**ANALYSIS OF FIVE-SPOT TRACER TESTS TO
DETERMINE RESERVOIR LAYERING**

SUPRI-TR-8

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by

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ABSTRACT

This report presents a computer algorithm for determining the degree of heterogeneity among the layers of a reservoir. The algorithm uses the equations developed by Brigham and Smith¹ that predict the behavior of a tracer slug flowing in a five-spot injection pattern.

To illustrate the use and potential problems in the application of this algorithm, examples are presented using five sets of simulated field test data. One example using actual field data is also presented.

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I. INTRODUCTION

The use of well-to-well tracers to detect reservoir heterogeneities is growing in importance in industry. In any fluid injection project, high permeability streaks in the reservoir can channel a disproportionate share of the injected fluid. In waterfloods, such streaks decrease the vertical sweep efficiency of the flood and cause early breakthrough of water in the producing wells. The initial result is an increase in the operating costs to handle the extra water. More importantly, this can lead to a decrease in ultimate oil recovery as these added costs force stoppage of the project before flooding of all the reservoir layers is complete. In enhanced recovery projects that use polymers, surfactants, or micellar fluids, these streaks can dramatically boost costs by causing inefficient use of these expensive chemicals. Therefore, before starting any fluid injection project, it is important to determine the existence of reservoir heterogeneities that can cause channeling so that corrective actions can be taken. Field tests with well-to-well tracers and the proper analysis of the test data provide one possible means of making that determination.

O. R. Wagner² has presented a list of the specific types of information obtainable from well-to-well tracer tests. This list includes:

(1) volumetric sweep--the volume of fluid injected until breakthrough of tracer in the producer gives an indication of the sweep efficiency. A small volume would be indicative of a fracture or high permeability streak.

(2) identification of offending injectors--by using different tracers in different injectors in the pattern, one can determine which injector is causing early breakthrough in specific producers.

(3) directional flow trend--again by using different tracers in different injectors, one can spot the preferential direction of flow from injectors to producers.

(4) delineation of flow barriers--the failure of a tracer to travel from an injector to a producer would indicate that a flow barrier exists between the two wells.

(5) relative velocities of injected fluids--if different fluids are tagged with different tracers and then injected down the same well, the arrival times of the different tracers at a producer would give an indication of the relative velocity of their respective fluids.

(6) evaluation of sweep improvement treatments--use of tracers before and after a treatment can give an indication of the effectiveness of the work.

D. E. Baldwin, Jr.,³ developed a computer model to predict the produced concentration profile of a tracer as it moved through a five-spot pattern. He divided the five-spot pattern into a series of diverging-converging radial elements between the injector and the producer. Each element had an included angle of one degree. The flow rates in each element were adjusted so that the overall fractional flow behavior would match that of a five-spot pattern. He then derived the mathematical equations that described the dispersion of the tracer as it moved down each element.

Raimondi et al.⁴ presented an approximate solution of the dispersion problem for diverging radial flow. W. E. Brigham and D. H. Smith applied Raimondi's solution to a five-spot pattern. Brigham later revised this work to account for diverging-converging radial flow. Because these revisions were never published, Brigham and Smith's original paper has been included in this report as Appendix A, with handwritten revisions included.

Tracer test data usually comes in the form (or can be put in the form) of "tracer concentration at the producing well" versus "total volume of fluid injected into the pattern." Brigham and Smith's work provides a dimensionless solution to the problem of quantifying the behavior of tracer flowing in a single layer in a five-spot pattern. The algorithm presented in this paper applies this dimensionless, single-layer solution to field test data coming from real multi-layer reservoirs.

II. METHOD OF ANALYSIS

Brigham and Smith derived an integral equation to relate the tracer concentration at the producing well to the volume of fluid injected into one layer.

$$\frac{C}{C_{mp}} (PV) = \int_a^b \frac{dF_D}{dQ_i} e^{-\frac{3L(Q_i - PV)^2}{\pi^2 \alpha}} dQ_i \quad (1)$$

where C = concentration measured in the producing well

C_{mp} = the layer's peak tracer concentration in the reservoir in the vicinity of the producing well

PV = displaceable pore volumes of fluid injected

L = distance from the injector to the producer, ft

α = dispersion constant defined as equal to $2\pi\phi rK/Q$ in units of ft

F_D = fraction of injected fluid in the producing stream

Q_i = displaceable pore volume, a variable of integration in units of pore volume in Eq. 1

ϕ = porosity, fraction

K = dispersion coefficient, ft^2/sec

Q = volumetric injection rate, $\text{ft}^3/\text{sec-ft}$

r = distance away from injector, ft

a,b = limits of integration that depend on the injection pattern used

It should be noted that the dispersion "constant," α , is constant only if the dispersion coefficient, K, is assumed to be proportional to fluid velocity. To show this, assume:

$$K = \text{velocity} \cdot \text{constant} \quad (2)$$

Since

$$\text{velocity} = \frac{Qh}{2\pi r h \phi} = \frac{Q}{2\pi r \phi} \quad (3)$$

Then

$$K = \frac{Q}{2\pi r \phi} \cdot \text{constant} \quad (4)$$

The dispersion "constant" can then be expressed as:

$$\alpha = \frac{2\pi \phi r K}{Q} = \frac{2\pi \phi r}{Q} \cdot \frac{Q}{2\pi r \phi} \cdot \text{constant} = \text{constant} \quad (5)$$

For a five-spot pattern, Brigham and Smith empirically related F_D and Q_i by the equation:

$$\log \left[\log \left(\frac{1.07}{1.07 - F_D} \right) \right] = -.0410 + .581 \log(Q_i - .72) \quad (6)$$

for $.72 < Q_i < 2.29$

Taking this equation, solving for F_D , and then differentiating with respect to Q_i yields:

$$\frac{dF_D}{dQ_i} = 1.302728 \left[(Q_i - .72)^{-.419} \right] \left[10^{-.909913[(Q_i - .72)^{.581}] } \right] \quad (7)$$

We can substitute Eq. 7 into Eq. 1 to get an expression applicable to a five-spot pattern:

$$\frac{C}{C_{mp}} (PV) = \int_{.72}^{2.29} 1.302728 \left[(Q_i - .72)^{-.419} \right] \left[10^{-.909913[(Q_i - .72)^{.581}] } \right] \exp \left[-\left[\frac{3(Q_i - PV)^2}{\pi^2} \right] \frac{L}{\alpha} \right] dQ_i \quad (8)$$

The results of solving Eq. 8 numerically are shown in Tables I and II, and graphed on Figs. 1 and 2. Table II and Fig. 2 are included to show that they essentially match the hand-calculated numbers of C/C_{mp} shown in Fig. 5 of Brigham and Smith's paper. The numbers in Tables I and II were calculated using an adaptive quadrature subroutine based on the eight-point Newton-Cotes integration formula.

To apply Brigham and Smith's solution, we make the same assumptions they did:

- (1) the permeability and porosity are homogeneous within each layer,
- (2) the dispersion coefficient, K , is proportional to the velocity of the fluid (so that α is constant),
- (3) the mobility ratio is one;
- (4) the water saturation is constant in each layer.

We also make the following additional assumptions to apply this single-layer solution to our multi-layer system:

- (1) there is no crossflow between layers,
- (2) the dispersion constant, α , is the same for all layers,
- (3) the distribution of fluid injected into each layer is proportional to the kh of each layer,
- (4) the distribution of the tracer material is also proportional to the kh of each layer.

To develop the analysis algorithm for a multi-layer system, consider layer j in our model (Fig. 3). If the injection rate into layer j , i_j , is proportional to $(kh)_j$, then

TABLE I: DIMENSIONLESS SOLUTION FOR QUANTIFYING TRACER BEHAVIOR IN A FIVE-SPOT PATTERN

L/ALPHA= PV	500.0 C/Cmp	1000.0 C/Cmp	5000.0 C/Cmp	10000.0 C/Cmp	50000.0 C/Cmp
0.500	0.9747E-04	0.7740E-05	0.2146E-30	0.2428E-62	0.0000E 00
0.510	0.1916E-03	0.2860E-04	0.1479E-27	0.1153E-56	0.0000E 00
0.520	0.3657E-03	0.5876E-06	0.7517E-25	0.2979E-51	0.0000E 00
0.530	0.6776E-03	0.1972E-05	0.2820E-22	0.4192E-46	0.0000E 00
0.540	0.1219E-02	0.6241E-05	0.7806E-20	0.3212E-41	0.0000E 00
0.550	0.2130E-02	0.1861E-04	0.1594E-17	0.1340E-36	0.0000E 00
0.560	0.3615E-02	0.5229E-04	0.2403E-15	0.3044E-32	0.0000E 00
0.570	0.5960E-02	0.1385E-03	0.2672E-13	0.3765E-28	0.0000E 00
0.580	0.9542E-02	0.3457E-03	0.2193E-11	0.2536E-24	0.0000E 00
0.590	0.1484E-01	0.8137E-03	0.1328E-09	0.9297E-21	0.0000E 00
0.600	0.2244E-01	0.1806E-02	0.5933E-08	0.1856E-17	0.0000E 00
0.610	0.3296E-01	0.3782E-02	0.1956E-06	0.2018E-14	0.0000E 00
0.620	0.4706E-01	0.7472E-02	0.4759E-05	0.1194E-11	0.1875E-64
0.630	0.6533E-01	0.1394E-01	0.3244E-06	0.3848E-09	0.6516E-52
0.640	0.8819E-01	0.2454E-01	0.4556E-05	0.6751E-07	0.1083E-40
0.650	0.1158E 00	0.4084E-01	0.4768E-04	0.6449E-05	0.8621E-31
0.660	0.1480E 00	0.6424E-01	0.3714E-03	0.1080E-05	0.3283E-22
0.670	0.1840E 00	0.9560E-01	0.2158E-02	0.3362E-04	0.5981E-15
0.680	0.2229E 00	0.1347E 00	0.9384E-02	0.5799E-03	0.5214E-09
0.690	0.2631E 00	0.1800E 00	0.3068E-01	0.5579E-02	0.2175E-04
0.700	0.3027E 00	0.2284E 00	0.7595E-01	0.3029E-01	0.1035E-03
0.710	0.3397E 00	0.2755E 00	0.1439E 00	0.9477E-01	0.1369E-01
0.720	0.3724E 00	0.3168E 00	0.2126E 00	0.1776E 00	0.1153E 00
0.730	0.3988E 00	0.3482E 00	0.2516E 00	0.2148E 00	0.1232E 00
0.740	0.4180E 00	0.3668E 00	0.2493E 00	0.1922E 00	0.8086E-01
0.750	0.4291E 00	0.3721E 00	0.2199E 00	0.1528E 00	0.6295E-01
0.760	0.4321E 00	0.3650E 00	0.1848E 00	0.1238E 00	0.5273E-01
0.770	0.4277E 00	0.3484E 00	0.1561E 00	0.1054E 00	0.4578E-01
0.780	0.4168E 00	0.3254E 00	0.1352E 00	0.9267E-01	0.4064E-01
0.790	0.4007E 00	0.2995E 00	0.1200E 00	0.8309E-01	0.3663E-01
0.800	0.3809E 00	0.2733E 00	0.1085E 00	0.7549E-01	0.3338E-01
0.810	0.3588E 00	0.2486E 00	0.9915E-01	0.6926E-01	0.3069E-01
0.820	0.3358E 00	0.2266E 00	0.9144E-01	0.6402E-01	0.2841E-01
0.830	0.3129E 00	0.2074E 00	0.8489E-01	0.5953E-01	0.2645E-01
0.840	0.2909E 00	0.1910E 00	0.7924E-01	0.5563E-01	0.2474E-01
0.850	0.2704E 00	0.1770E 00	0.7429E-01	0.5221E-01	0.2324E-01
0.860	0.2516E 00	0.1650E 00	0.6992E-01	0.4917E-01	0.2190E-01
0.870	0.2346E 00	0.1546E 00	0.6602E-01	0.4646E-01	0.2070E-01
0.880	0.2195E 00	0.1455E 00	0.6251E-01	0.4401E-01	0.1962E-01
0.890	0.2060E 00	0.1375E 00	0.5934E-01	0.4179E-01	0.1863E-01
0.900	0.1939E 00	0.1303E 00	0.5645E-01	0.3977E-01	0.1774E-01
0.910	0.1832E 00	0.1238E 00	0.5381E-01	0.3792E-01	0.1692E-01
0.920	0.1736E 00	0.1179E 00	0.5138E-01	0.3622E-01	0.1616E-01
0.930	0.1650E 00	0.1125E 00	0.4914E-01	0.3465E-01	0.1546E-01
0.940	0.1572E 00	0.1076E 00	0.4707E-01	0.3320E-01	-0.5615E-05 *
0.950	0.1501E 00	0.1030E 00	0.4514E-01	0.3185E-01	0.1422E-01
0.960	0.1435E 00	0.9879E-01	0.4336E-01	0.3059E-01	0.1366E-01
0.970	0.1375E 00	0.9487E-01	0.4168E-01	0.2941E-01	0.1313E-01
0.980	0.1320E 00	0.9120E-01	0.4012E-01	0.2831E-01	0.1264E-01
0.990	0.1268E 00	0.8778E-01	0.3865E-01	0.2728E-01	0.1161E-04
1.000	0.1220E 00	0.8457E-01	0.3727E-01	0.2631E-01	0.1175E-01

