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MODELING OF TIME-VARIANT CONCRETE
PROPERTIES AT ELEVATED TEMPERATURES

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SUMMARY AND CONCLUSIONS

The elevated-temperature, time-variant concrete property equations used by a previous version of SAFE-CRACK (Ref. 19) were evaluated via comparison to the Portland Cement Association (PCA) data for Hanford concrete (Refs. 1 and 9). For some properties, the agreement was reasonably good, while for others there were significant differences. Since the previous SAFE-CRACK equations for Hanford concrete properties were developed before completion of the PCA study, the property equations were re-evaluated, modified where necessary, and fitted to the full PCA data base. Statistical methods were applied to produce expressions describing the uncertainties in the modified equations. Uncertainty expressions for both expected property behavior (confidence bands) and ranges of property behavior (tolerance bands) were developed. Specific comments are made below for the various properties investigated.

Modulus of Elasticity and Compressive Strength - The basic forms of the property equations (linear function of log time) used by the previous version of SAFE-CRACK were retained. The property equations were enhanced to allow both the slope and intercept of the linear functions (of log time) to depend on the temperature and nominal initial strength of the concrete.

Splitting Tensile Strength - The same basic form of equation (linear function of log time) utilized for modulus of elasticity and compressive strength was adopted. The dependence of the slope and intercept (of the linear function of log time) on temperature and nominal initial strength was determined by the tensile strength data. This differs from the previous SAFE-CRACK approach of assuming the concrete splitting tensile strength to be 10% of the compressive strength.

Creep Strain - The exponential type of equation previously used by SAFE-CRACK was found to significantly over-predict concrete creep strains when compared with the PCA measurements. The Pacific Northwest Laboratory (PNL) has developed a logarithmic equation in this study that fits the PCA data well. Nevertheless, the limited nature of the PCA data suggests that the SAFE-CRACK creep equations be based on a wider data base. This was done by Anatech in Ref. 17. Comparison showed inconsistencies between the PCA and other data, but adjusting for time-dependent elastic properties removed the

inconsistencies. Moreover, much of the over-prediction relative to the logarithmic equation is removed, particularly at extrapolated times. The creep equations resulting from Ref. 17 are considered in agreement with the data that are available in the 250-450 F range.

The PCA data for modulus of elasticity, compressive strength, and splitting strength are based on constant temperature tests with the following ranges:

Temperature:	250, 350, and 450 F
Initial nominal strength:	3000 and 4500 psi
Time at temperature:	3-10 to 920-1300 days.

The property equations fitted to these data have time-degradation terms which Anatech phased out linearly in the range of 250 to 200 F to obtain time-independent data at lower temperatures. This extrapolation in temperature is considered reasonable.

SAFE-CRACK applications involve variable temperature histories, while the PCA data and resulting property equations developed by PNL are for constant temperature histories. Anatech's incremental analysis method (Ref. 17) for adapting the constant temperature property equations to variable temperature histories is based on certain assumptions (see Appendix B), which appear to be reasonable for the concrete properties involved. Hence, we conclude that the incremental analysis method is a viable way to predict concrete properties for variable temperature histories. Anatech also applies the incremental analysis approach to the concrete property uncertainty expressions (95% confidence and 95%/95% tolerance bands) developed herein. Although this does not provide for transferring the 95% confidence and 95%/95% tolerance interpretations from constant temperature to variable temperature histories, it appears to be a viable approach to handling the uncertainty. Linearizations of the hyperbolic confidence and tolerance bands given in Appendix A are provided in Appendix C as a means of simplifying the incremental analysis approach for the uncertainty expressions.

Extrapolations to 50 years for the concrete modulus of elasticity, compressive strength, and splitting tensile strength property equations presented in this report are concluded as being reasonable, based on the

adequacy of fit of the equations to PCA data and the theoretical support for no more than a gradual deterioration of concrete with exposure to elevated (250 F to 450 F) temperatures. Physical and chemical characteristics of concrete undergo a gradual, continuing degradation at elevated temperatures (250 F to 450 F). Rates of degradation are not available from current literature on concrete chemistry. Therefore, the property degradation equations presented herein are empirical mathematical expressions based on the PCA Hanford concrete data. To allow for uncertainties in the PCA data and the fitting process, it is recommended that the uncertainty expressions (confidence and tolerance bands) be used in place of the fitted equations where appropriate to do so. Specific recommendations on the use of the uncertainty expressions are given in Chapter 8.

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MODELING OF TIME-VARIANT CONCRETE PROPERTIES AT ELEVATED TEMPERATURES

1.0 INTRODUCTION AND PURPOSE

Analysis and evaluation for Hanford reinforced concrete waste storage tank structures has made use of the finite element computer code SAFE-CRACK. The concrete properties versus elevated temperature equations previously used in SAFE-CRACK (Ref. 19) were developed with test results from, but before the completion of the Portland Cement Association (PCA) study on Hanford concrete properties (Refs. 1 and 9).

In 1986, Rockwell Hanford Operations, the site contractor responsible for operation of the tanks (at that time), requested that Pacific Northwest Laboratory (PNL) evaluate the concrete property equations in SAFE-CRACK relative to the complete PCA data base (Refs. 1 and 9). The initial work and recommendations were issued in a draft report (Ref. 16), which established that: 1) the SAFE-CRACK property equations should be modified to provide a better fit to the PCA data base, 2) uncertainty in the modified property equations should be accounted for, 3) the modified concrete property equations could be extrapolated out to 50 years provided appropriate statistical conditions are met, and 4) physical characteristics of concrete such as chemical reactivity, crystalline changes, bonding, etc., supported the long-term extrapolation.

In 1987, Kaiser Engineers Hanford (KEH), the on-site architect-engineer currently designing the 241-AQ tanks, requested that PNL continue the evaluation, and develop modified concrete property equations and uncertainty expressions based on the PCA Hanford concrete data base. This report summarizes and documents the results of the evaluation and development efforts.

