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Summary

The momentum spread required to stabilize longitudinal modes in a proposed high-current betatron is computed.

The High-Current Betatron

A conventional betatron has been proposed for the acceleration of multikiloampere currents of electrons.¹ Stable operation is achieved by injecting current at sufficiently high energy that the space-charge-induced tune shift is dominated by image effects, which are compensated by external control of wall currents. High injection energy also permits the suppression of unstable longitudinal modes by momentum spread.

Specific beam parameters are: injection energy $(\gamma-1)mc^2 = 50$ MeV, current $I \leq 10$ kA, and emittance $\epsilon_v = \epsilon_H = 5 \times 10^{-4}$ m-r. The machine circumference $C = 18.0$ m contains three 120° bends of radius $R = 2.0$ m, separated by straight sections of length $L_s = 1.8$ m. The straight sections contain the fast kickers and septa used for single turn injection and extraction (see Figure 1). Chamber half height and half width are respectively $h = .2$ m and $w = .3$ m. The beam's half height,

$$b = (C\epsilon_v/2\pi v_v)^{1/2} = 4.4 \text{ cm}, \quad (1)$$

is calculated using a vertical tune $\nu_v = .75$. This tune yields the desired quarter-wavelength betatron oscillation between kicker and septum. The beam's half width at 10 kA is approximately 20 cm, due primarily to the dispersion of orbits of differing momenta.

Stabilization of longitudinal modes is achieved by introducing a rather large spread of momentum prior to injection:

$$(\delta P/P_0)^2 \propto |(Z_n/n)(I/\gamma)n^{-1}|, \quad (2)$$

where Z_n is the complex impedance for mode number n and η is the compaction factor. This formula is valid for pencil-like beams in which the perturbed longitudinal field does not vary appreciably over the transverse profile. However, in the present application, the required full width spread is estimated to be $\delta P/P_0 = .10$ for 10 kA current; this results in a disk-like structure comparable in width to the conducting chamber.

Dispersion Relation

The standard formalism for longitudinal modes of a pencil-like, low current beam is employed.² For a perturbation of the form

$$\delta I \propto \exp i(n\omega - \omega t), \quad (3)$$

Fig. 1. Injection and Extraction for Betatron

the dispersion relation is

$$1 = (U + iV) \int dx \frac{dG/dx}{x-y} \quad (4)$$

where

$$U + iV = i \left(\frac{Z_n}{n} \right) \left(\frac{q I}{2\pi c P_0} \right) \left(\frac{1}{-n} \right) \quad (5)$$

$$x = \frac{P - P_0}{P_0}, \quad y = \frac{\omega - \omega_0}{\omega_0} \quad (6)$$

Here $G(x)$ is the distribution of momentum deviation, normalized to unity, and ω_0 is the angular frequency for momentum P_0 . Since ω is considered to have a positive imaginary part and n must be negative, the quantity y/n is treated as having negative imaginary part. Factors of relativistic β are set equal to unity.

The longitudinal impedance,

$$Z_n = -C \delta E_n / \delta I_n \quad (7)$$

is in general a function of both ω and n . However, its dependence on ω may be replaced by the approximate value at $n\omega_0$ unless a near cancellation occurs.

Momentum Compaction

The vertical tune $\nu_y = .75$ is realized when the field index in the bends (n) satisfies the relation

$$\tan(2\pi \sqrt{n}/3) = 2R/(\sqrt{n} L_s) \quad (8)$$

The coherent image of the displaced beam has negligible effect during the short (one turn) extraction time. We find $n = .3845$ and

$$(-n)^{-1} = \frac{1-n}{L_s/c} = .8800 \quad (8)$$

Note that, although the straight sections occupy only 30 percent of the circumference, they increase n^{-1} by over a factor of two from the limiting case [$L_s = 0$, $n = \nu_y^2 = 9/16$, $(-n)^{-1} = 7/16$]. The horizontal tune is $\nu_H = .9593$, which is uncomfortably close to unity.

Impedance

Several sources of longitudinal impedance have been considered and their respective contributions to Z_n estimated.

Subrelativistic effect. A low current beam of radius b transported in a straight pipe of radius a has long wavelength impedance due to the incomplete cancellation of inductive and capacitive effects

$$\frac{Z_n}{n} \approx i \frac{Z_0}{\gamma} \left[\log \left(\frac{a}{b} \right) + \frac{1}{4} \right] \quad (10)$$

where $Z_0 = 377 \Omega$. This contribution is clearly insignificant ($\leq 10^{-1} \Omega$ for 50 MeV electrons). However, since the factor γ^{-2} results from the difference $1 - (\omega b / \omega_0 a)^2$, a small deviation

of ω from $n\omega_0$ appreciably changes its value. It is readily shown that for currents $I \geq (17/\gamma) \text{ kA}$ there is complete stabilization even with no momentum spread (provided there is no other contribution to Z_n).