*Apparent numerical error

FIG. DIMENSIONLESS SOLUTION FOR QUANTIFYING TRACER BEHAVIOR IN A FIVE-SPOT PATTERNS

TABLE II: DIMENSIONLESS SOLUTION FOR QUANTIFYING TRACER BEHAVIOR IN A FIVE-SPOT PATTERN

L/ALPHA = PV	417.0 C/Cmp	1250.0 C/Cmp	4170.0 C/Cmp	12500.0 C/Cmp	41700.0 C/Cmp
0.500	0.3618E-03	0.1956E-06	0.4313E-25	0.0000E 00	0.0000E 00
0.510	0.6378E-03	0.1002E-05	0.1004E-22	0.3218E-71	0.0000E 00
0.520	0.1097E-02	0.4759E-05	0.1815E-20	0.1875E-64	0.0000E 00
0.530	0.1841E-02	0.2094E-04	0.2545E-18	0.5112E-58	0.0000E 00
0.540	0.3015E-02	0.4743E-06	0.2770E-16	0.6516E-52	0.0000E 00
0.550	0.4820E-02	0.1843E-05	0.2339E-14	0.3885E-46	0.0000E 00
0.560	0.7520E-02	0.6658E-05	0.1534E-12	0.1084E-40	0.0000E 00
0.570	0.1145E-01	0.2233E-04	0.7802E-11	0.1413E-35	0.0000E 00
0.580	0.1703E-01	0.6956E-04	0.7080E-09	0.8622E-31	0.0000E 00
0.590	0.2471E-01	0.2012E-03	0.9438E-08	0.2460E-26	0.0000E 00
0.600	0.3503E-01	0.5407E-03	0.2244E-06	0.3283E-22	0.0000E 00
0.610	0.4848E-01	0.1350E-02	0.4142E-05	0.2049E-18	0.4675E-65
0.620	0.6554E-01	0.3134E-02	0.2442E-06	0.5981E-15	0.1698E-53
0.630	0.8657E-01	0.6766E-02	0.2753E-05	0.8166E-12	0.4887E-43
0.640	0.1117E 00	0.1359E-01	0.2523E-04	0.5214E-09	0.1115E-33
0.650	0.1409E 00	0.2541E-01	0.1806E-03	0.1557E-06	0.2016E-25
0.660	0.1738E 00	0.4425E-01	0.1012E-02	0.2176E-04	0.2889E-18
0.670	0.2096E 00	0.7189E-01	0.4444E-02	0.4461E-05	0.3281E-12
0.680	0.2472E 00	0.1090E 00	0.1535E-01	0.1529E-03	0.2953E-07
0.690	0.2854E 00	0.1546E 00	0.4187E-01	0.2514E-02	0.4485E-06
0.700	0.3227E 00	0.2053E 00	0.9073E-01	0.2011E-01	0.3130E-03
0.710	0.3573E 00	0.2559E 00	0.1570E 00	0.8018E-01	0.1924E-01
0.720	0.3879E 00	0.3003E 00	0.2227E 00	0.1674E 00	0.1212E 00
0.730	0.4132E 00	0.3332E 00	0.2614E 00	0.2029E 00	0.1339E 00
0.740	0.4320E 00	0.3509E 00	0.2635E 00	0.1734E 00	0.8943E-01
0.750	0.4439E 00	0.3530E 00	0.2385E 00	0.1344E 00	0.6918E-01
0.760	0.4487E 00	0.3416E 00	0.2043E 00	0.1092E 00	0.5785E-01
0.770	0.4467E 00	0.3206E 00	0.1736E 00	0.9349E-01	0.5020E-01
0.780	0.4387E 00	0.2944E 00	0.1501E 00	0.8245E-01	0.4454E-01
0.790	0.4255E 00	0.2669E 00	0.1328E 00	0.7404E-01	0.4013E-01
0.800	0.4083E 00	0.2408E 00	0.1196E 00	0.6733E-01	0.3658E-01
0.810	0.3884E 00	0.2177E 00	0.1092E 00	0.6180E-01	0.3362E-01
0.820	0.3668E 00	0.1979E 00	0.1005E 00	0.5715E-01	0.3113E-01
0.830	0.3445E 00	0.1812E 00	0.9328E-01	0.5316E-01	0.2898E-01
0.840	0.3225E 00	0.1672E 00	0.8702E-01	0.4969E-01	0.2710E-01
0.850	0.3013E 00	0.1554E 00	0.8156E-01	0.4664E-01	0.2545E-01
0.860	0.2813E 00	0.1453E 00	0.7673E-01	0.4394E-01	0.2398E-01
0.870	0.2628E 00	0.1365E 00	0.7243E-01	0.4151E-01	0.2267E-01
0.880	0.2460E 00	0.1287E 00	0.6857E-01	0.3933E-01	0.2148E-01
0.890	0.2307E 00	0.1218E 00	0.6508E-01	0.3735E-01	0.2041E-01
0.900	0.2170E 00	0.1155E 00	0.6190E-01	0.3555E-01	0.1943E-01
0.910	0.2047E 00	0.1099E 00	0.5900E-01	0.3390E-01	0.1853E-01
0.920	0.1937E 00	0.1047E 00	0.5633E-01	0.3238E-01	0.1770E-01
0.930	0.1837E 00	0.1000E 00	0.5387E-01	0.3098E-01	0.1693E-01
0.940	0.1747E 00	0.9567E-01	0.5160E-01	0.2968E-01	-0.1624E-04*
0.950	0.1666E 00	0.9166E-01	0.4948E-01	0.2847E-01	0.1557E-01
0.960	0.1591E 00	0.8793E-01	0.4752E-01	0.2735E-01	0.1495E-01
0.970	0.1523E 00	0.8447E-01	0.4568E-01	0.2630E-01	0.1438E-01
0.980	0.1460E 00	0.8123E-01	0.4397E-01	0.2531E-01	0.1384E-01
0.990	0.1402E 00	0.7820E-01	0.4236E-01	0.2439E-01	0.1334E-01
1.000	0.1348E 00	0.7536E-01	0.4084E-01	0.2352E-01	0.1287E-01

*Apparent numerical error

FIG. 2: DIMENSIONLESS SOLUTION FOR QUANTIFYING TRACER BEHAVIOR IN A FIVE-SPOT PATTERN

FIG. 3: MULTI-LAYERED, FIVE-SPOT MODEL ASSUMED BY THE ALGORITHM

$$i_j = i \cdot \frac{(kh)_j}{\Sigma kh} \quad (9)$$

where i is the total injection rate.

The total fluid injected into layer j , V_j , is

$$V_j = i_j \cdot \text{TIME} = i \cdot \frac{(kh)_j}{\Sigma kh} \cdot \text{TIME} = V \cdot \frac{(kh)_j}{\Sigma kh} \quad (10)$$

where V is the total injection volume in barrels.

The pore volume injected into layer j , PV_j , can be then described as

$$PV_j = \frac{V_j \cdot 5.615}{A(h\phi)_j S_w} = \frac{5.615}{A(h\phi)_j S_w} \cdot V \cdot \frac{(kh)_j}{\Sigma kh} = \frac{5.615}{AS_w} \cdot \frac{k_j}{\phi_j \Sigma kh} \cdot V \quad (11)$$

where A = injection pattern area, sq ft

$(h\phi)_j$ = porosity-thickness of layer j , ft

S_w = water saturation, fraction

We can assume that the arrival of a concentration peak in the field data corresponds with the arrival of a concentration peak from one of the layers. As will be discussed later, this is only an approximation of the actual arrival of a layer's peak, but it will still give reasonably accurate answers.

The location of the peak in the dimensionless solution PV_p , varies with L/α . As shown in Fig. 4, the following empirical equation relates PV_p and L/α .

$$PV_p = .72 + .580541 (L/\alpha)^{-.430043} \quad (12)$$

FIG. 4: EMPIRICAL FIT TO RELATE L/α WITH LOCATION OF PEAK IN DIMENSIONLESS SOLUTION

This equation was derived by fitting a least-squares log-log straight line through the PV_p versus L/α data and by noting that PV_p approaches .72 when L/α approaches infinity. Knowing L/α , we can now calculate PV_p . If α is assumed to be the same for each layer, then this PV_p is the same for each layer also.

