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A MODEL FOR TURBULENT MOMENTUM AND
HEAT TRANSPORT IN LARGE ROD BUNDLES

by

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Components Technology Division

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NOMENCLATURE

A_x	Cross-sectional area of a subchannel (or computational cell) upstream or downstream from blockage	T_w	Wall temperature
A_0	Open area of a subchannel (or cell) at blockage	U, V, W	Velocity component in x, y, and z directions, respectively
a, b, c	Coefficients associated with friction formula	U_s, V_s, W_s	Effective wall slip velocity in x, y, and z directions, respectively
c_μ	Turbulent viscosity coefficient	U_τ	Friction velocity
c_l	Turbulence length-scale coefficient	V_{sc}, W_{sc}	Velocity at gap in y and z directions, respectively (Sec. III only)
c_p	Specific heat at constant pressure	W_p	Wetted perimeter
c_R	Drag coefficient	W_p^*	Heat-transfer perimeter
c_s	Shape coefficient	x, y, z	Cartesian-coordinate system
c_B	Form-loss conversion coefficient	α	Local porosity, ratio of fluid volume to total volume
d	Rod diameter	$\tilde{\alpha}$	Heat-transfer coefficient between tube and fluid
D_h	Equivalent hydraulic diameter	β	Dimensionless volumetric expansion coefficient
f_x, f_y, f_z	Friction factors in x, y, and z directions, respectively	ϵ	Dissipation rate of turbulence kinetic energy due to viscosity
F_x	Force per unit length in x direction	λ	Molecular thermal diffusivity
g	Gravitational acceleration	λ_t	Turbulent thermal diffusivity
h	Fluid enthalpy	μ	Viscosity
k	Fluid turbulence kinetic energy	μ_{eff}	Effective viscosity
$\hat{k}$	Thermal conductivity of fluid	μ_t	Turbulent viscosity
l	Turbulent length-scale	ν	Kinematic viscosity
n	Number of subchannels per unit area	ρ	Fluid density
P	Fluid static pressure	$\bar{\rho}$	Average fluid density over subchannel
P_k	Production rate of turbulence kinetic energy due to shear	ρ_w	Fluid density at wall
P_y, P_z	Pitch in y and z directions, respectively	τ	Shear stress
P_{yz}	Mean pitch, $(P_y P_z)^{1/2}$	τ_w	Wall shear stress
$\dot{q}_w''$	Heat-transfer rate per unit surface area of tube from tube to fluid	$\bar{\tau}_w$	Average wall shear stress
$\dot{q}_{y,d}''$	Diffusive transport of heat in y direction	Pr	Molecular Prandtl number
$\dot{q}_{x,t}''$	Turbulent transport of heat in x direction	Pr_t	Turbulent Prandtl number
R_x	Drag force in x direction	Re_x	Axial-flow Reynolds number, $\rho U D_h \alpha / \mu$
S_B	Generation rate of turbulent kinetic energy due to baffle	Re_y	For subchannel, $\rho V (P_z - d) / \mu$; for quasi-continuum, $\rho V P_z \alpha / \mu$
SG	Generation rate of turbulent kinetic energy due to gravity	Re_y^*	For subchannel, $\rho V d / \mu$; for quasi-continuum, $\rho V d \alpha / \mu (1 - d / P_z)$
T	Mean fluid temperature in subchannel	Re_{yz}^*	Crossflow Reynolds number, $\rho (V^2 + W^2)^{1/2} d \alpha / \mu [1 - d / (P_y P_z)^{1/2}]$
$\tilde{T}$	Mean fluid temperature at a given elevation		

A MODEL FOR TURBULENT MOMENTUM AND HEAT TRANSPORT IN LARGE ROD BUNDLES

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ABSTRACT

A quasi-continuum model for turbulent momentum and heat transport in large rod bundles has been developed. This model has been derived from a subchannel analysis and adapted to a quasi-continuum form by introducing concepts of porosity and distributed resistance. The effects of turbulent kinetic-energy generation due to shear, viscosity, diffusion, geometric effects, buoyancy, and Reynolds number are explicitly included. The proposed model of turbulence is relatively simple, yet it is believed to provide a framework for taking account of important turbulent mechanisms in rod bundles.

I. INTRODUCTION

The rod bundle is perhaps the most important geometry in heat-exchange equipment. Dependent on the application, the external flow may be directed either predominantly parallel to the axis of the rods (or tubes) or predominantly normal to the tube axes. In other applications, a mixed type of flow is produced through the use of baffles or other devices to regulate the flow direction.

A feature common to all applications is the complexity of the topography and of the resultant flow structure. Until recently, this complexity has meant that designs had to be evolved purely from experimental testing. In the last decade, however, digital computers have become available with a sufficiently large core and a sufficiently fast execution time to allow at least limited computational analyses of rod-bundle geometries. Here, for example, we may mention the THI3D,¹ COBRA,² and SABRE³ codes for three-dimensional analysis, and the MVARR II Code,⁴ which provides a two-dimensional treatment. (The MVARR II Code is a modified version of the general-purpose VARR II Code.⁵) In these schemes, no attempt is made to resolve the flow in the vicinity of the tubes or walls. Either the flow domain is subdivided into a number of subregions (usually an area "enclosed" by a number of rods or tubes) and the flow within each subregion is treated essentially by a one-dimensional analysis;^{1,2} or the tubes are considered to be absent, but their effects are accounted for by introducing porosity and appropriate resistance coefficients into a continuum analysis.⁴ Here, these approaches are referred to as "subchannel" and "quasi-continuum" models, respectively.

If the number of tubes under consideration is small (≤ 100) and relatively closely packed, the "subchannel" model is usually preferable because it allows a clearer description of the various mechanisms of momentum and heat exchange (though uncertainties still remain in modeling the mechanisms, especially the assumptions used in the formulation of the transverse momentum equations). For large numbers of tubes, computer storage limitations dictate that the subchannels be interpreted as the complicated space between clusters of tubes--perhaps a dozen or more. In these circumstances, the advantage mentioned above disappears and because of the much simpler internal organization that it allows, a quasi-continuum approach with volume porosity and distributed resistance is more attractive.

