Magnetic Measurements on the
X-17 Superconducting Wiggler

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INTRODUCTION

In this report, magnetic field measurements for the superconducting wiggler magnet to be installed in the straight section of the x-ray ring illuminating beam line X17 are described and experimental results are given. A general multipole expansion for the integrated field is derived using the theory of analytic functions. The theory is applied to the measurement of multipole coefficients. Qualitative features of the wiggler's behaviour, such as quenching characteristics and hysteresis, are also discussed.

Two different kinds of measurements were made. First, a rotating long coil was used to measure properties of the integrated field, and second, the field variation along the axis of the wiggler was measured using a Hall probe.

One of the main purposes of the field measurements was to determine the leading coefficients in a multipole expansion of the the transverse magnetic field, integrated along the axis of the wiggler. With these coefficients available, it is possible to study the effects of the wiggler on the dynamics of the electron beam motion, and in principle to compensate for the effects of the wiggler by using magnetic elements external to the wiggler.

1. THE MULTIPOLE EXPANSION - THEORY

Consider the integrated magnetic field and vector potential defined according to

$$\vec{b} = \int_{-\infty}^{\infty} \vec{B} \, dz \quad (1)$$

$$\vec{a} = \int_{-\infty}^{\infty} \vec{A} \, dz \quad (2)$$

where $\vec{B}$ and $\vec{A}$ are the ordinary magnetic field and vector potential, respectively.

Using equations (1) and (2), the relation $\vec{B} = \nabla \times \vec{A}$, and the fact that the field and potential vanish at $z = \pm\infty$, one can show that

$$b_z = \frac{\partial a_z}{\partial y}, \quad (3)$$

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$$b_y = - \frac{\partial a_z}{\partial x}. \quad (4)$$

In a source free region, Ampere's law gives

$$\nabla_t \times \vec{b}_t = 0. \quad (5)$$

where $\nabla_t \equiv \nabla - \hat{z} \partial / \partial z$, and $\hat{z}$ is the unit vector in the z - direction.

This means that a potential function ϕ can be defined according to

$$\vec{b}_t = \nabla_t \times \vec{a}_z = - \nabla_t \phi, \quad (6)$$

which in turn leads to

$$b_x = \frac{\partial a_z}{\partial y} = - \frac{\partial \phi}{\partial x}, \quad (7)$$

$$b_y = - \frac{\partial a_z}{\partial x} = - \frac{\partial \phi}{\partial y}. \quad (8)$$

Equations (7) and (8) are the Cauchy - Riemann conditions implying that the complex valued potential ψ defined by

$$\psi = a_z + i \phi \quad (9)$$

is an analytic function of the complex variable

$$z = x + iy. \quad (10)$$

The distinction between the complex variable z and the real variable z that is used in equations (1) and (2) should be clear from the context.

Because ψ is analytic, it can be expanded in a Taylor - Maclaurin series in the following way:

$$\psi = \sum_{n=0}^{\infty} c_n z^n = \sum_{n=0}^{\infty} a_n r^n e^{i(n\theta + \alpha_n)} \quad (11)$$

where the definitions

$$c_n = a_n e^{i\alpha_n} \quad (12)$$

and

$$z = r e^{i\theta} \quad (13)$$

have been used.

From equations (7) and (8) it is seen that

$$i \frac{d\psi}{dz} = i \sum_{n=0}^{\infty} n c_n z^{n-1} = b_x - i b_y. \quad (14)$$

The complex numbers c_n are the multipole coefficients for the integrated field.

1.1. APPLICATION OF THE THEORY

Consider now two infinitely long conducting wires oriented normal to the $x-y$ plane, with transverse displacements given by $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$. The two wires are assumed to be joined at infinity, forming a loop. The magnetic flux cutting the plane defined by the loop, applying Stoke's theorem to equation (6), is just

$$\Phi = a_z(z_2) - a_z(z_1) = \text{Re} [\psi(z_2) - \psi(z_1)]. \quad (15)$$

where "Re" indicates that the real part is to be taken.

If the loop is now rotated about the z - axis with angular frequency ω so that the coordinates vary according to $z_i = r_i e^{i(\omega t + \beta_i)}$, a voltage will be induced, given by

$$\begin{aligned} V &= -M \frac{d\Phi}{dt} = -M \frac{d}{dt} \text{Re} \left\{ \sum_{n=0}^{\infty} c_n [z_2^n - z_1^n] \right\} \\ &= -iM\omega \text{Re} \left[\sum_{n=0}^{\infty} n c_n (r_2^n e^{in\beta_2} - r_1^n e^{in\beta_1}) e^{in\omega t} \right]. \end{aligned} \quad (16)$$

where M is the number of turns in the loop.

