ICRF MODELLING

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ABSTRACT

This lecture provides a survey of the methods used to model fast magnetosonic wave coupling, propagation, and absorption in tokamaks. The validity and limitations of three distinct types of modelling codes, which will be contrasted, include discrete models which utilize ray tracing techniques, approximate continuous field models based on a parabolic approximation of the wave equation, and full field models derived using finite difference techniques. Inclusion of mode conversion effects in these models and modification of the minority distribution function will also be discussed. The lecture will conclude with a presentation of time-dependent global transport simulations of ICRF-heated tokamak discharges obtained in conjunction with the ICRF modelling codes.

SECTION I: INTRODUCTION

An assessment of the degree to which supplementary plasma heating is required to reach ignition in a tokamak fusion reactor has become an integral part of future device design studies because it is possible that ignition cannot be achieved with ohmic heating alone. The most promising candidate for providing auxiliary power to the plasma is the resonant absorption of fast magnetosonic waves near the fundamental cyclotron layer of the minority species or the second harmonic cyclotron layer of the majority species in a two-ion component plasma.¹⁻³ This method is favored for a number of reasons, including experimental success with the method on devices such as PLT,⁴ TFR,⁵ JET,⁶ and ASDEX,⁷ relatively straightforward technological requirements, good centralized power deposition characteristics, and moderate accessibility needs for the wave launcher and power source. To date, good qualitative agreement has been found between certain theoretical models and

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the existing data base⁸⁻¹³ for power levels up to 5 MW in medium-sized devices. Evaluation of the performance of the method at power levels up to 30 MW and in large machine sizes will be possible over the next few years when experiments on JET⁶ and JT-60¹⁴ are completed. With preliminary design work on the next generation of devices commencing, it is imperative that modelling codes, which synthesize theoretical treatments of the various processes involved in an ICRF-heated tokamak discharge, be developed to the point where they can provide quantitative agreement with the existing data base over a wide range of parameters. The models may then be utilized in design efforts with more confidence.

Ideally, a modelling code encompasses all physics necessary to quantitatively reproduce experimental results, while at the same time remains sufficiently numerically tractable to warrant its development and use. A fully self-consistent simulation of an ICRF-heated discharge, starting from the Vlasov-Maxwell system of equations and including effects of plasma equilibrium, stability, transport, and wave coupling, propagation, and absorption is beyond the current capability both of theory and computational physics. Nevertheless, because of large differences in the spatial-temporal scales for the different processes involved, significant progress towards achieving a realistic simulation can be made by reducing the problem into various subregimes. The dominant physical effects in each subregime may be treated in detail, while the remainder are treated approximately. A global time-dependent simulation is then reconstructed by interfacing the individual subcomponents.

Because transport and global stability time scales, τ_E , are typically much longer than the wave propagation and absorption time scales, τ_w , and the collisional time scales, τ_c , for collisional relaxation of the particle velocity distribution functions, it is natural to model the ICRF-heated discharge using three different time regimes. For short time scales on the order of τ_w , power deposition profiles may be computed assuming the background plasma is in a local equilibrium. For longer scales τ_c , collisional transfer of power from the heated species to the background species is determined. On the longest time scales on the order of τ_E , the power absorption is treated as a known source function and the discharge is allowed to relax via various transport mechanisms. The deposition profiles may then be recomputed in time as the plasma parameters evolve on the transport time scales.

Turning now to a determination of the power absorption profiles for each species, the basic scheme consists of a wave launcher at the vacuum vessel wall which excites fast magnetosonic waves in the adjacent plasma edge regions. The excited waves then penetrate to the cyclotron layer located in the high density region near the magnetic axis where resonant absorption of the waves by the plasma ions may occur. In Fig. 1, reproduced from Bhatnagar et al.¹⁵, a local evaluation of the fast wave root of the WKB hot plasma dispersion relation is displayed for a JET discharge. A natural separation of the wave physics into three distinct regions is evident. For minor radii in Fig. 1 greater than about 1.2 m, a thin layer at the plasma edge exists in which the fast wave is mainly evanescent. Over the bulk of the discharge, say for $0.5 < x < 1.2$, a propagation region exists in which the wavelength of the mode evolves slowly as the wave penetrates towards the magnetic axis. Finally, a thin absorption/mode conversion layer is evident at small minor radii in which the wavelength varies rapidly, and cyclotron absorption and mode conversion to the ion Bernstein branch are both possible. Based on the nature of the wave dispersion characteristics in the WKB limit, a modelling code should address different physics issues in each of the regions depicted in Figs. 2 and 3.

Region I consists of the antenna-plasma coupling region. The edge plasma is characterized by low densities and temperatures and may be influenced by recycling of hydrogenic and impurity species off the wall, so a cold plasma model is appropriate. Of prime importance in this region is the determination of the power distribution and wave number spectrum of the fast waves which are coupled through the vacuum/evanescent layers. An understanding of this region would permit optimization of the wave launcher for maximum plasma heating efficiency, minimal parasitic loading by the slow wave, and minimal generation of impurities via contact between the plasma and the launcher or excitation of energetic particles at the plasma edge. In general, the edge region is poorly understood from a theoretical viewpoint and underdiagnosed from an experimental viewpoint.

Region II encompasses the WKB propagation region. Because there is little difference between the fast wave root obtained from the hot plasma or cold plasma dielectric tensors, a cold plasma model is generally sufficient to model this region. Furthermore, because changes in the wavelength are both generally small relative to the intrinsic gradient scale lengths of the

equilibrium, the wave propagation may be successfully treated using methods based on the WKB, or geometrics optics, approximation. The important effects to be addressed during the propagation phase include refraction of the wave due to gradients in the plasma density and magnetic field profiles, and geometric focussing induced by the finite size of the wavefronts launched by the antenna at the edge.

Region III is the diffraction-limited propagation region in which the launched wavefront has been focussed down into such a compact spatial extent that diffractive effects become significant. A cold plasma model is still probably acceptable for this region, since the wave has not yet encountered the absorption/mode conversion layer. However, WKB approximations are becoming invalid here and full wave solutions are probably required. Furthermore, nonlinear effects may beg. to influence the heating process in this region as incident power levels, and corresponding intensity levels in this region, are increased. It is encouraging that such effects do not appear to be significant in experiments to date at power levels up to 5 MW in PLT.⁸

Finally, the ion cyclotron absorption/mode conversion layer is designated as Region IV in Fig. 2. A kinetic, full wave treatment is necessary to determine the power partition among the plasma species in the presence of cyclotron absorption processes at the fundamental or higher harmonic frequencies, mode conversion to and subsequent damping on the ion Bernstein wave, and reflection from and tunnelling of power through the layer.¹⁶⁻²⁰ A one-dimensional treatment may be adequate, especially if the layer is thin and the vertical extent of the incident wavefront is confined to the central regions of the discharge where the equilibrium profiles tend to be flat. However, two-dimensional effects have not yet been studied so it is difficult to address their impact. Velocity space evolution of the plasma distribution functions, which occurs on the intermediate time scales, τ_c , in this region as a result of the heating, can also affect the power partitioning among the various species through quasilinear effects^{13,20-22} and should be included in any simulation of this region.

