A Method of Measuring Directed Electron
Velocities in Flowing Plasma Using the
Incoherent Regions of Laser Scattering

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ABSTRACT

With the presumption that a shifted Maxwellian velocity distribution adequately describes the electrons in a flowing plasma, the details of a method to measure their directed velocity are described. The system consists of a ruby laser source and two detectors set 180° from each other and both set at 90° with respect to the incident laser beam. The lowest velocity that can be determined by this method depends on the electron thermal velocity. The application of this diagnostic to the measurement of flow velocities in plasma being lost from the ends of θ -pinch devices is described.

INTRODUCTION

The magnitude and direction of the flow velocity of a plasma can be inferred from such quantities as impact pressure [1] or the position of luminous fronts [2]. However, direct means of measuring this velocity have not been well developed. One method of directly determining this velocity for the electron component of a flowing plasma can be derived from the scattered signal in a conventional Thomson scattering experiment, provided the apparatus and plasma satisfy certain constraints. Such constraints as they relate to a novel diagnostic procedure will be outlined here.

THEORY

To develop a set of general equations illustrating this diagnostic, it is useful to refer to Figure 1. A general expression for the scattered signals P_s , that holds for electron temperatures up to around 1 keV is [3]

$$P_s d\lambda d\Omega = \frac{P_i r_o^2 n_e d\Omega L}{2\sqrt{\pi} \sin^2 \frac{\theta}{2}} \frac{c}{a} \frac{d\lambda}{\lambda_i} \left[1 - \frac{3.5\Delta\lambda}{\lambda_i} + \frac{c^2 \Delta\lambda^3}{4a^2 \lambda_i^2 \sin^2 \frac{\theta}{2}} \right] \exp\left[-\frac{c^2}{4a^2 \sin^2 \frac{\theta}{2}} \left(\frac{\Delta\lambda}{\lambda_i} + \frac{v_{oa} \pm v_{ob}}{c} \right)^2 \right] \quad (1)$$

where the plus sign refers to detector No. 2 and the minus sign to detector No. 1. Here $d\lambda$ is the wavelength passband of the filter or monochromator behind which the detector sits, $d\Omega$ is the solid angle viewed by the detector, P_i is the incident laser power, r_o is the classical radius of an electron, n_e is the electron density, L is the scattering volume length, c is the speed of light, θ is the angle between incident and scattered directions, a is the electron thermal speed or $\sqrt{T_e/m_e}$, λ_i is the incident laser wavelength, $\Delta\lambda$ is $\lambda_i - \lambda_s$ with λ_s being the center wavelength of the detection passband, and v_{oa} and v_{ob} are the electron directed velocities along the $\hat{a}$ and $\hat{b}$ axes, respectively. Equation (1) also presumes that a shifted Maxwellian adequately describes the electron population in the scattering volume. For the analysis that follows, a 90° scattering arrangement is used. Performing the subtraction of detector No. 2's output from that of detector No. 1 produces

$$(P_{s1} - P_{s2}) d\lambda d\Omega = A \exp\left[-\frac{c^2}{2a^2} \left(\frac{\Delta\lambda}{\lambda_i} + \frac{v_{oa} - v_{ob}}{c} \right)^2 \right] \left\{ \exp\left[\frac{cv_{ob}}{a} \left(\frac{\Delta\lambda}{\lambda_i} + \frac{v_{oa}}{c} \right) \right] - 1 \right\} \quad (2)$$

where

$$A = \frac{P_{foe}^2 d\Omega L}{\sqrt{2\pi}} \frac{c}{a} \frac{d\lambda}{\lambda_1} \left[1 - \frac{3.5\Delta\lambda}{\lambda_1} + \frac{c^2 \Delta\lambda^3}{2a^2 \lambda_1^2} \right] \quad (3)$$