Irrespective of which of these basic approaches is adopted, the usefulness of any computer model of flow through rod bundles will depend essentially on the realism of the model adopted for heat and momentum transport by turbulent mixing. The subject of turbulence modeling has undergone rapid development in the last few years. Extensive accounts of these developments are available in the books and articles by Launder and Spalding,^{6,7} Launder,⁸ and Bradshaw,⁹ and a further review is not attempted here. Suffice it to say that, for geometries simpler than the tube bank, much progress has been made toward evolving models of general validity. That is, models consisting of a single set of differential and algebraic equations may be applied to predict the level of turbulent mixing in many different kinds of flow. The most important element in this approach toward a "universal" turbulence model has been the inclusion of transport effects on turbulence. Typically, in currently used turbulence models, between one and ten partial-differential equations describing the turbulence transport are solved simultaneously with the mean-flow conservation equations.

The benefits of these advances in turbulence modeling have begun to be felt in rod-bundle geometries. Carajilescov and Todreas,¹⁰ using the model of Launder and Ying,¹¹ have successfully predicted the axial and secondary flow patterns that arise for fully developed flow in a single subchannel. Massey¹² has applied the two-equation model of Jones and Launder¹³ to compute the case for which flow is purely normal to the tube axis and the flow pattern between one row of tubes and the next is exactly repetitive.

The above two examples are ones in which, because of the imposed constraints, attention could be limited to a small section of the bundle--small enough to allow a complete continuum, finite-difference analysis of the flow in question. The same degree of detail is not possible, of course, in an analysis of the whole rod bundle. One must, of necessity, draw on simple porosity and distributed-resistance concepts that, from the very earliest computer-based treatments of flow in rod bundles, have been at the heart of the models of heat and momentum exchange. Nevertheless, in devising the resistance coefficients, we can draw many lessons from continuum turbulence models while retaining the main idea of turbulence transport by the flow in simplified form.

The present report presents a model of turbulence for single-phase flows in rod bundles. In line with the above discussion, we have attempted to draw on the best of contemporary turbulence-model practices to provide a description that will apply to the greatest possible range of problems. Because of lack of experimental data, in several places guesses have had to be made at the magnitude of certain coefficients; sometimes even the form of a correlation is speculative. These uncertainties can be removed only by application of the model to predict various types of rod-bundle flow and its successive refinement.

The basic model is developed in Sec. II based on a subchannel approach. This section begins with a discussion of the scope of the model and the strategy to be used. Thereafter, the effects of turbulent exchange in each of the four mean-flow conservation equations (i.e., the momentum equations and the energy equation) are considered in turn. The section concludes with an examination of the factors affecting the transport of turbulence kinetic energy, a quantity of significant importance in determining the effective turbulent transport coefficients,

As discussed above, for most applications the numerical-solution scheme will need to be based on a quasi-continuum rather than a subchannel treatment because of the large numbers of tubes involved. Section III therefore transforms the formulas developed in Sec. II into a system of partial differential equations for use in the quasi-continuum framework. In Sec. IV, we discuss the stages of further work by which the reliability and the applicability of the current model may be improved. Finally, a simplified algebraic turbulence model for large rod bundles is presented in the appendix.

II. A SUBCHANNEL MODEL FOR MOMENTUM AND HEAT TRANSPORT

The formulas developed in this section will be evolved by reference to a simple subchannel partially enclosed by rods or tubes as shown in Fig. 1.

Fig. 1

Subchannel Arrangements for (a) Staggered (triangular), and (b) In-line Tubes (square)

In most regions of flow, the flow direction is predominantly parallel with the axis of the tubes but, as indicated in Fig. 1, there may be substantial local crossflow velocities.

The present concept of modeling transport rates across the boundaries of the subchannel takes the view that these rates will also be affected by the local level of turbulence kinetic energy. The scheme proposed therefore requires the solution of transport equations for the velocity component parallel

with the tube axis, U ; the velocity components normal to the tube axis, V and W ; the mean temperature, T (or the mean enthalpy, h); the turbulence kinetic energy, k ; and the static pressure, P . The static-pressure distribution may be regarded as determined by the continuity equation, and no "modeling" assumptions are needed; therefore no further consideration will be given to this quantity.

Although it is now common practice in continuum turbulence models to take account of the transport of turbulence length-scale and its energy,^{6,7} we do not propose such a scheme in the present treatment. Given the rather closely packed tube arrangements that occur in nearly all practical applications, the local magnitude of the length-scale at any point in the flow will be determined, above all else, by the distance of a point in the flow from the neighboring tube surfaces. The effects of convective transport of length-scale between adjacent subchannels will be hardly significant.

Subsequent sections focus attention, in turn, on the conservation equations for each of the primary dependent variables, suggesting approximations for the direct and indirect effects of turbulence. Convective transport and time-dependent contributions are treated directly in Sec. III. Only diffusive and source and sink terms are considered here. In discussing the contribution of buoyancy effects, we will assume that the axis of the tubes is vertical.

A. Axial (U) Momentum Transfer

Here we need to account for the effects of drag at the surface of the tubes; the diffusion of momentum between adjacent subchannels due, principally, to turbulence agitation; and blockage effects due to baffles or grids.

1. Drag at Tube Surface

The force per unit length opposing the axial motion is given by

$$F_x = \bar{\tau}_{w,x} W_p, \quad (1)$$

where W_p is the wetted perimeter of the tube or wall surface bounding the subchannel (see Fig. 1), and $\bar{\tau}_{w,x}$ is the average wall shear stress at the tube or wall surface. The latter quantity is expressed by way of a subchannel friction factor as

$$\bar{\tau}_{w,x} = \rho U |U| f_x, \quad (2)$$

where the magnitude sign ensures that, in regions where the axial flow reverses its direction, the shear stress does too. The axial friction factor is assumed expressible as a function of the subchannel Reynolds number Re_x through

$$f_x = aRe_x^b + c, \quad (3)$$

and the constants a , b , and c are evaluated according to whether the flow is laminar or turbulent. For the present, we recommend values listed in Table I.