Taking the total injection volume at which a peak occurs at the producing well and this calculated value for PV_p , we can then use Eq. 11 and solve for

$$\frac{k_j}{\phi_j \Sigma kh} = \frac{PV_p}{V_{pj}} \frac{AS_w}{5.615} \quad (13)$$

If there are n peaks, we can infer n layers and calculate a separate value that is characteristic of each layer. Once we know this value of $k/\phi \Sigma kh$ for a layer, then we can use Eq. 11 to calculate PV_j for any V we desire. Knowing PV_j , we can calculate a value for C/Cmp for that layer at that V.

Equation 15 in Brigham and Smith's paper relates the Cmp for a layer to the weight of tracer entering the layer.

$$m = 400 h \phi S_w Cmp L^{1.5} \alpha^{.5} \quad (14)$$

where Cmp is in units of weight fraction and m is the weight of tracer in pounds.

Rearranging this equation and converting Cmp to units of parts per million, we get for layer j:

$$Cmp_j = \frac{m_j}{.0004 (h \phi)_j S_w L^{1.5} \alpha^{.5}} \quad (15)$$

For any value of C_j/C_{mp_j} that we obtain from the dimensionless solution, we can calculate the tracer concentration in the fluid entering the producing well from layer j as:

$$C_j = \frac{C_j}{C_{mp_j}} \cdot C_{mp_j} = \frac{C_j}{C_{mp_j}} \cdot \frac{m_j}{.0004 (h\phi)_j S_w L^{1.5} \alpha^{.5}} \quad (16)$$

If the amount of tracer in layer j , m_j , is assumed to be proportional to kh , then:

$$m_j = m \cdot \frac{(kh)_j}{\Sigma kh} \quad (17)$$

where m is the total amount of tracer injected into the reservoir in pounds.

C_j can then be expressed as:

$$\begin{aligned} C_j &= \frac{C_j}{C_{mp_j}} \cdot \frac{m}{.0004 (h\phi)_j S_w L^{1.5} \alpha^{.5}} \cdot \frac{(kh)_j}{\Sigma kh} \\ &= \frac{C_j}{C_{mp_j}} \cdot \frac{m}{.0004 S_w L^{1.5} \alpha^{.5}} \cdot \frac{k_j}{\phi_j \Sigma kh} \end{aligned} \quad (18)$$

Upon entering the wellbore, this fluid from layer j is mixed with fluids from the other layers. This mixing gives us the overall concentrations seen in the wellbore. If the amount of fluid entering the producing wellbore from each layer is proportional to the kh of each layer, then the overall concentration can be expressed as a weighted sum:

$$\begin{aligned}
 C &= \sum C_j \cdot \frac{(kh)_j}{\sum kh} \\
 &= \sum \frac{C_j}{C_{mp_j}} \cdot \frac{m}{.0004 S_w L^{1.5} \alpha^{.5}} \cdot \frac{k_j}{\phi_j \sum kh} \cdot \frac{(kh)_j}{\sum kh}
 \end{aligned} \tag{19}$$

Since $k_j/\phi_j \sum kh$ was calculated in Eq. 13, and C_j/C_{mp_j} can be calculated from the dimensionless solution, the unknowns of this equation are the terms $(kh)_j/\sum kh$. If we take n values of C from our field data, we can write n equations in the form of Eq. 19. For an n layer system, this means we will have a system of n equations and n unknowns. In our algorithm, we will use the concentration values at the n peaks found in the field data. However, any n points could have been used. Letting C_{p_i} (for $i = 1, \dots, n$) be the data points we pick out from our field, we can rewrite Eq. 19 as:

$$C_{p_i} = \sum \left(\frac{C_j}{C_{mp_j}} \right)_i \frac{m}{.0004 S_w L^{1.5} \alpha^{.5}} \frac{k_j}{\phi_j \sum kh} \cdot \frac{(kh)_j}{\sum kh} \tag{20}$$

$$\text{Setting } a(i,j) = \left(\frac{C_j}{C_{mp_j}} \right)_i \frac{m}{.0004 S_w L^{1.5} \alpha^{.5}} \frac{k_j}{\phi_j \sum kh}$$

$$x(j) = (kh)_j / \sum kh$$

$$b(i) = C_{p_i}$$

We now have an $n \times n$ matrix equation of the form $\vec{b} = A\vec{x}$.

After solving this matrix equation, we have a value for $(kh)_j/\sum kh$ as well as $k_j/\phi_j \sum kh$ to characterize each layer. We can divide one by the other to get $(\phi k)_j$.

$$(\phi h)_j = \frac{(kh)_j}{\Sigma kh} \div \frac{k_j}{\phi_j \Sigma kh} \quad (21)$$

The ratio $kh/\Sigma kh$ can be thought of as a term describing the layer's relative conductance, and ϕh can be thought of as describing the layer's capacity.

Attached as Appendix B is a computer program to carry out this algorithm. Given the necessary input data, it will calculate values for $kh/\Sigma kh$ and $h\phi$ for each layer. It will then take these values and generate expected values for overall concentration versus total volume injected. These values can be compared with the field data to judge the accuracy of the analysis.

The necessary input data are:

- (1) L = distance between injector and producer, ft,
- (2) α = dispersion constant defined in the Brigham and Smith paper, ft,
- (3) n = number of layers expected,
- (4) m = total weight of tracer injected, lbs,
- (5) S_w = water saturation in the reservoir, fraction,
- (6) area = area of injection pattern, sq ft,
- (7) $V_{\min}, V_{\max}$ = this sets the range at which the "overall concentration" versus "total injection volume" values will be calculated for comparison with the field data. The program will generate 51 values between $V_{\min}$ and $V_{\max}$, inclusive,
- (8) $V_p(j)$ = location of each peak in the field data in units of bbls,
- (9) $C_p(j)$ = height of each peak in the field data in concentration units of parts per million.

The following flowchart summarizes the steps taken by this program.

FLOWCHART FOR COMPUTER ALGORITHM

A

CALCULATE EACH ELEMENT OF THE MATRIX:

$$a(i,j) = \frac{C}{C_{mp}} (i,j) \cdot \frac{m}{.0004 \cdot S_w \cdot L^{1.5} \cdot \alpha \cdot .5} \cdot \frac{k_j}{\phi_j \Sigma kh}$$

USE THE HEIGHT OF EACH FIELD DATA PEAK [CP(i)] AS THE ELEMENTS OF VECTOR $\vec{b}$; USE $kh_j / \Sigma kh$ AS THE ELEMENTS OF THE UNKNOWN VECTOR $\vec{x}$; THEN SOLVE THE MATRIX EQUATION $\vec{b} = A\vec{x}$ USING SUBROUTINES "DECMPI" AND "SOLVE1": THIS WILL GET VALUES FOR $kh_j / \Sigma kh$.

CALCULATE $h\phi$ FOR EACH LAYER:

$$(h\phi)_j = \frac{(kh)_j}{\Sigma kh} \div \frac{k_j}{\phi_j \Sigma kh}$$

TO GENERATE CONCENTRATION VS INJECTION VOLUME DATA:

CALCULATE V AT WHICH CONCENTRATION VALUES ARE DESIRED, STARTING WITH V_{min} AND ENDING WITH V_{max}

CONVERT V TO PV FOR EACH LAYER BY:

$$PV_j = V \cdot \frac{k_j}{\phi_j \Sigma kh} \cdot \frac{5.615}{AREA \cdot S_w}$$

CALCULATE C_j / C_{mpj} FOR EACH LAYER USING PV_j AND SUBROUTINE "QUANC8"

B

C

(B)

CALCULATE OVERALL CONCENTRATION BY:

$$C = \sum \frac{C_j}{C_{mp_j}} \cdot \frac{m}{.0004 S_w L^{1.5} \alpha^{.5}} \cdot \frac{k_j}{\phi_j \sum kh} \cdot \frac{(kh)_j}{\sum kh}$$

GET NEW V

(C)

III. APPLICATION AND DISCUSSION OF RESULTS

Figures 5, 6, and 7 show the results of applying this computer program to three different sets of simulated field data for a two-layer reservoir. The solid line on each figure represents the simulated data and the triangular points show the match obtained by the program. The input values for $kh/\Sigma kh$ and $h\phi$ used to generate the simulated data are shown on Table III. For comparison, Table III also shows the corresponding values of $kh/\Sigma kh$ and $h\phi$ calculated by the program.

As these results show, the program was able to accurately calculate $kh/\Sigma kh$ and $h\phi$ values for each layer. The farther apart the peaks in the data were, the more accurate the analysis. But even in the worst case of Example 3, where one peak is almost hidden by the other, the error in $kh/\Sigma kh$ and $h\phi$ is under five percent.

This error that results when the peaks are close together can be attributed to the assumption made in the algorithm that a peak in the field data corresponds with the arrival to the wellbore of a peak from one of the layers.

When the peaks are far apart, the shape of the overall concentration curve is dominated by the tracer in only one layer at a time. In this case, the peak in the overall curve corresponds extremely closely with the arrival of a peak in one of the layers. However, in the case where the peaks are close together, the shape of the overall curve can be affected by more than one layer.

TABLE III: RESULTS OF ANALYZING SIMULATED FIVE-SPOT TRACER TEST DATA

EXAMPLE	LAYER	PARAMETERS USED TO GENERATE SIMULATED DATA					RESULT OF ANALYSIS	
		K(md)	h(ft)	ϕ	kh/ Σ kh	h ϕ	kh/ Σ kh	h ϕ
1	1	1,000	1	.30	.182	0.30	.1811	0.298
	2	500	9	.20	.818	1.80	.8182	1.800
2	1	1,000	3	.30	.462	0.90	.4618	0.900
	2	500	7	.18	.538	1.26	.5373	1.256
3	1	750	4	.25	.500	1.00	.4733	0.952
	2	500	6	.18	.500	1.08	.4908	1.055
4	1	1,000	1	.35	.233	0.35	.2327	0.350
	2	750	1	.30	.174	0.30	.1732	0.297
	3	400	2	.18	.186	0.36	.1860	0.358
	4	500	1	.25	.116	0.25	.1180	0.252
	5	250	5	.14	.291	0.70	.2925	0.703
5	1	1,000	1	.40	.286	0.40	.2861	0.401
	2	500	1	.28	.143	0.28	.1369	0.267
	3	250	8	.15	.571	1.20	.5765	1.209

In all above cases:

Total mass of tracer injected, m = 200 lbs
 Distance between injector and producer, L = 233.3 ft
 Area of pattern, A = 108,858 ft²
 Dispersion constant, α = 0.05 ft
 Water saturation, S_w = 0.55

FIG. 5: RESULTS OF ANALYZING DATA OF EXAMPLE 1

FIG. 6: RESULTS OF ANALYZING DATA OF EXAMPLE 2

A more analytical way of looking at this effect is to note that when two equally weighted peaking curves are added together, the peaks in the resulting curve appear at the locations where the slopes of the individual curves are equal in magnitude but opposite in sign. If the peaks are close together, the locations of these equal but opposite slopes will be away from the location of the individual peaks. This same general idea controls the location of the peaks in the case of adding the unequally weighted curves of a tracer test.

