2 \int \sum_\ell \frac{T_\ell}{(\hat{s}+\ell)^2} d^3v - i\tau\theta \int \sum_\ell \frac{T_\ell}{\hat{s}+\ell} d^3v \\ & + \frac{1}{|\Gamma\phi_1|^2} \frac{\omega_p^2 k M^2}{\Omega^2 \epsilon^2 e^2} \frac{\tilde{A}}{1+\tilde{B}\tilde{U}} \frac{dU}{dx} \int \sum_n T_n d^3v \\ & + [1 - i\tau(R - 2/s)] [\rho_0 \theta - \frac{1}{|\Gamma\phi_1|^2} \frac{\omega_p^2 k M^2}{\Omega^2 \epsilon^2 e^2} \frac{\tilde{A}}{1+\tilde{B}\tilde{U}} \frac{dU}{dx} \rho_1] \\ & - i\tau \left[\frac{2\rho_0 \rho_1}{s} + \frac{\lambda'M}{\lambda e s} (2\rho_1 + \rho_1^* - \frac{1}{|\Gamma\phi_1|^2} \frac{\omega_p^2 M^2}{\epsilon^2 e^2} \frac{\tilde{A}\tilde{U}}{1+\tilde{B}\tilde{U}}) \right. \\ & \left. + \frac{\lambda''' M^3}{2\lambda e^3 s} \right]. \end{aligned} \quad (57)$$

Here

$$\rho_0 = \theta^2 + \frac{\lambda'M\theta}{\lambda e} - \frac{\lambda''M^2}{2\lambda e^2} + \frac{\lambda'M^3 \omega_p^2 \tilde{A}\tilde{U}}{|\Gamma\phi_1|^2 2\lambda e^3 \epsilon^2 (1+\tilde{B}\tilde{U})} \quad (58)$$

$$\rho_1 \equiv \theta + i(\tau/s)(2\theta - \lambda'M/\lambda e) \quad (59)$$

$$T_{\ell} \equiv T_{\ell}^{(1)} + T_{\ell}^{(2)} + T_{\ell}^{(3)}$$

$$T_{\ell}^{(1)} \equiv \frac{1}{|\Gamma\phi_1|^2} \frac{\omega^2 M^2 J_{\ell} \tilde{A} \hat{a}}{\epsilon^2 (1 + \tilde{B} \tilde{U})} \frac{\partial F_0}{\partial v_1^2} \\ \left[2 \frac{dU}{dx} \left(\frac{J_{\ell}}{z} + i \tau J_{\ell}' \right) + \hat{a} \frac{d^2 U}{dx^2} \left(\frac{3}{2} J_{\ell} + J_{\ell}'' \right) \right] \quad (60)$$

$$T_{\ell}^{(2)} \equiv - \frac{k^2}{\Omega^2} \sum_n \{ J_n J_{\ell} [I_m Q_n \left(\frac{\epsilon}{k} w_1 + \frac{n}{z} w_5 \right) + \text{Re} Q_n \left(\frac{\epsilon}{k} w_2 + \frac{n}{z} w_4 \right)] \\ - \frac{J_n}{z} [\text{Re} Q_n (J_{\ell} V_1)' - \text{Im} Q_n (J_{\ell} V_2)'] + \frac{n J_n J_{\ell}}{z} \\ [W_1 \text{Im} Q_n + W_2 \text{Re} Q_n] - \frac{\tau J_n}{z(s+n)} \text{Re} K_n (J_{\ell} W_1)' \} \quad (61)$$

$$T_{\ell}^{(3)} \equiv \frac{\tau \theta}{\pi} \sum_n \frac{H_n J_n J_{\ell} W_1}{s+n} - \rho_0 F_0 J_{\ell}^2 \quad (62)$$

$$T_n \equiv \frac{K_n J_n^2}{(\hat{s}+n)^2 [(\hat{s}+n)^2 - 1]} \quad (63)$$

$$W_q = \sum_m \frac{(-1)^m (J_m)^{\alpha(i)r}}{(m+n)^2 + \tau^2} [(m+n) (J_{\ell-m-n} - J_{\ell+m+n}) \\ + i \tau (J_{\ell-m-n} + J_{\ell+m+n})^{\beta}] \quad (64)$$

$$V_q \equiv \sum_m (-1)^m J_m (J_{\ell-m-n} + p J_{\ell+m+n}) i^{r-1} \quad (65)$$

where

α = unity if $q=1,2,5,6$

= prime symbol if $q=3,4$

β = unity if $q=1,2,3,4$

= prime symbol if $q=5,6$

$$\begin{aligned}
r &= 1 \quad \text{if } q \text{ is odd} \\
&= 0 \quad \text{if } q \text{ is even} \\
p &= 1 \quad \text{if } q \text{ is odd} \\
&= -1 \quad \text{if } q \text{ is even.}
\end{aligned}$$