TABLE I. Coefficients in Eq. 3

a	b	c	Range of Re_x
8.0	-1.0	0	<940 ^a
0.07	-0.32	0.0007	>940

^aThe transition value of 940 is chosen to ensure that f_x is a single-valued one-sided continuous function. The original proposed transition value for Re_x between laminar and turbulent regime is 1500.

The Reynolds number Re_x is based on the mean subchannel axial velocity U and the hydraulic diameter D_h (equal to four times the subchannel area divided by the wetted perimeter). For the turbulent flow regime, Eq. 3 gives levels of friction factor about 15% higher than for fully developed flow in a round tube. This higher value is intended to allow for the effects of turbulence-induced secondary flows that will be absent in the round tube. It must be said that Eq. 3 and Table I provide only a rough guide to the correct value of f_x . In laminar flow, friction factors are not well correlated by use of the hydraulic diameter. Thus the coefficient a will depend on the detailed topography of the subchannel. No published results are known that display, say, the dependence of this coefficient on the pitch-diameter ratio of the pins. The task of calculating these quantities from a finite-difference (or finite-element) solution of the flow is entirely straightforward, however, and should be undertaken as a preliminary task if substantial laminar-flow regions are expected.

A further weakness of Eq. 3 is that no effects of crossflow are included. Their omission reflects the absence of any firm documentation as to their magnitude (or even their sign), rather than any certainty that they are negligible. Clearly this is an area in which we should hope, in due course, to devise improvements. A further promising line for development appears to be the specification of $\bar{\tau}_{w,x}$ in terms of modified friction factors and Reynolds numbers, using the square root of the turbulence kinetic energy as a characteristic velocity scale. The advantage of such a formulation is the prospect that it will allow account to be taken of extra high turbulence energy levels in raising the level of friction factor. The details are discussed further in Sec. IV.

2. Momentum Diffusion between Subchannels

There will be a diffusive transport of momentum between subchannels due to variations in bulk velocity between one subchannel and the next. We assume that the rate of transport per unit area is equal to

$$\mu_{\text{eff}} \frac{\partial U}{\partial y}, \quad (4)$$

where the effective viscosity is related to the turbulence energy and length scale by

$$\mu_{\text{eff}} = \underbrace{c_{\mu} \rho k^{1/2} \ell}_{\mu_t} + \mu, \quad (5)$$

and

$$\ell = c_{\ell} D_h. \quad (6)$$

Based on experience with continuum turbulence modeling, the turbulent-viscosity and length-scale coefficients (c_{μ} and c_{ℓ}) should take values of about 0.1 and 0.4, respectively, at high Reynolds number. The importance of diffusive transport in the overall momentum balance will depend to a large extent on the spacing between the tubes. In closely packed bundles, its significance will be small compared with the drag of the tubes themselves.

3. Blockage due to Baffles or Grids

No entirely general rule can be given. In most cases, however, the blockage or baffles will be blunt with a drag coefficient of about unity. We assume the drag force in any subchannel can be represented as

$$R_x = c_R \rho U^2 A_x (A_x/A_0 - 1)^2, \quad (7)$$

where A_x is the cross-sectional area of the subchannel upstream or downstream from the blockage, A_0 is the corresponding open area at the blockage, which will be less than A_x and U is the upstream velocity. Where the baffles extend over only a portion of the cross section, A_0 will equal A_x in the unblocked section, and the associated drag is zero. The drag coefficient c_R should be about 0.5 for blunt orifices.

Of course, the above force is included in the momentum balance only at positions where the baffles are located.

4. Buoyancy Effects

Under conditions of low flow rates, buoyancy effects may modify the streamwise momentum balance in numerous ways. In the following

discussion of these effects, we assume that the flow is vertically downward and that the tubes are cooler than the fluid around them (circumstances applicable to the CRBRP-IHX). It is easy to infer the corresponding effects when the tubes are hot and/or the flow is vertically upward.

a. Effect of Density Variation among Subchannels at a Given Elevation. When, at any horizontal section through the flow domain, there are large variations in the fluid temperature between subchannels, the hotter fluid will receive a net upthrust while the colder fluid will receive a net downthrust. The appropriate x-direction body force per unit volume in a particular subchannel is

$$\bar{\rho} g \beta (\bar{T} - T), \quad (8)$$

where $\bar{T}$ is the mean fluid temperature at a given elevation, β is the volumetric expansion coefficient,

$$\beta \equiv - \frac{1}{\rho} \frac{\partial \rho}{\partial T} \Big|_P,$$

and g is the gravitational acceleration.

b. Effect of Density Variation within a Subchannel. Within any subchannel there will be a tendency for cold fluid near the tube surface to descend faster than the bulk fluid. Since, in the absence of buoyancy, the cooler fluid is also the slower moving, the effect of buoyancy will be to make the velocity more uniform over the subchannel. Thus, for a given value of mean subchannel velocity, the momentum flux will be reduced. At present, however, it is not clear how to quantify this effect. In any event, it probably will be of small importance compared with the effects discussed next.

