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RESEARCH ON THE "HYLIFE" LIQUID-FIRST-WALL CONCEPT
FOR FUTURE LASER-FUSION REACTORS

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NOMENCLATURE

A_o	=	Half-amplitude of the nozzle excitation
b	=	Sheet jet thickness (2h)
c_r	=	Wave speed
f	=	Forcing frequency
g	=	Acceleration of gravity
h	=	Sheet jet half-thickness (b/2)
k	=	Wave number, $2\pi/\lambda$
LT	=	Linear temporal stability theory
m	=	Mass
p	=	Pressure
PT	=	Particle trajectory theory
r	=	Radius
r_d	=	Ratio of ambient density to liquid density, ρ_a/ρ
t	=	Time
TT	=	Trajectory theory
U	=	Vertical component of mean jet velocity
v	=	Transverse component of mean jet velocity
$\hat{We}$	=	Weber number, $\rho U_o^2 h_o / \sigma$
w	=	Sheet jet width
x	=	Vertical coordinate
y	=	Transverse coordinate
β	=	Coefficient in trajectory theory
δ	=	Differential amount
η	=	Wave amplitude in linear theory
λ	=	Wave length
ρ	=	Density
σ	=	Surface tension coefficient
ω	=	Complex frequency

Subscripts

a	=	ambient
c	=	critical
i	=	imaginary part
o	=	at the nozzle exit (initial value)
r	=	real part
σ	=	surface tension

EXPERIMENTAL INVESTIGATION OF THE STABILITY OF LIQUID SHEET JETS

R. K. Takahashi and M. A. Hoffman

ABSTRACT

It has been proposed to protect the structural walls of a future laser fusion reactor with a curtain or wall of liquid lithium jets. As part of the investigation of this concept, experiments have been performed on planar sheet water jets issuing vertically downward from slit nozzles. The nozzles were subjected to forced harmonic excitation to simulate the vibrational environment of the laser fusion reactor, and experiments were run at both 1 atm and 0.2 atm pressure.

Linear temporal stability theory is shown to predict the onset of the unstable regime and the spatial growth rates reasonably well for the cases where the amplitudes of the nozzle excitation are not too large and the waveform is nearly sinusoidal. However, the amplitudes are not well predicted by linear theory and a simplified trajectory theory is described which does predict the wave envelope amplitudes very well. For larger amplitude nozzle excitation, the waveform becomes highly nonlinear and non-sinusoidal and resembles a sawtooth waveform with the sheet jets displaying a remarkable resistance to breakup; these latter experimental results can only be partially explained by existing theories at the present time.

EXPERIMENTAL INVESTIGATION OF THE STABILITY OF LIQUID SHEET JETS

R. K. Takahashi* and M. A. Hoffman**

1. INTRODUCTION

The study of planar or sheet jets of liquid is of interest in atomization studies for droplet production, in fuel injection and combustion studies, in coating and spraying processes and more recently in connection with a new laser fusion reactor design concept [1,2]. In this latter application, which motivated this present research, it has been proposed to use near-vertical liquid lithium jets inside the fusion reactor in front of the inner walls of the reactor vessel. The liquid lithium would be thick enough (on the order of half a meter) to absorb the X-ray and ion energy as well as a good part of the energy of the neutrons released in the fusion reactions. In this way, the solid structure would be protected from erosion, embrittlement and severe neutron damage, and its useful life increased significantly. The lithium jets act as a "self-healing" liquid first wall of the reactor vessel. One of the most important potential problem areas in this application of liquid jets arises because the reactor vessel and jet nozzles will probably be subject

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to rather large amplitude vibrations due to the "ringing" of the entire structure caused by the pulsed nature of the laser-fusion microexplosions. The objective of this study was to determine whether forced vibration of the nozzle would cause jet instability and breakup within the typical fall distance in the reactor (on the order of 5 to 10 m). Breakup might allow neutrons, ions, and X-rays to stream directly through to the solid walls and seriously shorten their useful life.

Our initial studies of annular liquid jets [3] and thin liquid films on the walls [4] indicated that additional experimentation with sheet jets (and also round jets) would provide a valuable data base for the design of a liquid lithium curtain for a future fusion reactor.