Ensuring that the laser beam is along a direction where the flow velocity is small, it can reasonably be expected that

$$\frac{v_{oa}}{c} < 0.1 \frac{\Delta\lambda}{\lambda_1} \quad (4)$$

and, hence, this term will be dropped. For simplification, presume that

$$\frac{cv_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} < 0.1 \quad (5)$$

such that expansion of the exponential is permitted. This need not be the case, but it does allow a certain transparency in the argument, and with the exception of high velocities compared to the electron thermal speed, this condition will not be violated. Accordingly, equation (2) reduces to

$$(P_{s1} - P_{s2}) d\lambda d\Omega = A \exp \left[\frac{-c^2}{2a^2} \left(\frac{\Delta\lambda}{\lambda_1} + \frac{v_{oa} + v_{ob}}{c} \right)^2 \right] \frac{cv_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} \quad (6)$$

Finally, if equation (6) is normalized with respect to detector No. 2 output, the velocity can be isolated as

$$\frac{P_{s1} - P_{s2}}{P_{s2}} = \frac{cv_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} \quad (7)$$

Also, since the current, I_s , produced by the detectors is directly proportional to P_s , equation (7) can be rewritten as

$$\frac{I_{s1} - I_{s2}}{I_{s2}} = \frac{cv_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} \quad (8)$$

Thus, the velocity in the $\hat{b}$ direction can be arrived at when the electron temperature, T_e , at the scattering volume is known. The temperature can be obtained either from independent diagnostics or through a simultaneous measurement of the Thomson scattered wavelength spectrum from the same plasma volume. The dependence of the difference signal on the electron thermal speed can be noted.

SIGNAL TO NOISE RATIO CONSIDERATIONS

With the signal level established, it is useful to determine the signal to noise ratio for this experimental arrangement and to derive an optimum condition. To avoid a profusion of symbols it is useful to note that in most applications $\Delta\lambda \ll \lambda_i$, and, hence, the terms with $\Delta\lambda$ can be ignored in equation (1) when compared to unity. There are two basic noise sources in this system: (1) noise resulting from the photo-emissive process of the photomultiplier, and (2) bremsstrahlung. The photomultiplier noise is first examined. The current, I , developed in such a tube with a gain, G , and quantum efficiency of η sitting behind a narrow band optical filter of half width $d\lambda$ centered on wavelength λ_s is

$$I_{sig} = \frac{P_i r_o^2 n L d\Omega d\lambda e T}{\sqrt{2} v h a} \eta G \exp\left[-\frac{c^2}{2a^2} \left(\frac{\lambda_i - \lambda_s}{\lambda_i}\right)^2\right] \quad (9)$$

where T is the overall transmission of the system and the flow velocity has been ignored. Due to the Poisson statistics of the photoemissive process, a noise term of

$$I_n = \sqrt{I_{sig} e G / \tau} \quad (10)$$

arises with τ being the length of time of the incident laser pulse. After

being processed by the differential amplifier, this noise is doubled.

Therefore, the signal to noise ratio is

$$\frac{S}{N} = (I_{sig_1} - I_{sig_2}) / 2\sqrt{I_{sig}} eG/\tau \quad (11)$$

$$\frac{S}{N} = \frac{c(\lambda_1 - \lambda_s)v_{ob}}{2a^2\lambda_1} \sqrt{\tau I_{sig}/eG} \quad (12)$$

This ratio can be maximized through suitable adjustment of λ_s . When this is done, the value of λ_s is found to be

$$\lambda_s = \lambda_1 \left(1 - \frac{a}{c\sqrt{2}}\right) \quad (13)$$

This reduces equation (12) to

$$\frac{S}{N} = \frac{v_{ob}}{a\sqrt{2}} \sqrt{\frac{w_1 r_o^2 n_e L d \Omega T d \lambda_n}{\sqrt{2\pi} h a}} e^{-1/2} \quad (14)$$

Note that it is now possible to return to equation (5) and use equation (13) to reduce the condition permitting expansion of the exponential to

$$\frac{v_{ob}}{\sqrt{2} a e} < 0.1$$

Returning again to noise considerations, the bremsstrahlung contribution is now examined. Both detectors view the same plasma volume, and hence, the bulk of the plasma noise signal should cancel out, but there are still the fluctuations of the bremsstrahlung again produced by the photoemissive process left. In this analysis the bremsstrahlung contribution shall be handled such that it can be simply appended to equation (14).

