**PORTIONS
OF THIS
DOCUMENT
ARE
ILLEGIBLE**

CONF - EIC 3.10 - - 24

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36

MASTER

TITLE USE OF ZERNIKE POLYNOMIALS AND INTERFEROMETRY IN THE OPTICAL DESIGN
AND ASSEMBLY OF LARGE CARBON-DIOXIDE LASER SYSTEMS

AUTHOR(S) V. K. Viswanathan

SUBMITTED TO SPIE 25th Anniversary Celebration
San Diego, California
August 24-28, 1981

By acceptance of this article the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution or to allow others to do so for U.S. Government purposes. The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

REPRODUCTION OF THIS DOCUMENT IS UNLIMITED

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

Use of Zernike polynomials and interferometry in the optical design and assembly
of large carbon-dioxide laser systems*

V. K. Viswanathan

University of California, Los Alamos National Laboratory
MS 532, P.O. Box 1663, Los Alamos, NM 87545

Abstract

This paper describes the need for non-raytracing schemes in the optical design and analysis of large carbon-dioxide lasers like the Gigawatt,¹ Gemini,² and Helios³ lasers currently operational at Los Alamos, and the Antares⁴ laser fusion system under construction. The scheme currently used at Los Alamos involves characterizing the various optical components with a Zernike polynomial set⁵ obtained by the digitization⁶ of experimentally produced interferograms of the components. A Fast Fourier Transform code⁷ then propagates the complex amplitude and phase of the beam through the whole system and computes the optical parameters of interest. The analysis scheme is illustrated through examples of the Gigawatt, Gemini, and Helios systems. A possible way of using the Zernike polynomials in optical design problems of this type is discussed. Comparisons between the computed values and experimentally obtained results are made and it is concluded that this appears to be a valid approach. As this is a review article, some previously published results are also used where relevant.

Introduction

This paper starts with a brief description of the Helios and Gemini laser systems, and the Gigawatt Test Facility (GWTF) as examples of large carbon-dioxide lasers at Los Alamos. The layout, aberration correction mechanisms, sources of optical degradation, and the optical parameters of interest are then discussed using Helios as an example. Next, the optical design, engineering, and analysis schemes are explored. Examples of the scheme used at Los Alamos are given using the Gemini, Helios, and the GWTF laser systems. The usage of Zernike polynomials to understand and improve the optical performance of these systems is discussed. Comparisons are made between the results computed from the analysis and those experimentally obtained. They compare favorably and it is concluded that this is a valid approach for these types of systems.

Description of the large, Los Alamos carbon-dioxide laser systems

The large Los Alamos CO₂ lasers typically consist of a master-oscillator/power-amplifier configuration. A subnanosecond pulse is switched out of the TEA oscillator by an electro-optic switch. This pulse is directed through preamplifiers, beam splitters, and a multipass amplifier. The amplified beams then enter the target chamber and are focused onto a target.

The Helios system⁸ is somewhat typical of the Los Alamos CO₂ laser fusion systems, especially from the point of view of the optics. The Helios optical system is schematically shown in Fig. 1. It contains more than 100 optical elements in 8 identical paths, each about 120 m long. Figure 2 shows the optical schematic of one of the eight paths in the Helios system. (The other seven beams are optically similar.) A nominal nanosecond pulse is switched out of a TEA oscillator through a three-stage CdTe Pockels cell array, which acts as an electro-optic switch. It then undergoes three stages of preamplification and beam splitting before entering the final amplifier. Figure 3 shows the optical schematic and the beam path through one of the final amplifiers all the way to the target plane. Optically, the final amplifier is a triple-pass 17X afocal off-axis Gregorian telescope. A 100 mJ, 2-cm-diam, collimated beam enters the triple-pass amplifier and increases in energy to nearly 3 J in the first pass. It then strikes a turning mirror and is focused at the spatial filter. The beam then diverges and is deflected by another flat turning mirror and its energy in this second diverging pass reaches nearly 100 J. Double passing the saturable absorber cell and the salt window (element No. 5 in Fig. 3) reduces the energy to about 150 J. The roughly 34-cm-diam collimated beam then makes a third amplification pass and the energy reaches nearly 1200-1400 J. The collimated beam is then brought to focus by an off-aperture parabola resulting in an f/2.4 beam at the target. Throughout most of the system, the beam size is ~2-2.5 cm in diameter.