To lowest order in τ , the expressions for A , C and E reduce to those in (I), taking the zero electron gyroradius limit. In particular,

$$A_0 = s \left(\sum_n \frac{J_n^2 H_n}{(s+n)^2} - \frac{\epsilon k}{\omega^2} \right) \quad (66)$$

where the subscript zero denotes the leading term in τ . Unlike the case in (I), this expression no longer vanishes. Using the linear dispersion relation, this can be written as

$$A_0 = \theta(1-s) \int \frac{F_0 J_n^2}{s+n} d^3 v - \frac{M}{e} \frac{\lambda'}{\lambda} . \quad (67)$$

Note that to lowest order, we have a real (local) equation

$$A_0 \frac{\delta \omega^2}{\omega} + C_0 |\Gamma \phi_1|^2 + E_0 \Delta = 0 . \quad (68)$$

To obtain a determinate set, one must continue to first order. The first order terms are all imaginary and if we divide by $i\tau$, the resulting equation is

$$A_1 \frac{\delta \omega^2}{\omega} + C_1 |\Gamma \phi_1|^2 + E_1 \Delta = 0 \quad (69)$$

where A_1 , C_1 and E_1 are all real. One may note a clear analogy to the pair of equations, (18) and (19) which determined the linear stability boundaries.

It is not difficult to estimate the size of these coefficients, assuming $T_+/T_- \gg 1$. One obtains

$$A_0 \approx \frac{M}{T_-} \quad (70)$$

$$E_0 \approx \frac{M}{T_+} \quad (71)$$

$$C_0 \approx \frac{M}{T_+} \left(\frac{e^2}{T_+ T_-} \right) (ka)^2. \quad (72)$$

Care must be taken in estimating ρ_0 (which arises from the expression for the electron density) since the leading terms in $(T_+/T_-)^2$ cancel identically. One also finds, quite easily that A_1, E_1, C_1 have the same order of magnitude as A_0, E_0, C_0 . Hence each saturation equation has the rough form

$$O(1) \frac{\delta\omega^2}{\Omega} + O(ka)^2 \left| \Gamma \frac{e\phi_1}{T_+} \right|^2 + O\left(\frac{T_-}{T_+}\right) \Delta = 0$$

where the order of each coefficient is indicated. A rough estimate of the result is then

$$\left| \frac{\Gamma e\phi_1}{T_+} \right|_{II}^2 \approx (\epsilon a)^2 \frac{T_-}{T_+} \Delta \quad (73)$$

$$\left(\frac{\delta\omega^2}{\omega} \right)_{II} \approx - \frac{T_-}{T_+} \Delta. \quad (74)$$

Combining this with the estimate of ϕ_0^2 in Eq. (34), we obtain

$$\left(\frac{e\phi_0^2}{T_+} \right)_{II} \approx (\epsilon a)^2 \Delta. \quad (75)$$

We will see qualitative agreement between these estimates and the numerical results obtained in the next section.

For the model used in (I), a similar rough estimate is possible, however, the λ'' and λ' terms are absent and the resulting estimate of C_0 is smaller by T_-/T_+ . The resulting estimate of the fluctuation amplitude is then

$$\left| \frac{\Gamma e \phi_1}{T_+} \right|_I^2 \approx (\epsilon a)^2 \Delta. \quad (76)$$

A comparison with the results in Tables I-III of (I) shows that this is a reasonable estimate and that the calculated values of $|\Gamma|^2/\Delta$ do vary as $(\epsilon a)^2$. It is also interesting to combine this with the estimate for $(\phi_0^2)_I$ given in Eq. (35). We obtain

$$\left(\frac{e \phi_0^2}{T_+} \right)_I \approx -(\epsilon a)^2 \Delta. \quad (77)$$

Incidentally, these estimates show that the real parameter of smallness in this problem is not Δ but $(\epsilon a)^2 \Delta$, which indicates that it is legitimate to use our results even when Δ itself is of order unity.