Returning now to the longer transport and equilibrium time scales, a global time-dependent simulation of an ICRF-heated discharge may now be obtained by interlocking a calculation of the spatial-temporal evolution of the plasma equilibrium quantities with a reevaluation of the instantaneous power depositions. Effects on the transport and equilibrium caused in

particular by changes in the electron and ion thermal conductivities because of the production of non-Maxwellian, anisotropic tails on the ion distribution functions and direct electron heating by the wave are not well understood theoretically. Experimental evidence that indicates that such modifications are important include observations of a decrease in the overall energy confinement time during the RF pulse,^{8,11} a dramatic enhancement of the sawtooth amplitude in the central electron temperature measurements during the RF pulse,⁸ and direct observation of the tail formation during second harmonic hydrogen¹⁰ or deuterium majority/hydrogen minority heating²² experiments. Theoretical models addressing these issues need to be developed and incorporated into modelling codes.

An all-inclusive modelling code does not yet exist. However, significant progress has been made in treating many aspects of the heating process and will be reviewed in the remainder of this lecture. Only those models which attempt to treat more than one aspect of the problem, for example, wave propagation and absorption, will be discussed. Detailed analyses which study only one particular feature, such as coupling or absorption, are available in the literature. Cold plasma models of the coupling region with or without realistic models for the launching structure have been developed by Theilhaber and Jacquinot,²³ Ram and Bers,²⁴ Fortgang, Colestock and Hwang,²⁵ among others. One-dimensional kinetic models of the absorption/mode conversion region have been developed by Swanson,^{16,17} Colestock and Kashuba,¹⁸ and are reviewed in an upcoming article by Swanson.²⁶ Fokker-Planck calculations of both the velocity and configuration space evolution of the particle distribution functions and subsequent plasma response have been performed by Stix,²⁰ Scharer and Jacquinot,²⁷ Kerbel and McCoy,²¹ and Hammett.²²

As a matter of nomenclature, "n"-dimensional models will refer to the number of dimensions in which the plasma equilibrium is allowed to be inhomogeneous while two-dimensional models retain both radial and poloidal variations. Thus, one-dimensional models retain only radial inhomogeneities. Since tokamak equilibria are generally assumed to be axisymmetric, the wave structure in the toroidal direction can usually be constructed from decoupled solutions of the wave equation obtained for each toroidal harmonic excited by a wave launcher of finite toroidal extent. However, full three-dimensional treatments may be necessary if kinetic effects couple the behavior of the fields for different toroidal mode numbers.

Models for the fast wave fields and corresponding power deposition profiles in tokamaks, which will be reviewed here, include three basic categories. One category encompasses methods which generate global full field solutions for waves excited by an antenna in or near a plasma by solving the Maxwell equations as a driven boundary value problem.²⁸⁻³⁷ The plasma is modelled as a lossy dielectric medium of varying complexity which is surrounded by a conducting vacuum vessel wall. A second category consists of geometric optics solutions, in which the launched wave fields are discretized and ray tracing techniques are utilized to determine the power flow from the plasma edge to the absorption layer.^{13,15,38-40} Power deposition profiles are then generated by using the wave characteristics near the absorption region in conjunction with linear, quasilinear, or kinetic absorption models. The final category consists of parabolic approximation methods⁴¹⁻⁴² in which continuous representations for the fields obtained in the propagation regions of the discharge are interfaced with full field solutions in the coupling and absorption regions to provide global power deposition profiles. The fields in each region are generated by making appropriate approximations to the Vlasov-Maxwell system. To date, only those ICRF field models based on the geometric optics approach have been successfully integrated with an MHD equilibrium and transport code.^{13,40} This lecture will conclude with a presentation of time-dependent simulations of ICRF-heated discharges in PLT generated using one of these codes.

SECTION II: GLOBAL FULL FIELD SOLUTION METHODS

In an effort to gain an understanding of the interplay between wave coupling, propagation, and absorption, the global structure of the wave fields has been studied by solving the Maxwell equations for a plasma dielectric medium which is bounded by a conducting vacuum vessel wall and driven by a current source. This method is particularly well suited for the range of plasma parameters in which wave damping is weak, allowing for the possibility of coupling to an eigenmode of the system. It is also appropriate for the study of low density, small- to moderate-sized devices in which the fast mode wavelength approaches the size of discharge, thereby invalidating other treatments based on WKB approximations. Since the field solutions are generated by solving a boundary value problem, it is possible to retain effects caused by realistic equilibrium geometry, such as toroidal

corrections^{31,35-37} and high beta modification^{35,37} of the wave propagation as well as the structure of the eigenmodes of the system. Focussing effects induced by the finite poloidal and toroidal extent of the launcher as well as refraction and wave coherence effects are also naturally incorporated in the solutions. By calculating the local power absorption in these models and relating it, via Poynting's Theorem, to the impedance of the wave launcher, it may be possible to optimize the design of the launcher to achieve maximum coupling to and heating of the plasma.

The wave equation describing the structure of the wave fields may be derived from the Maxwell equations,

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J}_{\text{ext}} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}, \quad (1)$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \quad (2)$$

and

$$\vec{D} = \bar{\bar{K}} \cdot \vec{E}, \quad (3)$$

where $\bar{\bar{K}}$ is the dielectric tensor describing the response of a magnetized plasma to the applied RF fields and $\vec{J}_{\text{ext}}$ is the current source modelling the antenna. Combining Eqs. (1-3) and assuming a harmonic time dependence at a frequency ω , i.e., $\partial/\partial t \rightarrow -i\omega$, the wave equation may be written as

$$\nabla \times \nabla \times \vec{E} - \omega^2/c^2 \bar{\bar{K}} \cdot \vec{E} = \frac{4\pi i\omega}{c^2} \vec{J}_{\text{ext}}. \quad (4)$$

The plasma response is usually modelled in the cold plasma limit using the Lorentz force equations,

$$n_j m_j \frac{\partial \vec{v}_j}{\partial t} = n_j q_j \left(\vec{E} + \frac{\vec{v}_j}{c} \times \vec{B} \right) - \nu_j m_j \vec{v}_j \quad (5)$$

where j denotes a particular plasma species. Wave damping is simulated in a heuristic fashion through the inclusion of the collisional term, $\nu_j m_j \vec{v}_j$. This term also effectively removes the singularity and zero present in the usual cold plasma dispersion relation at the two-ion hybrid and ion cyclotron resonances. In conjunction with Ohm's Law, $\vec{J} = \bar{\bar{\sigma}} \cdot \vec{E}$, where $\bar{\bar{\sigma}}$ is the

conductivity tensor, Eq. (5) leads to the collisionally modified cold plasma dielectric tensor, $\bar{K}$, where

$$\bar{K} = \begin{bmatrix} K_{\perp} & -iK_x & 0 \\ iK_x & K_{\perp} & 0 \\ 0 & 0 & K_{\parallel} \end{bmatrix}, \quad (6)$$

and the elements of $\bar{K}$ are given by²⁸:

$$K_{\perp} = 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2 - \Omega_j^2}, \quad (7)$$

$$K_x = \sum_j \frac{\Omega_j}{\omega} \frac{\omega_{pj}^2}{\omega^2 - \Omega_j^2}, \quad (8)$$

$$K_{\parallel} = 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2}. \quad (9)$$

In Eqs. (7)-(9), the cyclotron frequency of the j th species is given as $\Omega_j = q_j B / m_j^* c$ and the plasma frequency of the j th species is given by $\omega_{pj}^2 = 4\pi q_j^2 n_j^2 / m_j^*$, where $m_j^* = m_j(1 + i\nu/\omega)$. The dielectric tensor has been written in a local cartesian coordinate system aligned with the z -axis along the equilibrium magnetic field. It is necessary to rotate the tensor from the local frame to the coordinate system chosen for the laboratory frame in which the wave equation is solved.