To illustrate this point, the simulated overall concentration field data of Example 3 have been redrawn as the solid curve in the middle in Fig. 8. The concentration profiles for each of the two layers have been included at the bottom of the same figure. Note that the peaks of the individual curves are at 15,650 and 16,950 barrels, but that the peaks in the overall curve are at 15,775 and 16,850 barrels. The peaks in the individual concentration profiles appear to have "shifted" in location after being combined to get the overall curve.

When the data in Example 3 were analyzed using 15,775 and 16,850 barrels as the location of the peaks in the individual layers instead of 15,650 and 16,950 barrels, a noticeable error resulted in the calculation of $kh/\sum kh$ and $h\phi$. The error is also demonstrated by the difference between the field data curves and the curves outlined by the triangular points in Fig. 8. These outlined curves are the concentration profiles calculated by the algorithm for the individual layers and for the overall system.

To get a more accurate calculation of $kh/\sum kh$ and $h\phi$, we could:

(1) look at Fig. 7 and see that the true location of the peaks in the individual layers must be farther apart than we had assumed before,

FIG. 8: CONCENTRATION PROFILES FOR EXAMPLE 3

(2) guess a true location for each of the peaks. The guesses could be guided by trying to determine where the slopes of the individual curves would be equal in magnitude and opposite in sign,

(3) rerun the program using the new guesses,

(4) compare the new calculated overall concentration values with the field data.

We could continue this process until we got an acceptable match with the field data.

Further work can be done on this problem by finding an analytical method or a computerized iterative method to locate the true individual peak locations.

To demonstrate the use of this program with a system of more than two peaks, Example 4 on Table III shows the results of a hypothetical five-layer problem. The program included as Appendix B in this paper is written to handle up to a five-layer system. It can be modified to handle any multi-layer system by simply changing the dimension of the array and the vector variables.

The results for Example 4 are plotted on Fig. 9. The program gives fairly accurate calculations for $kh/\Sigma kh$ and $h\phi$ for each layer, but, like Example 3, the close bunching of the peaks causes some error in the results.

One potential problem in analyzing field data is in determining the number of layers in the system. In cases where two peaks are very close together, one peak may be "hidden" in the data. Such a case is shown as Example 5 in Fig. 10. At first glance, there appears to be only two layers indicated by the two field data peaks. Upon applying the algorithm

FIG. 9: RESULTS OF ANALYZING DATA OF EXAMPLE 4

FIG. 10: RESULTS OF ANALYZING DATA OF EXAMPLE 5

using these two peaks and assuming two layers, the triangular points of the figure show the resulting match. The large gap in the match around 15,000 barrels suggests the existence of another peak, and hence another layer. As a first guess, we could assume that the third peak is located near the point where the separation of the match is the greatest. Re-running the program assuming a third peak at 15,300 barrels resulted in the circular match points shown in Fig. 10. The process could be repeated by adjusting the guesses of all three peak locations until an acceptable match is obtained.

As a last example, Fig. 11 shows the results of applying the algorithm to an actual tracer test. The field data used here have been presented in Brigham and Smith's paper as the ammonium thiocyanate tracer test in Well D. It is redrawn as the solid curve on Fig. 11.

From the solid curve, peaks were picked at 2,650 barrels at 30 ppm, 2,980 barrels at 28.5 ppm, 3,240 barrels at 31.5 ppm, and 3,620 barrels at 27.5 ppm. As described in the Brigham and Smith paper, before the field data could be analyzed it had to be adjusted to account for the production of fluid from outside the injection pattern. The well produced an average of 240 BWPD, and it was estimated that the injector contributed 205 BWPD into the quadrant of the pattern drained by this producer. The other 35 BWPD must have been coming from outside the pattern. This outside water diluted the tracer entering the wellbore by a factor of 205/240. The concentrations given in the field data therefore had to be multiplied by a factor of 240/205 in order to make the concentrations compatible with the model assumed by the algorithm. The field data gave the location of the peaks as a function of fluid produced by the well instead of as a function of fluid injected into the

FIG. 11: ACTUAL FIELD DATA (AMMONIUM THIOCYANATE TRACER DATA FROM WELL D IN BRIGHAM AND SMITH'S PAPER [Ref. 1])

pattern. The "volumes injected" were determined by multiplying the "volumes produced" by 205/240. After adjustment, the peak locations and heights were input as 2,264 barrels at 35.1 ppm, 2,545 barrels at 33.4 ppm, 2,768 barrels at 36.9 ppm, and 3,092 barrels at 32.2 ppm.

Since we are considering only Well D, the pattern area input was one-fourth of two and one-half acres, or 27,225 sq ft. Also, since the injected fluid was not distributed equally among the producers, the value input for m , the amount of tracer material, had to be adjusted to an "effective" value. If the injection fluid were equally distributed and Well D received exactly one-fourth of the 600 BWPD injected, or 150 BWPD, then the "effective" value for m for Well D's system would be 200 lbs. However, since Well D received 205 BWPD of the injected fluid instead of 150 BWPD, it received a larger share of the tracer material. Its "effective" m was greater than 200 lbs by a factor of 205/150. The value of m used for this analysis was thus set at 273 lbs. The value of α used was 0.05 and the water saturation was 0.55.

The results of this analysis are shown on Fig. 11 and Table IV. Four permeability streaks are assumed to be causing the concentration profile seen in the field data. As shown on the table, these four streaks are calculated to have $kh/\Sigma kh$ values of 0.1002, 0.0664, 0.0697, and 0.0587, and $h\phi$ values of 0.1157, 0.0862, 0.0983, and 0.0925. Since the four $kh/\Sigma kh$ values total 0.295, this indicates that these four streaks make up only 29.5% of the total kh of the reservoir.

In their paper, Brigham and Smith report that the reservoir has a thickness of twelve feet, an average permeability of 1,500 md, and an average porosity of 0.26. If we assume that this average porosity value applies to each layer, we can use these reservoir parameters and the

TABLE IV: RESULTS OF ANALYZING ACTUAL FIELD DATA (WELL D AMMONIUM THIOCYANATE TRACER TEST DESCRIBED IN THE BRIGHAM AND SMITH PAPER [REF. 1])

<u>LAYER</u>	<u>kh/Σkh</u>	<u>hφ</u>
1	.1002	.1157
2	.0664	.0862
3	.0697	.0983
4	.0587	.0925
	<u>.2950</u>	

Assuming: average permeability = 1,500 md
total thickness = 12 ft
average porosity = .26 (assumed same for all layers)

Then: $\Sigma kh = 1,500 \text{ md} \cdot 12 \text{ ft} = 18,000 \text{ md-ft}$

$$kh_i = \frac{kh_i}{\Sigma kh} \cdot \Sigma kh = \frac{kh_i}{\Sigma kh} \cdot 18,000 \text{ md-ft}$$

$$h_i = \frac{(h\phi)_i}{\phi_i} = \frac{h\phi}{.26}$$

$$k_i = \frac{(kh)_i}{h_i}$$

<u>LAYER</u>	<u>kh(md-ft)</u>	<u>h(ft)</u>	<u>k(md)</u>
1	1,804	.445	4,050
2	1,195	.332	3,600
3	1,255	.378	3,320
4	1,057	.356	2,970
		<u>1.511</u>	

kh/ Σ kh and $h\phi$ numbers to obtain the permeability and thickness of each streak. We find that these four streaks have permeabilities ranging from 2,970 md to 4,050 md and that they total about one and one-half feet in thickness. This compares reasonably well with the analysis performed by Brigham and Smith that found three streaks totaling about two and one-half feet thick and ranging from 2,980 md to 4,110 md.

IV. CONCLUSIONS

As a result of this work, the following conclusions were reached:

(1) The location of the peaks in Brigham and Smith's dimensionless solution for tracer flow in a five-spot injection pattern can be empirically related to L/α by:

$$PV_p = .72 + .580541 (L/\alpha)^{-.430043}$$

(2) The algorithm presented in this paper can calculate $kh/\Sigma kh$ and $h\phi$ values for a multi-layered reservoir from the type of data obtained from five-spot tracer tests.

(3) The farther apart the peaks in the field data, the more accurate the analysis.

(4) Cases where the peaks are close together result in noticeable errors because the location of a peak in the field data does not quite correspond to the arrival of a peak in one of the layers. However, of the cases examined here, the errors calculated for $kh/\Sigma kh$ and $h\phi$ were less than five percent in the worst case. Rerunning the program several times with adjusted peak locations will probably allow one to locate the correct values.

(5) The method can be applied to locate additional peaks (and therefore additional layers) that can be obscured by the overall concentration data.

(6) The algorithm can be applied to actual field data if the data are first put in a form that is compatible with the model assumed by the algorithm.

V. NOMENCLATURE

- A = area of injection pattern, sq ft
- a, b = limits of integration for Brigham and Smith's solution integral (Eq. 1) that depend on injection pattern used
- C = overall tracer concentration measured in producing well, parts per million
- C_j = tracer concentration of fluid from layer j as it enters producing wellbore, ppm
- C_{mp_j} = layer j 's peak tracer concentration in the reservoir in the vicinity of the producing well, ppm
- C_{p_i} = i th concentration peak in the overall field data, ppm
- $\left(\frac{C_j}{C_{mp_j}} \right)_i$ = dimensionless concentration ratio for layer j when the i th peak in the field data arrives at the producing well
- F_D = fraction of displacing fluid in the producing stream
- h_j = thickness of layer j , ft
- i = total injection rate in pattern area, any consistent units
- i_j = injection rate into layer j , any consistent units
- K = dispersion coefficient, ft^2/sec
- k_j = permeability of layer j , any consistent units
- L = distance between injector and producer, ft
- m = total weight of tracer injected, lbs
- m_j = weight of tracer injected into layer j , lbs
- n = number of layers
- PV_j = displaceable pore volumes injected into layer j
- PV_P = pore volumes injected when peak in dimensionless solution (C/C_{mp}) occurs
- Q = volumetric injection rate, $\text{ft}^3/\text{sec-ft}$

Q_i = variable of integration in Eq. 1 (location volume on F_D curve),
units of effective pore volume

r = radial distance from injector, ft

S_w = water saturation, fraction

V = total volume of fluid injected into pattern area, barrels

V_{p_j} = volume of fluid injected when j th peak in field data appears
at producing well, barrels

α = dispersion constant defined as $2\pi\phi r K/Q$, ft

ϕ_j = porosity of layer j , fraction

Σ = saturation of terms from $j=1, \dots, n$

VI. REFERENCES

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APPENDIX A

"PREDICTION OF TRACER BEHAVIOR IN FIVE-SPOT FLOW"

by W. E. Brigham and D. H. Smith

Note: This paper was originally written assuming diverging radial flow between the injection and producing wells. The handwritten equations are the revisions later made by W. E. Brigham to account for diverging-converging radial flow.