VI. MODELING THE PR-6 EXPERIMENTAL DATA

The principal objective of this low-beta single-mode nonlinear saturation calculation is to determine the extent to which one can reproduce the experimental results obtained on the PR-6 device. We concentrate on the measurements shown in Fig. 2 of Ref. 3, which show the time variation of the ion density, the fluctuating potential, and the floating (D.C.) potential.

Note that the instability sets in at about $t = 150 \mu\text{secs}$, as evidenced by a rise in the level of oscillation and of the floating potential and onset of an enhanced plasma loss rate. We shall make no attempt to model the behavior before onset. In a recent publication, Kanaev⁽⁵⁾ has shown that cold gas was flowing into the system until just before the instability onset, and that this is undoubtedly the reason for the time delay before the instability begins. Our procedure will be to use the observed density at onset together with a representative value of the magnetic field ($B = 10 \text{ kG}$) to fix an initial value $(\omega_p/\Omega)_+$. We also use a fixed estimate of the density gradient length ($\epsilon \approx 1/R = 0.2 \text{ cm}^{-1}$) and assume an initial ion temperature T_{+i} to determine the value of ϵa_h , where $a_h = v_h/\Omega$. This now locates an initial point on the linear stability diagram [Fig. 2 of (I)]. Assuming that the ion distribution function is of the Subtracted Maxwellians form, and taking $Mv_h^2 = 2T_{+i}$, one then determines a particular value of $\eta = \eta_i$ such that the stability boundary passes directly through the initial point. We simultaneously, of course, determine the corresponding initial values of k_i and ω_i .

One now assumes that the ion distribution remains in the Subtracted Maxwellians form with fixed Mv_h^2 and that only the parameter η varies in time. Of course, it would require solution of a very complicated Fokker-Planck equation to obtain a realistic estimate of the time variation of η , or to see if the assumption of a Subtracted Maxwellians form is justified. In lieu of this, we use a simplified version of a model due to Marx,⁽⁶⁾ which uses energy balance to estimate the time variation of η . Denoting the total kinetic energy of the particles of a given species by E , we have the following expression for the total

energy loss rate:

$$\frac{dE}{dt} = \bar{E}_\ell \frac{dN}{dt} + N \left(\frac{d\bar{E}}{dt} \right)_c \quad (78)$$

where $\bar{E}_\ell$ is the average energy of the particles lost from the mirror trap and where $(d\bar{E}/dt)_c$ represents the rate of change of $\bar{E}$ as a result of collisions with the other species. Since $E = N\bar{E}$, this may be rewritten as:

$$\frac{d\bar{E}}{dt} = \left(\frac{d\bar{E}}{dt} \right)_c + \frac{\bar{E}_\ell - \bar{E}}{N} \frac{dN}{dt}. \quad (79)$$

We assume that the ions lose energy to the electrons by classical drag and that this is given by the Spitzer formula

$$\left(\frac{d\bar{E}}{dt} \right)_c = -\nu_d (\bar{E}_+ - \bar{E}_-) \quad (80)$$

where

$$\nu_d = \frac{8}{3} (2\pi m)^{1/2} \frac{z^2 N_e^4 \ln \Lambda}{M T_-^{3/2}}.$$

By conservation then,

$$\left(\frac{d\bar{E}}{dt} \right)_c = \nu_d (\bar{E}_+ - \bar{E}_-). \quad (81)$$

It should be noted that for the Subtracted Maxwellians, the average ion kinetic energy, $\bar{E}_+$, is:

$$\bar{E}_+ = \frac{M}{2} v_h^2 \left(\lambda^2 + \frac{3}{2} \right) = T_{+1} \left(\lambda^2 + \frac{3}{2} \right) \quad (82)$$

where we keep the parallel component of the kinetic energy fixed at $T_{+1}/2$.