Equation (16) is the working equation to be used for the measurement of the multipole coefficients c_n for the superconducting wiggler. It relates the c_n to the Fourier components of the voltage induced in a rotating coil.

2. GENERAL DISCUSSION

The superconducting wiggler magnet is composed of 5 poles powered in series (i.e. the main windings) having a peak field near 6 Tesla, and two end poles also powered in series (end windings) with a peak field near 3 Tesla. The strength of the end poles is varied to yield zero net dipole field integral. The wiggler poles and supporting structure are made of iron. An iron field clamp is attached to each end of the wiggler to control fringe fields. The wiggler period is 17.4 cm, yielding a peak K value near 100.

The design current in the main windings needed to reach 6 Tesla is 233 amp. In practice, a maximum main winding current of 229 amp. was observed several times immediately prior to a quench. This was done with approximate dipole compensation. For normal operation, a maximum current of 215 amp. with a peak field of 5.2 Tesla can be used reliably without the risk of a quench occurring. High field operation of the wiggler necessitates the use of slow ramp rates (e.g. 0.2 amp./sec. or less), because a quench tends to occur sooner when faster rates are used. A peak field of 4.7 Tesla can be reached using a ramp rate of 1.0 amp./sec. in the main winding, but slower rates must be used at higher fields.

At peak fields below approximately 1 Tesla, the fields inside the wiggler become difficult to reproduce due to iron dominated hysteresis effects. In addition, the inductance of the wiggler becomes very large at low currents making the hysteresis loop power supply dependent. When the wiggler is shut off the remnant fields depend in a critical way on how the power supply handles the inductive load (e.g. a unipolar supply will shut itself off at some point leading to exponential decay thereafter, while a bipolar supply could drive the wiggler to zero at some fixed ramp rate.) The power supplies used for the magnet measurements were unipolar. This made the wiggler difficult to ramp down at low current. The zero field operation of this magnet will have to be implemented following installation in the accelerator, and using the electron beam as a probe.

3. LONG COIL MEASUREMENTS

Figure 1 is a schematic diagram of the experimental arrangement used for measuring the integrated multipole field coefficients of the superconducting wiggler magnet. A long coil (1.1176 meters, 20 turns) was inserted along the axis of the wiggler and rotated at constant angular velocity (period = 3.500 seconds). The length of the coil was greater than that of the wiggler so that the coil ends were in an essentially field free region. A cross-section of the long coil is shown in figure 2. Actually, three sets of windings, each with 20 turns, were used to give independent measurements of the field components; a diameter winding, a radius winding, and a 30 degree winding. The diameter winding is coplanar with the rotation axis with leads extending out on the surface of the long coil and returning along the opposing diameter, i.e. $\beta_2 = \beta_1 + \pi$, $r_2 = r_1$, in the notation of formula (16). The radius winding, also coplanar with the rotation axis, has leads extending out on the long coil surface and returning along the rotation axis, i.e. $r_2 = \text{constant}$, $r_1 = 0$. The 30 degree winding lives on the long coil surface, with $\beta_2 = \beta_1 + \pi/6$. The Fourier components of the time-varying voltage measured across these coils during rotation yielded the integrated multipoles, according to (16).

Calibration of the long coil windings was done using a large dipole magnet with an accurately known field (1.8046 Tesla). The $n=1$ Fourier components of the induced voltages gave a direct measure of the sensitivity of the windings to the dipole field. The calibrations for higher multipoles were inferred from theoretical considerations.

The long coil measurements were originally made with the coil centered along the axis of the wiggler, and again after translating the coil by 5 mm in the x-direction (see figure 1 for axes definitions). This was done to provide an additional set of independent measurements of the field.

With the coil centered, values for the current in the wiggler end correction windings necessary to zero the dipole component of the integrated field were found for a number of main winding currents. This determined an operating curve, shown in figure 3. The data of figure 3 is listed in table 1, where it is seen that the operating currents depend on the history of the magnet, e.g. whether the current was increased from smaller values or decreased from larger values. It was also observed that the operating point was a function of the rate at which the wiggler was ramped. Any operating curve is therefore incomplete without specifying the ramp cycle which was used.