Extensive studies of the breakup of fan-shaped sheet jets and sprays have been performed by Dombrowski and his colleagues in connection with spraying and atomization studies [e.g., see Reference 5]. However, for our particular geometry of a nearly-constant-width sheet jet, only two stability analyses have been uncovered in our literature search which are directly relevant. In 1953, Squire [6] published a theoretical study of the linear temporal stability of a horizontal sheet jet. Two years later, Hagerty and Shea [7] independently published a very similar theoretical analysis along with some experimental data directly relevant to our study. Their experiments employed a very thin, horizontal sheet of water with a thickness of only about 1.6 mm and with jet velocities of about 3 to 5 m/s. The jet nozzle was vibrated transverse to the flow direction at frequencies up to about 200 Hz. Measurements of the exponential growth rates were made from flash photographs of the edge view of the sheet jets and the growth rate data agreed very well with the predictions of the linear theory.

In the present experimental study we have extended the work of Hagerty and Shea in several directions in order to better simulate the laser fusion reactor environment. We have employed:

- (a) Thicker sheet water jets
- (b) Large amplitude forced excitation up to several hundred Hz.
- (c) Vertical (downward) jet orientation.
- (d) Experiments at 0.2 atm pressure in a vacuum vessel as well as at one atmosphere.

In Section 2, we briefly review the key results of the linear temporal stability theories of References [6] and [7]. Then in Section 3, we develop an alternative simplified spatial growth rate theory referred to as the "trajectory theory" which was developed in order to explain the large-amplitude, nonlinear, forced-excitation data. The experimental apparatus is described in Section 4, and the results are compared to the various theories in Section 5.

2. REVIEW OF LINEAR TEMPORAL STABILITY THEORY

The theories developed in References [6] and [7] are essentially equivalent and both are basically extensions of the theories of Rayleigh [8] and Lamb [9] for the temporal growth of waves on a single free surface to the present case of two free surfaces. We will henceforth refer to the linear temporal stability theories of References [6] and [7] as simply the "linear theory" (LT). In developing the linear theory, it is assumed that the effect of gravity is negligible so that the mean sheet thickness, b , and velocity, U , stay constant at the initial values b_0 and U_0 , respectively. In addition, it is assumed that both the liquid and the surrounding gas are inviscid and that the surface tension coefficient between the two fluids is constant. In addition, we will assume here that the surrounding fluid, air, is at rest.

The wave amplitude, η , is assumed to be much less than $h_0 = b_0/2$, so that the interface boundary conditions can be applied at $\pm h_0$ as part of the linearization of the problem. The geometry of the sheet with two types of waves on it is shown in Figure 1. For case (a), the waves on the two surfaces are in phase, and this is referred to as the sinuous or "snake" mode. For case (b), the waves are 180° out of phase, and this is referred to as the dilation or "sausage" mode. Reference [7] proves that only these two cases are permitted by the linear theory, and that the sinuous waves of case (a) always have a higher growth rate in the unstable regime and hence will tend to be the dominant waves observed. The linear theory assumes a wave amplitude of the form

$$\eta = A_0 e^{i(kx - \omega t)} \quad (1)$$

where the wave number, k , is real and ω is complex ($\omega = \omega_r + i \omega_i$) in the case of temporal instabilities. This implies waves which move downward with a wave speed $c_r = \omega_r/k$ and grow in time everywhere along the sheet with a growth rate ω_i .

The solution for the growth rates is obtained by solving Laplace's Equation for the velocity potential in each region subject to a linearized kinematic constraint at the interface and a linearized pressure boundary condition. As mentioned previously, the boundary conditions are applied at $y \approx \pm h_o$ as part of the linearization of the problem.

Solution of the linearized equations and boundary conditions leads to the following characteristic equation:

$$\rho \left(\frac{\omega}{k} - U_o \right)^2 \tanh kh_o + \rho_a \left(\frac{\omega}{k} \right)^2 - \sigma k = 0 \quad (2)$$

It can be seen that this is a quadratic equation in $\omega(k)$ which is easily solved.