To complete the description of the time variation of η , we assume that mainly the coldest ions are lost, and thus $\bar{E}_{+l} \ll \bar{E}_+$. For most purposes $\bar{E}_{+l}$ may be neglected compared to $\bar{E}_+$. The electrons leaving the trap must have a kinetic energy $E_- \geq e\phi_0$. We will assume again that those with the lowest possible energy constitute the bulk of the exciting population and take $\bar{E}_{-l} \approx e\phi_0$. Finally, the ion density loss rate is taken in the form

$$\frac{dN}{dt} = -p v_d N \quad (83)$$

where p is a numerical factor which adjusts for geometrical effects, direct mirror scattering loss, charge exchange, etc. [This corresponds to choosing $\gamma = p v_d$]. We adjust p to get a best fit to the data and find that a value near 10 gives a satisfactory match. This also has some relation to the observation in Ref. 3 that the density loss rate exceeds the classical by one order of magnitude.

One is now able to integrate forward in time, determining ϕ_0 at each time step by the requirement of equal loss rates of electrons and ions. When one carries through this procedure, one obtains a description of how η , and hence the instability boundary curve varies in time. We also determine the variation of N and T_- . The result is that the instability boundary on the ω_p/Ω , ϵa diagram moves downward (toward more loss-cone like, or more unstable distributions) faster than the point p representing the plasma. The plasma becomes unstable and one can predict the level of oscillations, $|\Gamma\phi_1|^2$, and the floating potential, $\phi^0 + \phi_0^2$, from a knowledge of Δ . In essence, we are assuming that the nonlinear saturation occurs on a time scale which is very short compared to the scale described here. Hence, the plasma passes through a series

of saturated states as η varies.

The method we use is described further in Fig. 1. If the relative situation is as shown at some particular time $t=T$ after onset of instability, we define Δ as $\Delta \equiv [(\epsilon a)_h - (\epsilon a)_c]/(\epsilon a)_h$. We also determine the wavenumber and linear frequency of the saturated mode from the corresponding critical values of $z_c (=k_c v_h/\Omega)$ and $s_c (= \omega_c/\Omega)$ at point c on the neutral stability curve for $\eta = \eta(t)$. The nonlinear equations given in Sect. V then yield $|\Gamma\phi_1|^2$, $\delta\omega^2$ and ϕ_0^2 . The total floating potential is the sum of ϕ_0^2 and ϕ_0^2 . All of these results are summarized in Tables I-IV.

In all cases, we choose $\bar{N} = 1.2 \times 10^{12} \text{ cm}^{-3}$ at $t = 150 \text{ } \mu\text{sec.}$ in conformity with the observations in Ref. 3, and the initial electron temperature is assumed to be 10 ev. Table I contains the results for $T_{+i} = 150 \text{ ev.}$, $p = 5$ and $\bar{E}_{-l} = e\phi_0$. Note that the electron temperature and wavelength increase in time, in qualitative agreement with experiment. The observed wavelength increases from 4 mm to 9 mm over the duration of the experiment while our predicted wavelengths are consistently smaller by a factor of 2 or 3. The predicted oscillation amplitude is smaller than the observed peak value by a factor of 2.

The ion temperature was chosen to be 400 ev. in the runs summarized in Tables II-IV. It should be noted that the value of T_{+i} ($\approx 150 \text{ ev.}$) given in Ref. 3 is based on extrapolation of the measured ion energy spectrum to low values [see Fig. 1 of Ref. 3]. The actual spectrum resembles a two or even a three-temperature distribution. In light of Kanaev's⁽⁵⁾ observations, there may be some validity to using the extrapolation of the hot tail of the distribution in Fig. 1 of Ref. 3. This

yields 400 ev as an estimate.

Table II ($T_{+i} = 400$ ev, $p = 5$, $\bar{E}_\ell = e\phi_0$) shows that the resulting predicted amplitude is slightly larger than the observed value at its peak at $t = 250$ μ sec. The agreement in wavelength prediction is only slightly improved. Note that the electrons now heat at a faster rate and that the predicted increase in T_- by a factor of 3.5 is consistent with experiment. In our loss model, the ratio of $e\phi_0$ and T_- is constant, so ϕ_0 also increases by this factor.

It is interesting to note that our model suggests that the major part of the observed increases in plasma potential and electron temperature are due to classical transfer processes and not to the instability fluctuation levels. Note that ϕ_0^2 is much smaller than ϕ_0 . This differs from the conjecture of Kanaev and Yushmanov about a self-amplifying process. They attributed the observed factor of four increase in the electron temperature to be due to heating by the unstable fluctuations. In their picture, the electrons are heated by the instability fluctuations, resulting in an increased ambipolar potential. The increased potential then ejects more low energy ions thus driving the plasma more unstable and leading to more electron heating. We believe the observation can be accounted for by classical processes. One should also take note of the observation shown in Fig. 3 of Kanaev.⁽⁵⁾ In Fig. 3a and 3b, one may see that a rapid rise in the ambipolar potential begins at least 30 μ secs. before onset of the instability. This is under conditions where the cold plasma flow is cut off at the very beginning of free decay.

