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PION NUCLEON INTERACTION AT LOW ENERGY[†]

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The period from the mid 1950's to the late 1960's saw intensive developments in the field of πN interaction. The classic work of Chew and Low¹ was followed by developments in dispersion theories,² current algebras, PCAC, Chiral symmetry,³ etc. During the last six or seven years there has been a resurgence of interest in πN interactions at low energies inspired entirely by the growth of various pion experimental facilities which have provided increasingly precise data on pion nucleus scattering and reactions. To understand the interaction of a pion with a nucleus we require the knowledge of πN scattering amplitudes for π and N 4-momenta which are off the mass shell, but real. These amplitudes can be determined only with the help of a dynamical theory. It should be noted that the usual dispersion theories deal with amplitudes where the 4-momenta are on the mass shell and, in general, complex. Since the work of Chew and Low there has not been a great deal of work in particle physics attempting to explain the low energy πN scattering data in terms of a dynamical theory. So when the need arose the intermediate energy theorists had to take up the task.

In the literature there are several papers which deal with the S and P wave πN scattering amplitudes in separable form. Some of these use separable potentials;⁴ others are extensions of the static theory of Chew and Low.⁵

In this talk I will describe a theory of πN interaction at low energy developed by Barry Cammarata and myself. An early version of the theory was presented at the First Meson Nucleus Conference at Carnegie-Mellon University.⁶

I will try to convey the spirit of our approach by analogy with an unusual approach to potential scattering problem. Let t and v be the kinetic and potential energy operators, respectively. Let $|p\rangle$, $|q\rangle$ be plane wave states and $|\psi^{(+)}(n)\rangle$ be a scattering eigenfunction of $H = t + v$ with eigenvalue E_n . The half-off-shell scattering amplitude $\langle n | T(E_n) | p \rangle = \langle \psi^{(+)}(n) | v | p \rangle$ satisfies the Low equation

$$\langle k | T(E_k) | p \rangle = \langle k | v | p \rangle + \sum_n \frac{\langle n | T(E_n) | k \rangle^* \langle n | T(E_n) | p \rangle}{E_k - E_n + i\eta}.$$

If one knew all the potential matrix elements $\langle k | v | p \rangle$ one could consider solving the nonlinear integral equation. In a field theoretic Low expansion of the scattering amplitude the so-called seagull terms play the role of potential matrix elements. If we assume a Lagrangian we can write down, in principle, the form of the seagull terms with a few unknown parameters, such as coupling constants, form factor masses, etc. But there are serious practical problems. For

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example, consider a Lagrangian involving nucleons, a scalar-isoscalar meson (σ meson) and pions with pseudoscalar coupling. We are all aware of the pair suppression mechanism in the isoscalar amplitude where the huge repulsive contribution of $-\frac{g_\pi^2}{M}$ from the Z-graphs

is almost completely cancelled by $\frac{g_{\sigma NN} g_{\sigma \pi \pi}}{m_\sigma^2}$, the attractive contribution from the σ meson exchange. The remainder, presumably of the same order as the physical amplitude, is about $\frac{1}{100}$ th of $\frac{g_\pi^2}{M}$. It

will be very difficult for any practical calculation to reproduce this kind of result. We avoid these dangers by taking a different path. Suppose we have a way of knowing all elements of a fully off-shell T matrix, $\langle \vec{k} | T(E_0) | \vec{p} \rangle$ for some value of E_0 which may be quite unphysical. Then not only can we write down an integral equation

$$\langle \vec{k} | T(E_k) | \vec{p} \rangle = \langle \vec{k} | T(E_0) | \vec{p} \rangle + \sum_n \langle \vec{k} | T^\dagger(E_n) | n \rangle \langle n | T(E_n) | \vec{p} \rangle \left\{ \frac{1}{E_k - E_n + i\eta} - \frac{1}{E_0 - E_n + i\eta} \right\} \quad (1)$$

without any reference to the potential matrix elements but we gain the additional advantage that the equation is once-subtracted, thus more rapidly convergent.

In our theory⁷ a soft pion limit plays the role of $T(E_0)$. This requires two conjectures.

Conjecture 1: Strong PCAC.

$$\phi_\alpha = \frac{\sqrt{2}}{f_\pi} \frac{\partial A_\mu^\alpha}{\partial x_\mu},$$

with $f_\pi = 0.939 m_\pi^3$, is the canonical pion field.