For the case where the air-to-liquid jet density ratio, $r_d \equiv \rho_a/\rho$, is much less than $\tanh kh$, as is the case here, the solution indicates that the sheet is unstable for frequencies less than a critical frequency:

$$f_c \approx \frac{\rho_a U_o^3}{2\pi \sigma}$$

where we find that the wave speed, $c_r \approx U_o$; the corresponding temporal growth rate is given by:

$$\omega_i \approx \sqrt{\frac{k^2 U_o^2 r_d - k^3 \sigma / \rho}{\tanh kh_o}} \quad (3a)$$

In non-dimensional form:

$$\frac{\omega_i}{2\pi f} \approx \sqrt{\left[\frac{r_d U_o}{2\pi f h_o} - \frac{1}{\hat{We}} \right] \left[\frac{kh_o}{\tanh kh_o} \right]} \quad (3b)$$

where the jet Weber number is based on the sheet half thickness, h_o :

$$\hat{We} \equiv \frac{\rho U_o^2 h_o}{\sigma}$$

and where we have used the following relation to obtain Equation (3b):

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c_r} \approx \frac{2\pi f}{U_o}$$

For the special case where $kh_o \ll 1$, the last factor under the radical, $kh_o/\tanh kh_o$, is essentially unity; this is usually a valid approximation for the longer wave lengths (i.e., small wave numbers) of the waves predicted for thin sheet jets. However, it is not always valid for our experimental conditions.

These theoretical results are plotted in Figure 2 for three typical pressures (1.0, 0.67 and 0.20 atm) at a velocity of 5.0 m/s for water jets from one of the two nozzles used in our experiments (nozzle #2). We see that both the growth rates at a given frequency and the critical frequency decrease significantly as the ambient air pressure is reduced. In a vacuum the sheet jet should be stable to all disturbance frequencies according to the linear theory. Also shown by the dashed curve in Figure 2 are the predicted growth rates for a thicker water jet from nozzle #1 at a velocity of 1.65 m/s and 1 atm pressure. The values from this figure will be compared to the experimental results in Section 5.

In the actual experiments, the waves grow in space rather than in time. The temporal growth rates, ω_i , are often converted to spatial growth rates, k_i , for comparison with experiments by assuming [e.g., 10,11]:

$$k_{i(\text{spatial})} = \frac{\omega_{i(\text{temp})}}{c_r} \approx \frac{\omega_{i(\text{temp})}}{U_o} \quad (4)$$

where the wave speed, c_r , is approximately equal to the mean jet velocity, U_o , in our case. Using this transformation, the linear theory predicts a spatially-growing, exponential wave envelope for the unstable cases given by:

$$\eta \approx A_o e^{\omega_i x / U_o} \quad (5)$$

where A_o is the maximum forcing amplitude of the nozzle for a harmonic nozzle motion given by:

$$y_o = A_o \sin(2\pi f t) \quad (6)$$

In our search of the published literature, we have not yet uncovered a general set of requirements for this transformation to be valid, although Yih [12] and Betchov and Criminale [13] seem to imply that it is only expected to be accurate in the neighborhood of the neutral stability point where $\omega_i \Rightarrow 0$ (i.e. where $f \Rightarrow f_c$) in the general case. However, it has been shown to be a reasonably good approximation over the entire range of frequencies in several specific instances [e.g., 11,14,15] where the jet (or liquid film flow) velocity is nearly uniform in the region of the wave growth.

A more direct way to obtain the spatial growth rates is to assume that k is complex and ω is real and this time solve the characteristic equation (Equation 2) for $k(\omega)$. The equation is nonlinear in k and must be solved numerically, in general. This work is now in progress and will be published soon [16]. Preliminary results indicate that the transformation given by Equation (4) is very accurate for our case when $r_d \ll 1$ and $\hat{We} \gg 1$.

3. SPATIAL TRAJECTORY THEORY

In our experiments, we purposely imposed rather large amplitude forced excitation on the sheet jet nozzles to simulate the worst situation for the laser-fusion application. The resulting wavy motion of the sheet jets was of very large amplitude, with peak amplitudes often an order of magnitude larger than the jet thickness as illustrated in the typical photograph of the edge view of the sheet jet shown in Figure 3. Clearly, the linear stability theory results are suspect under these conditions, and will be shown to only partially agree with the experimental results. As a result, it was decided to try to develop a simple first approximation to a nonlinear spatial stability theory.

3.1 General Case of Increasing $U(x)$:

The basic idea of our approach is illustrated in Figure 4. We consider the fluid particle, δm , which is at the crest of the wave shortly after the flow leaves the nozzle. It is then assumed that the wave speed, $c_r(x)$, is approximately equal to the vertical component of the local jet

velocity, $U(x)$. This assumption implies that a fluid particle which starts in the wave crest stays in the wave crest, and that its trajectory traces out the envelope of the waves.

A simple experiment was performed to verify this approximation, since it is one of the key assumptions of the trajectory theory. A dimple was made in the wave crests on one side of the flow with a pointed object as the waves passed a point a few centimeters below the nozzle exit. By viewing this dimple with a strobe it was ascertained that the dimple did stay in the wave crests on the side where the dimple was originally produced, even for very large amplitude waves.