It was possible to return the field to a previous state, for example after a power down, by running the wiggler through a single hysteresis loop. Further cycling was not necessary because the operating point at high field serves as a reference point; for high field levels a large fraction of the iron in the wiggler is saturated. At high peak fields, the integrated dipole component of the field could be made to return to within plus or minus 40 Gauss-cm. of previous values. This may be compared with a typical x-ray ring steering coil peak dipole component of 6400 Gauss-cm.. At lower field levels (e.g. 1 Tesla peak), the repeatability was significantly worse due to iron dominated hysteresis effects.

3.1. Z - SCANS

The measured values of the multipole components should in principle have been independent of the location of the long coil along the axis of the wiggler (i.e. the z - axis of figure 1). Small nonuniformities of the long coil windings along their length caused the measured values to oscillate as the long coil was moved along the z - axis. The amplitudes of these oscillations were observed to be large enough to require that systematic corrections be applied to the data.

As an example, the results of a z -scan for the sextupole ($n=3$) component of the field as measured by the radius coil are presented in figure 4. Illustrated are the real and imaginary parts of the $n=3$ Fourier component of the voltage measured across the coil as a function of vertical position of the coil in inches (the wiggler was suspended vertically inside the cryostat). The point $z=13$ corresponds roughly to the situation where the end of the long coil is adjacent to the end of the wiggler, and similarly for the point $z=30$. At $z=22$ the coil was centered vertically in the wiggler.

From the figure one obtains a good value for the sextupole component (both amplitude and phase), an estimate of the systematic uncertainty in that value, and a systematic correction to apply to data measured at fixed z , indicated by the phasor originating from the point $z=22$. Systematic corrections for the different multipoles and coils were scaled according to the peak field inside the wiggler, as later measured using a Hall probe. This analysis was performed on

the first three multipoles as measured by the three long coil windings at two transverse positions ($x=0$ and $x=5$ mm.). The z - scans were done at the high field operating point of table 1.

Results for the measured values of the multipole coefficients $c_2 = a_2 e^{i\alpha_2}$ and $c_3 = a_3 e^{i\alpha_3}$ from z -scans with the coil located at $x = 0$ and $x = 5$ mm. are summarized in table 2. The magnitude of the quantity $2c_2$ is the integrated magnetic field gradient B' , and the magnitude of $6c_3$ can be interpreted as the integrated curvature of the field, B'' , using appropriately chosen axes. A phase reference was obtained from a long coil measurement made with the wiggler in an uncompensated condition, i.e. with the main windings at full current and the end windings shut off, producing a large dipole field oriented along the y - axis of figure 1. The phases α_i were determined by defining $\alpha_1 = 0$.

The value of the $n=3$ component can be inferred from the $n=2$ components measured at two different transverse coil locations according to

$$6c_3(0 < x < 5\text{mm.}) = \frac{2c_2(x = 5\text{mm.}) - 2c_2(x = 0)}{5\text{mm}}, \quad (17)$$

which states that the rate of change of the quantity $2c_2$ in the interval $0 < x < 5$ mm. is equal to the value of $6c_3$ somewhere within that interval (i.e. the mean value theorem). The values for $6c_3$ inferred in this way from the data of table 2 are $3.73 - 0.14$ i Tesla/meter for the radius coil, and $3.66 - 0.11$ i Tesla/meter for the 30 degree coil, showing reasonable agreement with the directly measured values.

The dipole component c_1 at $x = 0$, as measured by the long coil winding having the smallest systematic uncertainty, was within 700 Gauss-cm. of zero for all points on the ramp cycle. Table 3 gives rough values for the rate of change of the magnitude of c_1 when the current I_{end} in the wiggler end winding is varied. The final zeroing of the dipole component will have to be done when the wiggler is installed, using the numbers in table 3 as a guide. External steering elements can also be used for fine tuning the optics.

3.2. RESULTS - MULTIPOLES 2 AND 3 VS. EXCITATION

Values for the quantities $n! c_n$ for $n = 2, 3$ at transverse positions $x = 0$ and $x = 5$ mm. are given in tables 4 - 13. This data was taken at a fixed z - position, and has been corrected for systematic errors as described in the previous section. Each table corresponds to a single multipole component as measured by a particular long coil winding (the diameter coil cannot measure even multipoles because of symmetry). The column in the tables labelled "ramp" indicates the immediate history of the given operating point, i.e. whether the wiggler was on

the up or down side of the ramp cycle. Note that results for different long coil windings are in good agreement with each other.