*Work performed under the auspices of the U.S. Department of Energy

However, during the second pass through the final amplifier, the beam diameter increases to nearly 34 cm and retains this size until it is focused on the target to a spot ~120 μ m in diameter.

The Gemini laser fusion system is a 1-TW, two-beam system. Figure 4 shows the schematic of this system and Fig. 5 shows the optical schematic of the final optical and the target chamber area of the Gemini system. A nominal nanosecond pulse is switched out of the TEA oscillator. This pulse undergoes amplification to about 1 J after passing through the two preamplifiers. It is then split into two beams, which are amplified to full power in a large electron-beam-controlled, high-pressure, dual-beam-module amplifier. This beam is brought to focus at the target plane by an off-aperture parabola.

Figures 6 and 7 show the schematic and the various optical elements in the RIO Lumonics/GWTF system. Typically, a 70 ns, CO₂ laser pulse from the oscillator goes through a three-stage electro-optic switch and a 0.2-1 ns pulse (adjustable) emerges. It then makes four passes through the amplifier and emerges with a 400 mJ energy. The beam then goes through a spatial filter, isolator, and a beam expander section (the details are shown in Fig. 7) before going through the Lumonics 600 amplifier. The beam diameter at the output end of the electro-optic switch is 1 cm, the beam diameter at the output of the Lumonics 600 is 6 cm, and the energy is 3-6 J.

Taking the Helios system as an example of these types of systems, referring to Fig. 2, we find that the optical layout consists of basically three regions. The first part consists of 1X afocal chains using either refracting elements or slightly off-axis mirrors. The beam diameter in this region is roughly 2 cm. The second part, which consists of the triple-pass power amplifier, is a triple-pass 17X Gregorian off-axis telescope. After the second pass through this system, a roughly 34 cm collimated beam with the full energy emerges. The third part, which is the target chamber area of the system, has turning mirrors and the focusing off-aperture parabola. Consequently, while parts of the system deal with off-axis beams, the total system is a zero-field-angle system. The design of these systems results in a nominally diffraction-limited system, and the saturating property of the final amplifier results in a tilted plane wave (spatial) emerging from the final amplifier. Unless additional approaches for improving beam quality, such as adaptive optics, phase conjugation, etc., are used (they currently are not used in these systems), the use of clear apertures, spatial filters, and long-distance diffraction propagation are the only mechanisms used for aberration corrections. These aberrations could arise from defective components, alignment errors, etc. These are discussed in greater detail in the next section. The systems resemble conventional electro-optical systems that form an aerial image in the focal plane and also have a lot in common with typical laser systems. The correspondence with conventional image forming systems arises from the fact that there are afocal and focusing subsections that constitute the optical elements. They resemble typical laser systems in that spatial filters are extensively used to clean up the beam. In most conventional optical systems, the emphasis is on optimization of the optical transfer function properties and the tolerance analysis is based on fairly well understood departures from the nominal situation for the optical components and the various misalignments. In these systems, as the ultimate purpose is the coupling of the laser energy to the target, quantities such as the Strehl ratio, irradiance distribution, and encircled energy distributions are of interest.

Discussion of analysis techniques

The major departures from conventional systems in the case of these large laser systems include: (1) use of clear apertures, diffraction propagation over large distances, and use of spatial filters to correct aberrations in the beam train; (2) use of novel optical components such as 16- to 18-inch diam sapphire windows and single-point diamond-turned large optics, which are not made by conventional techniques and hence exhibit different kinds of imperfections; (3) the saturating properties of the final amplifier and other possible non-linear effects, which can alter the phase and intensity distribution in the beam.

Codes based on raytracing are not capable of handling propagation of beams through tight spatial filters, among other things. The state-of-the-art novel optical components like the large H₂I windows (which contribute most to the degradation of the beam) and the diamond-point turned mirrors have to be properly accounted for. Representation of these surfaces by ispheric terms or trigonometric functions, or a combination of these, appears inadequate. It is not readily obvious how one can easily and effectively represent them using spline-fit techniques. Existing programs which are capable of handling some of these peculiar difficulties are proprietary and prohibitively expensive to use, and, in addition, do not appear capable of running in a streamlined fashion.