A second basic assumption is that the particle which ends up in the wave crest is the one which leaves the nozzle with the maximum initial transverse velocity, v_o (as the vibrating nozzle passes through the vertical axis), where $v_o = 2\pi f A_o$.

The basic equations for the trajectory of the fluid particle in the wave crest are:

$$\ddot{x} \delta m = g \delta m \quad (7)$$

$$\ddot{y} \delta m = \delta F_a - \delta F_\sigma \quad (8)$$

where $\bar{y}$ is the mean wave crest amplitude measured to the mean jet thickness (as shown on Figure 4). The aerodynamic force, δF_a , due to the air pressure difference across the wavy jet has been estimated using the solution for the pressure distribution on a wavy wall (e.g. Ref. [17]). Taking twice the pressure at the wavy wall peaks we obtain:

$$\delta F_a = \left[\frac{4\pi r_d f U}{b} \right] \bar{y} \delta m \quad (9)$$

This is clearly a first approximation valid only when the wave shape is close to sinusoidal and when the wave amplitude is not so large as to cause nonlinear effects or separation to come in. However, one of the potential advantages of this simple trajectory theory is that the estimate can be improved for large amplitude, nonlinear waves by using results of experiments and/or theoretical calculations of the separated flow over a large amplitude wavy wall; work is now in progress on this improvement.

The surface tension force, δF_σ , on the fluid particle in the wave crest is dependent on the local radius of curvature of the wave:

$$\delta F_\sigma = \left(\frac{\sigma}{r_L} + \frac{\sigma}{r_R} \right) \delta A_s \quad (10)$$

where r_L and r_R are the radii of curvature at the crest on the left and right surfaces, respectively, and δA_s is the surface area of one side of the fluid element. As a first approximation, both radii of curvature have been taken equal to the mean radius of curvature at the crest of a sine wave:

$$\frac{1}{\bar{r}} = \frac{d^2 \bar{y}}{dx^2} = \left(\frac{4\pi^2 f^2}{U^2} \right) \bar{y} \quad (11)$$

Putting this in Equation (10) we obtain:

$$\delta F_\sigma \approx \frac{2\sigma}{\bar{r}} \delta A_s \approx \left(\frac{8\pi^2 f^2 \sigma}{\rho b U^2} \right) \bar{y} \delta m \quad (12)$$

This estimate can be improved for the larger amplitude, nonlinear waves by taking account of the non-sinusoidal shape of the wavy jet. For example, it is possible to measure the mean radius of curvature at the wave crests

from the photographs when the waves have a more zig-zag or saw-tooth-like waveform as was often the case for large amplitude forced excitation (such as shown in the photograph in Figure 5).

For the present paper, we will use the simpler linearized estimates of δF_a and δF_σ given by Equations (9) and (12) (assuming a near-sinusoidal waveform). Substituting these results in Equation (8) gives

$$\frac{\ddot{y}}{y} - \beta^2 \frac{y}{y} = 0 \quad (13)$$

where

$$\beta(t)^2 \equiv \frac{4\pi r_d f U(t)}{b(t)} - \frac{8\pi^2 f^2 \sigma}{\rho b(t) U(t)^2} \quad (14)$$

It should be noted that both the vertical jet velocity, U , and the jet thickness, b , are functions of the vertical coordinate x or equivalently functions of the fall time, t . Using the continuity equation and neglecting the change in the sheet jet width, w , due to the surface tension forces acting on the edges of the sheet, we can evaluate $b(t)$:

$$b(t) = \frac{b_o U_o}{U(t)} \quad (15)$$

where from Equation (7), the velocity is given by:

$$U(t) = U_o + gt \quad (16)$$

Using these relations, Equation (13) can be seen to be a linear second order differential equation with time-dependent (or space-dependent) coefficients. Equation (13) has been solved numerically for our specific nozzle geometries and flow conditions and the results will be compared to experiments in Section 5.