Self consistency of the data in tables 4 - 13 can be checked using equation (17). A comparison of the directly measured values for $6c_3$ with those inferred from equation (17) is made in tables 14 and 15 for the radius and 30 degree coil, respectively. The fourth and fifth columns give the real and imaginary parts of $6c_3$ computed from equation (17). The sixth and seventh columns show the real and imaginary parts of the average of the directly measured values for $6c_3$ using

$$\overline{c_3} = \frac{c_3(x=0) + c_3(x=5mm.)}{2} \quad (18)$$

Agreement is good, including the sign reversal of c_3 at low field levels.

4. HALL PROBE RESULTS

For these measurements, a Hall probe was mounted on the end of a stainless steel tube. The tube had a series of dowel pin holes positioned along its length at 0.25 inch intervals to facilitate systematic movements of the probe. A small hose was run down to the probe through which heated nitrogen was delivered, for the purpose of temperature stabilization. Also, a temperature sensor was mounted near the probe.

The sensitivity of the Hall probe was tested against an NMR probe, the latter serving as an absolute reference. Results are shown in Figure 5 where the difference between measured field values for the Hall and NMR probes is plotted against the NMR value. The data indicate that the Hall probe gives a measured field which is within 1 percent of the true field, for the range between 6 and 60 kilogauss. This is in agreement with the Hall probe specifications.

In figure 6 are data showing the variation of wiggler peak pole tip field with the current in the wiggler main winding. The upper curve shows the average of the 5 interior poles, and the lower shows the average of the two end pole fields. The wiggler was compensated to zero dipole field integral for each current level. The curves are well approximated by straight lines, except at low field levels.

The z - coordinate was scanned with the Hall probe yielding data shown in figure 7. The probe's angular position was fixed prior to the scan by adjusting it to give maximal measured field at the center pole. Plotted are normalized field strength vs. position. Two sets of data are superimposed; data at high field (5.22 Tesla), and low field data (1.15 Tesla). The high field data is very nearly sinusoidal, however, the low field data deviates from this behaviour in that the region between poles is almost linear, lacking the curvature of a sinusoid. This effect can be made quantitative using Fourier analysis.

Shown in figures 8 and 9 are the Fourier transforms of the two curves of figure 7, defined by

$$F(k) = \text{Re} \int_{-\infty}^{\infty} dz B(z) e^{ikz} \quad (19)$$

The imaginary part of the transform is very small, and has a non-zero value only because of small asymmetries in the data.

The transformed curves are characterized by a large peak at $k = 2\pi/\lambda_{wig} = 36 \text{ m}^{-1}$. The major difference between figures 8 and 9 is the existence of a significant third harmonic peak in the low field data near $k = 108 \text{ m}^{-1}$. This is a manifestation of the non-sinusoidal behaviour described above.

In figures 10 and 11 are shown the second integrals of the two curves of figure 7, yielding curves proportional to the transverse particle trajectory. The amplitude of the oscillations is $K \lambda_{wig} / 2 \pi \gamma$, which translates into .10 mm at 1.15 Tesla (figure 10), and .47 mm at 5.22 Tesla (figure 11), using $\gamma = 5000$.

Small variations in pole tip field from pole to pole can lead to quite observable effects in the calculated trajectory. At high field (figure 11), the dominant effect is a decrease in oscillation amplitude at the center pole. In general, the center pole tended to have a lower peak field than its neighbors. It should be noted that the temperature during the scans tended to drift toward higher values as the probe was withdrawn from the cryostat. This is discussed in the following section.

4.1. TEMPERATURE RELATED EFFECTS

The specifications for the Hall probe's temperature stability are that

$$\left| \frac{1}{B} \frac{dB}{dT} \right| < .005\%/^{\circ}\text{C} \quad (20)$$

with a zero drift less than .13 Gauss / $^{\circ}\text{C}$.

The temperature drift at the Hall probe was less than $\pm 3.5^{\circ}\text{C}$ for the peak field measurements of figure 6, and $\pm 5^{\circ}\text{C}$ for the z - scan data of figure 7. This implies that the Hall probe stability was better than $\pm .018\%$ and $\pm .025\%$ respectively, with a zero drift better than .46 and .65 Gauss respectively. The percentages amount to ± 9 and ± 12.5 Gauss respectively at 5 Tesla.