As a consequence, it became necessary to consider a custom-tailored approach based on a combination of programs. The first of these is CDPL[®] (Computer Determined Fringe

Locator) developed by W. S. Hall of Los Alamos National Laboratory. The second is a version of FRINGE⁶ (the generic name for an interferometric analysis code originating from the Optical Sciences Center, University of Arizona) developed by J. Loomis. The third of these codes is LOTS⁷ (Laser Optical Transport System) which was developed by George Lawrence while he was at the Optical Sciences Center at the University of Arizona, and at Los Alamos. The next section describes these codes as well as the procedures used in the analysis of large laser systems.

Brief description of the procedure and the various programs used in the analysis

From the previous discussion, it is clear that the proper representation and characterization of the optical components in the system was essential to analyze and understand the optical performance of the system. It also became obvious that the diffraction propagation of a coherent beam had to be taken into account, and representation of each element by a method which enabled a good connection with the well known aberrations in conventional optical design, engineering, and manufacturing is desirable. One practical interface is to deal with the optical path differences (OPD) introduced by each optical element. However, just a map of the OPD in many instances is not enough to provide the physical insight as to the role that components is playing in the overall optical performance of the system. On the other hand, use of a polynomial set like the Zernike polynomials results in an understanding of the role played by each component.¹⁰ Consequently, the choice was a method which could use either the OPD (which is capable of better accuracy) or the Zernike polynomial set (which provides better insight).

In practice, the program FRINGE fits this scheme perfectly. Fizeau or Twyman-Green interferograms at 0.633 μm wavelength are made of the actual manufactured components. These are digitized using the program CDFL⁹ and serve as input data to FRINGE. FRINGE uses a global polynomial to represent the fringe data, which has the general form:

$$Z = a_0 + a_1 f_1(x,y) + a_2 f_2(x,y) + \dots + a_n f_n(x,y),$$

where n is the number of terms and $f_n(x,y)$ is the Zernike polynomial in two dimensions. The maximum number of terms currently used in the program is 36. The fitting is done using the method of least squares and the minimization results in a system of linear equations. These are solved by the use of a modified Gram-Schmidt method of constructing orthogonal polynomials. Zernike polynomials at 10.6 μm are computed for each component and stored in a file called ARR. This serves as input to the diffraction propagation program LOTS.

LOTS has been designed to compute the performance of the Los Alamos CO₂ laser fusion systems in terms of beam quality and energy. The laser pulse is treated as a two-dimensional complex amplitude distribution. The principal operations performed on this distribution include: (1) diffraction propagation; (2) propagation through aberrated optical components, which can be represented by the Zernike polynomial set data reduced from interferograms or in the form of random wavefronts of specified statistics; (3) propagation through spatial filters; (4) propagation through nonlinear amplification and absorption regions; and (5) propagation through clear apertures of arbitrary shape.

In principle, it is similar to the system-optical-quality code originally developed by Siegman and Sziklas to deal with the propagation of high-energy laser beams. However, LOTS is custom tailored to the Los Alamos CO₂ laser fusion systems and the version discussed here uses a 64 x 64 matrix and runs in about 52,000 octal words vs. an estimated minimum of 140,000 octal words for most systems optical quality codes. LOTS runs through one entire leg of the Helios system, consisting of ~100 optical components, in about 20 seconds on the CDC6600. This low core and fast execution time makes it quite convenient to use. A larger version which allows higher resolution of diffraction detail by the use of very large matrices is currently in the process of being implemented at Los Alamos. LOTS propagates the complex valued wavefront using diffraction calculations and represents the laser pulse as a two-dimensional complex array. It should be pointed out that the finite temporal width of the pulse is not treated, since only the spatial variations in the wavefront are of interest in optical design and engineering situations. The wavefront computations proceed from the oscillator through the various optical components (as represented by their respective Zernike polynomial set or as random wavefronts of specified statistics), spatial filters, amplifying media, (saturating gain effects are computed using the Franz-Rundvik equations or the results of the Los Alamos rate equation codes), saturable absorbers, and clear apertures as they sequentially occur in the actual system, thus directly simulating the optical train from end to end. The Strehl ratio, the enclosed energy and irradiance distributions can be printed out at any of the stations in the chain. The intensity distribution of the laser beam is displayed by isometric plots or by gray scale maps. The phase of the wavefront can be displayed in the form of interferograms or phase maps.