c. Effects of Buoyancy on Friction Factor. The turbulent viscosity variation in the immediate vicinity of the wall is extremely sensitive to variations in shear stress across the sublayer and transition region, particularly when the stress decreases with distance from the wall. In many flows, the sublayer is so thin that shear stress variations across it are truly negligible; one then obtains a flow pattern that is usually referred to as the "universal" near-wall structure. Important departures from this form may occur, however, and, as Hall and Jackson have shown,¹⁴ buoyancy effects are one of the phenomena that can modify the near-wall flow structure. Very close to the wall, convective transport of momentum may be neglected. The stress at a distance y from the wall is then given by

$$\tau = \tau_w + \int_0^y (\bar{\rho} - \rho) g \, dy. \quad (9)$$

Thus, for a cold wall ($\rho > \bar{\rho}$), the shear stress decreases. Modifications to the near-wall structure appear to become substantial when the shear-stress change

across the buffer region exceeds about 5% of the wall stress. By making a number of simplifications, we deduce from Eq. 9 that we shall observe significant effects when

$$\left(1 - \frac{T_w}{T}\right) \frac{g\nu}{U_\tau^3} \approx 2 \times 10^{-3}, \quad (10)$$

where T is the mean temperature in any subchannel, T_w is the tube-wall temperature in that subchannel, U_τ is the friction velocity ($= \sqrt{\tau_{w,x}/\rho_w}$), and ν is the kinematic viscosity of the fluid.

No correlation of the effect of the buoyancy parameter on f_x appears to be available. For extensive computations where the buoyancy parameter is of the magnitude (or greater than) indicated by Eq. 10, it would be desirable to devise such a correlation.

d. Direct Effects of Buoyancy on Turbulent Stresses in Fully Turbulent Region. The main effect here will be expressible in terms of the so-called flux Richardson number, defined as $-S_G/P_k$. Both S_G and P_k appear in Sec. D below in connection with turbulent kinetic energy transport. It is provisionally assumed that, for vertical flow, we may retain Eqs. 4 and 5, but that the level of kinetic energy in Eq. 5 will be altered by buoyant action as discussed later.

B. Transverse (V and W) Momentum Equations

In the momentum equations, the major process requiring modeling is the drag due to the tubes themselves. Characterizing this resistance is one of the major uncertainties in the present (or any other) treatment. In the absence of better information, we propose that, initially, the correlations adopted in the SABRE computer program³ be used here. Thus, for the y direction,

$$\tau_{w,y} = f_y \rho V |V|, \quad (11)$$

where the friction factor f_y is taken as the largest of

$$f_y = 3 \text{Re}_y^{-1} \left(\frac{P_z - d}{P_z - 0.93d} \right)^2, \quad (12)$$

$$f_y = 0.6 \frac{P_z}{d} \left[0.25 + \frac{0.118}{(P_z/d - 1)^{1.08}} \right] \text{Re}_y^{*-0.15}, \quad (13)$$

and

$$f_y = 3 \left(\frac{P_z - d}{P_z - 0.93d} \right)^2 \left[\frac{\mu}{\mu_{\text{eff}}} \text{Re}_y \left(1 + \frac{2.16 D_h}{P_z - d} \right) \right]^{-1}, \quad (14)$$

where the Reynolds number $Re_y \equiv \rho V(P_z - d)/\mu$ and $Re_y^* \equiv \rho Vd/\mu$. Equation 12 was reportedly devised for fully developed laminar crossflow; Eq. 13 for strong crossflow, such as may occur near a baffle; and Eq. 14 for weak crossflow where the flow is turbulent. (Equation 13 differs from the form quoted in Ref. 3. This reference may have misinterpreted friction-factor formulas quoted by McAdams.¹⁵) Cross velocities V and W are those measured in the throat (gap) between adjacent tubes.

Exactly parallel formulas to Eqs. 12-14 may be written for the z direction.

C. Thermal-energy Equation

Kinetic-energy and viscous heating contributions are assumed negligible. To devise an appropriate form of this equation, we need expressions for (1) the rate of heat transfer between the pins/tubes and the fluid, (2) lateral heat transport from and to fluid in adjacent subchannels, and (3) axial heat transport. (Note: Since we are dealing with liquid metals, the Peclet number may be low, necessitating provision for the inclusion of axial heat transport.) Next, we consider the modeling of each of these heat transports.

1. Heat Transfer between Tubes and Fluid

We assume that

$$\dot{q}_w'' = -\tilde{\alpha}(T - T_w), \quad (15)$$

where $\dot{q}_w''$ is the heat-transfer rate per unit surface area of tube from the tube to the fluid, and $\tilde{\alpha}$ is the heat-transfer coefficient, obtained from either

$$\begin{aligned} \frac{\tilde{\alpha} D_h}{\hat{k}} = & 0.25 + 6.2(P_{yz}/d) + [0.032(P_{yz}/d) - 0.007](Re_x Pr)^{0.8-0.24(P_{yz}/d)} \\ & - 0.15(P_{yz}/d - 0.85)^{-2}, \end{aligned} \quad (16)$$

or

$$\frac{\tilde{\alpha} D_h}{\hat{k}} = 4.03 + 0.23(Re_{yz}^* Pr)^{0.67}, \quad (17)$$

where $\hat{k}$ is the thermal conductivity of the fluid, P_{yz} the mean pitch which we can take to be $(P_z P_y)^{1/2}$, and Re_{yz}^* is the crossflow Reynolds number $\rho(V^2 + W^2)^{1/2}d/\mu$. Equation 16 is a modified form of Graber and Rieger's proposal¹⁶ for flow predominantly parallel with the tube axis; the final term has been added in the present study to compensate for the too high values of $\tilde{\alpha}$ produced by the original formula at small values of P_{yz}/d and low Peclet numbers. Equation 17, quoted by El-Wakil,¹⁷ applies when crossflow is predominant. The larger of the values of $\tilde{\alpha}$ given by these two formulas should be selected.

When there is significant, but not dominant crossflow, there is considerable uncertainty in estimating heat-transfer rates from the above formulas.