The effect of the thermal expansion of the stainless steel tube was much larger than any thermal drifts intrinsic to the Hall probe. The thermal expansion coefficient of stainless is about $1.7 \times 10^{-5} / ^{\circ}\text{C}$. At any given time, between 60 and 130 cm. of the tube was inserted into the wiggler cryostat. The largest field gradient dB_y/dz is equal to $5.2 \text{ Tesla} \times 2\pi/\lambda_{wig} = 188 \text{ Tesla / meter}$. This means that in the worst case, an error in positioning due to thermal expansion

can lead to a field measurement error of the order of $1.7 \times 10^{-5} / ^\circ \text{C} \times 1.3$ meters $\times 188$ Tesla/meter $= \pm 42$ Gauss / $^\circ \text{C}$, or, about 0.08 % of the peak field / $^\circ \text{C}$. This effect was in fact measured to be 38 Gauss / $^\circ \text{C}$ at full field (5.2 Tesla peak) at a point in the wiggler where the field took on a value of 1 Tesla, i.e. the gradient was near its maximum.

5. BACKGROUND FIELD MEASUREMENT

After the completion of the measurements on the superconducting wiggler, the long coil was reinserted into the empty cryostat to determine the magnitude of any remnant fields. This was done at room temperature.

An integrated dipole field of 101. Gauss-cm. was measured, to be compared with the systematic uncertainty of 700 Gauss-cm. stated previously. The integrated quadrupole field $2 a_2$ was found to be 11.5 Gauss, and the sextupole term $6 a_3$ was .28 Tesla/m. These numbers are to be compared with the measured uncertainties stated in table 2. No attempt was made to compensate for the background field in the preceding analysis.

6. REFERENCES

- [1] A. Luccio, "MAGNETIC MEASUREMENTS ON THE NSLS SUPERCONDUCTING WIGGLER", NSLS TECHNICAL NOTE #186 (1984)
- [2] A. Luccio, "PRELIMINARY MEASUREMENTS OF INTEGRATED MULTIPOLES OF THE SUPERCONDUCTING WIGGLER", NSLS TECHNICAL NOTE #233 (1985)

FIGURE 1.

NSLS
Superconducting
Wiggler Magnet
Measurement

Field
Clamp
(Iron)

Long Coil

Iron Pole Tips

0.609 m

Superconducting
Coil Pack

* REV.

SPEED: rpm

SCALE: 4:1

WINDING NUMBER	TYPE OF WINDING	ANGULAR OFFSET	NUMBER OF TURNS	ELECTRICAL RESISTANCE	WINDING RADIUS (cm)	WINDING LENGTH (cm)
① 1	RADIUS DIPOLE	0°	20		0.4204	111.76 * (44.0 INCHES)
② 2	DIPOLE (Diameter coil)	45°	20		0.8966	111.76 * (44.0 INCHES)
③ 3	TANGENTIAL DIPOLE (30° coil)	165°	20		0.8966	111.76 * (44.0 INCHES)

Figure 2. Cross Section of Long Coil.

Figure 3.

11-5 component

Radius coil

T 8 717. 1/m / Volt.

Figure 4.

Figure 5. Hall probe comparison with NMR reference.

Figure 6. Wiggler peak field vs. current.

Figure 7. Wiggler field profile at high and low field levels.

Figure 8. Fourier analysis of low field profile.

Figure 9. Fourier analysis of high field profile.

Figure 10. Second Integral of Normalized Field at 1.15 Tesla.

Figure 11. Second Integral of Normalized Field at 5.22 Tesla.

$I_{main} (A)$	$\frac{dI_{main}}{dt} (A / s)$	$I_{end} (A)$	$\frac{dI_{end}}{dt} (A / s)$
20.	start	-4.6	start
189.82	1.0	-88.594	-0.5
214.80	0.2	-100.785	-0.1
189.82	-0.2	-88.091	0.1
149.88	-1.0	-68.454	0.5
99.92	-1.0	-43.219	0.5
49.972	-0.5	-17.836	0.25
20.009	-0.2	-4.486	0.1
49.973	1.0	-18.087	-0.5
99.923	1.0	-43.592	-0.5
149.88	1.0	-68.829	-0.5
189.82	1.0	-88.594	-0.5

Table 1. Ramp cycle yielding zero dipole field integral.