Conjecture 2: Notion of σ commutator.

$$i \int d^4 y \delta(x_0 - y_0) [A_0^\alpha(x), \partial^\mu A_\mu^\beta(y)] = \delta_{\alpha\beta} \sigma(x).$$

The locality of $\sigma(x)$ follows from microcausality. The statement that it is an isoscalar field is the conjecture.

It is very important to note that the foregoing conjecture implies that

$$\int d^4 y \delta(x_0 - y_0) [A_0^\alpha(x), A_\mu^\beta(y)]$$

is a conserved vector isovector current. We may assume, if we wish, that this is proportional to the vector current, i.e.,

$$\int d^4 y \delta(x_0 - y_0) [A_0^\alpha(x), A_\mu^\beta(y)] = \zeta \{ i \epsilon_{\alpha\beta\lambda} V_\mu^\lambda(x) \}.$$

But our conjecture says nothing about the constant ζ , which in current algebra equals one. Thus the notion of σ commutator is compatible with current algebra but does not imply it completely. We do not assume current algebra and, therefore, we can use it as an external test of our theory.

When the notion of PCAC was first introduced a popular accompanying assumption was that the matrix elements of the axial vector current or of its divergence are slowly varying functions of the relevant momentum variables. We should emphasize that we make

no such assumption.

The soft pion limit can be used to eliminate not all the seagull terms, but only the isoscalar part. So a conjecture about the isovector part must be made.

Conjecture 3: The seagull terms are purely isocalar.

The third conjecture does not allow a term in the underlying Lagrangian involving the coupling of a vector-isovector field [ρ meson] to the hadronic isospin current. Thus the ρ meson must appear as a two pion resonance and not as a canonical field. In other words, we cannot draw a Feynman diagram with a ρ meson exchange.

While in our approach it is not necessary to spell out what the underlying Lagrangian is we may note that the Gell-Mann-Levy σ model⁸ is completely compatible with our conjectures, and, as we shall see shortly, with some of our results. Several years back Basdevant and Lee⁹ had shown how the ρ and the f resonances appear from the Gell-Mann-Levy σ model.

Finally, our theory gives an integral equation which is shown below pictorially.¹⁰

$$\begin{aligned}
 & \text{Diagram: } \begin{array}{c} k', \alpha \\ \swarrow \\ \text{---} \circ \text{---} \\ \searrow \\ k, \beta \end{array} \quad \begin{array}{c} p_i \\ \swarrow \\ \text{---} \circ \text{---} \\ \searrow \\ p_f \end{array} \\
 & = \left[\left(\frac{\sqrt{2}}{f_\pi} \right)^2 m_\pi^2 (t - m_\pi^2) \langle p_f | \sigma(0) | p_i \rangle \delta_{\alpha\beta} + \dots \right] \\
 & + \left[\begin{array}{c} \text{Diagram 1: } \begin{array}{c} \swarrow \quad \searrow \\ \text{---} \circ \text{---} \end{array} \\ \text{Diagram 2: } \begin{array}{c} \swarrow \quad \searrow \\ \text{---} \circ \text{---} \end{array} \\ \text{Diagram 3: } \begin{array}{c} \swarrow \quad \searrow \\ \text{---} \circ \text{---} \end{array} \end{array} + \dots \right] \\
 & + \text{CROSSED TERMS} - \text{SOFT LIMITS} \\
 & + \begin{array}{c} \text{Diagram 4: } \begin{array}{c} \swarrow \quad \searrow \\ \text{---} \circ \text{---} \end{array} \end{array} + \text{CROSSED TERMS} - \text{SOFT LIMITS}
 \end{aligned}$$

Fig. 1: Once-subtracted Low expansion.

The analogy between this integral equation and Eq. (1) is transparent. The first group of terms on the right-hand side make up the soft pion limit. Since pions can be absorbed and emitted and can dissociate into $\bar{N}N$ pair we get the middle group of terms. The last group involve integrals over the πN scattering amplitudes and their complex conjugates. The first two groups are the driving terms for the integral equation. In the work on low energy S wave phase shifts that we did two years back we did not include S wave inelasticity.^{7,10} The nucleon pole terms have only one unknown ingredient, namely, the πN form factor for $t < 0$. Note that our

assumption of PCAC fixes $g_\pi(0) = \frac{\sqrt{2}m_\pi^2}{f_\pi} g_A(0)$ which is the Goldberger-Treiman¹¹ relation. The nucleon pole terms play a relatively minor rôle and so we do not determine the form factor for $t < 0$ very well.