THIS IS A PREPRINT --- SUBJECT TO CORRECTION

PREDICTION OF TRACER BEHAVIOR IN FIVE-SPOT FLOW

By

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ABSTRACT

Equations are developed to predict the time of tracer breakthrough, the peak concentration of the tracer, and the general form of the breakthrough curve in a 5-spot flood. It is shown that these results depend on the amount of stratification of the reservoir, the volumes injected and produced, the natural dispersion coefficient of the tracer in the reservoir, the amount of tracer injected, plus all the reservoir volume parameters (i.e. well spacing, porosity, thickness).

Many laboratory data are available on the breakthrough characteristics of a 5-spot flood, also much data is available on the natural linear dispersion coefficients of reservoir rock. To derive the equations, these data were combined and several assumptions were made. It was also necessary to graphically differentiate the breakthrough data. Thus it should be recognized that the final equations likely have some error. However, this should not invalidate their use, for the method and logic behind the derivation are sound, and thus the form of the final equations should be close to correct. In this paper, the prediction equations are used in a reverse sense. That is, the detailed tracer production history from a field test is used to estimate permeability variation in a 5-spot.

INTRODUCTION

Tracers have been used for many years in

reservoir floods to help the operating engineer understand the flow characteristics. Generally this use has been entirely qualitative. The results of time-of-flight, peak concentrations at the producing wells, concentration history, and directional flow have been used only to substantiate that channelling does nor does not exist. No attempts have been made to predict quantitatively the tracer breakthrough behavior that might be expected from different reservoir characteristics.

In the past couple of years it has become apparent that some prediction technique could be well used to supplement the other tools available to the reservoir engineer. It is becoming increasingly important that maximum recovery be obtained from a flood, and any quantitative information about the reservoir can be of help in achieving this maximum. The work reported here is a step in this direction -- an attempt to quantify the tracer behavior.

Equations are developed to predict the time of tracer breakthrough, the peak concentration of the tracer, and the general form of the breakthrough curve in a 5-spot flood. Tracer production history from a field test are compared to the behavior as predicted by the equations. As more field data become available, and more sophisticated derivations are made, the equation constants can be adjusted and better predictions and

interpretations will be possible in the future.

These calculations were directed for use of tracers where the mobility ratio is near unity. In general, this is valid in many waterfloods and almost all gas cycling projects. The method of approach, however, can be used whenever the reservoir coverage and dispersion characteristics can be approximated. In systems where the mobility ratio is inherently unfavorable and fingering is predominant, the approach cannot yet be used because of the limited knowledge of dispersion in these systems. Thus, at present, this approach would be only partially successful in miscible flooding calculations.

DEVELOPMENT OF EQUATIONS

This section of the paper covers the method of calculating tracer behavior in a homogeneous 5-spot system when the mobility ratio is 1:1. Two characteristics affect the breakthrough history of the tracer; the mixing due to dispersion, and the pattern sweep efficiency. These effects are treated separately and then mathematically combined.

Dispersion or Mixing

When one material miscibly displaces another in a porous medium, mixing occurs in a manner similar to Fick diffusion, except the constant of mixing is changed. In radial flow this equation can be expressed as follows:

$$\frac{\partial C}{\partial r} + \frac{2\pi\phi r}{Q} \frac{\partial C}{\partial t} = \frac{2\pi\phi r K}{Q} \frac{\partial^2 C}{\partial r^2} \quad (1)$$

where C = Concentration (any consistent units)
K = Dispersion coefficient - analogous to the Fick diffusion coefficient (ft²/sec)
r = Radius (ft)
t = Time (sec)
Q = Volumetric injection rate (ft³/sec-ft)

For convenience in writing, the term, $Q/2\pi\phi$, can be called Q' . Another simplification can be made if the dispersion coefficient is assumed proportional to velocity. Experimental evidence shows this is a close approximation. Then the term, $2\pi\phi r K/Q$ (or rK/Q'), can be replaced by α (a constant for any given porous medium). The resulting equation is

$$\frac{\partial C}{\partial r} + \frac{r}{Q'} \frac{\partial C}{\partial t} = \alpha \frac{\partial^2 C}{\partial r^2} \quad (2)$$

Raimondi et al² have shown that this equation can be solved if one realizes that the term on the right hand side of the equation is quite small compared to the other terms. The value of the second partial can then be approximately calculated by assuming the right hand side is zero.

$$\frac{\partial C}{\partial r} + \frac{r}{Q'} \frac{\partial C}{\partial t} \approx 0 \quad (3)$$

The expression for the second partial then becomes,

$$\frac{\partial^2 C}{\partial r^2} \approx -\frac{1}{Q'} \frac{\partial C}{\partial t} + \frac{r^2}{(Q')^2} \frac{\partial^2 C}{\partial t^2} \quad (4)$$

and Equation 2 can be replaced by,

$$\frac{\partial C}{\partial r} + \frac{r}{Q'} \frac{\partial C}{\partial t} \approx -\frac{\alpha}{Q'} \frac{\partial C}{\partial t} + \alpha \frac{r^2}{(Q')^2} \frac{\partial^2 C}{\partial t^2} \quad (5)$$

In Equation 5 there are two multipliers on $\partial C/\partial t$: r/Q' and α/Q' . Experimental data on C (Ref. 1) shows that it is always considerably smaller than one foot; so it can properly be neglected compared to r, and Equation 5 can be simplified to:

$$\frac{\partial C}{\partial r} + \frac{r}{Q'} \frac{\partial C}{\partial t} = \alpha \left(\frac{r}{Q'}\right)^2 \frac{\partial^2 C}{\partial t^2} \quad (6)$$

The equation can be further simplified by changing variables.

Define:

$$\tau = t - \frac{r^2}{2Q}$$

and

$$\rho = \frac{r^3}{3}$$

The resulting expression is:

$$\frac{\partial C}{\partial \rho} = \frac{\alpha}{Q'^2} \frac{\partial^2 C}{\partial \tau^2} \quad (7)$$

When one fluid is displacing another in an infinite, radial porous medium, the solution to Equation 7 is:

$$\begin{aligned} \frac{C}{C_o} &= \frac{1}{2} \operatorname{erfc} \left(\frac{\tau}{2\sqrt{\alpha\rho/Q'^2}} \right) \\ &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q't - r^2/2}{2\sqrt{\alpha r^5/3}} \right) \end{aligned} \quad (8)$$

When a tracer is injected, it is necessary to include both the front and back edges of the tracer slug. The solution will then contain two terms of the form similar to Equation 8. For symmetry it is convenient to have the reference point at the middle of the tracer slug. If the tracer is injected for a time, t_1 , the reference points will be at $t \pm t_1/2$, and the result is;

$$\frac{C}{C_o} = \frac{1}{2} \operatorname{erfc} \left(\frac{Q'(t+t_1/2) - r^2/2}{2\sqrt{\alpha r^3/3}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q'(t-t_1/2) - r^2/2}{2\sqrt{\alpha r^3/3}} \right) \quad (9)$$

We wish to know the peak concentration, or midpoint concentration, C_{mp} , of the tracer slug. At this point $Q t = r^2/2$, and the result is,

$$\frac{C_{mp}}{C_o} = \operatorname{erf} \left(\frac{Q' t_1}{4\sqrt{\alpha r^3/3}} \right) \quad (10)$$

The term $Q' t$, is a measure of the amount of tracer injected. For easier visualization of the variables it might be simpler to look at this term in relation to a width of tracer slug. If the slug moved out into the reservoir without mixing, its undiluted "width", W , would be expressed as $rW = Q t$, and Equation 10 could be written,

$$\frac{C_{mp}}{C_o} = \operatorname{erf} \left(\frac{\sqrt{3} W}{4\sqrt{\alpha r}} \right) \quad (11)$$

By comparison, for dispersion of a tracer in a linear flow for a distance, L , the result is,

$$\frac{C_{mp}}{C_o} = \operatorname{erf} \left(\frac{W}{4\sqrt{\alpha L}} \right) \quad (12)$$

so the radial and linear equations differ only in the factor $\sqrt{3}$.

When the argument of the error function is less than about 0.1, the function becomes linear, with the constant 1.13. In tracer work, the desired output concentration is almost always less than one tenth of the input concentration, so Equation 11 can be simplified to,

$$\frac{C_{mp}}{C_o} = \frac{1.13 \sqrt{3} W}{4 \sqrt{\alpha r}} \quad (13)$$

Flow in a 5-spot is not radial, but it is possible to approximate the geometry of a 5-spot system with a radial model. If a circle is drawn with its center at the injection well and the circumference cutting the four surrounding producing wells, and then the circle is folded back on itself as shown in Figure 1, the shape of the front is very similar to the actual front in a 5-spot displacement at breakthrough. Of great importance is that the length of the front thus formed is very close to the length of the actual front in 5-spot flow. And a study of Equation 13 shows that the front length is the most important parameter affecting the mid-point concentration.

The mass of tracer injected is equal to the perimeter times: width, height, porosity saturation, concentration and density. For a waterflood tracer this is

$$m = 2\pi L W h \phi S_w C_o (62.4) \quad (14)$$

Where L = Distance from injector to producer (ft)
 C_o = Injected concentration (weight fraction)

Equations 13 and 14 can now be combined to determine the amount of tracer to inject for a desired peak concentration, remembering that r , in Equation 13 is now the interwell distance, L .

$$m = 800 h \phi S_w C_{mp} L^{1.5} \alpha^{.5} \quad (15)$$

Areal Sweep Effects

Equation 15 gives us the concentration of the tracer flowing in the reservoir. However, this is not the same as the concentration flowing out of the wellbore, for the geometry of the 5-spot system causes the tracer front to be diluted at the wellbore. This can be explained as follows.

Assume we have a homogeneous 5-spot filled with fluid "A" and inject fluid "B" which has the same mobility. Also assume no mixing takes place at the front (later the areal sweep effects will be combined with the mixing effects). Thus "B" is injected as a sharp front and will remain as a sharp front. The resulting breakthrough curve looks like curve, 1, in Figure 2a. Note that the percent "B" rises after breakthrough, but it requires a large injection volume before the producing concentration climbs to 100 percent "B". The "B" that is flowing in the reservoir, however, is at 100 percent concentration.