To briefly summarize the procedure, the optical components are characterized interferometrically. Fizeau or Twyman-Green interferograms of the components are made at 633 nm wavelength. These are digitized using CDFL. The Zernike polynomial coefficients at 10.6 μm are generated using FRINGE, and these are used to characterize the optical path difference introduced at each manufactured surface. The wavefront is propagated from end to end using LOTS, taking diffraction, nonlinear effects, and OPD modifications introduced by each component into account. The various parameters of interest, such as the Strehl ratio, the irradiance and encircled energy distributions, the amplitude and phase of the wavefront, etc., are computed and displayed as desired.

Examples of analysis and comparison with experiments. In this section, we present the results of analysis and experiments for the Gemini and Helios systems. We present an analysis of parts of the GWTF to show how this approach can be used to examine the classical aberrations present in these types of systems. Finally, using the Gemini power amplifier and the target system as an example, we show how examination of the Zernike polynomial set for individual elements can be used to understand and improve the performance of these systems.

Figure 8 shows the results of the analysis and the experiments¹¹ on the Gemini laser fusion system. As this figure shows, the Strehl ratios for both the beams agree and the encircled energy values agree also to within the uncertainties of the experimental techniques and the analytical computations. Figure 9 shows the results of the computations and experiments on two of the eight beams of the Helios laser system. Curve A represents the diffraction limited performance. Curve B represents the performance (computed) that can be expected if each of the optical components contributed a random peak-to-valley error of 0.1 λ (at 10.6 μm). All the components in Helios were expected to meet this specification goal, but the large salt windows in their mounts and a few other large components actually did not meet this goal. Curves C and D represent the computational and experimentally obtained values for one of the better beams in Helios.¹² Curves E and F represent the computed and experimental values for one of the worst beams in Helios. It must be stressed that these experimental results represent only the oscillator being fired. However, an experiment in the Gemini laser using a hole grating by J. V. Parker, J. Liberman, and V. K. Viswanathan,¹³ as well as computational analysis showed that the effects of firing the final amplifier altered the results to a negligible extent. So, these can be taken as values for the whole laser at full power. These results also show reasonable agreement between analysis and experiments.

The examples of Gemini and Helios show the analysis of complete systems. We now show how sub-system analysis is done. The first example deals with the analysis of parts of the GWTF system (Figs. 6 and 7).

A surrogate laser (CO₂ cw laser) was used along with a focusing lens to produce Smartt interferograms of the various subsystems in the RIO/Lumonics system. The aberrations due to the laser and the focusing lens were subtracted in the computer analysis to compute the contributions of each subsegment. Table 1 shows the results obtained with the RIO, Lumonics, and the test lens. The sum of the third order spherical aberration and coma contribution for the individual systems (RIO, Lumonics, Testlens) is almost equal to the whole system aberration. The coma and astigmatism need not add up properly, because their azimuthal angles can be different. Nevertheless, this shows that this method appears to work very well indeed. Table 2 shows the typical results from studying the stability of the various sections of the GWTF system. Among the major conclusions reached were: (1) The various parts of the RIO/Lumonics system appear to have reasonable optical stability. (2) The Lumonics appears to have the best and the RIO the worst optical quality. (3) Significant compensation of defects in optical quality takes place in the RIO/Lumonics system.

The final example deals with the usage of Zernike polynomials. This analysis was done on the Gemini system power amplifier and target system.¹⁰ In the analysis, the effect on windows W_1 and W_2 are combined and referred to as W_1 (Fig. 5). Table 3 gives a description in detail of the Zernike coefficients obtained as a result of interferogram reduction of actual components in their mounts.