Table III is for $T_{+i} = 400$ ev and $p = 10$. Due to the more rapid

loss of cold ions, the rate of increase of $\bar{E}_+$ is greater, while the electrons are cooled significantly. The parameter η becomes equal to 1 for $t > 250 \mu\text{sec}$, and our simple model of subtracted Maxwellians is then inadequate for further description of the evolution of η . Finally, Table IV illustrates the effect of changing $\bar{E}_{-l}$ to $2e\phi_0$ (for $T_{+1} = 400 \text{ ev}$ and $p = 5$). The major effect is a decreased heating of the electrons. With a lower electron temperature, the ion drag is increased and one obtains an increased ion loss rate and a smaller ambipolar potential. In both Table III and IV, one sees little change in the predicted oscillation amplitude from that given in Table II.

Note finally that the ion density decrease due to fluctuations n_0^2 is always negligible compared to $\bar{N}$ itself. This may be due, in part, to our model in which the ion loss term is not a function of the potential fluctuations. Inclusion of such a term might give a significant increase in the magnitude of n_0^2 , and should also increase the "classical" decay rate. One might then require values of p which are smaller than 10. It would not be too difficult to extend the model to include such terms.

VII. SUMMARY

We have calculated the single-mode time-asymptotic nonlinear saturation amplitude of the drift cyclotron loss-cone instability. In our model, there is a uniform magnetic field, and a low beta plasma with mirror losses and beam sources. The general theory was determined, and then specialized to an ion equilibrium distribution function in the form of a Subtracted Maxwellians. These results were then compared with experimental observations on the PR-6 device.

The comparison assumed that the ion distribution function remained in the Subtracted Maxwellians form, but that the parameter η , which is related to the temperature of the "hole population" varied in time. Choosing η initially so that the plasma goes unstable at the observed density and scale length, one then determines how η varies from a simple energy argument, using an adjusted density decay rate. This, in turn, allows one to calculate the manner in which the plasma moves into the unstable regime. One can then calculate the time history of the saturated potential oscillation amplitude, the floating potential, the nonlinear frequency shift, and the single-mode's wavelength and frequency. Rather good agreement was found with the observations, especially in light of the complex field geometry existing in the actual experiment. The best fit did seem to require assumption of an initial ion temperature (≈ 400 ev) substantially higher than that reported (150 ev) in the original paper by Kanaev and Yushmanov.⁽³⁾ However, subsequent measurements by Kanaev⁽⁴⁾ do cast some doubt on the firmness of their estimate.

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APPENDIX A: LINEAR DISPERSION' RELATION FOR THE SUBTRACTED MAXWELLIANS

The linear dispersion relation for the Subtracted Maxwellians is:

$$\begin{aligned}
 & \frac{\epsilon}{ks} + \frac{i\tau}{s} \left[\frac{2\epsilon}{ks} + \frac{Mv_h^2}{z_h^2 T_-} \left(1 + \frac{T_-}{e\phi_0} \right) \right] + \frac{1}{z_h^2 (\eta^2 - 1)} \left\{ \left(2 - \frac{\epsilon \eta^2 z_h^2}{k\hat{s}} \right) {}_2F_2(\eta^2 z_h^2, \hat{s}) \right. \\
 & \quad \left. - \left(2 - \frac{\epsilon z_h^2}{k\hat{s}} \right) {}_2F_2(z_h^2, \hat{s}) - \frac{i\tau\epsilon}{ks\hat{s}} [\eta^2 z_h^2 {}_2F_2(\eta^2 z_h^2, \hat{s}) - z_h^2 {}_2F_2(z_h^2, \hat{s})] \right\} \\
 & \quad - \left(\frac{\Omega}{\omega_p} \right)^2 = 0
 \end{aligned} \tag{A1}$$

where η is the parameter which occurs in the Subtracted Maxwellians distribution function, and