Now consider a case where a slug of "B" is injected and this in turn is followed by "A". Again assume no dispersion occurs and slug "B" remains sharp and undiluted. The breakthrough curve for the front of the "B" slug will remain the same, and the breakthrough curve for "A" following "B" will be identical in shape--merely lagging the first curve as shown in curve 2, Figure 2a. At any point in the production history, the concentration of "B" being produced is equal to the vertical distance in the "B" area of Figure 2a, and the resulting "B" concentration production curve looks like Figure 2b. Notice that the peak producing concentration of "B" is considerably less than 100 percent, even though the concentration of "B" flowing in the reservoir is 100 percent.

When the tracer becomes dispersed (or mixed) in the reservoir it is convenient to think of the tracer being composed of many small slugs, each with a different concentration. The highest concentration is in the center and progressively lower values are ahead and behind. This concept is shown in Figure 2c. In this case the "B" output concentration curve can be calculated by vertically summing the concentration contributions of each small segment, and the resulting curve looks something like Figure 2d. The general shape of this curve is similar to the undiluted slug of Figure 2b, but it is less sharply peaked; and, of course, has a lower concentration due to the mixing.

The problem now is to develop these qualitative concepts into numbers and equations. For the 5-spot areal sweep information the data of Dyes, Caudle, and Erickson³; Caudle and Witte⁴; and Fay and Prats⁵ were used. Since these data differed slightly from each other, they were

averaged and the resultant curve plotted in Figure 3. This curve could not be conveniently fit to an equation, so it was differentiated graphically, plotted as the solid stair-step line of Figure 4, and empirically fit to the following equation.

$$\log \left\{ \log \left(\frac{1.07}{1.07 - F_D} \right) \right\} = -.0410 + .581 \log(Q_i -.72) \quad \text{for } .72 < Q_i < 2.29 \quad (16)$$

Where F_D = Fraction of displacing fluid in the producing stream
 Q_i = Volume injected and produced (effective pore volumes)

This is plotted as the dashed line of Figure 4.

Combining the Breakthrough and Dispersion Data

Equations 11 or 13 predict the concentration at the midpoint, but the concentration ahead of and behind the midpoint is also needed. This is an e^{-x^2} type term, which can be extracted from Equations 11 or 13 as follows,

$$\frac{C_1}{C_{mp}} = e^{-\frac{3(\Delta r)^2}{4\alpha L}} \quad (17)$$

Where C_1 = Concentration at some point r_1 near the midpoint, L
 $\Delta r = L - r_1$

Since Equation 16 is in pore volume units, and we wish to combine it with Equation 17, it will be necessary to change Equation 17 to pore volume units. The volume of a tracer segment is $2\pi\phi\Delta r Lh$ and one pore volume is $2\phi L^2h$, so an increment in pore volume is,

$$\Delta(PV) = \frac{2\phi\pi\Delta r Lh}{2\phi L^2h} = \frac{\pi\Delta r}{L} \quad (18)$$

and the concentration profile becomes,

$$\frac{C_1}{C_{mp}} = e^{-\frac{3L(\Delta PV)^2}{4\pi^2\alpha}} \quad (19)$$

Assume now that a tracer slug has been injected, it is followed by a volume of water (V_i) and the midpoint of the tracer is along the front at V_i . Equation 19 states that the tracer is distributed everywhere in the reservoir at a concentration proportional to $e^{-(\Delta PV)^2}$. According to Equation 16, any tracer at a location 2.29 or more pore volumes out from the injection well will have already been produced, and any tracer at a location less than .72 pore volume has

not yet started to be produced. The rest of the tracer is being produced at a rate depending on its location. To determine the total tracer production it is necessary to sum all contributions from .72 to 2.29 pore volumes, thusly.

$$\frac{C(V_i)}{C_{mp}} = \int_{.72}^{2.29} \frac{dF_D}{dQ_i} e^{-\frac{3L(Q_i - V_i)^2}{4\pi^2\alpha}} dQ_i \quad (20)$$

Where $C(V_i)$ = Producing concentration as a function of volume injected.

F_D = Fraction displacing fluid.

V_i = Volume injected (pore volumes).

Q_i = Location volume (pore volumes).

This is the same "summing" used earlier to get from Figure 2c to Figure 2d.

Unfortunately, Equation 20 cannot be integrated analytically due to the complex relationship between F_D and Q ; so it was integrated numerically, giving C versus V_i for various values of L/α . The results of these calculations are plotted in Figure 5. Note that, for all spacings and dispersion coefficients, the peak concentration producing from the well is considerably less than the peak concentration in the reservoir.

The peak concentrations of Figure 5 were plotted on log-log paper in Figure 6. Empirically they were found to fit a straight line with the following equation.

$$\frac{C_p}{C_{mp}} = 2.63 \left(\frac{L}{\alpha}\right)^{-.235} \quad (21)$$

Where C_p = Peak concentration produced from the reservoir.

Equation 21 can now be combined with Equation 15 to give us a working equation for 5-spot geometry.

$$m = 304h\phi S_w C_p \alpha^{-.265} L^{1.735} \quad (22)$$

For convenience, it is easier to use the concentration, C_p , in parts per million. This is the usual method of expressing tracer concentration. The inter-well distance is more easily handled in hundreds of feet. With these changes, the equation becomes,

$$m = .90h\phi S_w C_p \alpha^{-.265} L^{1.735} \quad (23)$$

Where C_p = Peak concentration producing from the well (ppm)
 L = Distance from injector to producer (hundreds of feet)

Effect of Outside Wells

Only the flow in the pattern has been included so far in the calculations. In a developed 5-spot pattern, each producing well receives fluids from four wells. Thus, if the pattern is balanced, the tracer is further diluted by a factor of four.

If the pattern is not balanced, the flow system is not so simple, for the basic sweep pattern will be changed from that used to arrive at Equation 23. The effect of the sweep pattern change cannot be properly calculated because data are not available. However, the equations can be partially corrected by accounting for the total volumes injected and produced. This is shown in the following example.

For instance, assume that an injection well is taking 1000 B/D, and it is surrounded by four producers making 600 B/D each. If the reservoir is liquid filled and there is dynamic balance of flow, each producer will get 250 B/D from the injector and 350 B/D from outside the pattern. Thus the actual peak concentration is smaller than calculated from Equation 23 by the ratio 250/600.

If the pattern is balanced, each producer will make 1000 B/D. Each producer will be getting 250 B/D from the injector and 750 B/D from outside the pattern. The peak concentration will be smaller by the ratio 250/1000.

These examples illustrate the idea, but they are considerably simpler than the usual field case.

Stratification of Reservoir

All the previous calculations were based on a homogeneous reservoir. Unfortunately, such does not exist. But it is possible to make some allowance for the heterogeneity of the actual reservoir with some further assumptions.

One of the most common methods of handling inhomogeneity is to mathematically divide the reservoir into layers, assign a permeability to each layer, and assume no cross flow. We should not believe these assumptions too wholeheartedly, but this model has a history of passable reservoir predictions to back up its usage.

If a reservoir is divided into layers, the amount of tracer going into each layer is proportional to its permeability. So, from Equation 15, the concentration flowing in that layer will also be proportional to its permeability. At the producing wellbore the tracer from a given layer will be diluted by flow coming in from other layers. The dilution is proportional to the kh product of the layer compared to the total kh of the reservoir. So the resulting producing concentration is as follows

$$C_i = C_p \frac{k_i}{k_{avg}} \left(\frac{h_i k_i}{\sum h_i k_i} \right) = C_p \frac{h_i}{h} \left(\frac{k_i}{k_{avg}} \right)^2 \quad (24)$$

C_i = Concentration peak from the ith layer

C_p = Peak concentration if the reservoir is homogeneous (from Equation 23)

k_i = Permeability of the ith layer

k_{avg} = Average permeability of the reservoir

h_i = Height of the ith layer

h = Height of the reservoir

Since the flow rate through the ith layer is proportional to its permeability, the time to breakthrough of that layer is inversely proportional to its permeability, as follows.

$$t_i = t_{avg} \frac{k_{avg}}{k_i} \quad (25)$$

Where t_i = Time to breakthrough of the ith layer

t_{avg} = Time to breakthrough of the homogeneous reservoir

When Equations 23, 24 and 25 and Figure 5 are used in conjunction with the effect of the outside wells, the peak concentration, the concentration profile, and breakthrough time of each layer can be calculated for any given 5-spot condition. In actual practice the contribution from the most permeable layer, or layers, will be most important.

FIVE-SPOT FIELD TEST

A 2 1/2 acre inverted 5-spot having approximately 12 feet of net pay was chosen for a test. The average porosity was 26 percent and water saturation was 55 percent. From core data, the average air permeability of the sand was 1500 md with indications of a 5000 md streak approximately one foot thick. Laboratory mixing data on the core gave α of 0.05 ft. The orientation of the pattern wells is shown in Figure 7. Further data on the test is given in Reference 6.

Two hundred pounds of ammonium thiocyanate and 150 pounds of potassium iodide were injected simultaneously as tracers. During the test,

the injection well averaged 600 BHPD and the producing rates were as follows: Well A, 260 BHPD; Well B, 160 BHPD; Well C, 140 BHPD; and Well D, 240 BHPD; for a total of 800 BHPD. The data on breakthrough of the tracers is shown in Figures 8, 9 and 10 for Wells A, C and D. No tracer was ever detected from Well B.

Since no tracer was found in Well B, it was assumed that only fifty barrels per day of the injected water were moving toward this well. The remaining water was distributed among the other three wells according to their production rate; Well A 225 BHPD, Well C 120 BHPD, and Well D 205 BHPD. The total tracer produced from these wells indicates that this assumption is sound, for about equal volumes were produced from Wells A and D with considerably less from Well C.

The ammonium thiocyanate curve of Well D was chosen for detailed analysis. Since the injection well was down for about four days, it was felt that it would be meaningless to attempt to match the concentration curve during and after that period. So the tracer elution curve was approximated from the period of 2000 bbls to 4000 bbls production using Equations 23, 24 and 25 and Figure 5. The resulting match is shown in Figure 11.

Three permeable streaks were used to arrive at Figure 11: 4110 md, 0.99 ft thick; 3420 md, 1.00 ft thick; and 2980 md, .59 ft thick. Notice that the shape of the curve is quite close to the thiocyanate curve of Figure 10. Although the detailed spikes and peaks are missing, the curve basically consists of two major peaks as shown. On the other hand the potassium iodide curve of Figure 10 is basically a single broad hump covering the same production interval as the thiocyanate. So the "double peak" approximation of Figure 11 is not valid for the iodide. Since the percentage of iodide and thiocyanate recovered was virtually the same in all three wells, we must conclude the iodide curve is as valid as the thiocyanate curve. Rather than emphasizing the exact values calculated for the permeability streaks of Figure 11, the curves should be considered in a broader sense to conclude there is a zone about 2 1/2 feet thick which ranges from about 2 to 3 times the average permeability.