$x = 0 :$	Dia. coil	Radius coil	30 deg. coil
$B' = 2a_2$	-	$129. \pm 4.4$ Gauss	$130. \pm 3.6$ Gauss
α_2 (deg.)	-	78.4	78.5
$B'' = 6a_3$	$3.12 \pm .14$ T/m	$3.44 \pm .59$ T/m	$3.28 \pm .71$ T/m
α_3 (deg.)	7.1	7.1	3.4

$x = 5mm :$			
$B' = 2a_2$	-	$247. \pm 16.$ Gauss	$242. \pm 10.$ Gauss
α_2 (deg.)	-	30.6	30.3
$B'' = 6a_3$	$3.84 \pm .40$ T/m	$3.99 \pm .79$ T/m	$3.71 \pm .57$ T/m
α_3 (deg.)	6.9	5.2	5.3

Table 2. Z - scan results for $n = 2, 3$ multipole coefficients.

$I_{main} (A)$	$I_{end} (A)$	$\frac{\Delta a_1}{\Delta I_{end}}$ (Gauss-cm./Amp)
214.77	-100.	2750.
189.78	-88.	2700.
149.83	-69.	2450.
99.88	-43.	2600.
49.923	-18.	3000.
19.968	-4.5	7000.

Table 3. Variation of n=1 multipole with end correction current.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$2 a_2$ (Gauss)	α_2 (deg.)	Re	Im
214.77	-100.785	-	127.6	79.5	23.25	125.5
189.78	-88.94	u	107.3	81.3	16.23	106.1
189.78	-88.472	u	105.0	80.7	16.97	103.6
189.78	-88.467	u	105.6	80.0	17.36	104.0
149.84	-68.827	u	92.98	84.0	9.719	92.47
149.84	-68.456	d	104.9	85.7	7.865	104.6
149.84	-68.828	u	89.95	83.3	10.49	89.34
99.878	-43.592	u	68.72	83.5	7.78	68.28
99.80	-43.215	d	71.05	85.8	5.20	70.86
99.880	-43.215	d	70.24	85.5	5.51	70.02
49.928	-17.836	d	19.55	69.6	6.81	18.32
49.293	-17.784	d	18.65	71.7	5.86	17.71
49.930	-18.083	u	16.08	64.2	7.00	14.48
19.968	-4.783	-	4.89	244.3	-2.12	-4.41

Table 4. Radius coil measurement of $n=2$ multipole coefficient at $x=0$.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$2 a_2$ (Gauss)	α_2 (deg.)	Re	Im
214.77	-100.785	-	127.0	79.9	22.3	125.0
189.78	-88.94	u	106.7	81.5	15.8	105.5
189.78	-88.472	u	105.0	81.1	16.2	103.7
189.78	-88.467	u	104.7	80.8	16.7	103.4
149.84	-68.827	u	90.10	82.2	12.23	89.27
149.84	-68.456	d	101.78	85.9	7.28	101.52
149.84	-68.828	u	93.41	83.4	10.74	92.79
99.878	-43.592	u	66.12	83.9	7.03	65.75
99.80	-43.215	d	67.33	86.2	4.46	67.18
99.880	-43.215	d	67.23	85.9	4.81	67.06
49.928	-17.836	d	17.25	72.4	5.22	16.44
49.293	-17.784	d	17.77	71.4	5.67	16.84
49.930	-18.083	u	17.37	74.2	4.73	16.71
19.968	-4.783	-	8.23	264.3	-82	-8.19

Table 5. 30 degree coil measurement of $n=2$ multipole coefficient at $x=0$.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$6 a_3 (T/m)$	$\alpha_3 (deg.)$	Re	Im
214.77	-100.785	-	3.103	10.4	3.052	.560
189.78	-88.94	u	2.611	10.7	2.566	.485
189.78	-88.472	u	2.646	10.3	2.603	.473
189.78	-88.467	u	2.661	10.4	2.617	.480
149.84	-68.827	u	2.230	9.6	2.199	.372
149.84	-68.456	d	2.337	9.3	2.307	.378
149.84	-68.828	u	2.266	9.2	2.237	.362
99.878	-43.592	u	1.694	8.2	1.677	.242
99.80	-43.215	d	1.769	8.6	1.749	.265
99.880	-43.215	d	1.779	8.6	1.759	.266
49.928	-17.836	d	1.972	177.7	-1.971	.079
49.293	-17.784	d	1.955	177.3	-1.953	.092
49.930	-18.083	u	1.941	177.3	-1.939	.091
19.968	-4.783	-	1.652	178.6	-1.652	.040