2.0 THE PCA-DEVELOPED DATA BASE

The concrete property data used in the evaluation are from elevated temperature tests at the PCA's Construction Technology Laboratories, Skokie, Illinois (Refs. 1 and 9). The PCA conducted the tests over a five year period. The concrete specimens (standard ASTM size 6 in. x 12 in. cylinders) were prepared at the PCA laboratories using Hanford aggregates and ASTM C 150 Type II (low alkali) portland cement, both shipped to the PCA from Hanford. Two concrete mixture designs were tested; specified minimum 3000 psi and 4500 psi mixtures (compressive strength at 28 days). Specimens were continuously moist-cured at 70 F in a fog room for 28 days minimum until placed in the oven. They were heated to test temperature at a rate of 70 to 75 F per day and then maintained at 250 F, 350 F and 450 F for up to 920 days, removed from the oven, and tested while hot (Ref. 1). One set of specimens was continuously exposed to 350 F for 1300 days (Ref. 9). Some specimens were subjected to temperature cycles of 70 F to 350 F and back to 70 F, for up to 18 cycles of 14- and 28-day duration (Ref. 1).

The following properties were measured:

- modulus of elasticity
- compressive strength
- splitting tensile strength
- Poisson's ratio.

In addition, six 4500 psi cylinders were tested to obtain creep strain data at 250 F and 350 F for 650 days.

3.0 LITERATURE REVIEW AND GENERAL FINDINGS

This section discusses the general effects of elevated temperature on portland cement concrete. The results of the literature review were in good agreement with the PCA test data (Refs. 1 and 9), although the PCA test data had greater exposure times at temperature than any other study found.

3.1 EFFECTS OF ELEVATED TEMPERATURE ON CONCRETE PROPERTIES - GENERAL

Davis (Ref. 2) noted that "Exposure to temperatures greater than 70 or 80 F has a deteriorating effect on the physical properties of portland cement concrete. However, for constant exposure at temperatures up to 150 or 200 F, the loss in strength, if any, is quite small; and for temperatures as high as 500 to 600 F, the deterioration in structural properties is ordinarily tolerable."

Exposure of concrete to elevated temperatures will evaporate free water from the voids and stop the chemical process of hydration, i.e., curing. At temperatures above 200 F, some of the water of hydration is removed from the cured cement paste in addition to evaporable water. It is generally agreed that heating portland cement concrete to 800 F or above will completely dehydrate the cement paste. However, concrete at 800 F still has about 50% of its strength which is progressively reduced if the temperature is increased.⁽²⁾ At about 950 to 1100 F, the calcium hydroxide in cement paste decomposes to calcium oxide and water and the concrete loses most of its remaining strength. In the 600 to 1100 F range, concretes made with siliceous or limestone aggregates change to a pink or red color and from 1100 to 1650 F to a grey, probably friable and porous state. At higher temperatures, concrete changes to a buff and then a yellow color.⁽³⁾

In general, the effects of heating on the properties of concrete are less on concrete where the moisture is allowed to evaporate than on concrete in which the water is sealed in.^(a) In specimens where the water is free to evaporate, losses in compressive strength range from none to 30% for 200 F

(a) Unless otherwise noted, the discussion of elevated temperature effects on concrete in this report are confined to the case of unsealed concrete, where the water is able to evaporate or be driven out by heating.

exposure and from none to 67% at 500 F depending upon the length of heating and testing conditions (see Table 1 of Ref. 2). As pointed out in Ref. 1, Abrams found that concrete that had an applied stress during heating lost significantly less compressive strength than unstressed specimens. This phenomenon, also reported by Malhotra (Ref. 4), adds an unquantified conservatism to the design of concrete structures subject to heating.

Heating has a pronounced effect on the modulus of elasticity of concrete. Two available test results for concrete cured 90 days show the modulus reduced to 33% after exposure to 660 F for 106 days and 67% after exposure to 392 F for 106 days.⁽²⁾

Davis⁽²⁾ reported data which indicate that concrete tensile strength decreases with increasing elevated temperature values and exposure time. Davis further concluded that the ultimate bond strength of embedded, deformed steel reinforcing bars is little affected by the effects of elevated temperatures, and that it is conservative to apply elevated temperature test data from small concrete specimens to full-size structures.

TABLE 1. Compressive Strength of Heated Specimens Expressed as Percent of Strength of Unheated Specimens^(a)

Source	Ref ^a	Oven Temperature, deg F ^b											Comments
		200	300	400	500	600	800	900	1100	1300	1500	1700	
Hannant.....	[7]	70	80	...	...	...	...	...	...	...	...	...	Moisture allowed to evaporate.
		70	60	...	...	...	...	...	...	...	...	...	Moisture loss restricted.
Saemann.....	[8]	80	100	100	100	...	...	...	...	...	...	...	W/C = 0.48) Heated 24 hr and tested hot.
		87	100	100	100	...	...	...	...	...	...	...	W/C = 0.84)
Malhotra.....	[9]	97	95	88	75	~53	...	...	...	...	...	...	Heated under 1041 psi load and tested hot.
		94	87	77	58	...	...	...	...	...	...	...	Heated at zero stress and tested hot.
		75	66	53	33	...	...	...	...	...	...	...	Heated at zero stress & tested at about 75 F.
Miller.....	[19]	84	87	...	55	...	48	...	14	4	...	4	W/C = 0.90. Tested cold at 28 days.
Heiskill.....	[20]	100	...	...	85	...	...	...	79	56	46	...	28-day moist cure.
		100	...	...	66	...	...	...	63	42	28	...	7-day moist cure.
Binner.....	[21]	100	...	...	...	55	...	39	...	...	...	...	Heated 7 days.
		100	...	...	...	47	...	40	...	...	...	...	Heated 14 days.
Germany.....	[2]	...	...	...	...	88	...	50	...	18	...	...	7-day moist cure and 10 hr at 572 F.
Hanford.....	[6]	...	...	...	...	...	...	...	23 to 25	...	...	...	Grout with fine sand.
		...	...	...	...	...	...	...	47 to 92	...	...	...	Grout with coarser sand.
		93	71	90	...	60	...	...	...	...	...	...	Concrete K (see Table 2).
		...	...	...	...	77	...	...	...	...	...	...	Concrete G (see Table 4).

* From list of references.

^b Specimens were oven heated at the respective temperatures, cooled, and then tested at or near room temperature, except as noted.

(a) Ref. 2, page 453.