Figure 10 shows the results of the analysis for the system with the components as described in Fig. 5. The Strehl ratio is only 0.1, and 85% of the energy is contained in a diameter of 240 μm (nearly 4 Airy discs) and the optical performance of the system needs considerable improvement. Examining the Zernike coefficients in Fig. 3, the predominantly large values occur for the zero degree astigmatism coefficient for M5C (-0.2812) and M6 (-0.2245). M6 is a parabola and, in addition to manufacturing difficulties, being a powered element it is wise to avoid using it if possible because of its complex effects on the various other aberrations, in addition to the one we wish to correct. It looks as though we can improve matters if we can reverse the sign of the zero degree astigmatism, the flat element. In practice, this can be accomplished by proper choice of that element

Table 1. Typical results of analysis of Smartt interferograms

Description	RMS (in λ)	P-V	Strehl	70-80 Diameter (μ m)	3rd Order Aberrations (in λ units)
RTO (without four-pass)	0.32	1.4	0.01	240 308	SPH -4.3 COMA +0.2 AST +0.2
Lumonics	0.14	0.7	0.5	46 88	SPH +1.6 COMA +0.4 AST +0.2
Test Lens	0.13	0.6	0.5	51 82	SPH +1.5 COMA +0.1 AST 0.0
After Lumonics (whole system including test lens)	0.13	0.8	0.6	34 67	SPH -1.0 COMA +0.7 AST +0.2
					Adding: SPH: -1.2 COMA: +0.7 AST: +0.4 (cannot add COMA and AST properly)

Table 2. Typical results from studying stability of GWTF system

Description	RMS (in λ)	P-V	Strehl	70-80 Diameter (μ m)	3rd Order Aberrations (in λ units)
10 (without four-pass)					
(A) 9:30 a.m.	0.32	1.4	0.01	240 308	SPH -4.3 COMA +0.3 AST +0.7
(B) 11:30 a.m.	0.37	1.5	0.02	240 295	SPH -5.2 COMA +0.9 AST +0.7
Lumonics Amplifier					
(A) 3 p.m.	0.14	0.7	0.45	42 80	SPH +1.1 COMA +0.3 AST +0.2
(B) 3:30 p.m.	0.14	0.7	0.52	46 88	SPH +1.6 COMA +0.4 AST +0.2
After Lumonics (whole system)					
(A) 3 p.m.	0.13	0.8	0.62	34 67	SPH -1.0 COMA +0.7 AST +0.2
(B) 3:30 p.m.	0.12	0.8	0.63	34 67	SPH -0.7 COMA +0.2 AST +0.5
(C) 4 p.m.	0.10	0.6	0.75	37 65	SPH -0.6 COMA +0.5 AST +0.2

Table 3.

	M4	M5A	M5B	M1	M5C	M6
Distance to Next Element in CMS	577.215	112.755	55.88	55.88	254.	83.82
Zernike Coefficient Description						
1. $RCOS\theta$ - X Tilt	0	.0011	0	0	.0009	-.0003
2. $PSIN\theta$ - Y Tilt	.0024	.0004	.0048	.0024	.0004	.0029
3. $(2R^2-1)$ - Focus	-.0014	-.0008	-.0028	-.0014	.0002	-.0021
4. $R^2COS2\theta$ - 0° Astigmatism	.0212	-.0957	-.0424	-.0212	-.2812	-.2205
5. $R^2SIN2\theta$ - 45° Astigmatism	.0353	.003	.0706	.0353	.0404	-.0185
6. $(3R^2-2)RCOS\theta$ - X Coma	.003	-.0015	.006	.003	-.0048	-.0615
7. $(3R^2-2)PSIN\theta$ - Y Coma	-.0121	-.0148	-.0242	-.0121	.0205	-.1202
8. $(6R^4-6R^2-1)$ 3rd order Spherical	.013	-.134	.026	.013	-.0006	.011
9. $R^4COS3\theta$ - X Clover	-.015	-.0051	-.03	-.015	-.0064	-.0583
10. $R^4SIN3\theta$ - Y Clover	.0183	-.0159	.0366	.0183	.0002	
11. $(4R^4-3R^2)COS2\theta$ - X Sphere Astigmatism	-.0141	.0003	.0282	-.0141		
12. $(4R^4-3R^2)SIN2\theta$ - Y Sphere Astigmatism		-.0007				

or by a combination of rotating the element in the mount and distorting it in a controlled fashion.) If we indeed reverse the sign to +0.2812 to MSC and analyze the system again, we get the results shown in Fig. 11. The Strehl ratio has gone up to 0.65, and 85% of the energy is enclosed in 120 μ m diameter (2 Airy discs), a considerable improvement in Strehl, peak irradiance, and in intensity and encircled energy distributions. Thus, this approach of analyzing actual manufactured components in their mounts and adjusting them in the assembly of the system appears to show considerable promise for optimizing the optical performance of large and complex systems where components are novel and state-of-the-art. It should be emphasized that the Two-Beam System performance at present lies in between the levels represented in Figs. 10 and 11.