Table 6. Diameter coil measurement of $n=3$ multipole coefficient at $x=0$.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$6 a_3 (T/m)$	$\alpha_3 (deg.)$	Re	Im
214.77	-100.785	-	3.427	7.8	3.396	.465
189.78	-88.94	u	2.905	7.6	2.880	.384
189.78	-88.472	u	2.953	7.4	2.928	.380
189.78	-88.467	u	2.970	7.4	2.945	.383
149.84	-68.827	u	2.473	6.4	2.457	.276
149.84	-68.456	d	2.598	6.4	2.581	.290
149.84	-68.828	u	2.523	6.3	2.508	.277
99.878	-43.592	u	1.895	4.6	1.889	.152
99.80	-43.215	d	1.950	5.1	1.942	.173
99.880	-43.215	d	1.926	5.3	1.918	.178
49.928	-17.836	d	1.973	179.7	-1.973	.010
49.293	-17.784	d	1.966	179.7	-1.966	.010
49.930	-18.083	u	1.955	180.5	-1.955	-.017
19.968	-4.783	-	1.693	181.2	-1.693	-.035

Table 7. Radius coil measurement of $n=3$ multipole coefficient at $x=0$.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$6 a_3 (T/m)$	$\alpha_3 (deg.)$	Re	Im
214.77	-100.785	-	3.322	6.3	3.302	.365
189.78	-88.94	u	2.831	6.0	2.815	.296
189.78	-88.472	u	2.870	6.1	2.854	.305
189.78	-88.467	u	2.875	6.2	2.859	.310
149.84	-68.827	u	2.427	5.4	2.416	.228
149.84	-68.456	d	2.544	5.4	2.533	.239
149.84	-68.828	u	2.463	5.3	2.453	.228
99.878	-43.592	u	1.887	4.0	1.883	.132
99.80	-43.215	d	1.936	4.3	1.931	.145
99.880	-43.215	d	1.940	4.2	1.935	.142
49.928	-17.836	d	1.752	176.5	-1.748	.107
49.293	-17.784	d	1.746	176.7	-1.743	.101
49.930	-18.083	u	1.707	177.2	-1.705	.083
19.968	-4.783	-	1.500	178.3	-1.499	.044

Table 8. 30 degree coil measurement of n=3 multipole coefficient at x=0.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$2 a_2$ (Gauss)	α_2 (deg.)	Re	Im
214.81	-100.786	-	235.2	27.4	208.8	108.2
189.83	-88.093	d	208.1	30.5	179.3	105.6
189.83	-88.591	u	207.7	27.3	184.6	95.3
149.88	-68.455	d	174.4	31.2	149.2	90.3
149.89	-68.829	u	167.7	26.1	150.6	73.8
99.93	-43.217	d	124.9	28.0	110.3	58.6
99.93	-43.590	u	121.1	22.8	111.6	46.9
49.976	-17.836	d	94.4	157.8	-87.4	35.7
49.979	-18.087	u	89.8	160.4	-84.6	30.1
20.01	-4.783	-	81.2	174.5	-80.8	7.8

Table 9. Radius coil measurement of $n=2$ multipole coefficient at $x=5$ mm.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$2 a_2$ (Gauss)	α_2 (deg.)	Re	Im
214.81	-100.786	-	244.9	31.5	208.8	128.0
189.83	-88.093	d	216.7	34.1	179.4	121.5
189.83	-88.591	u	216.2	30.8	185.7	110.7
149.88	-68.455	d	181.7	34.6	149.6	103.2
149.89	-68.829	u	173.6	29.6	150.9	85.7
99.93	-43.217	d	129.4	29.6	112.5	63.9
99.93	-43.590	u	126.8	25.2	114.7	54.0
49.976	-17.836	d	91.6	152.5	-81.3	42.3
49.979	-18.087	u	85.3	155.2	-77.4	35.8
20.01	-4.783	-	77.8	172.4	-77.1	10.3