3.2 CHEMICAL AND PHYSICAL MECHANISMS AT ROOM AND ELEVATED TEMPERATURES

The chemical reactions in portland cement hydration are complex and there are many involved. Figure 1 lists the major chemical reactions for cement paste formation. These reactions are all a part of the hydration process in which the water becomes bound as water of hydration and water of crystallization. The hydration process continues in concrete for many years, provided that the concrete is kept wet or under high humidity, and at least at 40 F. The strength at 28 days is only a reference strength; the strength continues to gain indefinitely. This is shown by the continuous moist cure curve in Figure 2. Popovics (Ref. 5) notes that "The strength finally stops increasing after the concrete dries out completely."

As shown in Figure 2, the curing time prior to heating influences the "starting strength". That is, the longer the structure or specimen is cured, the higher its strength at the start of a heating cycle, with higher hypothesized strength at the end of the heat cycle. Structures such as the Hanford waste tanks will have been moist-cured for at least 28 days and, because of the relatively thick walls, residual curing or sealing compounds, and the fact that they are subsequently buried, the concrete could continue to gain compressive strength from cement hydration. Thus, the concrete in the waste tanks is adequately hydrated by the time heating begins. This is important for minimizing the deteriorating effects of heating on concrete.⁽⁶⁾

Exposure of concrete to temperatures above normal accelerates drying and removal of water required by the chemical processes of cement hydration. At temperatures above about 200 F, some of the water of hydration is removed from the hardened paste in addition to evaporable water which may be in the concrete.

3.3 CHEMICAL CHANGES ON HEATING

Differential thermal analysis (DTA) investigations⁽⁷⁾ provide a convenient illustration of the reactions that may occur when heating a full hydrated concrete specimen in air at atmospheric pressure. Figure 3 shows a

FIGURE 1. Major Chemical Reactions in Concrete Formation
(Ref. 10, p. 10)

FIGURE 2. Concrete Compressive Strength Versus
Curing/Heating Conditions

FIGURE 3. Representative DTA Diagram of Aged Portland Cement Paste (in Air, at Atmospheric Pressure) (Ref. 7, page 13)

DTA diagram for a representative aged portland cement paste.⁽⁷⁾ The initial portion of this diagram is due to the removal of free water from the cured cement paste. The breakdown of portland cement hydration products would account for the remaining endotherm diagram.

Lankard⁽⁷⁾ states that "concrete heated below 500 F at atmospheric pressure exhibits a partial dehydration of hydrated cement phases and a loss of evaporable water." The same phenomenon was shown in the PCA study by the reduced specimen weights after heating. This loss of both evaporable and non-evaporable water appears to be responsible for the majority of the strength loss and property degradation of concrete upon heating, including the gradual degradation with continued exposure to 250 to 500 F temperatures.

A reversible reaction of CaO plus H_2O to form calcium hydroxide, $\text{Ca}(\text{OH})_2$ also contributes to the degradation of properties. At temperatures around 1000 F, the calcium hydroxide decomposes rapidly to form CaO and H_2O . At temperatures in the 250 to 500 F range, the reaction is also able to occur but at a much reduced (barely perceptible) rate.

X-ray analysis of cured neat cement samples exposed to 250 and 500 F for 14 days showed increasing loss of Ca(OH)_2 with temperature. The results also indicated that in the period of time during which the sample is being heated and still retains free moisture, accelerated hydration of unreacted anhydrous cement can occur.⁽⁷⁾ This would normally occur in the first few weeks or months of heating.

It is the two reactions, loss of water (free water, water of hydration and water of crystallization) and the slow change of Ca(OH)_2 to calcium oxide and water, that are believed to be primarily responsible for a slow, continual degradation of concrete properties in the 250 to 500 F range.

3.4 DIMENSIONAL CHANGES DURING HEATING

Aged concrete that is subsequently heated in air will incur dimensional changes as a result of losing its evaporable and chemically combined (non-evaporable) water and from thermal expansion.

Loss of Free Water - The loss of the evaporable water produces an isotropic shrinkage phenomenon in the concrete that is termed "drying shrinkage." A complete desorption-readsorption cycle over the 100-0-100% relative humidity range produces a hysteresis in the shrinkage strain versus moisture loss curve which indicates that a portion of the initial drying shrinkage is irrecoverable.

The complete removal of evaporable water as a result of heating can result in a shrinkage of over 2% in the volume of the cement paste. However, the presence of aggregate limits the shrinkage of concrete to about one tenth of that.⁽⁷⁾

Loss of Chemically Combined Water from the Hydrated Cement - Removal of chemically combined water from hydrated cement by heating also results in overall shrinkage of the cement. Removal of such water from tobermorite gel causes a considerable decrease in the interlayer dimension. Both bonded and dissociated (OH) water are driven from the hydrated paste during heating in air to 500 F.⁽⁷⁾

The shrinkage accompanying water losses causes cracking of the cement paste and subsequently the concrete itself.

Thermal Expansion - On initial heatup of saturated concrete, both the aggregate and cement phase will exhibit an expansion. Values reported in the literature for the thermal expansion coefficient of concrete range from about 3×10^{-6} in. per in. per F to as high as 18×10^{-6} in. per in. per F, although the majority fall in the 3 to 6×10^{-6} in. per in. per F range.⁽⁷⁾ The PCA study (Ref. 1) obtained a thermal expansion coefficient of 3.3×10^{-6} in. per in. per F for Hanford concrete.

3.5 PHYSICAL CHANGES DURING HEATING

Lankard⁽⁷⁾ and the PCA⁽¹⁾ made fractographic/petrographic analyses of concrete specimens after heating to temperatures up to 500 F. Both studies found evidence of cement paste microcracking leading to loss of bond to the aggregate. The microcracking occurred primarily along a paste-aggregate interface, but also extended into the paste.⁽⁷⁾ PCA⁽¹⁾ noted that the cooling to room temperature (to allow examination) may have been responsible for part of the observed paste-to-aggregate bond loss along with the shrinkage accompanying the desiccation at temperature. Specimens that were rapidly cooled from 500 F by water quenching showed significantly more disruptive cracking than specimens that were slowly cooled to room temperature.⁽⁷⁾ This microcracking is due, in part, to the differences in thermal expansion among the various mineralogical constituents in concrete, and partly to the existence of thermal gradients.