3.2 Special Case of Constant U

For the special case of high initial jet velocities ($U_0 \gtrsim 5$ m/s), the velocity and jet thickness changes in the fall times of the experiments were very small. In the limiting case of constant $U = U_0$ and $b=b_0=2h_0$, β becomes a constant, designated β_0 :

$$\beta_0 = 2\pi f \sqrt{\frac{r_d U_0}{2\pi f h_0} - \frac{1}{\hat{W}_e}} \quad (17)$$

Comparison of this result with Equation (3b) reveals that β_0 equals ω_i , the growth rate from the linear temporal stability theory when β_0 is real (i.e., when ω is complex) for the case where $kh_0 < 1$. Thus real values of β_0 correspond to the case of the unstable sheet jet where the aerodynamic destabilizing force exceeds the stabilizing surface tension force ($f < f_c$).

The solution to Equation (13) for real β_0 requires an assumption about the initial conditions for the fluid particle which ends up in the wave crest. As mentioned previously, we have assumed that this is the fluid particle which leaves the nozzle with the maximum initial transverse velocity, $v_0 = 2\pi f A_0$ (i.e., the velocity of the nozzle as it crosses the center-line during its forced harmonic motion. The solution then becomes:

$$\bar{y} = \frac{\pi f A_0}{\beta_0} \left[e^{\beta_0 x/U_0} - e^{-\beta_0 x/U_0} \right] \quad (18)$$

where we have used $t = x/U_0$ for this constant velocity jet case.

After a short distance, the second negative exponential term becomes negligible compared to the first exponential term, and the solution to the wavy jet envelope approaches a simple exponential which has the same growth rate as predicted by the spatial interpretation of the linear temporal stability theory (Equation 4). However, the envelope amplitude predicted by Equation (18) is $(\pi f/\beta_0)$ times larger than that given by the linear theory (Eq. 5). The implications of this result will be discussed more fully in Section 5.

It should be noted that by hindsight, we realized that we could obtain essentially the same results from the linear temporal stability theory by replacing the initial condition of Equation (1) by $\eta(t=0)=0$ and

$$\dot{\eta}(t=0) = v_0(t=0) e^{i(kx-\omega t)}$$

where v_0 is given by the time derivative of Equation (6):

$$v_0(t) = \dot{y}_0 = 2\pi f A_0 \cos(2\pi f t)$$

The interpretation of these initial conditions for a temporal instability on a sheet jet is that of initially perfectly flat sheet surfaces with an initial spatially-sinusoidal velocity distribution of the surfaces. Since this is a rather unusual initial condition for a linear stability analysis, it was only perceived as the appropriate one which would give the correct transformation to our spatially growing wave situation after the simple trajectory theory gave such good agreement with the experimental results.

4. EXPERIMENTAL APPARATUS AND PROCEDURE

A schematic diagram of the experimental apparatus is shown in Figure 6. The water source is a 7.62 cm (3 inch) industrial-grade water supply with a flow rate capability of about 13 litres/s (200 GPM). A 5.08 cm (2 inch) pipe about 10 meters long connects the industrial water supply to the apparatus. A standard ASME-type orifice is installed in this pipe about one meter downstream of the main ball valve. The pipe contains a section of crush-proof, flexible hose for vibration isolation and several sections of clear cast-acrylic pipe to allow viewing of the flow (and, in particular, to view any bubbles present).

The flow pipe enters the vacuum vessel vertically and is attached to the various nozzles used in the experiments by means of a custom-made flexible silicone rubber coupling which provides a smooth transition on the inner surface. The nozzles are fabricated from 5.08 cm (2 inch) cast acrylic tubes heated and pressed at one end to form a constant-width slit exit. A typical nozzle is shown in Figure 7.

The vacuum chamber is constructed from a 30 cm diameter cast-acrylic cylinder about 120 cm high. The flat acrylic end plates have O-ring grooves machined in them as well as holes for the many feedthroughs. Due to the large amount of water vapor produced by the sheet water jets during the subatmospheric experiments, a water-jet ejector pump was selected for the vacuum pumping. With the present jet-pump water supply of about 0.8 litres/s (12 GPM), we have been able to pump down to about 0.2 atm.

The nozzles are vibrated transversely by means of an electromechanical shaker. The shaker is mounted outside the vacuum tank and is attached to the nozzle by means of a steel rod which passes through a specially designed bellows-type feedthrough into the vacuum chamber (see Ref. [18] for details).