Accordingly, the ratio of the bremsstrahlung signal to the laser signal can be shown to be [3]

$$R = \frac{2.09 \times 10^{-36} \tau_n e^{2\Delta\lambda_{1/e}} V_p}{W_i r_o^2 \lambda_i^2 T_e^{1/2} 4\pi L} \quad (15)$$

where V_p is the volume of plasma viewed by the detector, W_i is the incident laser energy and

$$\Delta\lambda_{1/e} = \frac{\sqrt{2a}}{c} \quad (16)$$

Taking the square root of this and adding the factor of 1 to that result gives an estimate by which equation (14) should be divided to account roughly for the bremsstrahlung noise addition. The end result becomes

$$\frac{S}{N} = \frac{V_{ob}}{a\sqrt{2}} e^{-1/2} \sqrt{\frac{W_i r_o^2 n_e^2 L d\Omega T d\lambda \eta / \sqrt{2\pi} h a}{1/(1+R)}} \quad [1/(1+R)] \quad (17)$$

COMPARISON WITH ALTERNATE TECHNIQUE

It is worthwhile at this point to compare this system with the technique of directly determining the shift of the peak of the scattered spectrum [4]. That method [4] can be simplified by subtracting the outputs of two detectors, where one sees the full spectrum on the high side of 694.3 nm and the other sees the low side of 694.3 nm; this is shown in Figure 2. For ease of analysis, a square response is presumed for the filters. To simplify matters the effect

of the blue shift [4] in the scattered spectrum is ignored, although if this manner of measuring the directed velocity is to be employed the scattered signal shown in equation (1) must be used. Accordingly, the result after subtracting the two signals is

$$I_{sig_1} - I_{sig_2} = \frac{P_i r_o^2 n_e L d \Omega e T \eta G}{\sqrt{2\pi} h a} \left[\int_{\lambda_1}^{\infty} e^{-(c^2/2a^2) [(\lambda_1 - \lambda_s)/\lambda_1 + (v_{oa} - v_{ob})/c]^2} d\lambda_s - \int_{-\infty}^{\lambda_1} e^{-(c^2/2a^2) [(\lambda_1 - \lambda_s)/\lambda_1 + (v_{oa} + v_{ob})/c]^2} d\lambda_s \right] \quad (18)$$

Now, presuming that

$$\frac{v_{oa} \pm v_{ob}}{c} \ll 1, \quad (19)$$

equation (18) can be reduced to

$$I_{sig_1} - I_{sig_2} = \frac{P_i r_o^2 n_e L d \Omega e T \eta G}{\sqrt{2\pi} h a} \frac{\sqrt{2} a \lambda_1}{c} \frac{2v_{oa}}{c} \quad (20)$$

Accordingly, a signal to noise ratio just accounting for the photoemissive process is

$$\frac{S}{N} = \frac{2v_{oa}}{2c\sqrt{\pi}} \sqrt{2\pi P_i r_o^2 n_e L d \Omega T \eta \lambda_1 / hc} \quad (21)$$

Now for the same velocity, laser, and detection parameters a ratio of the two signal to noise ratios can be made in order to compare the two techniques:

$$\frac{\left(\frac{S}{N}\right)_{\text{two halves}}}{\left(\frac{S}{N}\right)_{\text{two detectors}}} = .00694 T_e^{3/4} \quad (22)$$

where a filter passband, $d\lambda$, of 1.0 nm has been used in equation (14). This indicates that for electron temperatures up to 755 eV the technique described in this paper (illustrated by Figure 1) has a better signal to noise ratio and accordingly can resolve lower directed velocities than the alternate method shown above (illustrated by Figure 3). However, above an electron temperature 755 eV, the latter technique possesses the advantage.