Upon examination of the form of Eq. (6), it becomes clear why the cold plasma limit is generally taken when global solutions to the wave equation are generated. In particular, the elements of the cold plasma dielectric tensor are independent of the wavelength of the modes and hence contain no derivative operators which would raise the order of the system of differential equations to be solved. Introduction of kinetic effects would increase the order of the system, corresponding physically to the inclusion of other modes such as ion Bernstein waves. Despite the difficulties encountered when kinetic effects are retained, some progress has been made in this area and will be discussed briefly further on. The main objection to the cold plasma model is that it inadequately deals with the absorption and energy partition processes among the plasma species and that it is incapable of modelling mode conversion

phenomena. However, in the weak damping limit in which the details of the absorption and mode conversion processes may only be important locally within the absorption layer, then the global solutions for the wave fields obtained using a heuristic model for the damping may be sufficiently accurate to be used in optimization studies of the design of the wave launcher.

Global solutions to the wave equation, Eq. (4), in a tokamak equilibrium with the plasma response modelled by the collisional cold plasma dielectric tensor, Eq. (6), may be numerically generated through the use of finite difference or finite element techniques. Specification of the model is completed by the constraints imposed by the proper boundary conditions. These boundary conditions include the familiar condition on the tangential component of the electric field on the conducting wall,

$$\vec{E}_{\text{Tan}} = 0, \quad (10)$$

and the usual jump and continuity conditions on the fields at an interface between two different dielectric media⁴³ (in this case, between the plasma and any adjacent vacuum regions),

$$(\vec{D}_2 - \vec{D}_1) \cdot \hat{n} = 4\pi q_{\text{surface}}, \quad (11)$$

$$\hat{n} \times (\vec{E}_2 - \vec{E}_1) = 0, \quad (12)$$

$$\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \frac{4\pi}{c} \vec{J}_{\text{surface}}, \quad (13)$$

$$\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0, \quad (14)$$

where $\hat{n}$ is a unit normal directed out of Region I into Region II and $\vec{B} = \mu \vec{H}$. If a coordinate system is chosen with the origin at the magnetic axis, then regularity of fields at origin must also be required. As a side note, instead of treating the current source modelling the antenna as a driving term in Eq. (4), one may instead treat it as a boundary surface upon which the conditions in Eqs. (10)-(14) must be satisfied.

The wave equation as it stands consists of three coupled second order partial differential equations for the wave components, $\vec{E}_1$ and $\vec{E}_2$. Considerable simplification of the system may be achieved if the component

governing the fast wave can be decoupled approximately from those describing the slow wave. The key approximation which allows this decoupling is that $E_{\parallel}$ is typically very small for the fast mode throughout most of the plasma. Physically, this is true because electrons streaming along magnetic field lines effectively short out any parallel electric field. Mathematically, this can be verified by noting, from Eqs. (6)-(9), that whereas $K_{\perp} \sim K_X \sim O(c^2/v_A^2)$, where v_A^2 is the local Alfvén speed, $K_{\parallel} \sim O(m_i/m_e c^2/v_A^2) \gg K_{\perp}$, K_X . As a result, from the parallel component of the wave equation, $K_{\parallel} E_{\parallel} \sim O(1)$, hence $E_{\parallel} \sim O(m_e/m_i) \ll 1$. In the limit $m_e/m_i \rightarrow 0$, the fast wave is described by the perpendicular components of the wave equation in which the terms proportional to $E_{\parallel}$ have been neglected. This approximation breaks down in the very low density regions at the plasma edge where the fast wave is evanescent and in the thin absorption/mode conversion layer in the plasma core, and thus may lead to an incorrect calculation of the wave coupling and corresponding antenna impedance. Since the antennas used in current experiments are protected by Faraday shields, which effectively prevent the launching of any RF wave fields with a nonzero toroidal component, coupling to the slow wave via finite $E_{\parallel}$ effects is minimized, though not eliminated because of the presence of the poloidal magnetic field. In modelling codes, breakdown of the approximation can be avoided by maintaining a sufficiently high plasma density out to the conducting wall boundary so coupling to the slow wave is prevented. However, if experimental results should ever indicate that such coupling is important, then it would be necessary to solve for all three components of the wave fields in order to model the antenna impedance more accurately.

Poynting's theorem is used in these codes to determine the power deposition profiles and to calculate the corresponding effective antenna impedance. Letting $\vec{S} = 1/\mu_0 \vec{E} \times \vec{H}^*$ denote the Poynting flux (and switching to MKS units to calculate powers in watts), then the condition for energy conservation may be written:

$$\nabla \cdot \vec{S} = -\vec{J} \cdot \vec{E} - \frac{\partial}{\partial t} \left[\frac{B^2}{2\mu_0} + \frac{\epsilon_0 E^2}{2} \right]. \quad (15)$$

The divergence of $\vec{S}$ represents power flow from the antenna to the plasma, while the term $\vec{J} \cdot \vec{E}$ represents energy absorption in the plasma. The third term represents a change in the field energy content with time. Following the

development in Stix²⁸ and averaging over a wave period leads to an expression for the power absorption per unit volume in the plasma:

$$\frac{\partial W(\vec{r})}{\partial t} = \frac{1}{2} \omega \epsilon_0 \vec{E}^* \cdot \vec{K}^A \cdot \vec{E}, \quad (16)$$

where $\vec{K}^A$ denotes the antihermitian part of the dielectric tensor. In the collisional cold plasma model, $\vec{K}^A$ is nonzero because of the frictional drag force assumed in Eq. (5). The loading resistance, $R_L(\omega)$, as seen by the antenna in an ideal system in which there are no power losses to the walls, etc. is related to terms of the power absorbed by the plasma as

$$R_L(\omega) = \frac{2 \int d^3r \frac{\partial W(\vec{r})}{\partial t}}{|I|^2}, \quad (17)$$

where I is the total current carried by the antenna. Similarly, the antenna impedance, $Z_A(\omega)$, may be determined using the ohmic power dissipated by the interaction of the source current and the wave fields.

$$Z_A(\omega) = \frac{2 \int d^3r \frac{1}{2} \langle \vec{J}_{\text{ext}} \cdot \vec{E} \rangle}{|I|^2}. \quad (18)$$

The real part of the antenna impedance in the ideal system is equal to the loading resistance, a consequence of conservation of energy in the system. The imaginary part of the impedance corresponds to circulating power which is not absorbed by the plasma medium. It is interesting to note that the calculation of $R_L(\omega)$ and $Z_A(\omega)$ depends on a knowledge of the actual current flowing in the antenna. In existing models for the global structure of the wave fields, this current is assumed to be a known quantity (constant current approximation). In fact, the amount of current flowing can be influenced by the existing plasma conditions and should be calculated self-consistently with the fields in order to better understand the relationship between wave coupling and absorption.