2. Heat Diffusion between Subchannels

Diffusive transport of heat is represented in an exactly parallel way to that of momentum diffusion:

$$\dot{q}_{y,d}'' = -c_p \left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{\partial T}{\partial y}, \quad (18)$$

where Pr is the molecular Prandtl number, and Pr_t the so-called turbulent Prandtl number, which, for the present, is obtained from the following proposal of Nijssing and Eiffel:¹⁸

$$Pr_t = 0.8 [1 - \exp(-6 \times 10^{-5} Re_x Pr^{1/3})]^{-1}. \quad (19)$$

3. Axial Diffusion of Heat

Under conditions of fairly rapid change in the streamwise direction (such as just upstream or downstream of a baffle), axial heat diffusion may be significant. There are two contributions to include, those due to molecular and turbulent transport. The former is created by axial temperature gradients; the latter is predominantly due to gradients of temperature in the y and z directions. Experiments in pipe flow suggest that the overall magnitude of the turbulent component of axial heat flux is about the same as the heat flux from the tubes. The sign is such that when the heat transfer is from the tubes to the fluid, the streamwise turbulent flux is in a direction opposite from the mean motion.

The pipe-flow experiments cited above relate to flows in which the Prandtl number is of the order of unity or greater. Strictly, the streamwise heat flux will be proportional to the fraction of the heat flux from the tubes due to turbulent motion, not molecular conduction. In the present formulation, the equation for $\dot{q}_w''$ is a global estimate with no clear distinction between turbulent and molecular contributions. For the present, we assume that, for flow through a heated circular tube,

$$\dot{q}_{x,t}'' = -\dot{q}_w'' \frac{\lambda_t}{\lambda + \lambda_t},$$

where λ and λ_t are the molecular and turbulent thermal diffusivities over the subchannel, or, in terms of mean Prandtl numbers,

$$\dot{q}_{x,t}'' = -\dot{q}_w'' / [1 + (\mu Pr_t / \mu_t Pr)]. \quad (20)$$

A plausible generalization of this formula for flow in subchannels is

$$\dot{q}_{x,t}'' = \frac{W^*}{\pi D_h} \tilde{\alpha} (T - T_w) / [1 + (\mu Pr_t / \mu_t Pr)]. \quad (21)$$

For a circular tube of diameter d , $D_h = d$ and $W_p = \pi d$, so Eq. 21 reduces to Eq. 20. When the pin spacing is small, we shall likewise find that the quantity $W_p / \pi D_h$ is near unity; however, as the pin spacing increases, D_h increases, reducing the magnitude of the mean axial heat flux. It is important to note that, while $\dot{q}_{x,t}''$ may be large, it is its variation with x that determines whether the term contributes significantly to the local enthalpy balance.

D. Turbulence Kinetic-energy Equation

This equation is needed only for cases in which the flow is turbulent. The subchannel balance equation for this quantity may be expressed literally as

$$\begin{aligned} \text{Net Convection of} &= \underbrace{\text{Production}}_{P_k} - \underbrace{\text{Dissipation}}_{\epsilon} + \text{Gain by Diffusion from} \\ \text{Turbulence Energy} &\quad \text{due to Shear} \quad \text{due to Viscosity} \quad \text{Adjacent Subchannels} \\ &+ \text{Miscellaneous Sources.} \end{aligned} \quad (22)$$

The processes on the right-hand side of Eq. 22 are approximated as discussed below.

1. Production due to Shear

In continuum form, this process is represented as

$$P_k = \mu_t \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right). \quad (23)$$

A simple analog of Eq. 23 appropriate to a subchannel treatment is

$$c_s \mu_t \frac{(U - U_s)^2 + (V - V_s)^2 + (W - W_s)^2}{D_h^2}, \quad (24)$$

where U_s , V_s , and W_s are effective wall "slip" velocities designed to account for the fact that the flow in the viscous sublayer does not contribute to the production of turbulence energy. In practice, these slip velocities depend on the subchannel Reynolds number. Initially, however, it appears advisable to use a constant value, which we take to be 50% of the corresponding subchannel velocities U , V , or W (implying that 50% of the velocity change occurs across the viscous sublayer--a reasonable rough estimate).

The coefficient c_s is included to account for the replacement of the x_i derivatives in Eq. 23 by the hydraulic diameter. Its magnitude will be of order unity. However, it will probably be somewhat larger, since most of the energy production occurs near the wall, where velocity gradients are in fact steeper than implied by Eq. 24.

2. Dissipation due to Viscosity

As in continuum models, we take the turbulence-energy dissipation per unit volume ϵ , to be equal to

$$\epsilon = \rho k^{3/2}/\ell. \quad (25)$$

In a sense, this may be regarded as defining the length scale, ℓ .

3. Turbulence-energy Diffusion

This will be represented by the gradient-type transport formulas used earlier to characterize the diffusion of heat and axial momentum:

$$\text{Diffusion rate of } k \text{ in } y \text{ direction} = \mu_{\text{eff}} \frac{\partial k}{\partial y}. \quad (26)$$

For closely packed tubes of practical interest, this process will be of negligible importance.

4. Miscellaneous Effects

a. Baffle Generation. The most reasonable simple proposition appears to be that a fixed proportion of the mean total energy lost in passing through the baffle is converted to turbulence (the remainder being directly dissipated to heat). Thus, we assume that the baffle loss should be expressed as

$$S_B = c_B \rho U^3 (A_x/A_0 - 1)^2 \text{ per unit cross-sectional area.} \quad (27)$$

This will produce a step change of turbulence kinetic energy. A little care is needed in transforming this equation to a quasi-continuum form for solution. The form loss conversion coefficient c_B should be of the order of 0.2, suggesting that about half the pressure loss goes into turbulence-energy generation.

b. Buoyancy Generation. For the case where the x axis is vertical with positive direction downward, the generation rate due to gravitational effects may be represented exactly as

$$S_G = - \frac{\dot{q}_{x,t}''}{c_p} \beta g \text{ per unit volume,} \quad (28)$$

where the turbulent heat flux is given by Eq. 21.

c. Reynolds-number Effects. All the above coefficients should strictly become functions of the turbulence Reynolds number ($\rho k^{1/2} \ell / \mu$) when the value of this Reynolds number falls below about 100. Initially, however, it is proposed that the Reynolds-number influence should be absorbed solely in the viscosity coefficient c_μ , and that the largest of the following mean-flow Reynolds numbers should be used as the basis for fixing viscous effects:

$$\frac{\rho U D_h}{\mu}; \quad \frac{\rho V (P_z - d)}{\mu}; \quad \frac{\rho W (P_y - d)}{\mu}.$$

As a starting point for explorations, it is proposed that the quantity c_μ should be unaffected by viscosity when the Reynolds number is greater than 2000, and that it should decrease linearly to zero for a Reynolds number of 1000. For values of Reynolds number below 1000, the function c_μ is zero.