PROPOSED APPLICATION TO THETA PINCH ENDLOSS

For the application considered here, measurement of the endloss velocity in a 50 cm theta pinch, the technique described here is superior. The physical layout of the proposed experiment is presented in Figure 3. This placement of detectors is necessary, or they would be overloaded with bremsstrahlung if placed along the axis. Detectors which could be used are two RCA C70042K photomultiplier tubes. They have a quantum efficiency, (η) of 6% at 694.3 nm with a current gain, G , of 8×10^4 . The plasma is presumed to have the following characteristics:

Electron Temperature	$T_e = 100 \text{ eV}$
Electron Density	$n_e = 10^{16} \text{ cm}^{-3}$
Ion Temperature	$T_i = 100 \text{ eV}$
Plasma Column Radius	$r_c = 1.0 \text{ cm}$

The laser and associated optics are presumed to have the following properties:

Wavelength Passband	1.0 nm
Overall Transmission	0.1
Laser Energy	10 J
Laser Pulse Length	20 nsec

Focused Beam Radius	0.1 cm
Scattering Length	1 cm
Solid Angle of Detectors	2×10^{-2} ster

Two practical considerations should be noted from the above parameters. First, with the pulse length of 20 nsec, only a 50 MHz bandwidth is required of the differential amplifier. Second, the solid angle viewed by the detectors will be set by the input lens of the detection scheme. This can be seen in the optical arrangement shown in Figure 4. The signals from the two tubes are

$$I_{sig_1} = \frac{W_i}{\tau} \frac{d\lambda d\Omega n_e G Tr_o^2 L}{\sqrt{2\pi} h a} \exp\left[-\frac{c^2}{2a^2} \left(\frac{\lambda_i - \lambda_s}{\lambda_i} + \frac{\sqrt{2} v_{oz}}{c}\right)^2\right] \quad (23)$$

$$I_{sig_2} = \frac{W_i}{\tau} \frac{d\lambda d\Omega n_e G Tr_o^2 L}{\sqrt{2\pi} h a} \exp\left[-\frac{c^2}{2a^2} \left(\frac{\lambda_i - \lambda_s}{\lambda_i} - \frac{\sqrt{2} v_{oy}}{c}\right)^2\right] \quad (24)$$

In taking measurements around the region at the end of the theta pinch coil, it is safe to presume that $v_{oy} \approx 0$, then

$$I_{sig_1} - I_{sig_2} = 61.3 \text{ ma} \exp\left[-\frac{c^2}{2a^2} \left(\frac{\lambda_i - \lambda_s}{\lambda_i}\right)^2\right] \left\{ \exp\left[-\frac{c v_{oz}}{a^2} \left(\sqrt{2} \frac{\lambda_i - \lambda_s}{\lambda_i} - \frac{v_{oz}}{c}\right)\right] - 1 \right\} \quad (25)$$

Using equation (13) to determine λ_s , equation (24) can be shown to give a current of

$$I_{sig_2} = 22.6 \text{ ma} \quad (26)$$

with the Poisson noise current associated with it of

$$I_{\text{noise}} = 0.12 \text{ ma} \quad (27)$$

Presuming the flow is roughly $v_{\text{oy}} \sim \sqrt{T_1/M_1}$, an expected difference signal of

$$I_{\text{sig}_1} - I_{\text{sig}_2} = 1.49 \text{ ma} \quad (28)$$

can be derived. Accordingly, with just the Poisson noise, a signal to noise ratio of

$$\frac{S}{N} = 4.4 \quad (29)$$

is expected. Finally, adding in the bremsstrahlung contribution reduces the signal to noise ratio to

$$\frac{S}{N} = 4.13 \quad (30)$$

CONCLUSIONS

These calculations indicate that with the plasma parameters used above, it is possible to investigate velocities below the electron thermal speed and still maintain a reasonable signal to noise ratio. The lowest velocity that can be examined for given plasma and laser parameters when bremsstrahlung is neglected is given by

$$v_{\text{ob}} = 2 \frac{S}{N} \sqrt{\pi/2} (ha) / (W_1 r_o^2 n_e L_d T d \lambda n) \quad (31)$$

Here the interplay between the lowest velocity measurable and the plasma density and temperature is clearly visible. Using equation (27), it can be quickly ascertained whether this type of diagnostic would measure a given velocity.

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Figure 1. Geometrical Arrangement

Figure 2. Alternative Method of Velocity Determination

Figure 3. End Loss Velocity Experimental Arrangement

Figure 4. Optical Arrangement