Since the curves for Well A are similar to Well D, the conclusions should be the same for this quadrant.

The injection well was down during the period when Well C was peaking out. Since this undoubtedly disrupted the flow pattern (notice the drop in tracer concentration in all three wells during this period), no analysis

was attempted on Well C.

It is obvious that a Northwest-Southeast permeability trend exists in this pattern. This could possibly have been expected from the differing production rates of the four wells; but it was dramatically pointed out by these tracer results. This is especially interesting in view of the fact that the core analysis data indicated no such permeability trend.

CONCLUSIONS

Equations have been derived for use in predicting tracer flow behavior in reservoirs. The specific equations were developed to handle 5-spot flow of water tracers at a 1:1 mobility ratio. Although these conditions limit the scope of the equations; the method of approach is general enough to be used wherever areal coverage, dispersion and flow pattern is known or can be approximated.

Using the equations and the detailed tracer production history, the degree of stratification in a reservoir can be calculated.

The field test data indicate that the stratification and producing concentration values calculated from these equations are realistic, giving some confidence in the use of these equations in reservoir analysis.

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NOMENCLATURE

C = Concentration (any units are permissible as long as they are consistent - generally weight fraction or ppm are used)
 C_i = Peak concentration due to flow from the i th layer
 C_l = Concentration at r_l , near the midpoint L
 C = Injected concentration
 C_P^0 = Peak concentration flowing from a homogeneous reservoir
 $C(V_i)$ = Producing concentration as a function of volume injected (V_i)
 F_D = Fraction of displacing fluid in the producing stream
 h = Height of reservoir (ft)
 h_i = Height of the i th layer (ft)
 K = Dispersion coefficient for miscible displacement in a porous medium (ft^2/sec)
 K_{avg} = Average permeability of the reservoir (md)

k_i = Permeability of the i th layer (md)
 L = Total length of displacement, also distance from injector to producer (ft)
 m = Amount of water tracer injected (lb)
 PV = Effective pore volumes (dimensionless)
 Q = Volumetric injection rate ($\text{ft}^3/\text{sec-ft}$)
 $Q_i = Q/2\pi\phi$
 Q_i = Location volume on the F_D curve (effective pore volumes)
 r = Radius (ft)
 $\Delta r = L - r_l$
 r_l = Radius near the tracer midpoint, L (ft)
 S_w = Water saturation (fraction)
 t_l^w = Time for injection of a tracer slug (sec)
 t_i = Time for breakthrough of the i th layer (sec)
 t_{avg} = Time for breakthrough of a homogeneous reservoir (sec)
 V_i = Volume injected (effective pore volumes)
 W = Width of tracer slug (ft)
 α = Dispersion constant rK/Q' (ft)
 ϕ = Porosity (fraction)

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Figure 1 APPROXIMATION OF FIVE-SPOT GEOMETRY USING A CIRCLE

Figure 2a

Figure 2c

Figure 2b

Figure 2d

Figure 2 EFFECTS OF AREAL SWEEP ON BREAKTHROUGH

Figure 3 5-SPOT SWEEP EFFICIENCY FOCIILITY RATIO 1.1

Figure 4 5-SPOT BREAKTHROUGH CURVE

Figure 5 EFFECT OF SPACING AND DISPERSION COEFFICIENT ON BREAKTHROUGH HISTORY

Figure 6 EFFECT OF DISPERSION AND SPACING ON PEAK PRODUCING CONCENTRATION

TRACER TEST PATTERN

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11. CALCULATED APPROXIMATION OF WELL D AMMONIUM THIOCYANATE BREAKTHROUGH CURVE

APPENDIX B

COMPUTER PROGRAM TO ANALYZE FIELD-TYPE DATA FROM

FIVE-SPOT TRACER TESTS TO DETERMINE THE DEGREE OF RESERVOIR LAYERING


```
//DATANL JOB 'SC$B93','YUEN',CLASS=Q
/*JOBPARM FORMS=1481,DEST=SELF
/*LIST
// EXEC WATFIV
//GO.SYSIN DD *
$WATFIV
```

JOB 605

```
C THIS PROGRAM ANALYZES FIELD-TYPE DATA FROM A FIVE-SPOT TRACER TEST
C AND DETERMINES THE DEGREE OF RESERVOIR LAYERING.
```

```
C PREPARED BY DEXTER L. YUEN AS PARTIAL FULFILLMENT OF THE
C REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN PETROLEUM
C ENGINEERING, STANFORD UNIVERSITY
```

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C AUGUST 1978
```

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C FUNDING FOR THIS WORK WAS PROVIDED BY THE STANFORD UNIVERSITY
C PETROLEUM RESEARCH INSTITUTE.
```

NUMENCLATURE

```
C ALPH = DISPERSION CONSTANT, FT.
C AREA = AREA OF INJECTION PATTERN, SQ.FT.
C C = OVERALL TRACER CONCENTRATION MEASURED IN PRODUCING WELL, PPM
C CAPAC(L) = H*POROSITY FOR LAYER L, FT.
C CNCL(L) = CONCENTRATION OF FLUID FROM LAYER L AS IT ENTERS THE
C WELLBORE, PPM
C CP(L) = HT. OF L TH CONCENTRATION PEAK IN THE OVERALL CURVE, PPM
C C$CMP = DIMENSIONLESS CONCENTRATION RATIO OBTAINED FROM
C SOLVING BRIGHAM AND SMITH'S INTEGRAL EQUATION
C CP$CMP = PEAK CONCENTRATION RATIO
C EL = DISTANCE FROM INJECTOR TO PRODUCER, FT.
C ELALPH = EL/ALPH RATIO
C M = TOTAL WEIGHT OF TRACER INJECTED, LBS.
C N = NUMBER OF LAYERS
C PERM(L) = KH/TOTAL KH FOR LAYER L
C PV = PORE VOLUMES INJECTED INTO A SPECIFIC LAYER
C SW = WATER SATURATION, FRACTION
C VMIN, VMAX = THIS SETS THE RANGE AT WHICH THE "OVERALL
C CONCENTRATION" VS. "TOTAL INJECTION VOLUME" DATA IS
C CALCULATED; 51 VALUES WILL BE GENERATED BETWEEN VMIN AND
C VMAX, INCLUSIVE.
C V = TOTAL VOLUME INJECTED, BARRELS
C ZZ = K/POROSITY*TOTAL KH
```

```
C DOUBLE PRECISION ELALPH, Z(5), ZZ(5), PV, PVP, C$CMP, CP$CMP
C DOUBLE PRECISION ERREST, FLAG, Y, M, W
C DOUBLE PRECISION A(5,5), LU(5,5), PERM(5), CAPAC(5)
C DIMENSION VP(5), CP(5), IPS(5), CNCL(5)
C EXTERNAL FUN
```

INPUT PARAMETERS

```
C READ,EL,ALPH
C READ,M,SW,AREA
C WRITE (6,100) M,SW,AREA
C READ,VMIN,VMAX
C READ,N
C WRITE (6,200)
```

```

      DO 1 L = 1,N
        READ,VP(L),CP(L)
        WRITE (6,201) L,VP(L),CP(L)
1 CONTINUE
C
C CALCULATE CONSTANTS
C
      IDIM = N
      ELALPH = EL/ALPH
      WRITE (6,300) EL,ALPH,ELALPH
      Y = M/(.0004*SW*(EL**1.5)*(ALPH**1.5))
      PVP = .720007+.560541*DU*(ELALPH**(-.430043DU))
C
C CALCULATE K/POROSITY*TOTAL K.H (=ZZ)
C
      DO 2 L = 1,N
        Z(L) = PVP/VP(L)
        ZZ(L) = Z(L)*AREA*SW/.5015
2 CONTINUE
C
C SET UP MATRIX ELEMENTS
C
      CALL QUANC8(FUN,.720007,DU,2.2900,1.0D-7,1.0D-7,CP$CMP,ERREST,
* NOFUN,FLAG,PVP,ELALPH)
      DO 5 L = 1,N
        DO 4 K = 1,N
          IF (L.EQ.K) GOTO 3
          PV = VP(L)*Z(K)
          CALL QUANC8(FUN,.720007,DU,2.2900,1.0D-7,1.0D-7,C$CMP,
* ERREST,NOFUN,FLAG,PV,ELALPH)
          A(L,K) = C$CMP*Y*ZZ(K)
          GOTJ 4
3 A(L,K) = CP$CMP*Y*ZZ(K)
4 CONTINUE
5 CONTINUE
C
C SOLVE MATRIX EQUATION TO GET KH/TOTAL KH (=PERM)
C
      CALL DECMPI(N,A,IDIM,LU,IPS,.650,.660)
      CALL SOLVE1(N,LU,IDIM,CP,PERM,IPS)
C
C CALCULATE H*POROSITY (=CAPAC)
C
      WRITE (6,400)
      DO 6 L = 1,N
        CAPAC(L) = PERM(L)/ZZ(L)
        WRITE (6,401) L,PERM(L),CAPAC(L)
6 CONTINUE
C
C GENERATE "OVERALL CONCENTRATION" VS. "TOTAL INJECTION VOLUME"
C VALUES BETWEEN VMIN AND VMAX
C
      WRITE (6,500)
      DELTV = (VMAX-VMIN)/50.
      DO 11 II = 1,51
        V = 0.
        V = VMIN+(II-1)*DELTIV
        DO 10 L = 1,N
          PV = V*Z(L)
          CALL QUANC8(FUN,.720007,DU,2.2900,1.0D-7,1.0D-7,C$CMP,