Table 10. 30 degree coil measurement of $n=2$ multipole coefficient at $x=5$ mm.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$6 a_3 (T/m)$	$\alpha_3 (deg.)$	Re	Im
214.81	-100.786	-	3.722	8.5	3.681	.550
189.83	-88.093	d	3.190	8.8	3.152	.488
189.83	-88.591	u	3.285	9.1	3.244	.520
149.88	-68.455	d	2.781	7.6	2.757	.368
149.89	-68.829	u	2.692	7.0	2.672	.328
99.93	-43.217	d	2.062	7.2	2.046	.258
99.93	-43.590	u	2.052	8.4	2.030	.300
49.976	-17.836	d	1.797	178.1	-1.796	.060
49.979	-18.087	u	1.734	179.4	-1.734	.018
20.01	-4.783	-	1.623	180.0	-1.623	0.00

Table 11. Diameter coil measurement of n=3 multipole coefficient at x=5 mm.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$6 a_3 (T/m)$	$\alpha_3 (deg.)$	Re	Im
214.81	-100.786	-	3.848	5.6	3.830	.375
189.83	-88.093	d	3.421	5.2	3.407	.310
189.83	-88.591	u	3.341	5.2	3.327	.303
149.88	-68.455	d	2.953	5.4	2.940	.278
149.89	-68.829	u	2.870	4.7	2.860	.235
99.93	-43.217	d	2.154	2.8	2.151	.105
99.93	-43.590	u	2.111	2.5	2.109	.092
49.976	-17.836	d	1.812	180.7	-1.812	-.022
49.979	-18.087	u	1.741	180.3	-1.741	-.009
20.01	-4.783	-	1.678	181.1	-1.678	-.032

Table 12. Radius coil measurement of n=3 multipole coefficient at x=5 mm.

$I_{main} (A)$	$I_{end} (A)$	Ramp	$6 a_3 (T/m)$	$\alpha_3 (deg.)$	Re	Im
214.81	-100.786	-	3.671	5.8	3.652	.371
189.83	-88.093	d	3.228	6.4	3.208	.360
189.83	-88.591	u	3.241	5.3	3.227	.299
149.88	-68.455	d	2.801	5.8	2.787	.283
149.89	-68.829	u	2.725	5.2	2.714	.247
99.93	-43.217	d	2.132	4.4	2.126	.164
99.93	-43.590	u	2.086	4.5	2.080	.164
49.976	-17.836	d	1.650	175.4	-1.645	.132
49.979	-18.087	u	1.580	175.2	-1.574	.132
20.01	-4.783	-	1.504	177.5	-1.503	.066

Table 13. 30 degree coil measurement of $n=3$ multipole coefficient at $x=5$ mm.

I_{main}	I_{end}	Ramp	$\text{Re } \frac{\Delta 2c_2}{\Delta x} \text{ (T/m)}$	$\text{Im } \frac{\Delta 2c_2}{\Delta x} \text{ (T/m)}$	$\text{Re } \overline{6c_3}$	$\text{Im } \overline{6c_3}$
214.7	-100.78		3.711	-.346	3.613	.420
189.8	-88.9	d	3.261	-.010	3.144	.347
189.8	-88.5	u	3.345	-.174	3.136	.343
149.8	-68.4	d	2.827	-.286	2.761	.284
149.8	-68.8	u	2.802	-.311	2.659	.256
99.9	-43.2	d	2.102	-.245	2.047	.139
99.9	-43.59	u	2.076	-.428	1.999	.122
49.9	-17.836	d	-1.884	.348	-1.893	-.006
49.9	-18.08	u	-1.832	.312	-1.848	-.013
19.9	-4.78		-1.574	.244	-1.686	-.034

Table 14. Consistency check between n=2 and n=3 data
for radius coil.

I_{main}	I_{end}	Ramp	$\text{Re } \frac{\Delta^2 c_2}{\Delta x} \text{ (T/m)}$	$\text{Im } \frac{\Delta^2 c_2}{\Delta x} \text{ (T/m)}$	$\text{Re } \overline{6c_3}$	$\text{Im } \overline{6c_3}$
214.7	-100.78		3.730	.060	3.477	.368
189.8	-88.9	d	3.272	.320	3.012	.328
189.8	-88.5	u	3.390	.146	3.043	.305
149.8	-68.4	d	2.846	.034	2.660	.261
149.8	-68.8	u	2.773	-.071	2.584	.238
99.9	-43.2	d	2.161	-.066	2.029	.155
99.9	-43.59	u	2.153	-.235	1.982	.148
49.9	-17.836	d	-1.730	.513	-1.697	.120
49.9	-18.08	u	-1.643	.382	-1.640	.108
19.9	-4.78		-1.526	.370	-1.501	.055

Table 15. Consistency check between n=2 and n=3 data
for 30 degree coil.

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