$${}_2F_2(\alpha^2, w) \equiv {}_2F_2(\frac{1}{2}, 1, 1+w, 1-w, -\alpha^2).$$

Using the identity,

$$M_0(\alpha^2, t) = \frac{\alpha^2}{2} \frac{\sin \pi t}{\pi t} {}_2F_2(\frac{1}{2}, 1, 1+t, 1-t, -\alpha^2),$$

the dispersion relation can be written as

$$\begin{aligned}
 & \frac{\epsilon}{ks} + \frac{i\tau}{s} \left[\frac{2\epsilon}{ks} + \frac{Mv_h^2}{T_-} \left(1 + \frac{T_-}{e\phi_0} \right) \frac{1}{z_h^2} \right] \\
 & \quad + \frac{2\pi\hat{s}}{z_h^2 (\eta^2 - 1) \sin(\pi\hat{s})} \left\{ \frac{2}{\eta^2 z_h^2} - \frac{\epsilon}{k\hat{s}} \right\} M_0(\eta^2 z_h^2, \hat{s}) \\
 & \quad - \left(\frac{2}{z_h^2} - \frac{\epsilon}{k\hat{s}} \right) M_0(z_h^2, \hat{s}) - \frac{i\tau\epsilon}{ks\hat{s}} [M_0(\eta^2 z_h^2, \hat{s}) - M_0(z_h^2, \hat{s})] - \left(\frac{\Omega}{\omega_p} \right)^2 = 0.
 \end{aligned} \tag{A2}$$

This dispersion relation is solved for a specific z_h , η , ω_p/Ω , Φ_0 , T_- and ϵa_h . A two dimensional Newton-Raphson technique is used to determine $\omega = \omega_r + i\omega_i$ such that the absolute value of the left hand side of Eq. (A2) is less than 1.0×10^{-6} .

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Table I. Comparison between calculations (linear and nonlinear) and PR-6 observations for $T_{+1} = 150$ ev, $p = 5$,
 $E_{-l} = e\phi_0$ and $\epsilon a_h = 0.034$.

Time (μsec)	$\bar{N}$ ($\times 10^{12}$)	$\frac{\omega_p}{\Omega} +$ ($\frac{p}{\Omega}$)	$\bar{E}_+$ (ev)	T_- (ev)	$\bar{\phi}_0$ (V)	η	Δ	z_c	s_c	ϵa_c ($\times 10^{-2}$)	ϕ^1 (V)	ϕ^{1^a} (V)	ϕ_0^2 (V)	$\bar{\phi}_0 + \phi_0^2$ (V)	$\bar{\phi}_0 + \phi_0^{2^a}$ (V)	$-n_0^2$ ($\times 10^6$)	$-\frac{\delta\omega}{\Omega}$ ($\times 10^{-2}$)	λ (mm)
150	1.2	15	230	10	33	0.19	0	12	0.58	3.6	0	2	0	33	45	0	0	0.89
200	0.91	13	279	15	49	0.60	0.41	6.3	0.52	2.4	2.1	5	0.5	50	60	1.0	0.55	1.7
250	0.78	12	308	17	58	0.75	0.39	5.5	0.51	2.4	3.8	10	1.2	59	85	2.5	1.5	2.0
300	0.69	11	331	19	64	0.84	0.35	5.0	0.51	2.5	4.3	10	1.4	66	90	2.8	1.5	2.1
350	0.64	11	350	21	69	0.91	0.32	4.7	0.50	2.6	4.8	8	1.6	71	75	2.9	1.7	2.3

^aThese are observed values as estimated from Fig. 2 of Ref. 3.

Table II. Comparison between calculations (linear and nonlinear) and PR-6 observations for $T_{+1} = 400$ ev, $p = 5$,
 $E_{-l} = e\phi_0$ and $\epsilon a_h = 0.059$.