```



```

C RESULT AN APPROXIMATION TO THE INTEGRAL HOPEFULLY SATISFYING THE CC000190
C LEAST STRINGENT OF THE TWO ERROR TOLERANCES. 00000200
C ERREST AN ESTIMATE OF THE MAGNITUDE OF THE ACTUAL ERROR. 00000210
C NOFUN THE NUMBER OF FUNCTION VALUES USED IN CALCULATION OF RESULT. 00000220
C FLAG A RELIABILITY INDICATOR. IF FLAG IS ZERO, THEN RESULT 00000230
C PROBABLY SATISFIES THE ERROR TOLERANCE. IF FLAG IS 00000240
C XXX.YYY, THEN XXX = THE NUMBER OF INTERVALS WHICH HAVE 00000250
C NOT CONVERGED AND 0.YYY = THE FRACTION OF THE INTERVAL 00000260
C LEFT TO DO WHEN THE LIMIT ON NOFUN WAS APPROACHED. 00000270
C 00000280
C 00000290
C DOUBLE PRECISION W0,W1,W2,W3,W4,AREA,X0,F0,STONE,STEP,COR1,TEMP 00000300
C DOUBLE PRECISION QPREV,QNOW,QDIFF,GLEFT,ESTERR,TOLERR 00000310
C DOUBLE PRECISION ORIGHT(3),F(16),X(16),FSAVE(8,30),XSAVE(8,30) 00000320
C DOUBLE PRECISION DABS,DMAX 00000330
C INTEGER LEVMIN,LEVMAX,LEVOUT,NOMAX,NOFIN,LEV,N,M,I,J 00000340
C *** STAGE 1 *** GENERAL INITIALIZATION 00000350
C SET CONSTANTS. 00000360
C 00000370
C 00000380
C LEVMIN = 1 00000390
C LEVMAX = 30 00000400
C LEVOUT = 6 00000410
C NOMAX = 5000 00000420
C NOFIN = NOMAX - 3*(LEVMAX-LEVOUT+2**((LEVOUT+1))) 00000430
C
C CALL TRAPS TO PREVENT INTERRUPTION OF EXECUTION BECAUSE OF
C EXPONENT UNDERFLOW (I.E., A NUMBER LESS THAN 10**(-75))
C
C CALL TRAPS(1,1,2000000000,1,1)
C
C TROUBLE WHEN NOFUN REACHES NOFIN 00000440
C 00000450
C 00000460
C W0 = 3956.700 / 14175.000 00000470
C W1 = 23552.000 / 14175.000 00000480
C W2 = -3712.000 / 14175.000 00000490
C W3 = 41984.000 / 14175.000 00000500
C W4 = -18160.000 / 14175.000 00000510
C 00000520
C INITIALIZE RUNNING SUMS TO ZERO. 00000530
C 00000540
C FLAG = 0.000 00000550
C RESULT = 0.000 00000560
C COR1 = 0.000 00000570
C ERREST = 0.000 00000580
C AREA = 0.000 00000590
C NOFUN = 0 00000600
C IF (A .EQ. B) RETURN 00000610
C 00000620
C *** STAGE 2 *** INITIALIZATION FOR FIRST INTERVAL 00000630
C 00000640
C LEV = 0 00000650
C NIM = 1 00000660
C X0 = A 00000670
C X(16) = B 00000680
C QPREV = 0.000 00000690
C F0 = FUN(VI,X0,ELALPH)
C STONE = (B - A) / 100.000 00000710
C X(8) = (X0 + X(16)) / 2.000 00000720
C X(4) = (X0 + X(8)) / 2.000 00000730

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	X(12) = (X(8) + X(16)) / 2.000	00000740
	X(2) = (X(0) + X(4)) / 2.000	00000750
	X(6) = (X(4) + X(8)) / 2.000	00000760
	X(10) = (X(8) + X(12)) / 2.000	00000770
	X(14) = (X(12) + X(16)) / 2.000	00000780
	DO 25 J = 2, 16, 2	00000790
	F(J) = FUN(VI, X(J), ELALPH)	
25	CONTINUE	00000810
	NOFUN = 9	00000820
C		00000830
C	*** STAGE 3 *** CENTRAL CALCULATION	00000840
C	REQUIRES QPREV, X0, X2, X4, ..., X16, F0, F2, F4, ..., F16.	00000850
C	CALCULATES X1, X3, ..., X15, F1, F3, ..., F15, QLEFT, QRIGHT, QNOW, QDIFF, AREA.	00000860
C		00000870
30	X(1) = (X(0) + X(2)) / 2.000	00000880
	F(1) = FUN(VI, X(1), ELALPH)	
	DO 35 J = 3, 15, 2	00000900
	X(J) = (X(J-1) + X(J+1)) / 2.000	00000910
	F(J) = FUN(VI, X(J), ELALPH)	
35	CONTINUE	00000930
	NOFUN = NOFUN + 8	00000940
	STEP = (X(16) - X(0)) / 16.000	00000950
	QLEFT = (W0*(F0 + F(8)) + W1*(F(1)+F(7)) + W2*(F(2)+F(6))	00000960
	+ W3*(F(3)+F(5)) + W4*(F(4)) * STEP	00000970
	QRIGHT(LEV+1) = (W0*(F(8)+F(16)) + W1*(F(9)+F(15)) + W2*(F(10)+F(14))	00000980
	+ W3*(F(11)+F(13)) + W4*(F(12)) * STEP	00000990
	QNOW = QLEFT + QRIGHT(LEV+1)	00001000
	QDIFF = QNOW - QPREV	00001010
	AREA = AREA + QDIFF	00001020
C		00001030
C	*** STAGE 4 *** INTERVAL CONVERGENCE TEST	00001040
C		00001050
	ESTERR = DABS(QDIFF) / 1023.000	00001060
	TOLERR = DMAX1(ABSERR, RELERR*DABS(AREA)) * (STEP/STONE)	00001070
	IF (LEV .LT. LEVMIN) GO TO 50	00001080
	IF (LEV .GE. LEVMAX) GO TO 52	00001090
	IF (NOFUN .GT. NOFIN) GO TO 50	00001100
	IF (ESTERR .LE. TOLERR) GO TO 70	00001110
C		00001120
C	*** STAGE 5 *** NO CONVERGENCE	00001130
C	LOCATE NEXT INTERVAL.	00001140
C		00001150
50	NIM = 2*NIM	00001160
	LEV = LEV+1	00001170
C		00001180
C	STORE RIGHT HAND ELEMENTS FOR FUTURE USE.	00001190
C		00001200
	DO 52 I = 1, 8	00001210
	FSAVE(I, LEV) = F(I+8)	00001220
	XSAVE(I, LEV) = X(I+8)	00001230
52	CONTINUE	00001240
C		00001250
C	ASSEMBLE LEFT HAND ELEMENTS FOR IMMEDIATE USE.	00001260
C		00001270
	QPREV = QLEFT	00001280
	DO 55 I = 1, 8	00001290
	J = -I	00001300
	F(2*J+18) = F(J+9)	00001310
	X(2*J+18) = X(J+9)	00001320
55	CONTINUE	00001330

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      GO TO 30
C
C   ***   STAGE 6   ***   TROUBLE SECTION
C   NUMBER OF FUNCTION VALUES IS ABOUT TO EXCEED LIMIT.
C
60 NOFIN = 2*NOFIN
   LEVMAX = LEVOUT
   FLAG = FLAG + (B - XC) / (B - A)
   GO TO 70
C
C   CURRENT LEVEL IS LEVMAX.
C
62 FLAG = FLAG + 1.00D0
C
C   ***   STAGE 7   ***   INTERVAL CONVERGED
C   ADD CONTRIBUTIONS INTO RUNNING SUMS.
C
70 RESULT = RESULT + QNOW
   ERREST = ERREST + ESTERR
   COR11 = COR11 + QDIFF / 1023.00D0
C
C   LOCATE NEXT INTERVAL.
C
72 IF (NIM .EQ. 2*(NIM/2)) GO TO 75
   NIM = NIM/2
   LEV = LEV-1
   GO TO 72
75 NIM = NIM + 1
   IF (LEV .LE. 0) GO TO 80
C
C   ASSEMBLE ELEMENTS REQUIRED FOR THE NEXT INTERVAL.
C
   QPREV = QRIGHT(LEV)
   X0 = X(16)
   F0 = F(16)
   DO 78 I = 1, 8
     F(2*I) = FSAVE(I,LEV)
     X(2*I) = XSAVE(I,LEV)
78 CONTINUE
   GO TO 30
C
C   ***   STAGE 8   ***   FINALIZE AND RETURN
C
80 RESULT = RESULT + COR11
C
C   MAKE SURE ERREST NOT LESS THAN ROUNDOFF LEVEL.
C
   IF (ERREST .EQ. 0.00D0) RETURN
82 TEMP = DABS(RESULT) + ERREST
   IF (TEMP .NE. DABS(RESULT)) RETURN
   ERREST = 2.00D0*ERREST
   GO TO 82
   END
C
C   FUN IS THE INTEGRAND
C
   FUNCTION FUN(VI,QI,ELALPH)
   DOUBLE PRECISION VI,QI,ELALPH,AA,BB,CC,FUN,EXP
   AA = (QI-.72)**(-.4.9)
   BB = 10.**((- .909913)*(QI-.72)**.581)

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      KP1 = K+1
      DO 16 I = KP1,N
        IP = IPS(I)
        EM = LU(IP,K)/PIVOT
        LU(IP,K) = EM
        DO 16 J = KP1,N
          LU(IP,J) = LU(IP,J) - EM*LU(KP,J)
C      INNER LOOP. USE MACHINE LANGUAGE CODING IF COMPILER
C      DOES NOT PRODUCE EFFICIENT CODE.
10    CONTINUE
17  CONTINUE
C    TEST FOR ZERO LAST PIVOT.
      IF(LU(IPS(N),N).EQ.0) RETURN 2
      RETURN
C    LAST CARD OF SUBROUTINE DECMP.
      END
C
C
C    SOLVE1 WAS OBTAINED FROM PROFESSOR JOEL H. FERZIGER OF THE
C    DEPARTMENT OF MECHANICAL ENGINEERING DURING THE ME 210A COURSE
C    HE TAUGHT IN FALL QUARTER, 1977
C
      SUBROUTINE SOLVE1(N,LU,IDIM,B,X,IPS)
      INTEGER N,IDIM,IPS(N)
      DOUBLE PRECISION LU(5,5), X(N)
      REAL B(N)
C    SOLVE A*X = B USING L*U FROM SUBROUTINE DECMP1
C    IPS IS THE ROW INTERCHANGE VECTOR FROM DECMP1
      INTEGER I,J,IP,IP1,IM1,NP1,IBACK
      REAL SUM
      NP1=N+1
C    FIND U*X = L(-1)*B
      X(1)=B(IPS(1))
      DO 2 I=2,N
        IP=IPS(I)
        IM1=I-1
        SUM=0.0
        DO 1 J=1,IM1
          SUM=SUM+LU(IP,J)*X(J)
1      X(I)=B(IP)-SUM
2      X(N)=X(N)/LU(IPS(N),N)
C    FIND X = U(-1)*L(-1)*B
      DO 4 IBACK=2,N
        I=NP1-IBACK
        I GOES (N-1),...,1
C      IP=IPS(I)
        IP1=I+1
        SUM=0.0
        DO 3 J=IP1,N
          SUM=SUM+LU(IP,J)*X(J)
3      X(I)=(X(I)-SUM)/LU(IP,I)
4      X(I)=(X(I)-SUM)/LU(IP,I)
      RETURN
C    LAST CARD OF SUBROUTINE SOLVE1
      END
$DATA
$STOP
//
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DECMP046
 DECMP047
 DECMP048
 DECMP049
 DECMP050
 DECMP051
 DECMP052
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 DECMP056
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 DECMP059
 DECMP060
 DECMP061