III. QUASI-CONTINUUM FORMULATION

In this section the subchannel model presented in Sec. II is adapted for use in a continuum-like solution scheme for the five dependent variables U , V , W , T , and k , which denote the three mean velocity components, the fluid temperature (or enthalpy), and the turbulence kinetic energy, respectively. The principles of the conversion are discussed in Sec. A below; the equations themselves and the associated auxiliary equations are presented in their entirety (though without discussion) in Sec. B. Section C provides a guide for promoting stability in the numerical-solution scheme.

A. Principles Applied in Transformation from Subchannel Model to Continuum Form

1. Porosity

The continuum computations are made by replacing the irregular open passages between tubes by a porosity α , which will vary in a continuous manner over the cross section. The local porosity is such that the ratio of flow volume to control volume is the same as in the subchannel model. This approach has been applied with considerable success in the analysis of structured or porous multiphase flow,¹⁹ for which the appropriately modified forms of the continuity and momentum equations are given by Wnek et al.²⁰ Alternatively, to represent certain of the processes of Sec. II, a volume porosity and a vectorial surface permeability could be employed.²¹ We have, for the present, retained the volume porosity only because our computational cell contains many subchannels or tubes; thus, it is not entirely clear how to obtain a representative surface permeability for the y and z directions. Moreover, in optimizing the empirical coefficients in the crossflow drag laws etc., the effects of non-isotropic surface permeability will become absorbed in the magnitudes of the optimized coefficients.

2. Axial Velocity

A consequence of the above choice is that the quasi-continuum axial velocities are the same as for the subchannel model. Thus no modifications are required to the formulas for the axial friction factor.

3. Lateral Velocities

In the subchannel analysis of Sec. II, the lateral velocities represented "in-the-gap" values. These may be significantly different from the averaged "quasi-continuum" values, particularly for conditions in which the spacing between tubes is small. Thus corresponding corrections need to be applied to the friction-factor and heat-transfer formulas involving lateral velocity components. The relation between the subchannel and the continuum velocities may be approximated by

$$V_{sc} = \frac{V\alpha P_z}{P_z - d}$$

and

$$W_{sc} = \frac{W \alpha P_y}{P_y - d},$$

where the y- and z-direction pitches are defined in Fig. 1, and the subscript "sc" denotes subchannel values.

4. Cluster of Subchannels

The action of forces and heat fluxes on each subchannel is converted to quasi-continuum form by multiplying the appropriate formulas in Sec. II by the number of subchannels per unit area, n (typical dimensions (metre)⁻²).

B. Complete Equation Set for Simulation of Flow in Rod Bundles

The following pages provide a complete statement of the system of differential and auxiliary equations that are held to characterize flow in rod bundles. The origin of the auxiliary equations is explained in Sec. II.

In Eqs. 29 and 30 below, two terms are outlined in dashes. This emphasizes the fact that these terms are included only over the forward step δx in which a baffle appears.

1. System Equations

x Momentum

$$\begin{aligned} \frac{D}{Dt} (\rho \alpha U) = & -\alpha \frac{\partial P}{\partial x} - (\rho |U| f_x W_{pn}) U + \rho \alpha g \beta (\tilde{T} - T) \\ & \left[-[c_{Rp} |U| (A_x/A_0 - 1)^2 / \delta x] U \right] \\ & + \alpha \left[\frac{\partial}{\partial x} \left(\mu_{eff} \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_{eff} \frac{\partial U}{\partial y} \right) + \frac{\partial}{\partial z} \left(\mu_{eff} \frac{\partial U}{\partial z} \right) \right]. \end{aligned} \quad (29)$$

y Momentum

$$\begin{aligned} \frac{D}{Dt} (\rho \alpha V) = & -\alpha \frac{\partial P}{\partial y} - \left[\rho |V| W_{py} \frac{\alpha^2 n}{(1 - d/P_z)^2} \right] V \\ & + \alpha \left[\frac{\partial}{\partial x} \left(\mu_{eff} \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_{eff} \frac{\partial V}{\partial y} \right) + \frac{\partial}{\partial z} \left(\mu_{eff} \frac{\partial V}{\partial z} \right) \right]. \end{aligned} \quad (30)$$

z Momentum

$$\begin{aligned} \frac{D}{Dt} (\rho \alpha W) = & -\alpha \frac{\partial p}{\partial z} - \left[\rho |W| W_{Pfz} \frac{\alpha^2 n}{(1 - d/P_y)^2} \right] W \\ & + \alpha \left[\frac{\partial}{\partial x} \left(\mu_{eff} \frac{\partial W}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_{eff} \frac{\partial W}{\partial y} \right) + \frac{\partial}{\partial z} \left(\mu_{eff} \frac{\partial W}{\partial z} \right) \right]. \end{aligned} \quad (31)$$

Thermal Energy (Enthalpy)

$$\begin{aligned} \frac{D}{Dt} (\rho \alpha c_p T) = & -[\tilde{\alpha}(T - T_W) W_{Pn}^*] \\ & + \alpha \frac{\partial}{\partial x} \left(\hat{k} \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial x} \left[\frac{\alpha \tilde{\alpha}(T - T_W)}{1 + (\mu Pr_t / \mu_t Pr)} \frac{W_P^*}{\pi D_h} \right] \\ & + \alpha \frac{\partial}{\partial y} \left(\hat{k} + \frac{c_p \mu_t}{Pr_t} \right) \frac{\partial T}{\partial y} + \alpha \frac{\partial}{\partial z} \left(\hat{k} + \frac{c_p \mu_t}{Pr_t} \right) \frac{\partial T}{\partial z}. \end{aligned} \quad (32)$$

The enthalpy (h) equation can be written by applying the relationship $c_p \Delta T = \Delta h$ in Eq. 32.