Our main driving terms are the σ commutator term, the main isoscalar mechanism, and the Z-graphs, the main isovector mechanism. The subtraction of the soft limits kills the huge isoscalar part of the Z-graphs, thus simulating pair suppression.

Evaluation of the Z-graphs require the quantity $g_\pi(t \geq 4M^2)$. At present it is not practical to calculate its value reliably. However, the various studies¹² of πN form factor assure us that $g_\pi(t \geq 4M^2)$ is not likely to be small compared to $g_\pi(0)$. Through a slight approximation we arrange matters such that we need to know only $|g_\pi(t \geq 4M^2)|$ which we parametrize as

$$|g_\pi(t \geq 4M^2)| = \frac{\bar{g}_\pi = |g_\pi(4M^2)|}{1 + \frac{t - 4M^2}{4 m_\sigma^2}}.$$

We parametrized the σ commutator matrix element as

$$\langle p_f | \sigma(0) | p_i \rangle = \frac{\sigma(\pi N)}{(1 - \frac{t}{m_1^2})(1 - \frac{t}{m_2^2})^2}.$$

Commonly the quantity $\sigma(\pi N)$ is referred to as the σ commutator.

Our S wave equations have another type of driving term which is not manifest in the pictorial representation. Our choice of the CM frame guarantees that the πN system in the right-hand cut is at rest. But the πN system is not at rest in the crossed term nor in the soft pion limits. Because of this the P wave (and, in principle, all higher waves) amplitudes do provide a small but non-negligible effect. The resulting driving term is parametrized in terms of the P wave cross sections and form factors by making a separability ansatz. The form factor is assumed to be of the form^{7,10} $k/(1+k^2/\mu^2)^{5/2}$.

Naturally, our low energy S wave work cannot fix the various form factors very precisely. In Refs. 7 and 10 we quoted the values $m_0 = 8.6 m_\pi$, $m_1 = 8.24 m_\pi$, $m_2 = 7.5 m_\pi$ and $\mu = 8 m_\pi$. But for any given set of values of the form factor masses the sensitivity of the S wave phase shifts on the main parameters $\sigma(\pi N)$ and $\bar{g}_\pi$ is very marked. In Fig. 2 we exhibit experimental and theoretical phase shifts. The solid lines are the energy dependent fits of

Zidell, Roper and Arndt;¹³ the flagged points are the energy independent fit phase shifts of Carter, Bugg and Carter.¹⁴ The dashed lines are theoretical calculations for three sets of $\sigma(\pi N)$ and $\bar{g}_\pi$, listed below.

Curve	$\sigma(\pi N)$	$\bar{g}_\pi$
a	25.0 MeV	11.85
b	25.5 MeV	11.69
c	26.0 MeV	11.54

The values were chosen such that they gave the same δ_{31} .

We draw two conclusions. First, the energy dependent fits are unacceptable at low energies. It is nearly impossible to reproduce theoretically the sharp change of

Fig. 2: Experimental and theoretical S wave phase shifts.

slope of δ_{11} . Second, as long as the true phase shifts are close to those of Carter, Bugg and Carter, the parameters $\sigma(\pi N)$ and $\bar{g}_\pi$ will have values close to the ones shown here.

We obtained the following scattering lengths, $a^{(+)} = \frac{1}{2}[a(\pi^- p) + a(\pi^+ p)] = -0.016 m_\pi^{-1}$ and $a^{(-)} = \frac{1}{2}[a(\pi^- p) - a(\pi^+ p)] = 0.079 m_\pi^{-1}$, which agrees exactly with the current algebra prediction of Weinberg.¹⁵

Current algebra provides another check of our theoretical work in the form of the following sum rule,

$$\left(\frac{\sqrt{2}}{f_\pi}\right)^2 \frac{m_\pi^4}{2} = \frac{1}{2M^2} |g_\pi(4M^2)|^2 + \frac{M}{6\pi^2} \int \frac{q^2 dq}{\omega(q)E(q)} \frac{|f_{11}(q,0)|^2 - |f_{31}(q,0)|^2}{[E(q) + \omega(q) - M]^2},$$

where $f_{2I,2J}(q,0)$ is a half-off-mass shell S wave scattering amplitude. We satisfy the sum rule within the numerical accuracy of our work. The sum rule may also be interpreted as providing a check on our value for $\bar{g}_\pi$.