Time (μsec)	$\bar{N}$ ($\times 10^{12}$)	$\frac{\omega}{\Omega} p_+$	$\bar{E}_+$ (eV)	T_- (eV)	$\bar{\phi}_0$ (V)	η	Δ	z_c	s_c	ϵa_c ($\times 10^{-2}$)	ϕ^1 (V)	ϕ^{1^a} (V)	ϕ_0^2 (V)	$\bar{\phi}_0 + \phi_0^2$ (V)	$\bar{\phi}_0 + \phi_0^{2^a}$ (V)	$-n_0^2$ ($\times 10^7$)	$-\frac{\delta\omega}{\Omega}$ ($\times 10^{-2}$)	λ (mm)
150	1.2	15	603	10	33	0.08	0	18	0.72	5.9	0	2	0	33	45	0	0	1.0
200	1.0	14	698	23	76	0.50	1.4	7.1	0.52	2.5	5.2	5	7.2	84	60	0.98	8.5	2.6
250	0.92	13	743	28	94	0.60	1.4	6.4	0.50	2.4	7.1	10	10	105	85	1.6	9.5	2.9
300	0.86	13	776	32	107	0.66	1.4	6.0	0.50	2.4	8.8	10	13	120	90	2.0	10	3.1
350	0.82	12	803	35	117	0.71	1.5	5.7	0.48	2.4	11	8	16	133	75	2.7	11	3.3

^aThese are observed values as estimated from Fig. 2 of Ref. 3.

Table III. Comparison between calculations (linear and nonlinear) and PR-6 observations for $T_{+1} = 400$ ev, $p = 10$, $E_{-l} = e\phi_0$ and $\epsilon a_h = 0.059$.

Time (μsec)	$\bar{N}$ ($\times 10^{12}$)	$\frac{\omega}{\Omega} + \frac{p}{\Omega}$	$\bar{E}_+$ (ev)	T_- (ev)	$\bar{\phi}_0$ (V)	η	Δ	z_c	s_c	ϵa_c ($\times 10^{-2}$)	ϕ^1 (V)	ϕ^{1^a} (V)	ϕ_0^2 (V)	$\bar{\phi}_0 + \phi_0^2$ (V)	$\bar{\phi}_0 + \phi_0^{2^a}$ (V)	$-n_0^2$ ($\times 10^6$)	$-\frac{\delta\omega}{\Omega}$ ($\times 10^{-2}$)	λ (mm)
150	1.2	15	603	10	28	0.08	0	18	0.72	5.9	0	2	0	28	45	0	0	1.0
200	0.85	13	835	23	63	0.77	0.79	5.4	0.55	2.1	16	5	17	80	60	4.3	25	3.4
250	0.74	12	947	28	78	0.93	1.63	4.8	0.53	2.2	22	10	25	103	85	9.7	29	3.9
300												10			90			
350												8			75			

^aThese are observed values as estimated from Fig. 2 of Ref. 3.

Table IV. Comparison between calculations (linear and nonlinear) and PR-6 observations for $T_{+1} = 400$ ev, $p = 5$,
 $E_{-2} = 2e\phi_0$ and $\epsilon a_h = 0.059$.

Time (μsec)	$\bar{N}$ ($\times 10^{12}$)	$\frac{\omega_p}{\Omega} +$ ($\frac{p}{\Omega}$)	$\bar{E}_+$ (eV)	T_- (eV)	$\bar{\phi}_0$ (V)	η	Δ	z_c	s_c	ϵa_c ($\times 10^{-2}$)	ϕ^1 (V)	ϕ^{1^a} (V)	ϕ_0^2 (V)	$\bar{\phi}_0 + \phi_0^2$ (V)	$\bar{\phi}_0 + \phi_0^{2^a}$ (V)	$-n_0^2$ ($\times 10^7$)	$-\frac{\delta\omega}{\Omega}$ ($\times 10^{-2}$)	λ (mm)
150	1.2	15	603	10	33	0.08	0	18	0.72	5.9	0	2	0	33	45	0	0	1.0
200	0.96	14	718	19	62	0.54	1.5	6.7	0.53	2.4	11	5	9.5	71	60	0.72	15	2.8
250	0.86	13	783	22	73	0.68	1.5	5.9	0.52	2.4	16	10	16	88	85	1.1	22	3.2
300	0.78	12	835	24	80	0.77	1.5	5.4	0.51	2.4	19	10	20	100	90	1.5	25	3.4
350	0.73	12	880	26	86	0.84	1.4	5.1	0.51	2.4	20	8	23	108	75	1.4	26	3.7

^aThese are observed values as estimated from Fig. 2 of Ref. 3.

FIGURE CAPTION

Fig. 1. Schematic of time variation of the stability boundary and of the point representing the plasma. The instability level is determined by $\Delta \equiv [(\epsilon a)_h - (\epsilon a)_c] / (\epsilon a)_h$.