The combination of loss of water from the concrete and cement paste microcracking during heating are believed to be the primary causes of reductions to the strength and modulus of elasticity. The changes in concrete modulus of elasticity from heating, which were proportionately greater than for other properties such as compressive and tensile strength, were believed by Lankard⁽⁷⁾ to be caused primarily by the loss of evaporable water.

4.0 MATHEMATICAL EXPRESSIONS FOR MODULUS OF ELASTICITY, COMPRESSIVE STRENGTH, AND SPLITTING TENSILE STRENGTH

This section presents mathematical expressions for modulus of elasticity, compressive strength, and splitting tensile strength developed using the PCA data of Refs. 1 and 9. Also presented are statistical expressions for the uncertainty in expected behavior of the properties (confidence bands), and for the uncertainty in the range of values of the properties (tolerance bands). These expressions are used to extrapolate concrete properties out to 50 years for the six combinations of mixture type (3000 and 4500 psi) and temperature (250, 350, and 450 F) considered in the PCA experiments. However, the expressions can accommodate intermediate values of these variables.

4.1 THE PCA DATA ON HANFORD CONCRETE

The PCA Hanford concrete test data are contained in Tables B-2 and B-3 of Ref. 1 and in Tables 2 and 3 of Ref. 9. These are data for modulus of elasticity, compressive strength, and splitting tensile strength measurements on 6 x 12 in. concrete cylinders exposed to elevated temperatures. Cylinders were made using two concrete mixtures (3000 or 4500 psi, nominal). They were moist-cured at 70 F for approximately 200 days, then heated and held at a constant temperature of 250, 350 or 450 F. The three concrete properties were measured at selected intervals over the elevated temperature test period.

Mixture type, temperature, and time-at-temperature are the three experimental variables which form the basis for the PCA test results. Variations not quantitatively accounted for, but which were present in the PCA Hanford concrete test data include:

- Within-batch variation. For almost all tests, each property was measured on two or three cylinders from the same batch for each mixture type x temperature x time-at-temperature combination. The measured property values varied significantly for specimens from the same batch.

- Batch-to-batch variation. Property measurements over time for a specific mixture type x temperature combination were typically made on cylinders from several different batches. There were 16 batches of 3000 psi concrete and 13 batches of 4500 psi concrete. The water/cement ratio, air content and cement content varied significantly between batches of the same nominal strength.
- Moist-cure time variation. Not all cylinders for a given mixture type x temperature combination had the same moist-cure time before being subjected to temperature. The curing time in the fog room before heating varied from 193 days to 283 days. Time in the fog room affects the initial strength of the specimens and hence may affect elevated temperature properties.

The above variations affected the concrete property data, and should be considered by the statistical analyses. The within-batch variation is best treated as random "noise", and can be estimated and accounted for in data analyses via statistical methods. The batch-to-batch variation can be treated as random noise, or as correctable differences. If the variation between batches in the PCA data is representative of the potential variation of waste tank concrete mixes, then it should be treated as random variation (and included with within-batch variation) in the estimate of the experimental error variance. If the batch-to-batch variation is not representative and/or results over time for a fixed-batch concrete are desired, then it would be better to try to adjust (correct) the property values for the batch-to-batch differences. Similarly, the moist-cure time variation can be treated as random noise or as correctable differences, depending on whether or not the variation is representative of that likely to occur in waste tank curing.

Several attempts were made to adjust the data for batch-to-batch and moist-cure time differences. However, the results met with only very limited success. The PCA experiments did not measure initial (after moist-cure but before temperature) values of the three properties for several cylinders from each batch, which would have been the best basis for adjusting batch-to-batch

differences. (a) Other attempts to adjust for batch-to-batch differences were limited by poor or insufficient data. Attempts to adjust for moist-cure time differences were similarly unsuccessful. The cure time and time-at-temperature data were partially confounded (increases in cure time correspond to increases in time-at-temperature), which limited adjustment efforts. Also, it is believed that variation in cure time from 193 to 283 hours has only a minor effect on concrete properties. Hence, it was decided to ignore moist-cure time differences in attempting to fit equations to the data.

The lack of success in adjusting for batch-to-batch differences prompted a review of the literature to gather information on typical random variation in concrete properties due to between- and within-batch variations. Table 2 (an adaptation of Table 3.5 of Ref. 12) presents typical overall (within- and between-batch) standard deviations for compressive strength. Approximate relationships such as:

$$\text{modulus of elasticity} = 57,000 \sqrt{f'_c}$$

and

$$\text{splitting tensile strength} = 10\% \text{ of compressive strength}$$

were used in combination with Table 2 to produce the approximate typical overall standard deviations for modulus of elasticity and splitting tensile strength given in Table 3. The contents of Tables 2 and 3 will be of use in Section 4.3.

4.2 SAFE-CRACK VERSUS PCA DATA

SAFE-CRACK used the following equations to predict modulus of elasticity and compressive strength in the most recent (1982) analysis of the 241-AP tank: (8)

-
- (a) PCA did measure compressive strength after approximately 880 days of moist-cure for two specimens from each of the batches used (unpublished data obtained from PCA). However, this data exhibited significantly higher within-batch variation than did the at-temperature compressive strength data. The limited nature of this data, its higher variability, and the presence of several suspect observations may have been the reasons for the lack of success in using it to adjust for batch-to-batch differences.

TABLE 2. Typical Overall (Within- and Between-Batch) Variation
in Compressive Strength for Different Levels of Control

<u>Class of Operation</u>	<u>Overall Variation</u>				
	<u>Standard Deviations for Different Levels of Control, psi</u>				
	<u>Excellent</u>	<u>Very Good</u>	<u>Good</u>	<u>Fair</u>	<u>Poor</u>
General construction testing	Below 400	400 to 500	500 to 600	600 to 700	Above 700
Laboratory trial batches	Below 200	200 to 250	250 to 300	300 to 350	Above 350

TABLE 3. Approximate Overall Variation in Modulus of Elasticity and
Splitting Tensile Strength for Different Levels of Control

<u>Class of Operation</u>	<u>Overall Variation</u>				
	<u>Standard Deviations for Different Levels of Control</u>				
	<u>Excellent</u>	<u>Very Good</u>	<u>Good</u>	<u>Fair</u>	<u>Poor</u>
A. <u>Modulus of Elasticity (10^6 psi)</u>					
General construction testing	Below .18	.18 to .22	.22 to .27	.27 to .32	Above .32
Laboratory trial batches	Below .09	.09 to .11	.11 to .14	.14 to .16	Above .16
B. <u>Splitting Tensile Strength (psi)</u>					
General construction testing	Below 40	40 to 50	50 to 60	60 to 70	Above 70
Laboratory trial batches	Below 20	20 to 25	25 to 30	30 to 35	Above 35

$$\hat{E} = 10^6 [6.0 - 0.007 T - 0.195 \log_e (t + 1)] \quad (1)$$

$$\hat{f}_c = 6000 - 270.0 \log_e (t + 1) \quad (2)$$

where t = time at temperature (days)

T = temperature (F)

$\hat{E}$ = predicted modulus of elasticity (psi)

$\hat{f}_c$ = predicted compressive strength (psi).