Modelling codes which solve for the global structure of the fast wave fields in tokamak equilibria of varying degrees of sophistication have been developed over the past few years by a number of different individuals²⁸⁻³⁷. Though none of the codes have been used extensively to simulate observations from any of the existing ICRF heating experiments in tokamaks, numerical results from the codes are useful for studying general characteristics of the

wave structure, absorption profiles, and loading impedance. Initial studies of the global nature of the fast wave fields in a tokamak were presented in 1982 by Colestock and Kluge²⁹ and in 1983 by Colestock and Kashuba.³⁰ Using a two-dimensional straight cylindrical tokamak model in which the axial field exhibited gradients in the form $B_0/(1 + r/R_0 \cos \theta)$, they studied the effects of changes in plasma composition and other operating parameters on the existence of strongly damped waves and weakly damped eigenmodes in a PLT-like device. A detailed study of the fast wave in a similar type of a two-dimensional tokamak was published by Itoh, Itoh, and Fukuyama.³¹ Their model included effects of finite poloidal and toroidal extent of the wave launcher, radial density gradients, and gradients in the equilibrium magnetic field, but neglected rotational transform effects and toroidal effects related to the shape of the vacuum vessel. Numerical results from their code for the case of waves launched on the low field side are reproduced in Fig. 4. Damping is fairly strong and localized along the hybrid layer, as indicated in Fig. 4a. Tunneling of the wave through to the high field side is observed in the plots of the real and imaginary parts of the field. More recently, effects related to a fully toroidal 2-D tokamak equilibrium,^{35,37,44} to fully three-dimensional equilibria,³⁶ and to the presence of the cold plasma slow wave³² have been included in individual codes of this genre.

The main limitation on this particular method of modelling the fast wave physics in tokamaks is a numerical one related to adequate resolution of the wave structure given the finite amount of storage space available on the largest and fastest computers. High resolution has been achieved in a code by M. Phillips,³² in which all three components of the wave electric field are generated on a grid with approximately 3200 grid points using finite difference techniques on a CRAY-XMP computer. Considering that it takes at least three grid points to resolve a sinusoidal wave structure, and that an $N_\theta \times N_\phi$ numerical grid must cover the spatial dimensions of a tokamak, i.e., an area of approximately a^2 square meters, then the minimum structure that can be resolved is given approximately as $a/(N_\theta/3)$. For a 57×57 grid in a reactor-sized device with an average minor radius of 1.5 meters, the wavelength resolution limit is thus approximately equal to 8 cm. An estimate of the wavelength of the fast magnetosonic wave obtained by evaluating the cold plasma dispersion relation²⁸ for reactor densities of around $6 \times 10^{14} \text{ cm}^{-3}$ indicates that the fast wave is barely resolvable in a reactor by the global

field codes. For these devices, we must turn to the geometric optics or parabolic approximation methods described in the next two sections. Of course, since $\lambda_1 \sim 1/\sqrt{n}$, where n is the typical plasma density, the fast mode is fully resolvable by the global field codes in smaller, lower density devices, such as PLT or TFR.

To conclude this section on global field solution methods for the ICRF fields in tokamaks, let us return briefly to the question of kinetic effects on the wave absorption and mode conversion. The wavelength resolution limit is the main reason why kinetic effects, which introduce the slow wave with an even shorter wavelength, are generally not included in the global field codes. Attempts by Fukuyama, Nishiyama, Itoh, and Itoh³³ and by Fukuyama, Goto, Itoh, and Itoh³⁴ to solve the higher order systems which describe the simultaneous presence of both the fast and the kinetic slow modes in one-dimensional and two-dimensional models, respectively, have met with some success. The main problem in formulating the solutions for the higher order system involves the identification of the additional boundary conditions needed to specify the slow mode. In particular, at some point in the plasma, the slow mode becomes unresolvable and the system reduces in order. The reader is referred to the papers by Fukuyama et al.^{33,34} to see one possible manner in which this difficulty may be addressed. Another possible method for including kinetic effects in global field codes has recently been formulated by Colestock.⁴⁵ His approach, which is described more fully in his paper for this conference, is to maintain the lower order system which describes propagation and absorption of the fast mode but to replace the collisional cold plasma dielectric tensor with a numerically derived tensor, which includes an approximate kinetic modification of the absorption and wave dispersion of the fast mode in the vicinity of the resonance zone. Though his approach will be unable to treat mode conversion effects, it may provide a better model for the calculation of the plasma loading for comparison with experiments.

SECTION III: GEOMETRICS OPTICS APPROACH

Global field representations described in the previous section for the ICRF waves in tokamaks are most useful in moderately sized devices, such as TFR or PLT, at modest density levels, and in the weak damping limit. For larger, denser devices, such as JET or TFTR, as the relative ratio of the fast mode wavelength to the machine size decreases, models which utilize geometric optics approximations to determine the power flow in the propagation region between the coupling region and the absorption layer (see Fig. 2) become more appropriate. In the geometric optics approximation, the wave equation is solved in the propagation region using WKB techniques, in which solutions are developed perturbatively in terms of a small expansion parameter. The wave electric field is assumed to be of the form:

$$\vec{E}(\vec{r}) = \vec{a}(\vec{r}) e^{i\psi(\vec{r})} \quad (19)$$

where $\vec{a}(\vec{r})$ is a slowly varying amplitude and $\psi(\vec{r})$ is a rapidly varying phase function. In this case, the expansion parameter is $1/L_g \nabla E_{\perp} \sim 1/k_{\perp} L \sim \delta \ll 1$, where $k_{\perp}$ is the perpendicular wave number of the fast mode and L_g is the characteristic scale length for equilibrium variations in the propagation region. The ramifications of this approach have been discussed extensively by Weinberg,⁴⁶ Bernstein,⁴⁷ and Brambilla and Cardinali⁴⁸ and the interested reader is referred to their papers for more details. Expanding $\vec{E}(\vec{r})$ in an asymptotic series in δ , substituting the expansion into the wave equation, and solving the resulting hierarchy of equations order by order leads to the well-known ray tracing equations for the wave phase-space evolution:

$$\frac{d\vec{r}}{dt} \equiv \vec{v}_g = - \frac{\partial D^H / \partial \vec{k}}{\partial D^H / \partial \omega} \quad (20)$$

$$\frac{d\vec{k}}{dt} = \frac{\partial D^H / \partial \vec{r}}{\partial D^H / \partial \omega} \quad (21)$$

$$\frac{d\psi}{dt} = \vec{k} \cdot \vec{v}_g \quad (22)$$

and the corresponding power transport equation,

$$\frac{d \ln P}{dt} = -2\vec{k}_{\perp} \cdot \vec{v}_g \quad (23)$$

In Eqs. (20)-(22), $\vec{r}$ and $\vec{k}$ denote configuration and wave vector space, $\vec{v}_g$ is the group velocity, D^H is the hermitian portion of the locally evaluated dispersion relation, P is power per unit volume, and k_i is the imaginary portion of the wave vector, $\vec{k}$, given by³⁸

$$k_i = \frac{\text{Im } D}{|\partial D / \partial \vec{k}|}, \quad (24)$$

where D is the full locally evaluated dispersion relation.

The implementation of Eqs. (20)-(24) in a geometric optics field model requires the assumption that the wave fields may be represented as a collection of noninteracting wave packets, whose power flows along the phase-space paths defined by Eqs. (20)-(22) and evolves according to Eqs. (23)-(24). To complete the model, initial starting conditions for the ray bundles must be specified just inside the propagation region, and power deposition profiles to each species must be calculated.