Our value of $\sigma(\pi N) \sim 25$ MeV is in sharp disagreement with the value of ~ 65 MeV obtained in recent years by various authors using dispersion relations method.^{16,17,18} But significantly our value is close to those obtained in the late 1960's¹⁹ using the Fubini-Furlan theory.²⁰ The dispersion relations method is based on an idea proposed by Osypowski²¹ and by Cheng and Dashen.²² It deals with the amplitude

$$\bar{C}^{(+)}(\nu, t) = A^{(+)}(\nu, t) + \frac{\nu}{1 - \frac{t}{4M^2}} B^{(+)}(\nu, t) - \frac{G_\pi^2}{M(1 - \frac{t}{4M^2})} \frac{\nu_B^2}{\nu_B^2 - \nu^2}$$

where A and B are the usual CGLN² amplitudes, with pions and nucleons in their mass shell. $\nu = (s-u)/4M$, $\nu_B = (t-2m_\pi^2)/4M$ and $G_\pi = g_\pi(m_\pi^2)$ is the πN coupling constant. The authors observed that

$$\bar{C}^{(+)}(0, 2m_\pi^2) = \left(\frac{\sqrt{2}m_\pi^2}{f_\pi}\right)^2 \sigma(\pi N) - \frac{m_\pi^2 G_\pi}{2M^3} + O(m_\pi^4), \dots \quad (2)$$

The left-hand side is an on-mass-shell amplitude for unphysical energy and momentum transfer and, therefore, can be determined from dispersion relations. The authors conjectured that the terms $\sim O(m_\pi^4)$ are negligible and so the above relation provides a method of determining $\sigma(\pi N)$. In practice there are three steps involved. First, one must determine a set of amplitudes $\bar{C}^{(\pm)}(\nu > m_\pi, t \leq 0)$ and $B^{(\pm)}(\nu > m_\pi, t \leq 0)$ for physical values of ν and t using fixed t dispersion relations in the variable ν .²³ It is in the course of this that one also determines the coupling constant $f^2 = \frac{G_\pi^2}{4\pi} \frac{m_\pi^2}{4M^2}$. In the second step one determines the values of $\bar{C}^{(+)}(0, t < 0)$. Finally $\bar{C}^{(+)}(0, 2m_\pi^2)$ is evaluated with some extrapolation technique.

The amplitudes $\bar{C}^{(+)}(0, t < 0)$ involve pure imaginary space momentum $\sqrt{(\frac{t^2}{16} - M^2 m_\pi^2)/(M^2 + m_\pi^2 - \frac{t}{2})}$ for small t . But we noted that as long as the magnitude of the space momentum is small it is in the domain of analyticity of the scattering amplitude as a function of the momenta. Thus we could use our theory to determine $\bar{C}^{(+)}(0, t < 0)$ for small t .

The quantity $O(m_\pi^4)$ in Eq. (2) can be determined only with a dynamical theory. We evaluated it and found it to contribute ~ 4 MeV to $\sigma(\pi N)$. While the error is small compared to the discrepancy between 25 MeV and 65 MeV it is totally unacceptable for a phase shift calculation.

In Fig. 3 we present $\bar{C}^{(+)}(0, t < 0)$ s determined by Langbein¹⁷ as illustrative of typical dispersion relation method calculations. The solid curves are our theoretical calculations.²⁴ The curve a is based on parameters quoted in our paper on S wave phase shifts. For a theoretical calculation, with all parameters fixed a priori, the agreement for small t is most encouraging. We then found that we can noticeably improve the fit by changing the P wave form factor mass μ from $8 m_\pi$ to $10 m_\pi$. To get the same low energy S wave phase shifts it was necessary to increase $\bar{g}_\pi$ to 11.9 and decrease $\sigma(\pi N)$ to 24.9 MeV. The curve b is due to this set of parameters.