These curves were plotted versus the PCA data to see how well they compared (Ref. 16). In some cases, the SAFE-CRACK predictions were close to the bulk of the data for a given mix type (3.0K or 4.5K) and temperature (250, 350, 450 F), while in other cases, the SAFE-CRACK predictions were above or below the bulk of the data.

Splitting tensile strength is used in SAFE-CRACK to calculate where concrete cracks will occur. SAFE-CRACK presently uses 10% of the compressive strength (f_c) for the tensile strength of concrete. The PCA values of f_c for 900 days heating at 250 and 350 F are of the order of 4500 psi and above for both the 3000 psi and the 4500 psi specimens; this would give a value of 450 psi for the tensile strength. However, the splitting tensile strength of the specimens at about 900 days is of the order of 400 psi. An empirical equation fitted to the actual test values should be used in the calculations.

It was concluded that the basic form

$$\text{Property} = a + b \log_e (t + 1) \quad (3)$$

of the SAFE-CRACK equations for modulus of elasticity, compressive strength, and splitting tensile strength was suitable for further evaluation, but that the dependence of the slope and intercept on mixture type and on temperature should be reviewed. The model development and fitting process is discussed in the following section.

4.3 FITTED EQUATIONS, CONFIDENCE BANDS, AND TOLERANCE BANDS

The general equation (3) was fit (using ordinary least squares) to the modulus of elasticity, compressive strength, and splitting tensile strength data for each combination of mixture type and temperature. The coefficient estimates are given in Table 6. A few outlying data points were discarded, and some observations for early time-at-temperatures were deleted. This was done to remove the effects of points that otherwise would have adversely affected the curve fitting. Table 4 lists the data for modulus of elasticity and compressive strength while Table 5 lists the data for splitting tensile strength. The data not used to develop the equations are enclosed in parenthesis.

Using the fitted coefficients from Table 6, the dependence of the slope and intercept of (3) on mix type (initial compressive nominal strength) and temperature was studied graphically. The graphical results suggested various forms of dependence, which were then tested statistically using multiple regression analyses and associated hypothesis tests. The final fitted equations for the three properties are:

$$\hat{E} = 5.3947 + 0.1233 S - 0.006751 T - 0.1786 \log_e (t + 1) \quad (4)$$

$$\begin{aligned} \hat{f}_c = & 4416.338 + 490.919 S - 4.714 TA - 230.241 \log_e (t + 1) \\ & + 1.273 TB \cdot \log_e (t + 1) \end{aligned} \quad (5)$$

$$\hat{f}_{st} = 448.1758 + 23.7436 S - 0.6077 TA - 18.4341 \log_e (t + 1) \quad (6)$$

where

- $\hat{E}$ = predicted value of modulus of elasticity (10^6 psi)
- $\hat{f}_c$ = predicted value of compressive strength (psi)
- $\hat{f}_{st}$ = predicted value of splitting tensile strength (psi)
- S = nominal initial compressive strength of Hanford concrete (10^3 psi)
- T = constant value of temperature (F)
- $TA = \max[0, T-350]$ (F)
- $TB = \max[0, 350-T]$ (F)
- t = time at constant temperature (days).

TABLE 4. Compressive Strength and Modulus of Elasticity Data Used for Model Fitting

OBS	MIX TYPE (PSI)	BATCH #	SPECIMEN #	DATE	AGE AT TEST (DAYS)	TIME IN FOG ROOM (DAYS)	TIME IN OVEN (DAYS)	TEMPERATURE AT TEST (DEGREES F)	COMPRESSIVE STRENGTH (PSI)	MODULUS OF ELASTICITY (10 ⁶ PSI)
1	3.0	3	1	761204	196	193	3	250	5600	3.82
2	3.0	3	5	761204	196	193	3	250	5580	4.03
3	3.0	3	3	751204	196	193	3	250	5580	3.86
4	4.5	3	3	761204	196	193	3	250	(5320)	(3.82)
5	4.5	3	6	761204	196	193	3	250	(5230)	(4.04)
6	4.5	3	1	761204	196	193	3	250	5685	(3.91)
7	3.0	8	3	761211	203	193	10	250	5420	3.66
8	3.0	8	5	761211	203	193	10	250	5420	3.71
9	3.0	8	1	761211	203	193	10	250	5420	3.71
10	4.5	8	1	761211	203	193	10	250	8920	4.36
11	4.5	8	6	761211	203	193	10	250	8780	4.04
12	4.5	8	3	761231	223	193	10	250	5295	4.11
13	3.0	7	1	761231	223	193	30	250	5250	3.82
14	3.0	7	5	761231	223	193	30	250	5250	3.84
15	3.0	7	3	761231	223	193	30	250	5250	3.14
16	4.5	7	4	761231	223	193	30	250	8055	3.31
17	4.5	7	3	761231	223	193	30	250	8030	3.42
18	4.5	7	1	761231	223	193	30	250	5170	(4.08)
19	3.0	4	1	760219	273	193	80	250	5160	3.13
20	3.0	4	3	760219	273	193	80	250	5160	3.06
21	3.0	4	6	760219	273	193	80	250	6290	3.24
22	4.5	4	3	760219	273	193	80	250	6290	3.40
23	4.5	4	4	760219	273	193	80	250	6020	3.42
24	4.5	4	1	760219	273	193	80	250	5710	3.34
25	3.0	5	3	760429	343	193	150	250	5610	3.15
26	3.0	5	6	760429	343	193	150	250	5610	3.09
27	3.0	5	2	760429	343	193	150	250	5610	3.13
28	4.5	5	3	760429	343	193	150	250	6180	3.41
29	4.5	5	5	760429	343	193	150	250	6180	3.18
30	4.5	5	1	760429	343	193	150	250	5250	3.12
31	3.0	6	3	760827	463	193	270	250	5080	3.09
32	3.0	6	5	760827	463	193	270	250	5080	3.00
33	3.0	6	1	760827	463	193	270	250	5080	2.84
34	4.5	6	3	760827	463	193	270	250	6020	3.16
35	4.5	6	6	760827	463	193	270	250	5990	3.07
36	4.5	6	8	760827	463	193	270	250	5990	3.68
37	3.0	9	14	770401	679	283	396	250	5190	3.17
38	3.0	9	20	770401	679	283	396	250	4990	3.08
39	3.0	9	17	770401	679	283	396	250	4990	3.15
40	4.5	8	26	770401	679	283	396	250	6470	3.36
41	4.5	8	28	770401	679	283	396	250	7020	3.48
42	4.5	8	16	770401	679	283	396	250	5300	3.41
43	3.0	9	25	770929	861	283	578	250	5080	3.07
44	3.0	9	28	770929	861	283	578	250	5080	3.03
45	3.0	9	23	770929	861	283	578	250	6620	2.99
46	4.5	8	10	770929	861	283	578	250	6425	3.32
47	4.5	8	27	770929	861	283	578	250	6425	3.21
48	4.5	8	22	770929	861	283	578	250	4530	3.21
49	3.0	8	21	780901	1198	276	922	250	4625	2.43
50	3.0	8	26	780901	1198	276	922	250	4625	2.60
51	3.0	8	30	780901	1198	276	922	250	4625	2.71
52	4.5	9	5	780901	1198	276	928	250	4625	(4.01)