Conclusions

The examples given in this article show that the analysis procedure described works well for the case of large CO₂ laser systems. It further shows that it is possible to use the conventional aberration analysis to understand and possibly improve the performance of even large complex systems described in the article.

References

1. The Gigawatt laser is a pulsed CO₂ laser currently operational at Los Alamos. It is being used for various experiments to determine parameters and solutions to problems of interest in the Helios and Antares lasers, as well as for other research work.
2. Gemini is the two-beam CO₂ laser fusion system currently operational at Los Alamos. It is being used for inertial confinement fusion research experiments.
3. Helios is an eight-beam, subnanosecond-pulse, 15-TW CO₂ laser and target facility for the study of laser fusion process and is currently operational at Los Alamos.
4. Antares is a 24-beam 40-kJ pulsed CO₂ laser currently under construction at Los Alamos.
5. An excellent description of the Zernike polynomials, their properties, and use in optics, can be found in Principles of Optics, by M. Born and E. Wolf (MacMillan Company, New York).
6. J. S. Loomis, "A Computer Program for Analysis of Interferometric Data: Optical Interferograms -- Reduction and Interpretation," ASTM, STP 566 (1978), pp. 71-86. See also, J. S. Loomis, "FRINGE User's Manual, Version 2," Optical Sciences Center, Tucson, Arizona (November 1976).
7. G. Lawrence and P. N. Wolfe, "Application of the Laser Optical Train Simulation (LOTS) Computer Code to Laser Fusion Systems and Other Physical Optics Problems," pp. 238-242; See also, V. K. Viswanathan, "Optical Design and Analysis of Carbon-Dioxide Laser Fusion Systems Using Interferometry and Fast Fourier Transform Techniques," pp. 158-164, both from the Proc. of the Los Alamos Conference on Optics '79, SPIE Volume 190 (1979).
8. R. L. Carlson, J. P. Carpenter, D. E. Casperson, R. B. Gibson, R. P. Godwin, R. F. Haglund, J. A. Hanlon, E. L. Jolly, and T. F. Stratton, "Helios: A 15-TW Carbon-Dioxide Laser Fusion Facility," Los Alamos National Laboratory (submitted to Journal of Quantum Electronics).
9. CDFL is the Computer Determined Fringe Locator code which uses merger criteria and automatically digitizes the fringes by proper sampling.
10. V. K. Viswanathan, "Unique Problems and A Possible Novel Optical Design Approach for the Design and Assembly of CO₂ Laser Fusion Systems," SPIE Proceedings, Vol. 147 (1978) on Computer-Aided Optical Design.
11. V. K. Viswanathan, J. J. Hayden, and I. Liberman, "Optical Performance of the Gemini CO₂ Laser Fusion System," Proc. of the Los Alamos Conference on Optics '79, SPIE Volume 190 (1979).
12. See article by I. Liberman in Digest of Topical Meeting on Inertial Confinement Fusion (Optical Society of America, 1980), Paper TuB/10.
13. See article by J. V. Parker, I. Liberman, and V. K. Viswanathan in Digest of Topical Meeting on Inertial Confinement Fusion (Optical Society of America, 1978), Paper ThD/13.

Fig. 1. Optical schematic of Helios laser.

Fig. 2. Optical schematic of one of the eight beams in Helios.

Fig. 3. Schematic of final amplifier-target region.

Fig. 4. Optical schematic of Gemini laser.

Fig. 5. Final amplifier-target region of Gemini.

Fig. 6. Schematic of GWT laser system.

Fig. 7. Optical schematic of GWT laser system.

Fig. 8. Experimental and computational results on Gemini system.

Fig. 9. Experimental and computational results on Helios system.

Fig. 10. Computational results with zero degree astigmatism coefficient = -0.2812.

Fig. 11. Computational results with zero degree astigmatism coefficient = +0.2312.