Resistive Wall. A thick chamber wall with conductivity $\sigma > 10^7 \text{ mho/m}$ contributes

$$\frac{Z_n}{n} \approx (1-i) \left(\frac{Z_0 c}{4\omega n^2 a} \right)^{1/2} \quad (11)$$

which has magnitude less than $.05 \Omega$ and may be neglected.

Curvature Effect at Long Wavelength. For mode numbers $n \ll n_0 = C/4h$ a small reactive component of impedance results from the chamber's curvature:

$$\frac{Z_n}{n} \approx i Z_0 \left(\frac{2h}{nR} \right)^2 \quad (12)$$

This contribution may be viewed as resulting from the slightly differing structures of the transverse electric and magnetic fields around the beam, resulting in the non-cancellation of the concomitant longitudinal electric field components. Again the absolute value ($\sim 1.5 \Omega$) is negligible.

Kickers and Septa. An extra structure such as the kicker magnet loop is a small inductance shunted by the drive circuit impedance. Since the rise time must be less than 10% of a full period, the circuit parameters are selected such that Z_n peaks at $n = 10$, with a value on the order of 20Ω for achievable pulse power. Actual experiments in a scale model ring are required to quantify the effect of the extraction slots.

Radiation Reaction. For modes with $n \gg n_0$ the perturbed beam radiates essentially as if the chamber were removed, with resultant impedance

$$|Z_n/n| \approx 300 n^{-2/3} \Omega$$

However the presence of chamber walls suppresses the emission of radiation as n approaches n_0 from above, such that a broad maximum of impedance is found near mode number $n \approx (C/2\pi h)^{3/2}$:

$$|Z_n/n|_{\max} \approx (300 \Omega) \left(\frac{2\pi h}{C} \right) = 21 \Omega \quad (13)$$

In the present study we adopt this value. A significant feature of the radiation-induced impedance is that it has maximum real and imaginary components of similar magnitude (although at somewhat different values of n).

Momentum Distribution

The distribution $G(x)$ is selected to stabilize all mode numbers with the minimum full width spread of momentum. A convenient choice is

$$G(x) \approx \frac{8}{3\omega a} (\Delta^2 - x^2)^{3/2} \quad (|x| < \Delta) \quad (14)$$

with full width $\Delta P = 2P_0 \Delta^4$. This yields a circular region of stability in the U - V plane, which

accommodates the complex structure of Z_n . The condition for stability is

$$\frac{p}{p_0} > 4 \left(u^2 + v^2 \right)^{1/4} = 4 \left[\left| \frac{Z_n}{21} \right| \frac{qI}{2\pi c p_0} \left(\frac{1}{-n} \right) \right]^{1/2} \quad (15)$$

$$= (.096) \left[\left| \frac{Z_n/n}{21} \right| \left(\frac{1}{10kA} \right) \left(\frac{50MeV}{E} \right) \left(\frac{1.14}{-n} \right) \right]^{1/2} \quad (16)$$

The desired distribution of momentum may be generated prior to injection by passing the beam through a thick graphite grating, with the distribution of thickness proportional to $G(x)$.⁵ The simplest grating would consist of closely spaced triangular ridges. However, this choice yields a flat top momentum distribution which can only stabilize modes with imaginary Z_n . A real (resistive) component of Z_n destabilizes longitudinal modes for either sign of n . The damping of these modes by momentum spread is sensitive to the degree of roundedness of $G(x)$ in addition to its full width; this consideration motivates the highly rounded distribution we select [Eq. 14].

Radial Width

The radial half width of the beam is

$$a = \frac{R\beta}{2(1-n)p_0} + \left(\frac{C_{cH}}{2\pi v_H} \right)^{1/2} = 19.5 \text{ cm}, \quad (17)$$

with only 3.9 cm due to emittance. Since a is of the order of magnitude as the chamber height ($h = 20$ cm), the perturbed field has appreciable structure over the beam's profile at high mode numbers ($n \gg n_0$). The naive picture of impedance derived for a pencil-like beam is inadequate in this limit and a two dimensional analysis of the coupled beam-field system is required for its further elucidation. It is plausible that the maximum impedance assumed in this study (Eq. 13) is an overestimated by a factor of two or greater since it is reached at mode numbers $n \geq 3 n_0 = 67$.

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