Geometric Optics (GO) field model codes have been developed by several different individuals.^{13,15,38-40} The ray tracing subroutine in any of the codes utilizes a cold, warm, or hot plasma dispersion relation to treat wave propagation in a two-dimensional equilibrium with no significant differences between the various codes. Though the use of the hot plasma dispersion relation yields different values for wave electric field polarizations than does the cold or warm plasma models in the vicinity of the absorption layer,¹⁶⁻¹⁹ the GO approximation breaks down in that region and a full field analysis should probably be implemented.

Implicit in each existing GO code is the assumption that full field solutions in the coupling region have been derived and may be used to choose initial ray conditions that accurately reflect the structure of the wave fields radiated into the plasma by the antenna. In practice, the actual interfacing of a full field solution model for the coupling region with a subroutine defining the initial conditions for the ray bundles has not yet been implemented in many of the codes, in part because codes for the coupling region are still under development. One approach to dealing with this difficult issue has been implemented by Bhatnagar et al.^{15,49} through the use of a one-dimensional radially inhomogeneous equilibrium model for the antenna-plasma coupling region. Poloidal variations in the equilibrium are ignored while the toroidal and poloidal variations in the wave field are represented as Fourier integrals over a range of $k_{\parallel}$ and k_y values in the form

$$E_Y(\vec{r}) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} E_Y(\vec{r}, k_{\parallel}, k_Y) e^{i(k_{\parallel} z + k_Y y)} dk_Y dk_{\parallel}, \quad (25)$$

where the $\hat{y}$ direction is identified with the poloidal direction. The terms proportional to $\partial E_Y / \partial y$ model the effects which may arise due to coupling to low poloidal mode number waves or to surface modes. The E_Y field is evaluated using a radiation boundary condition in the plasma, appropriate to the assumption of strong single pass absorption. This particular condition is still probably acceptable for the intermediate damping regime for wave components with high values of $k_{\parallel}$, since these components travel an appreciable distance in the toroidal direction without returning back to the launcher. Waves with low values of $k_{\parallel}$ that are not heavily damped tend to bounce numerous times between the edge region and the resonance layer, leading to the possibility of poloidal eigenmode excitation and subsequent modification of the coupling characteristics. A connection between the discrete ray representation for the fields and the asymptotic form of the full wave fields radiated by the antenna may be accomplished by expressing the radiated fields in WKB form inside the propagation region as:

$$E_Y(\vec{r}) = a(\vec{r}) e^{i\psi(\vec{r})}. \quad (26)$$

It is most convenient to perform the discretization process on wavefronts, or constant phase surfaces (CPS), defined by $\psi = \text{constant}$. The wave vectors for the rays on the CPS are identified with $\vec{k}_i = \nabla\psi(\vec{r})$, where the subscript "i" denotes any particular ray. This method yields reasonable agreement in a uniform plasma model between the constant phase surfaces constructed according to Eq. (26) and those reconstructed by evaluating starting parameters for rays on a CPS and ray tracing back towards the antenna source. Results of such a comparison are displayed in Fig. 5, reproduced from Bhatnagar et al.¹⁵

Since the model neglects two-dimensional equilibrium variations, it may lead to an inaccurate representation of the coupled spectrum in situations where the poloidal extent of the antenna is large and intersects the cyclotron layer in the edge region.

Models for evaluating the power deposition profiles vary widely among the existing codes. The linear damping model, as expressed in Eqs. (23)-(24), has been implemented by McVey³⁸ and Bhatnagar et al.¹⁵ to determine the direct

power flow from the wave to the plasma. Redistribution of the power flow to each species is further evaluated in the code by Bhatnagar, using a simplified version of a model developed by Stix.²⁰ Sample results of their code for a deuterium majority-hydrogen minority plasma under typical JET operating conditions are reproduced in Fig. 6. This approach does not address modifications of the power deposition caused by mode conversion and tunnelling phenomena or by tail formation in the minority ion species distribution function in velocity space as it is heated by the wave.

One approach for dealing with power deposition calculations in the presence of mode conversion and tunnelling has been implemented in a code developed by Brambilla.³⁹ Relative amounts of incident power that are reflected (R), transmitted (T), or mode converted to the ion Bernstein wave (C), are evaluated within the mode conversion layer using a fourth order one-dimensional full field model with no cyclotron dissipation processes included. Hence, the validity of the model is restricted to those plasma and wave parameters in which the Doppler-broadened cyclotron harmonic resonance layers are well separated from the mode conversion layer. In this model, the mode-converted power is assigned to the electron absorption profile, while a fraction of the reflected or transmitted power is assigned to the ions. The appropriate fraction is obtained by integrating the WKB power transport equation through the cyclotron resonance layer, using the hot plasma dispersion relation in a plane-layered limit. The full field treatment utilized in this preliminary version of the code should be replaced by a model which self-consistently treats the combined effects of mode coupling and cyclotron absorption on the wave polarization and damping, since absorption effects have been shown to have a significant impact on transmission and mode conversion coefficients.^{16-19,26}

Another method of including kinetic effects on the power deposition profiles has been formulated by C.K. Phillips, D.Q. Hwang, D.G. Swanson, and L. Ratzan.⁴⁰ A key assumption in the model is that the path of the power flow can be determined using ray tracing techniques while the power deposition profiles to each species must be determined using the full field model. In general, for a given initial ray launched from the coupling region, both reflection and tunnelling of the associated wave will occur when the ray impinges on the resonance layer. The path of power flow for the reflected ray is treated by the standard ray tracing equations using a warm fluid plasma dispersion relation, while the path for the transmitted wave is obtained by

including a small heuristic collision frequency in the dielectric tensor, as in Section II, to remove the singularity and zero in the dispersion relation in the absorption region. For each ray incident on the absorption layer, the amount of power carried by the reflected and transmitted rays, as well as the power absorbed locally via cyclotron damping processes and the power mode converted to the ion Bernstein wave are determined using a kinetic full field algorithm developed by Swanson.^{16,17} His algorithm self-consistently treats effects of localized first and second harmonic cyclotron damping, mode conversion to the ion Bernstein wave with oblique incidence of the incoming wave, finite $k_{\parallel}$, finite plasma beta, and one-dimensional gradients in the equilibrium magnetic field. Inputs to his code, which consist of local values for the densities and temperatures of each species, the magnetic field strength, the angle of oblique incidence in a poloidal plane, and the value of $k_{\parallel}$, are evaluated at the point where the transmitted ray intersects the cyclotron resonance layer. One-dimensional absorption profiles for each species are generated by this algorithm, which solves the inhomogeneous fourth order linear mode conversion-tunnelling equation,

$$\phi^{(iv)} + \lambda^2 z \phi'' + (\lambda^2 z + \gamma) \phi = g(z) \quad , \quad (27)$$

where $\phi \sim E_y$, $z \sim R - R_{\omega_{ci}}$, $R_{\omega_{ci}}$ is the major radius of the cyclotron resonance layer, and $g(z)$ contains the localized cyclotron dissipation terms. Details on the derivation of this equation and its coefficients as well as an analysis of its solutions can be found in the work by Swanson.^{16,17} Radial power deposition profiles for each ion species are determined by mapping the one-dimensional absorption profiles, obtained for each transmitted ray, back onto the flux surfaces according to the two-dimensional transmitted ray paths. The mode-converted power is assumed to damp rapidly on the electrons via Landau damping and thus is assigned to the electron absorption profile in the radial annulus containing the intersection of the transmitted ray and the cyclotron layer. Absorption profiles generated by the code for a deuterium majority-hydrogen minority plasma under typical PFT operating conditions are displayed in Fig. 7. Of particular interest is the fact that a significant enhancement of direct electron heating by the wave via the mode conversion process can be inferred from these results, which do not include electron Landau damping, by comparison with the amount calculated using a ray tracing/quasilinear damping model (to be described next) which includes only electron Landau damping of the fast wave branch (see Fig. 8).