TABLE 4. Continued

OBS	MIX TYPE (PSI)	BATCH #	SPECIMEN #	DATE	AGE AT TEST (DAYS)	TIME IN FOG ROOM (DAYS)	TIME IN OVEN (DAYS)	TEMPERATURE AT TEST (DEGREES F)	COMPRESSIVE STRENGTH (PSI)	MODULUS OF ELASTICITY (10 ⁶ PSI)
53	4.5	9	8	780901	1198	270	928	250	5855	3.29
54	4.5	9	25	780901	1198	270	928	250	5890	3.37
55	3.0	3	8	751205	197	193	4	350	(6100)	3.47
56	3.0	3	14	751205	197	193	4	350	(5780)	3.38
57	3.0	3	11	751205	197	193	4	350	.	3.53
58	4.5	3	8	751205	197	193	4	350	(5800)	4.04
59	4.5	3	14	751205	197	193	4	350	(5430)	3.33
60	4.5	3	11	751205	197	193	4	350	.	3.46
61	3.0	4	8	751211	203	193	10	350	(5150)	2.69
62	3.0	4	11	751211	203	193	10	350	(4740)	2.82
63	3.0	4	14	751211	203	193	10	350	.	2.73
64	4.5	4	8	751211	203	193	10	350	6520	3.09
65	4.5	4	11	751211	203	193	10	350	6330	3.40
66	4.5	4	14	751211	203	193	10	350	.	3.04
67	3.0	5	11	751231	223	193	30	350	5480	2.72
68	3.0	5	14	751231	223	193	30	350	5540	2.69
69	3.0	5	8	751231	223	193	30	350	.	2.11
70	4.5	5	8	751231	223	193	30	350	5280	2.75
71	4.5	5	11	751231	223	193	30	350	5875	2.77
72	4.5	5	14	751231	223	193	30	350	.	2.94
73	3.0	6	11	760219	273	193	80	350	4780	2.56
74	3.0	6	14	760219	273	193	80	350	4810	2.38
75	3.0	6	8	760219	273	193	80	350	.	2.61
76	4.5	6	11	760219	273	193	80	350	5430	2.74
77	4.5	6	14	760219	273	193	80	350	5275	2.72
78	4.5	6	12	760219	273	193	80	350	.	2.68
79	3.0	7	8	760429	343	193	150	350	4410	2.11
80	3.0	7	11	760429	343	193	150	350	4675	2.29
81	3.0	7	14	760429	343	193	150	350	.	2.17
82	4.5	7	8	760429	343	193	150	350	5490	2.34
83	4.5	7	10	760429	343	193	150	350	4890	2.28
84	4.5	7	15	760429	343	193	150	350	.	2.32
85	3.0	8	8	760827	463	193	270	350	4910	2.52
86	3.0	8	11	760827	463	193	270	350	4860	2.51
87	3.0	8	14	760827	463	193	270	350	.	2.52
88	4.5	8	8	760827	463	193	270	350	(8190)	2.78
89	4.5	8	11	760827	463	193	270	350	(8300)	3.14
90	4.5	8	14	760827	463	193	270	350	.	2.78
91	3.0	8	24	770401	679	276	403	350	4650	2.37
92	3.0	8	30	770401	679	276	403	350	4430	2.17
93	3.0	8	27	770401	679	276	403	350	.	2.20
94	4.5	8	18	770401	679	276	403	350	5450	2.43
95	4.5	8	26	770401	679	276	403	350	5180	2.48
96	4.5	8	27	770401	679	276	403	350	.	2.37
97	3.0	7	19	770930	861	276	585	350	4390	2.33
98	3.0	7	25	770930	861	276	585	350	4320	2.32
99	3.0	7	10	770930	861	276	585	350	.	2.18
100	4.5	7	24	770930	861	276	585	350	5100	2.32
101	4.5	7	25	770930	861	276	585	350	5500	2.34
102	4.5	7	17	770930	861	276	585	350	.	2.41
103	3.0	7	2	780901	1198	276	922	350	4525	2.46
104	3.0	7	21	780901	1198	276	922	350	4350	2.47