The water exits the nozzle and falls about 70 cm to a small pool at the bottom of the vacuum tank. For the atmospheric pressure runs, the water then simply flows out a drain pipe in the bottom end plate to a drainage sump (see Figure 6). For the subatmospheric tests, a helical-rotor water pump is used to pump out the water to the drainage sump and maintain a constant head of about 30 cm of water in the bottom of the vacuum tank. Although the water pump usually cavitates and vibrates severely, the use of a crush-proof, flexible hose between the vacuum tank and the pump isolates the experimental apparatus from these vibrations quite well. This open-loop arrangement was found to be much superior to a closed-loop arrangement originally tried. The main disadvantage of the open-loop arrangement is that we must use the industrial water supply which has substantial amounts of dissolved gases in it. This tends to create problems with bubble formation in the subatmospheric runs.

The mass flow rate is measured by the orifice plate in conjunction with an inclined manometer. All the flow rate calibration data in the range of the experiments fell on a straight line with a standard deviation of about $\pm 1.5\%$, at atmospheric pressure. However, for the subatmospheric runs, air bubbles produced both by a small amount of air leaking in through some of the subatmospheric pipe joints and by the dissolved gas coming out of solution caused larger uncertainties in the velocity measurement; we estimate that the jet exit velocities have an overall (one standard deviation) uncertainty of about $\pm 3\%$ for the 1 atm runs and about $\pm 5\%$ for the subatmospheric runs.

The vacuum tank pressure was measured roughly by a pressure gauge installed in the top end plate. It was also measured more precisely by a mercury manometer with an estimated uncertainty in the reading of $\pm 10\%$, due primarily to the fact that the vacuum chamber pressure fluctuated. The water temperature was measured to better than $\pm 1^\circ\text{C}$ by a mercury/glass thermometer. This was considered to be adequate accuracy for estimating the water density and surface tension coefficient.

The electromechanical shaker amplitude was measured by two independent accelerometers, one placed on the shaker face and the other mounted on the nozzle itself. The accelerometers were calibrated over the range of frequencies from 10 to 1000 Hz on a large electromechanical shaker with a built-in, calibrated accelerometer [18]. Based on these calibrations, the estimated uncertainty in the nozzle amplitude is $\pm 4\%$, due primarily to the limited number of digits on the digital voltmeter available at the time.

The basic wave amplitude data were obtained from photographs of the edge view of the wavy sheet jet such as shown in Figures 3 and 5. These photographs are actually the superposition of a relatively long exposure photo used to define the wave envelope and a short flash exposure to show up the wave form and permit measurement of the wave lengths.

For the larger vibrational amplitudes, where A_0 can also be read from the photos, the values of A_0 measured agreed with those calculated from the accelerometer readings to within about $\pm 5\%$. For smaller vibrational amplitudes, only the accelerometer values were used.

In reading the amplitude of the mean wave envelope, $\bar{y}$, at various vertical positions, it is necessary to subtract out the thickness of any fluid accumulations in the sheet edges as well as any fluid lumps or "blobs" near the edge regions of the wave crests, which were often present

at very large wave amplitudes and low jet velocities such as illustrated in Figure 5. Reasonably accurate corrections were possible after careful visual study of the jet wave shapes using a stroboscope. The estimated accuracy of the corrected mean wave amplitude data is ± 0.75 mm for Nozzle #1 ($b_o = 7.9$ mm) and ± 1.0 mm for Nozzle #2 ($b_o = 2.5$ mm). However, this is more like a bias error, since the eye can integrate and smooth the data read off the photos quite well. (The smaller scatter of the readings shown in Figures 8, 9 and 10 illustrates this point.)

5. DISCUSSION OF RESULTS

Representative experimental results for the mean wave envelope (excluding the fluid blobs) for Nozzle #2 (the thinner jet) at 1 atm and 0.2 atm pressure are shown in Figure 8. From Figure 2, we see that at 70 Hz, linear theory predicts that the jet should be unstable at 1 atm and stable at 0.2 atm. The experimental results shown in Figure 8 confirm these predictions as well as the predicted growth rates. However, the transformed linear theory for the unstable case (Equation 5) predicts a wave envelope amplitude which is much less than that observed. The linear theory is designated LT-A on Figure 8, where case A is for constant jet $U = U_0$ and $b = b_0$.

In contrast to the linear theory, the simple trajectory theory for constant U_0 and b_0 (Equation 18) labelled TT-A shows excellent agreement with the data on Figure 8 for the unstable case at 1 atm. The improved trajectory theory with time-varying (or equivalently, space-varying) U and b labelled TT-B shows even better agreement with the data. However, the difference between TT-A and TT-B is not very large for these high jet velocities of 5 m/s, as expected.