A final method for calculating the power deposition profiles has been formulated by Hwang and Karney et al.¹³ on the basis of quasilinear diffusion theory. Instead of using the linear WKB power transport equation given in Eqs. (23)-(24), they have derived a quasilinear power deposition equation which yields the flux-surface-averaged power, P , deposited to the various plasma species in the form

$$\frac{dP}{dt} = -P_d \left| v_{gr} \right| A, \quad (28)$$

where A is the area of the flux surface, v_{gr} is the radial component of the group velocity, and where P_d , defined as

$$P_d = - \sum_s \int d^3v n_s m_s \vec{v} \cdot \vec{D}_{QL,s} \cdot \frac{\partial f_s}{\partial \vec{v}}, \quad (29)$$

is the power absorbed per unit volume. In Eq. (29), subscript "s" denotes the different species, f_s is an isotropic velocity distribution function, n_s and m_s denote the density and mass of species s, and $\vec{D}_{QL,s}$ is the quasilinear diffusion tensor, defined in a paper by Kennel and Englemann.⁵⁰ This tensor includes cyclotron damping at all harmonics present and electron Landau damping effects in one convenient formulation. The reader is referred to Ref. 13 for further details on the derivation and application of the quasilinear power transport equation. An additional novel and potentially important feature of this model is that the quasilinear power deposition calculation may be iterated with a Fokker-Planck package which follows the velocity space evolution of the strongly heated minority species. Velocity space evolution of the minority species can lead to a significant broadening of the power deposition profiles. The Fokker-Planck package also provides a calculation of the power collisionally transferred from the energetic minority species to the remainder of the plasma species.

Power deposition profiles for each species computed using the quasilinear damping method are displayed in Fig. 8 for the same parameters treated in Fig. 7. Quasilinear broadening of the profiles is evident, as is direct electron heating via Landau damping for these parameters, as mentioned previously. The main deficiencies in this method include the neglect of kinetic effects on the wave polarizations and the neglect of the velocity space anisotropies which may arise because the ICRF fields preferential couple

energy into the perpendicular velocity components of the particles. As a final note, this model for the power deposition profiles has been successfully coupled with the BALDUR one-dimensional MHD equilibrium and transport code.⁵¹ Transport simulations of ICRF-heated discharges in PLT obtained using this code will be presented later on in Section V.

In conclusion, the geometric optics field models have been widely utilized to model ICRF power deposition in tokamaks because it is possible to include two-dimensional focussing effects caused primarily by the finite poloidal extent of the antenna without the vast computer memory requirements of the global full field solution methods described in Section II. However, the WKB assumptions upon which GO models are built are invalid in the regions where absorption processes are most important and in regions where intense focussing of the wave fields occurs. Attempts to connect the GO model of the wave propagation with a full field treatment in the absorption layer have been somewhat successful, though the connection methods are rather ad hoc in nature. In the next section, a brief discussion of a newer approach which combines the advantages of a full wave treatment with the advantages of WKB-like approximations will be presented.

SECTION IV: PARABOLIC APPROXIMATION METHODS

From the discussions thus far, it is clear that it is difficult to simultaneously model the strong focussing experienced by the fast wave as it propagates into the core region and the kinetic absorption/mode conversion processes which account for the transfer of power from the fast wave to the plasma. Global field solution methods accurately treat the wave focussing and associated coherent interference effects because continuous solutions of the full wave equation are generated. However, because of computer memory limitations and because of sizeable differences in the wavelengths between the fast and slow modes, the utility of these methods for properly including the kinetic effects remains uncertain. Geometric optics solution methods have been used in conjunction with full field models of both the edge and the absorption regions. However, because of the discrete nature of the ray tracing method and the limitations on the wavelength imposed by the WKB approximation, these methods are unable to obtain an accurate representation for the strongly focussed fields which impinge on the resonance layer.

The subject of this section will be an approach, recently formulated by C.K. Phillips, F.W. Perkins, and D.Q. Hwang,^{41,42} which utilizes a parabolic approximation to the wave equation in Region II of the plasma cross section (see Figs. 2-3) to effectively blend the full field nature of the solutions obtainable using global methods with the relevant WKB approximations of the geometric optics methods. Specifically, in Region II, the wave propagation proceeds primarily in the radial direction, if surface mode excitation, characterized by large poloidal wave numbers, is ignored. For a reactor-sized tokamak plasma, evaluation of the local cold plasma dispersion relation then indicates that $k_r \sim k_i \gg k_\theta$, $k_\parallel$ where k_r , k_i , k_θ , and $k_\parallel$ denote the radial, perpendicular, poloidal, and parallel components of the wave vector, respectively. This wave number ordering forms the physical basis for the parabolic approximation. The wave is assumed to propagate primarily in the radial direction with a slow amplitude variation induced by gradients in the equilibrium. Using this method, continuous solutions of an approximation of the full wave equation are derived which accurately model refractive and other focussing effects on the wave propagation in Region II. These parabolic solutions are then readily matched to full field solutions of the wave equation obtained in the remaining regions, using suitable approximations to the wave equation in each region. In particular, the diffraction-limited propagation region, Region III in Fig. 2, can be accurately treated using a cold plasma full wave model in which only gradients in the equilibrium magnetic field need be retained, since the wave has been focussed down into the central regions of the discharge where the equilibrium density profile is relatively flat. Similarly, the absorption/mode conversion layer may also be treated in the context of a one-dimensional equilibrium variation, but the inclusion of kinetic effects necessitates the use of the linear inhomogeneous fourth order mode conversion-tunnelling equation discussed earlier.¹⁶⁻¹⁸

Returning to the parabolic approximation for Region II, the basic method will be presented here in cylindrical geometry for a simplified tokamak equilibrium model in which focussing by finite poloidal extent of the antenna and refraction by radial gradients in the equilibrium are retained but complexities arising from the toroidal nature of the equilibrium and rotational transform and shear effects are ignored. Corrections to the propagation solutions in this region arising from poloidal field effects or toroidicity have been shown analytically to be insignificant in a large aspect

ratio, low beta, approximately circular tokamak.⁴² Cylindrical symmetry is assumed here; however, for D-shaped plasmas or plasmas which are highly noncircular, the derivation should be reformulated in orthogonal flux coordinates.