TABLE 4. Continued

OBS	MIX TYPE (PSI)	BATCH #	SPECIMEN #	DATE	AGE AT TEST (DAYS)	TIME IN FOG ROOM (DAYS)	TIME IN OVEN (DAYS)	TEMPERATURE AT TEST (DEGREES F)	COMPRESSIVE STRENGTH (PSI)	MODULUS OF ELASTICITY (10 ⁶ PSI)
105	3.0	7	27	780901	1198	276	922	350	4680	2.53
106	4.5	7	21	780901	1198	276	922	350	4770	2.58
107	4.5	7	22	780901	1198	276	922	350		2.62
108	4.5	7	27	780901	1198	276	922	350		2.58
109	3.0	4	20	751211	203	197	6	450	(4540)	2.39
110	3.0	4	23	751211	203	197	6	450	(4930)	2.26
111	3.0	4	17	751211	203	197	6	450		2.41
112	4.5	4	17	751211	203	197	6	450	5760	2.75
113	4.5	4	17	751211	203	197	6	450	5820	2.62
114	4.5	4	23	751211	203	197	6	450		2.69
115	3.0	5	20	751215	207	197	10	450	4995	2.28
116	3.0	5	23	751215	207	197	10	450	4830	2.33
117	3.0	5	17	751215	207	197	10	450		2.16
118	4.5	5	16	751215	207	197	10	450	5990	2.60
119	4.5	5	20	751215	207	197	10	450	5150	2.67
120	4.5	5	23	751215	207	197	10	450		2.13
121	3.0	6	17	751223	215	197	18	450	4540	2.23
122	3.0	6	23	751223	215	197	18	450	4320	2.09
123	3.0	6	19	751223	215	197	18	450		2.13
124	4.5	6	17	751223	215	197	18	450	5255	2.24
125	4.5	6	23	751223	215	197	18	450	5430	2.35
126	4.5	6	20	751223	215	197	18	450		2.08
127	3.0	7	17	760107	230	197	33	450	4280	2.06
128	3.0	7	23	760107	230	197	33	450	4455	2.06
129	3.0	7	20	760107	230	197	33	450		2.16
130	4.5	7	18	760107	230	197	33	450	5165	2.20
131	4.5	7	20	760107	230	197	33	450	5110	2.28
132	4.5	7	23	760107	230	197	33	450		2.29
133	3.0	8	17	760203	257	197	60	450	4780	2.34
134	3.0	8	22	760203	257	197	60	450	4630	2.17
135	3.0	8	24	760203	257	197	60	450		2.27
136	4.5	8	17	760203	257	197	60	450	5580	2.37
137	4.5	8	30	760203	257	197	60	450	5130	2.27
138	4.5	8	23	760203	257	197	60	450		1.93
139	3.0	9	5	760315	297	197	100	450	4350	2.08
140	3.0	9	8	760315	297	197	100	450	4415	2.04
141	3.0	9	11	760315	297	197	100	450		2.14
142	4.5	9	11	760315	297	197	100	450	5270	2.08
143	4.5	9	17	760315	297	197	100	450	5210	2.12
144	4.5	9	14	760315	297	197	100	450		2.06
145	3.0	9	25	760524	367	197	170	450	5140	1.91
146	3.0	6	25	760524	367	197	170	450	4560	1.84
147	3.0	4	24	760424	367	197	170	450		1.73
148	4.5	3	24	760524	367	197	170	450	4880	1.70
149	4.5	4	24	760524	367	197	170	450	5340	1.84
150	4.5	5	25	760524	367	197	170	450		1.94
151	3.0	3	20	760930	497	197	300	450	(5920)	2.14
152	3.0	3	23	760930	497	197	300	450	(5150)	(2.62)
153	3.0	3	17	760930	497	197	300	450		1.76
154	4.5	3	18	760930	497	197	300	450	4620	1.67
155	4.5	3	23	760930	497	197	300	450	4940	1.74
156	4.5	3	17	760930	497	197	300	450		

TABLE 4. Continued

OBS	MIX TYPE (PSI)	BATCH #	SPECIMEN #	DATE	AGE AT TEST (DAYS)	TIME IN FOG ROOM (DAYS)	TIME IN OVEN (DAYS)	TEMPERATURE AT TEST (DEGREES F)	COMPRESSIVE STRENGTH (PSI)	MODULUS OF ELASTICITY (10 ⁶ PSI)
157	3.0	3	10	770331	679	270	409	450	4720	1.91
158	3.0	3	12	770331	679	270	409	450	4590	1.84
159	3.0	3	27	770331	679	270	409	450	4590	1.90
160	4.5	3	27	770331	679	270	409	450	4850	1.78
161	4.5	3	21	770331	679	270	409	450	4950	1.77
162	4.5	3	22	770331	679	270	409	450	450	1.98
163	3.0	4	16	770929	861	270	591	450	3540	1.28
164	3.0	4	19	770929	861	270	591	450	3630	1.29
165	3.0	4	7	770929	861	270	591	450	4240	1.51
166	4.5	4	15	770929	861	270	591	450	4580	1.50
167	4.5	4	21	770929	861	270	591	450	4580	1.54
168	4.5	4	19	770929	861	270	591	450	4580	1.46
169	3.0	5	4	780901	1198	283	915	450	3850	1.55
170	3.0	5	9	780901	1198	283	915	450	3850	1.52
171	3.0	5	27	780901	1198	283	915	450	3805	1.42
172	4.5	6	2	780901	1198	283	915	450	4100	1.55
173	4.5	6	7	780901	1198	283	915	450	4100	1.40
174	4.5	6	28	780901	1198	283	915	450	4185	1.46
175	3.0	8	16	790913	1580	276	1304	350	4580	2.44
176	3.0	8	27	790913	1580	276	1304	350	4810	2.55
177	4.5	9	20	790913	1580	283	1297	350	4790	2.41
178	4.5	9	23	790913	1580	283	1297	350	4790	2.55
179	4.5	9	27	790913	1580	283	1297	350	4880	2.49

Note: The parentheses on certain compressive strength and modulus of elasticity values indicate that those data points were not used in fitting equations.