In view of the nice agreement for small t we are inevitably led to the conclusion that there is no real discrepancy between our value and the results of the dispersion relation methods. While the art of determining the best set of amplitudes from dispersion relation is at a very high level the art of determining the error bars on these amplitudes is far behind. The errors on $\sigma(\pi N)$

Fig. 3: $\bar{C}^{(+)}(0, t)$ amplitudes.

with the notion of pair suppression gives $\sigma(\pi N) \sim 26$ MeV. According to the σ model the σ commutator $\sigma = \sigma_{\text{field}} \frac{f_\pi}{\sqrt{2}}$. Thus $\sigma(\pi N) = \frac{f_\pi}{\sqrt{2}} \frac{g_{\sigma NN}}{m_\sigma^2}$. But pair suppression requires $g_{\sigma NN}/m_\sigma^2 = g_\pi/M^2$. Combining the two and using Goldberger-Treiman relation we have $\sigma(\pi N) = \frac{m_\pi^2 g_\pi(0)}{M} = 26$ MeV.

Up until now we have tested our theoretical amplitudes for small values of momenta only. It is possible that our successes have been spurious. Another source of worry is the value of $f^2 = \frac{G_\pi^2 m_\pi^2}{4 M^2}$ which according to dispersion theorists is 0.079 ± 0.001 .²⁶ As PCAC is the central assumption of our theory we are locked into a value ~ 0.0725 which includes a 3% enhancement due to form factor. However, there is a class of tests, to which we can subject our theory, which touches on both these issues.

Consider the unsubtracted forward dispersion relation

$$\frac{M+m_\pi}{Mm_\pi} a^{(-)} - \frac{2}{m_\pi^2} \frac{f^2}{1 - \frac{m_\pi^2}{4M^2}} = J = \frac{1}{2\pi^2} \int \frac{dk}{\omega} \sigma^{(-)}(\omega) \dots \quad (3)$$

and the once-subtracted relations

quoted in many papers are simply not based on careful analysis of the problem. For the evaluation of $\sigma(\pi N)$ considerable care must be exercised as the amplitude $\bar{C}^{(+)}(0, t)$ has a node very close to $t = 2m_\pi^2$.

We should also point out that any theory of πN interaction which claims to be reasonable should be capable of yielding values of $\bar{C}^{(+)}(0, t)$ for small t . In fact these amplitudes are of the "off-energy shell" category a la Blankenbecler and Sugar.²⁵ Thus dispersion relations provide a check on models of off-energy shell πN amplitudes. A parallel situation does not exist for NN amplitudes.

Before moving off this subject we may note that the σ model coupled

$$\frac{4\pi(M+m_\pi)}{Mm_\pi} a^{(-)} - \bar{c}^{(-)}(0,0) - \frac{8\pi f^2}{m_\pi^2} = I^{(-)} = \frac{2m_\pi^2}{\pi} \int \frac{dk}{\omega^3} \sigma^{(-)}(\omega)$$

$$M(1 - \frac{\pi}{4M^2})$$

$$\frac{4\pi(M+m_\pi)}{M} a^{(+)} - \bar{c}^{(+)}(0,0) + \frac{4\pi f^2}{m_\pi^2} = m_\pi I^{(+)} = \frac{2m_\pi^2}{\pi} \int \frac{dk}{\omega^2} \sigma^{(+)}(\omega).$$

$$M(1 - \frac{\pi}{4M^2})$$

It is useful to remind ourselves that f^2 is determined in the course of evaluation of the amplitudes $\bar{c}^{(\pm)}(\nu, t)$ and $B^{(\pm)}(\nu, t)$ using fixed t dispersion relations of which the above are special cases. Also, we may note that if $a^{(-)}$ was directly measurable one could use Eq. (3) to determine f^2 . Unfortunately, $a^{(-)}$ is not directly measurable.

From our low energy theory we can calculate²⁷ $a^{(\pm)}$ and $\bar{c}^{(\pm)}(0,0)$. As explained before $f^2 = 0.0725$ is practically a fixed parameter of the theory. Thus we have predictions for the cross section integrals via dispersion relations. We can then test our theory by comparing our results with those obtained by the dispersion relation methods. In Table I we present our theoretical results for parameter set b, described earlier and the results obtained by Höhler, Koch and Pietarinen.¹⁸

TABLE I

Integral	Ours (m_π^{-2})	Höhler, et al. (m_π^{-2})
J	-0.056	-0.053
I ⁽⁻⁾	-0.202	-0.206
I ⁽⁺⁾	-1.39	1.47

We consider the agreement eminently pleasing.

Work on the P wave is still in progress.²⁸ One of the things we have learnt is that the dynamical picture which emerges from a static theory is not the whole story. Details will be presented in a contributed paper.

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