In the usual fast wave limit, i.e., $m_e/m_i \rightarrow 0$ so $E_i \rightarrow 0$, the wave equation, Eq. (4), in a source-free region, reduces to:

$$\gamma_1 E_r - \frac{1}{r^2} \frac{\partial^2 E_r}{\partial \theta^2} + \frac{1}{r} \frac{\partial}{\partial \theta} \frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) - i \gamma_2 E_\theta = 0, \quad (30)$$

and

$$\gamma_1 E_\theta + i \gamma_2 E_r - \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) + \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial E_r}{\partial \theta} = 0, \quad (31)$$

where

$$\gamma_1 = k_z^2 - \omega^2/c^2 K_\perp \text{ and } \gamma_2 = \omega^2/c^2 K_x.$$

Using the parabolic approximation, in which $k_r \sim \partial/\partial r \gg k_\theta \sim \partial/\partial \theta$, Eqs. (30)-(31) may be solved iteratively for E_r and E_θ by treating poloidal derivative terms as small quantities and retaining terms only through order $\partial^2/\partial \theta^2$. Following this procedure, Eq. (30) reduces to an algebraic equation for E_r in terms of E_θ :

$$E_r = \frac{i \gamma_2}{\gamma_1} E_\theta - \frac{1}{\gamma_1 r^2} \frac{\partial^2}{\partial r \partial \theta} (r E_\theta) + \frac{i \gamma_2}{\gamma_1} \frac{1}{r^2} \frac{\partial^2 E_\theta}{\partial \theta^2}. \quad (32)$$

The remaining equation for E_θ may be separated into two parts using the ansatz $E_\theta(r, \theta) = a(r, \theta) u(r)$ where $a(r, \theta)$ is a slowly varying amplitude function which contains focussing and coherence effects while $u(r)$ is a rapidly varying radial waveform. Using this ansatz, the radial waveform, $u(r)$, is determined by a generalized Bessel-type equation:

$$\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} (r u(r)) + k_o^2(r) u(r) = 0, \quad (33)$$

where $k_o^2(r)$ is the local cold plasma dispersion relation. The remaining terms for $a(r, \theta)$ are written as a parabolic partial differential equation of the form:

$$\frac{\partial a}{\partial r} - i b(r) \phi(r) \frac{\partial a}{\partial \theta} + \frac{1}{2r^2} \phi(r) \frac{\partial^2 a}{\partial \theta^2} = 0, \quad (34)$$

where explicit forms for $\phi(r)$ and $b(r)$ are given in Ref. 41 and where the term $\partial^2 a / \partial r^2$ has been neglected because of the assumed slow variation of $a(r, \theta)$. Because the coefficients in Eq. (34) are θ -independent, by virtue of the equilibrium assumptions, closed form solutions for $E_\theta(r, \theta)$ are readily derived in the form:

$$E_\theta(r, \theta) = \sum_{i=1}^2 u^{(i)}(r) \sum_m a_m^{(i)}(r_0) \exp\{im\theta + m^2 F_2^{(i)}(r) - m F_1^{(i)}(r)\}. \quad (35)$$

The summation on (i) is over incident and reflected or transmitted waves, while the summation on (m) is over poloidal harmonics excited because of the finite poloidal extent of the launcher. Poloidal rotation of the wavefronts is represented by the integral function $F_1^{(i)}(r)$, while diffraction effects are contained in the integral function $F_2^{(i)}(r)$. Explicit forms for F_1 and F_2 , as well as a discussion of methods for evaluating Eq. (35) in the limits of single or multiple pass absorption scenarios are given in detail in Ref. 41.

Fast Fourier transforms provide the connection between the full field approximations derived in Regions I-IV. To date, the method has been used successfully to treat Regions II and III, with appropriate assumptions on the poloidal shape and power content of the wavefronts incident from the coupling region and with the assumption of single pass absorption, so only incident waves need be considered. Results for deuterium majority-hydrogen minority plasmas are depicted in Fig. 9 for PLT and in Fig. 10 for a compact ignition device. Because of the higher density and corresponding higher perpendicular index of refraction in the compact ignition device, the focal spot formed by the wavefronts incident on the absorption layer in the ignition device is much smaller than the one formed in PLT. By inference, the power deposition in larger, more dense devices should be more favorably concentrated in the central regions of the discharge. In order to complete this model for obtaining global power deposition profiles, the final connections must be made between the propagation solutions obtained thus far in Regions II and III with the kinetic solutions obtained in Region IV by Swanson^{16,17} and with cold

plasma full field solutions in the coupling region. These calculations are currently being completed and an evaluation of the advantages and disadvantages of this approach will then be ascertained.

SECTION V: TIME-DEPENDENT TRANSPORT SIMULATIONS OF ICRF-HEATED TOKAMAK DISCHARGES

To conclude this lecture, let us return to the question of time-dependent transport simulations of ICRF-heated discharges. An assumption implicit in all of the ICRF field and power deposition models discussed thus far is that the parameters characterizing the local tokamak equilibrium evolve on time scales, τ_E , which are much longer than the time scale, τ_c , for a collisional relaxation of the particles velocity space distribution functions, and the time scale, τ_w , for the wave propagation and absorption. To obtain global time-dependent simulations of ICRF-heated tokamak discharges for use in understanding present-day experimental results as well as for extrapolating these results for future device design studies, it is necessary to integrate the RF models into tokamak transport models. In this way, the interplay between the wave propagation and absorption and the transport properties of the discharge may be simulated by recomputing the RF fields and power deposition in time as the plasma parameters evolve according to the transport code.

The most sophisticated RF/transport code developed thus far consists of the ray tracing/quasilinear damping RF model developed by Hwang and Karney¹³ used in conjunction with the BALDUR one-dimensional MHD equilibrium and transport code developed by Singer, Post, Mikkelsen, Redi, McKenny et al.⁵¹ The basic philosophy assumed when using the BALDUR/RF code to simulate ICRF-heated discharges is that the power deposition profile produced by the RF modules is correct and that appropriate adjustments be made in the assumed forms of the transport coefficients in order to obtain the best fit to the experimental data. Simulations of minority heating experiments on PLT, at power levels of about 1 MW, obtained by Hwang and Karney et al.¹³ agree well with the experimentally measured density and temperature profiles, as discussed in their report.¹³

More recently, a newer version of the BALDUR code⁵¹ has been used in conjunction with the RF/quasilinear damping model of Hwang and Karney et al.¹³ to simulate the deuterium majority-helium-3 minority heating experiments on

PLT for RF power levels of 2.6 MW reported at the IAEA meeting in London in 1984 by Mazzucato et al..⁸ Experimental measurements of the line average electron density, the central ion temperature, and the electron temperature at different radial locations as a function of time are displayed in Figs. 11-13. Of particular interest are the large amplitude sawtooth oscillations observed in the T_e measurements at the center of the discharge. Simulation results for the time evolution of the central values of T_i and T_e are displayed in Fig. 14 while the spatial-time evolution of T_e is presented in Fig. 15. The assumed transport model is based on one used by the BALDUR transport modelling group⁵⁰ at PPPL to compare a variety of discharges in TFTR, JET, ASDEX, and PBX and as such is not intended to provide the best fit to the data in each case. In this model, the electron thermal conductivity is specified by

$$\chi_e = 4.4 \frac{r}{R} (n_e/10^{20})^{-7/8} \text{ m}^2/\text{sec} , \quad (36)$$

while ion thermal conductivity and particle diffusion coefficients are each set equal to $0.2 \times \chi_e$. A Ware pinch velocity is included for the deuterium, and an anomalous pinch velocity is included for the helium-3 minority and carbon impurity species. For the PLT simulation presented here, the ion thermal conductivity has been enhanced by an amount equal to three times the Chang-Hinton neoclassical value to obtain better agreement with the observed ion temperature rise during the RF pulse. Typically, the RF power deposition profiles for the ion species and the electrons that are computed and passed back to BALDUR are relatively peaked near the cyclotron layer at $r \approx 8-10$ cm but are spread over radii out to approximately 25 cm. The electrons typically receive about 0.2 MW, the ions about 2 MW, with the rest being lost in reflections at the wall. The calculated power deposition profile for the electrons does not provide a sufficient power input to the electrons within the $q < 1$ region of the discharge to simulate the observed amplitude of the sawtooth oscillations in the T_e measurements. However, the addition of another 0.1 MW to the electrons inside the $q < 1$ region, delimited by $0 < r < 10$ cm, yields the simulation results depicted in Figs. 14-15. This amount of additional power is quite consistent with the amount that might be expected to be transferred directly to the electrons via mode conversion processes.¹⁷ The ray tracing/mode conversion code, developed by Phillips, Hwang, Swanson, and Ratzan,⁴⁰ has been linked to the BALDUR code. However, in its present form,

it is only able to treat deuterium majority-hydrogen minority discharges. It is currently being modified to handle D-He³ discharges, and results based on its use will be presented in later reports.