Turbulence Energy

$$\begin{aligned} \frac{D}{Dt} (\rho \alpha k) = & \alpha \{ c_s \mu_t [(U - U_S)^2 + (V - V_S)^2 + (W - W_S)^2] / D_h^2 \\ & - \rho k^{3/2} / \ell \} + \left[c_B \rho |U|^3 (A_x / A_0 - 1)^2 / \delta x \right] \\ & - \frac{\alpha \tilde{\alpha}(T - T_W) \beta g / c_p}{1 + (\mu Pr_t) / (\mu_t Pr)} \frac{W_P^*}{\pi D_h} \\ & + \alpha \left[\frac{\partial}{\partial x} \left(\mu_{eff} \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_{eff} \frac{\partial k}{\partial y} \right) + \frac{\partial}{\partial z} \left(\mu_{eff} \frac{\partial k}{\partial z} \right) \right]. \end{aligned} \quad (33)$$

2. Auxiliary Equations and Definitions

$$\text{Friction factors: } f_x = a Re_x^b + c, \quad (34)$$

where

$$Re_x \equiv \rho U D_h / \mu,$$

and recommended values of the constants a , b , and c are listed in Table I. Factor f_y is the largest of the expressions

$$f_y = 3\text{Re}_y^{-1} \left(\frac{P_z - d}{P_z - 0.93d} \right)^2, \quad (35)$$

$$f_y = 0.6 \frac{P_z}{d} \left[0.25 + \frac{0.118}{(P_z/d - 1)^{1.08}} \right] \text{Re}_y^{*-0.15}, \quad (36)$$

and

$$f_y = 3 \left(\frac{P_z - d}{P_z - 0.93d} \right)^2 \left[\frac{\mu}{\mu_{\text{eff}}} \text{Re}_y \left(1 + \frac{2.16 D_h}{P_z - d} \right) \right]^{-1}, \quad (37)$$

where

$$\text{Re}_y \equiv \frac{\rho V (P_z - d) \alpha P_z / (P_z - d)}{\mu} = \frac{\rho V P_z \alpha}{\mu}$$

and

$$\text{Re}_y^* \equiv \frac{\rho V d \alpha}{\mu} P_z / (P_z - d) = \frac{\rho V d \alpha}{\mu (1 - d/P_z)}.$$

Exactly parallel expressions are used for f_z , replacing V by W and P_z by P_y in all the definitions.

The heat-transfer coefficient $\tilde{\alpha}$ is obtained from the larger of the estimates

$$\begin{aligned} \frac{\tilde{\alpha} D_h}{\hat{k}} &= (0.032 P_{yz}/d - 0.007) (\text{Re}_x \text{Pr})^{0.8-0.024(P_{yz}/d)} \\ &\quad + 0.25 + 6.2 P_{yz}/d - 0.15 (P_{yz}/d - 0.85)^{-2} \end{aligned} \quad (38)$$

and

$$\frac{\tilde{\alpha} d}{\hat{k}} = 4.03 + 0.23 (\text{Re}_{yz}^* \text{Pr})^{0.67}, \quad (39)$$

where P_{yz} is the mean pitch $(P_y P_z)^{1/2}$, Pr is the fluid Prandtl number, and Re_{yz}^* may be taken as

$$\frac{\rho (V^2 + W^2)^{1/2} d \alpha}{\mu [1 - d/(P_y P_z)^{1/2}]}.$$

The effective viscosity μ_{eff} is given by

$$\mu_{\text{eff}} = \underbrace{c_{\mu} \rho k^{1/2} \ell}_{\mu_t} + \mu, \quad (40)$$

where

$$\begin{aligned} c_{\mu} &= c_{\mu\infty} && \text{for } Re_{\text{max}} > 2000, \\ c_{\mu} &= c_{\mu\infty}(0.001 Re_{\text{max}} - 1) && \text{for } 10^3 \leq Re_{\text{max}} \leq 2 \times 10^3, \\ c_{\mu} &= 0 && \text{for } Re_{\text{max}} < 1000, \end{aligned}$$

and Re_{max} is the maximum of Re_x , Re_y , and Re_z .

The length-scale appearing in Eq. 40 and on the right-hand side of Eq. 33 is taken as linearly proportional to the hydraulic diameter of the tube; i.e.,

$$\ell = c_{\ell} D_h. \quad (41)$$

The turbulent Prandtl number is obtained from

$$Pr_t = 0.8[1 - \exp(-6 \times 10^{-5} Re_{\text{max}} Pr^{1/3})]^{-1}. \quad (42)$$

The various empirical coefficients appearing in the model should be assigned initial values as follows:

<u>Quantity</u>	<u>Initial Value</u>	<u>Quantity</u>	<u>Initial Value</u>
c_B	0.2	$c_{\mu\infty}$	0.1
c_{ℓ}	0.4	U_s	0.5U
c_R	0.4	V_s	0.5V
c_s	2.0	W_s	0.5W

C. Remarks on the Solution of the Equations

The system of differential Eqs. 29-33 is highly nonlinear and inter-coupled. Moreover, the equations are strongly influenced by source terms of either positive or negative sign. The dominance of the nonlinear sources makes the solution of the system of equations impossible, unless certain precautions are taken. In treating the source terms, we here note a simple practice that has been found helpful in speeding up convergence rates for such

equations. All the transport equations presented in Sec. B can be written in the common form

$$\frac{D}{Dt}(\rho \alpha \varphi) = \alpha \left[\frac{\partial}{\partial x} \left(\lambda_{\text{eff}} \frac{\partial \varphi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_{\text{eff}} \frac{\partial \varphi}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda_{\text{eff}} \frac{\partial \varphi}{\partial z} \right) \right] + S_{P\varphi} - S_{N\varphi}, \quad (42)$$

where φ represents the dependent variable in question, and $S_{P\varphi}$ and $S_{N\varphi}$ denote the positive and negative source terms, respectively. To secure stability, we must treat the sources differently according to their sign.