For the stable case at 0.2 atm, β^2 in Equation (13) for the trajectory theory becomes negative. This is due to the fact that the stabilizing surface tension force, δF_σ , acting on the fluid particle in the crest exceeds the destabilizing aerodynamic force, δF_a , for $f > f_c$. The stable solutions for both TT-A and TT-B can be seen to agree very well with the 0.2 atm data on Figure 8.

This excellent agreement between the data and the simple trajectory theory for Nozzle #2 was found to hold for a wide variety of initial amplitudes provided that A_0 was not too large compared to b_0 .

(typically $A_0 \lesssim 0.5 b_0$) and that the waveform was not too far from sinusoidal in shape. This appears to confirm the two key assumption made in developing the trajectory theory, namely:

- 1) Wave speed equals the jet speed so that a fluid particle which starts in the crest stays in the crest.
- 2) The fluid particle which ends up in the wave crest leaves the nozzle with a velocity $v_0 = 2\pi f A_0$.

For the original Nozzle #1, the initial jet thickness, b_0 , was 7.9 mm (compared to 2.5 mm for Nozzle #2). This large a thickness was chosen because one of the basic objectives of this research was to study thicker jets than employed by Hagerty and Shea (where b_0 was 1.6 mm). These thicker jets were limited to lower velocities (typically 1.65 m/s) because of the limitations on our water supply, so that spatial variations of U and b were expected to be much more important than for Nozzle #2. A typical waveform for large amplitude excitation of Nozzle #1 is shown in Figure 5. The waveform can be seen to be almost a sawtooth with fluid blobs projecting outward from the wave crests. These fluid blobs were primarily at the front and back edges of the sheet, with much smaller fluid accumulations along the entire wave crest. They stayed attached to the main waveform for tens of wavelengths and fell with the main waves at a velocity U . During the growth of these blobs, the sheet thickness varied continuously along the jet, being notably larger at the wave crests. The dramatic appearance of this highly nonlinear waveform when viewed with a strobe can be appreciated from Figure 5, and the unexpected stability against breakup into droplets cannot yet be fully explained.

Typical data for the mean envelope (excluding the fluid blobs) for the thicker jets for three initial amplitudes, A_0 , are shown in Figure 9. For $U_0 = 1.65$ m/s and $f = 70$ Hz, the linear theory (Figure 2) predicts

that these jets should be stable even at 1 atm. The data agree with this prediction, since it can be seen from both Figures 5 and 9 that the surface tension forces are pulling the mean wave envelope inward and trying to flatten the sheet.

From Figure 9, we see that the agreement between TT-A (the constant U_0 and b_0 trajectory theory) and the data is very poor, as expected, because U and b vary considerably as this low velocity jet accelerates downward. There appears to be reasonable qualitative agreement for TT-B, but the non-dimensionalized data exhibit a strong dependence on A_0 not predicted by the trajectory theory when we use the linearized surface tension and aerodynamic force terms.

We can only make the following qualitative observations about these nonlinear results. From Figure 9, we see that the dimensionless amplitudes, $\bar{y}/A_0$, decrease as A_0 increases, instead of staying nearly constant as they would for a strictly linear behavior. In fact, the actual value of the mean wave envelope, $\bar{y}$, appears to saturate at a maximum amplitude of between 12 and 13 mm as A_0 is increased to large values for this particular case. We are now investigating the nonlinear behavior further to see if this is a general result.

If there were no surface tension or aerodynamic forces on the fluid particle in the mean wave crest, it would follow a simple parabolic trajectory defined by:

$$y = v_0 = \text{constant}$$

$$U(t) = U_0 + gt = \text{constant}$$

These free particle trajectories are also plotted on Figure 9 for the limiting case PT-A where $U = U_0$ and $b = b_0$ and for case PT-B where U and b vary according to the above equations. These particle trajectories should form an outer bound to the wave envelopes, and the data points in Figure 9 can be seen to fall near or below this outer bound (within the reading accuracy of the photographs), as expected.

As a further test of our physical understanding of certain aspects of the nonlinear sawtooth waves, we plotted the dimensionless outer wave envelope, y'/A_0 , for the fluid blobs in Figure 10. It can be seen that the fluid blobs for all three initial wave amplitudes follow PT-B very closely up to high values of y'/A_0 of over 20. After this point, the surface tension forces due to the attachment of the blobs to the main sheet wave form tend to hold them back. This suggests that the fluid blobs are formed by excess fluid in the sheet edges which flows to the wave crests and tries to follow a pure free particle trajectory. It is remarkable that these long ligaments or lumps of fluid are observed to remain attached for tens of wavelengths (often all the way down to the pool in the bottom of our Plexiglas tank). Further research is in progress to try to clarify the behavior of the sheet jets in this nonlinear, nonsinusoidal regime because of its potential importance to the laser fusion application.