In conclusion, it is encouraging that qualitative, and in some cases, quantitative agreement can be found between numerical simulations and experimental observations of ICRF-heated discharges, despite the numerous shortcomings of the various models discussed in this lecture. In particular, certain aspects of the heating process are either treated in a rather simplified manner or else neglected altogether in the existing models. Finite temperature effects and more realistic models of the launching structures need to be considered in the antenna-plasma coupling region. Effects of the velocity space anisotropies induced by the heating mechanisms need to be included in treatments of the wave absorption, transport, and collisional relaxation processes. Two-dimensional equilibrium gradient effects may alter the absorption and mode conversion characteristics in the resonance layer. Nonlinear phenomena that may be driven by locally high values of the wave intensity produced by focussing effects need to be analyzed for the higher power experiments anticipated on JET and next generation devices. Finally, a theoretical analysis is needed to aid in the understanding of the scalings of global energy confinement times in auxiliary-heated tokamak discharges. Such analyses will hopefully lead to a better understanding of the basic physical phenomena which characterize RF wave propagation and absorption in tokamaks.

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FIGURE CAPTIONS

- Fig. 1 Typical dispersion curves for a deuterium majority-hydrogen minority plasma in JET, reproduced from Ref. 15. The plasma is characterized by the following parameters: $T_{e0} = T_H = T_D = 3$ keV, $n_{e0} = 5 \times 10^{13}$ cm⁻³, $n_H/n_D = 0.1$, $f = 47$ MHz, $B_0 = 34.5$ kG.
(a) Moderate $k_{\parallel}$, (b) large $k_{\parallel}$.
- Fig. 2 The plasma cross section is divided into four regions. Region I is the antenna-plasma coupling region. Region II is a propagation region in which the WKB approximation is valid. Region III is a propagation region in which diffraction effects are important because of significant focussing of the incident waves. Region IV consists of the mode conversion/cyclotron absorption layers, in which a kinetic full wave treatment of the fields is required to adequately describe the heating process.
- Fig. 3 Wave dispersion surfaces on a plasma cross section are depicted, indicating wave propagation and absorption characteristics for each of the four regions defined in Fig. 2.
- Fig. 4 Global full field solutions for the poloidal structure of a fast wave launched by an antenna on the low field side of the resonance (reproduced from Ref. 31). Characteristic plasma parameters include $n_H/n_D = 0.1$, $\omega/\Omega_D(0) = 1.75$, $\omega_{pD}^2/\Omega_D^2(0) = 10^3$, $\nu/\omega = 0.01$, $a/R = 0.2$, $a^2\Omega_D^2(0)/c^2 = 0.05$, $k_{\parallel}a = 2$.
(a) Power deposition profile, $\tilde{W}$; (b) Poynting flux; (c) Real (E_x); (d) Im (E_x); (e) Real (E_y); and (f) Im (E_y).
- Fig. 5 Comparison of the constant-phase surfaces obtained from the full wave solution with those reconstructed by ray tracing, for a uniform plasma in cartesian coordinates. The plasma center is located at $x = 0$; the B-field is parallel to the z-direction.
(Reproduced from Ref. 15.)
- Fig. 6 Linear plot of electron and deuteron power density profiles, corresponding to the JET parameters listed in Fig. 1, after redistribution of proton energy to other plasma species. For the CPS at $x = 0.9$ m: total power in electrons $n_e = 1.69$ MW, total power in deuterons $n_D = 1.07$ MW, for an input power of 3 MW coupled to the plasma by the antenna. For the CPS at $x = -0.6$ m: $n_e = 1.81$ MW, $n_D = 0.98$ MW. A part of the missing power ($k_{\parallel} = 0$

spectrum) is not absorbed and the other part which exists beyond the valid CPS is not even launched.

(Reproduced from Ref. 15.)

- Fig. 7 Direct wave power deposition profiles obtained using a 2-D ray tracing/1-D mode conversion code for a deuterium majority-hydrogen minority heating experiment for typical PLT parameters. Plasma parameters include $n_e(o) = 4.5 \times 10^{13} \text{ cm}^{-3}$, $n_H/n_D \approx 0.1$, $T_e = 1.8 \text{ keV}$, $T_D = T_H = 1 \text{ keV}$, $f = 30 \text{ MHz}$, $B_o = 1.96 \text{ T}$, $k_y R_o = 9$.
- Fig. 8 Direct wave power deposition profiles obtained using a 2-D ray tracing/quasilinear damping model for the same plasma conditions listed in Fig. 7.
- Fig. 9 Contours of the real part of B_z in PLT for a fast wave launched on the low field side of the resonance. The current distribution in the antenna varies as $\cos(\theta/2\theta_{\max})$, where $\theta_{\max} = 45^\circ$ to model the quarter-turn center-fed loop antenna.
- Fig. 10 Contours of the real part of B_z for typical parameters proposed for a compact ignition device. The wave is launched on the low field side of the resonance with $\theta_{\max} = 30^\circ$ (see Fig. 9) to model the fields radiated by an advanced waveguide coupler design.
- Fig. 11 Experimental measurements of line average electron density (dotted line) and total radiated power as a function of time for $P_{\text{rf}} = 2.6 \text{ MW}$ in a deuterium majority-helium minority plasma in PLT.
(Reproduced from Ref. 8.)
- Fig. 12 Time evolution of the central ion temperature derived from: charge exchange (squares), Doppler broadening of the Fe XXV line (circles), neutron emission (triangles) for $P_{\text{rf}} = 2.6 \text{ MW}$, $n_e(o) = 5 \times 10^{13} \text{ cm}^{-3}$, and $f = 30 \text{ MHz}$, for same experiment described by Fig. 11.
(Reproduced from Ref. 8.)
- Fig. 13 Time evolution of T_e derived from second harmonic electron cyclotron emission at different radial locations for $P_{\text{rf}} = 2.6 \text{ MW}$ and $\bar{n}_e = 3.3 \times 10^{13} \text{ cm}^{-3}$, for same experiment as in Fig. 11.
(Reproduced from Ref. 8.)
- Fig. 14 Simulation results for time evolution of $T_i(o)$ and $T_e(o)$ obtained from the BALDUR/RF code for experimental plasma parameters corresponding to Figs. 11-13.
- Fig. 15 Simulation results for the spatial and time evolution of T_e obtained from the BALDUR/RF code for experimental plasma parameters corresponding to Figs. 11-13.

Fig. 1

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PLASMA GEOMETRY

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

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Fig. 8

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Fig. 9

Fig. 10

Fig. 11

Fig. 12

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Fig. 14

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Fig. 15

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