Positive sources should be evaluated explicitly. That is, in calculating the value of φ during the $(n + 1)$ th iteration, values of dependent variables appearing in $S_{P\varphi}$ should be those evaluated at the n th iteration sweep. In contrast, negative sources should be linearized and treated semi-implicitly. That is, we rewrite the (magnitude) of the negative sources as

$$S_{N\varphi} = S'_{N\varphi} \varphi. \quad (43)$$

The quantity $S'_{N\varphi}$ is then evaluated explicitly (from the previous cycle of iteration), while the multiplier φ is evaluated implicitly, i.e., at the $(n + 1)$ th iteration.

For example, consider the turbulence-energy transport Eq. 33. In the shear-generation term (the first group of terms on the right-hand side of the equation), all quantities are evaluated at the n th iteration, since the term is endemically positive. The second term on the right-hand side is the viscous dissipation rate, which always acts to reduce the level of turbulence energy. This is treated as follows:

$$\frac{\rho k^{3/2}}{\ell} = \frac{\rho k^{1/2}}{\ell} k = \frac{\rho}{\ell} k_n^{1/2} k_{n+1}. \quad (44)$$

Sometimes a term may be positive or negative according to the conditions being simulated. The buoyancy term in Eq. 33 is one such example. Here a test should be applied to discover the sign of the term before deciding how to treat it numerically.

IV. DIRECTIONS FOR FURTHER IMPROVEMENT OF PHYSICAL MODEL

Although the framework of the model presented in this report should prove satisfactory for a number of years, there are numerous detailed refinements that it will be desirable to incorporate as soon as relevant information is available. Perhaps highest in priority is the largely unknown effect of crossflow on the axial and crossflow friction factors, and on the average Nusselt number at the tube surface. Although the information could be obtained entirely by experiment, a detailed finite-difference flow analysis appears to be a more attractive and economical method for producing most of the results. (A limited program of experiments would still be needed to validate the accuracy of the predictions.) It is emphasized that the technology for doing this already exists; relatively minor adaptations to the codes developed by Carajilescov and Todreas¹⁰ and by Massey¹² are all that is required.

Irrespective of the method of generating the information, we feel that it will be desirable to express friction factors and dimensionless heat-transfer rates in terms of Reynolds numbers containing the turbulence kinetic energy. Thus we should have relations of the form

$$\left. \begin{aligned} \frac{\tau_{w,x}}{\rho U k^{1/2}} &= f_1 \left(\frac{\rho k^{1/2} D_h}{\mu} \right) \\ \text{and} \\ \frac{\tilde{\alpha} D_h}{\hat{k}} &= f_2 \left(\frac{\rho k^{1/2} D_h}{\mu}, Pr \right) \end{aligned} \right\} \quad (45)$$

The potential advantage of such a recorrelation is that the functionals on the right side of Eq. 45 should possess wider generality than when the mean velocity is used in place of $k^{1/2}$. For example, since the decrease in f_1 with Reynolds number is weak, an extra high level of turbulence energy (perhaps caused by passage through a baffle) will have the effect of raising the wall shear stress, in accordance with experimental data. The derivation of such correlations--even for flow in a circular tube--is difficult because in most studies no measurements of the turbulence energy have been made. Moreover, the level of turbulence kinetic energy is highly non-uniform over the cross section.

In Secs. II and III we have let k represent some effective average level of turbulence energy over the cross section. It is by no means certain, however, that this effective value will correspond with any of the usual mathematical ways of defining the average value. So, even if the detailed point-to-point distribution of k in a tube or a subchannel is known, there remains the problem of relating it to the effective value used in the computations. Nevertheless, we feel that this is a problem worth solving in the near future.

A further potentially important weakness in the present characterization is the omission of any effect of buoyancy on the friction factor (see Sec. II. A. 4.a). The discussion in Sec. II. A.4 centered on the axial shear stress; it is certain, however, that if there are important effects on f_x , there will also be substantial influences on the crossflow friction factors and Nusselt number. Apart from the heat-transfer data of Hall and Jackson¹⁴ and one or two other studies of flows at states close to the critical point, no quantitative information seems to be available. Probably computer solution of the describing equations (using a turbulence model especially designed to account for low-Reynolds-number phenomena) represents the best route forward.

The above short survey has identified some of the ways in which our immediate proposals for characterizing the turbulent transport processes can be systematically refined or extended to improve the range of accurate predictions. The very first task however, should be the optimization of the empirical coefficients listed at the end of Sec. III. B.2. This can be achieved by a systematic program of predictions of flows for which well-documented, reliable experimental data are available.

APPENDIX

A Simplified Algebraic Turbulence Model for Large Rod Bundles

The transport equation for turbulence kinetic energy is replaced by the algebraic relationship that is prescribed by equating generative and de-structure agencies; that is, we drop convective and diffusive terms from Eq. 22. In place of Eq. 33, we have the implicit algebraic equation

$$k = c_s c_\mu \ell^2 [(U - U_s)^2 + (V - V_s)^2 + (W - W_s)^2] / D_h^2$$

$$+ \left[c_\beta \ell |U|^3 [(A_x/A_0) - 1]^2 / (\delta x k^{1/2} \alpha) \right]$$

$$- \frac{\tilde{\alpha} \ell (T - T_W) \beta g / (\rho c_p) W_p^*}{[1 + (\mu Pr_t) / (\mu Pr)] \pi D_h k^{1/2}} \quad (A.1)$$

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