6. CONCLUSIONS

Sheet jet experiments have verified that the predictions of linear theory are correct with regard to (1) the region of instability of the jets and (2) the linear growth rate when the jets exhibit a nearly uniform thickness and a nearly-sinusoidal growing waveform. However, the transformation of the linear temporal stability theory to predict the amplitude of the spatially growing waves yields too small values of the wave envelope amplitude unless an unusual initial condition is invoked. A simple spatial trajectory theory proposed agrees very well with the large, non-linear wave envelope data for both unstable and stable waves of nearly sinusoidal shape. For nonsinusoidal, sawtooth-like waveforms, none of the theoretical models fully explains the observed wave amplitudes, and further research in this area is recommended.

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FIGURE TITLES

1. Edge view of the two modes of instability predicted by linear temporal stability theory (LT) for a planar sheet jet subject to small perturbations, where (a) is the sinuous or "snake" mode and (b) is the dilation or "sausage" mode.
2. Growth rates for the sinuous mode predicted by the linear temporal stability theory (LT).
3. Edge view of a typical near-sinusoidal, exponentially-growing waveform of the sheet jet from Nozzle #2 with forced excitation of the nozzle at $f=70$ Hz ($U_0=4.75$ m/s, $A_0=1.06$ mm, $p_a=1.0$ atm).
4. Model used in the development of the simplified trajectory theory (TT) showing the motion of the fluid particle which is hypothesized to stay in the crest of the falling wave as it grows spatially to large nonlinear amplitudes.
5. Edge view of a typical sawtooth waveform of the sheet jet from Nozzle #1 with forced excitation at $f=70$ Hz ($U_0=1.65$ m/s, $A_0=1.27$ mm, $p_a=1.0$ atm).
6. Schematic diagram of the water flow system, vacuum tank apparatus and forced excitation instrumentation used in the sheet jet experiments.
7. Typical geometry of the slit nozzles fabricated from cast acrylic.
8. Typical experimental results for the mean envelope of the waveform (excluding any fluid blobs) for Nozzle #2 compared to the linear theory (LT) and the trajectory theory (TT).
9. Typical experimental results for the mean envelope of the waveform (excluding any fluid blobs) for Nozzle #1 compared to the trajectory theory (TT) and the free particle trajectory (PT).
10. Typical experimental results for the outer envelope of the waveform (including fluid blobs) for Nozzle #1 compared to the trajectory theory (TT) and the free particle trajectory (PT).

Figure 1: Edge view of the two modes of instability predicted by linear temporal stability theory (LT) for a planar sheet jet subject to small perturbations, where (a) is the sinuous or "snake" mode and (b) is the dilation or "sausage" mode.

Figure 2: Growth rates for the sinuous mode predicted by the linear temporal stability theory (LT).

Figure 3: Edge view of a typical near-sinusoidal, exponentially-growing waveform of the sheet jet from Nozzle #2 with forced excitation of the nozzle at $f=70$ Hz ($U_0=4.75$ m/s, $A_0=1.06$ mm, $p_a=1.0$ atm).

Figure 4: Model used in the development of the simplified trajectory theory (TT) showing the motion of the fluid particle which is hypothesized to stay in the crest of the falling wave as it grows spatially to large nonlinear amplitudes.

Figure 5: Edge view of a typical sawtooth waveform of the sheet jet from Nozzle #1 with forced excitation at $f=70$ Hz ($U_0=1.65$ m/s, $A_0=1.27$ mm, $p_a=1.0$ atm).

Figure 7: Typical geometry of the slit nozzles fabricated from cast acrylic.

Figure 8: Typical experimental results for the mean envelope of the waveform (excluding any fluid blobs) for Nozzle #2 compared to the linear theory (LT) and the trajectory theory (TT).

Figure 9: Typical experimental results for the mean envelope of the waveform (excluding any fluid blobs) for Nozzle #1 compared to the trajectory theory (TT) and the free particle trajectory (PT).

Figure 10: Typical experimental results for the outer envelope of the waveform (including fluid blobs) for Nozzle #1 compared to the trajectory theory (TT) and the free particle trajectory (PT).