TABLE 5. Splitting Tensile Strength Data Used for Model Fitting

OBS	MIX TYPE (PSI)	BATCH #	SPECIMEN #	DATE	AGE AT TEST (DAYS)	TIME IN FOG ROOM (DAYS)	TIME IN OVEN (DAYS)	TEMPERATURE AT TEST (DEGREES F)	SPLITTING TENSILE STRENGTH (PSI)
1	3.0	3	3	751204	196	193	3	250	502
2	3.0	3	7	751204	196	193	3	250	550
3	4.5	3	1	751204	196	193	3	250	547
4	4.5	3	7	751204	203	193	10	250	529
5	3.0	8	1	751211	203	193	10	250	476
6	3.0	8	4	751211	203	193	10	250	476
7	4.5	8	2	751211	203	193	10	250	588
8	4.5	8	3	751211	203	193	10	250	499
9	3.0	7	3	751231	223	193	30	250	434
10	3.0	7	4	751231	223	193	30	250	453
11	4.5	7	1	751231	223	193	30	250	498
12	4.5	7	9	751231	223	193	30	250	478
13	3.0	4	2	760219	273	193	80	250	441
14	3.0	4	5	760219	273	193	80	250	423
15	4.5	4	1	760219	273	193	80	250	497
16	4.5	4	6	760219	273	193	80	250	498
17	3.0	5	2	760429	343	193	150	250	441
18	3.0	5	7	760429	343	193	150	250	498
19	4.5	5	1	760429	343	193	150	250	497
20	4.5	5	6	760429	343	193	150	250	506
21	3.0	6	1	760827	463	193	270	250	412
22	3.0	6	4	760827	463	193	270	250	395
23	4.5	6	2	760827	463	193	270	250	471
24	4.5	6	8	760827	463	193	270	250	430
25	3.0	9	17	770401	679	283	396	250	451
26	3.0	9	29	770401	679	283	396	250	417
27	4.5	8	16	770401	679	283	396	250	460
28	4.5	8	26	770401	679	283	396	250	477
29	3.0	9	9	770929	861	283	578	250	484
30	3.0	9	23	770929	861	283	578	250	396
31	4.5	8	12	770929	861	283	578	250	504
32	4.5	8	22	770929	861	283	578	250	464
33	3.0	8	26	780901	1198	276	922	250	408
34	3.0	8	30	780901	1198	276	922	250	400
35	4.5	9	6	780901	1198	270	928	250	(373)
36	4.5	9	29	780901	1198	270	928	250	491
37	3.0	3	9	751205	197	193	4	350	495
38	3.0	3	11	751205	197	193	4	350	546
39	4.5	3	11	751205	197	193	4	350	498
40	4.5	3	16	751205	197	193	4	350	576
41	3.0	4	4	751211	203	193	10	350	450
42	3.0	4	14	751211	203	193	10	350	392
43	4.5	4	10	751211	203	193	10	350	516
44	4.5	4	14	751211	203	193	10	350	511
45	3.0	5	8	751231	223	193	30	350	478
46	3.0	5	12	751231	223	193	30	350	496
47	4.5	5	10	751231	223	193	30	350	470
48	4.5	5	14	751231	223	193	30	350	499
49	3.0	6	6	760219	273	193	80	350	380
50	3.0	6	6	760219	273	193	80	350	389
51	4.5	6	7	760219	273	193	80	350	427
52	4.5	6	12	760219	273	193	80	350	408

TABLE 5. Continued

OBS	MIX TYPE (PSI)	BATCH #	SPECIMEN #	DATE	AGE AT TEST (DAYS)	TIME IN FOG ROOM (DAYS)	TIME IN OVEN (DAYS)	TEMPERATURE AT TEST (DEGREES F)	SPLITTING TENSILE STRENGTH (PSI)
53	3.0	7	9	780429	343	193	150	350	376
54	3.0	7	14	780429	343	193	150	350	381
55	4.5	7	12	780429	343	193	150	350	393
56	4.5	7	15	780429	343	193	150	350	412
57	3.0	8	6	780827	483	193	270	350	409
58	3.0	8	14	780827	483	193	270	350	396
59	4.5	8	14	780827	483	193	270	350	424
60	4.5	8	15	780827	483	193	270	350	436
61	3.0	6	26	770401	679	278	403	350	436
62	3.0	6	27	770401	679	278	403	350	393
63	4.5	6	24	770401	679	278	403	350	433
64	4.5	6	27	770401	679	278	403	350	380
65	3.0	7	10	770930	861	278	585	350	343
66	3.0	7	24	770930	861	278	585	350	380
67	4.5	7	5	770930	861	278	585	350	459
68	4.5	7	17	770930	861	278	585	350	441
69	3.0	7	6	780905	1202	278	928	350	424
70	3.0	7	27	780905	1202	278	928	350	424
71	4.5	7	2	780905	1202	278	928	350	394
72	4.5	7	27	780905	1202	278	928	350	378
73	3.0	4	17	751211	203	197	6	450	393
74	3.0	4	18	751211	203	197	6	450	392
75	4.5	4	19	751211	203	197	6	450	469
76	4.5	4	23	751211	203	197	6	450	477
77	3.0	5	13	751216	207	197	10	450	425
78	3.0	5	17	751216	207	197	10	450	387
79	4.5	5	23	751216	207	197	10	450	424
80	4.5	5	27	751216	207	197	10	450	483
81	3.0	6	10	751223	215	197	18	450	392
82	3.0	6	19	751223	215	197	18	450	399
83	4.5	6	19	751223	215	197	18	450	440
84	4.5	6	20	751223	215	197	18	450	407
85	3.0	7	20	780107	230	197	33	450	387
86	3.0	7	22	780107	230	197	33	450	407
87	4.5	7	19	780107	230	197	33	450	436
88	4.5	7	23	780107	230	197	33	450	425
89	3.0	8	13	780203	257	197	60	450	396
90	3.0	8	24	780203	257	197	60	450	365
91	4.5	8	18	780203	257	197	60	450	428
92	4.5	8	23	780203	257	197	60	450	416
93	3.0	9	11	780315	297	197	100	450	393
94	3.0	9	12	780315	297	197	100	450	396
95	4.5	9	14	780315	297	197	100	450	396
96	4.5	9	24	780315	297	197	100	450	408
97	3.0	4	24	780524	387	197	170	450	349
98	3.0	6	16	780524	387	197	170	450	391
99	4.5	6	25	780524	387	197	170	450	410
100	4.5	6	29	780524	387	197	170	450	390
101	3.0	3	13	780930	497	197	300	450	433
102	3.0	3	17	780930	497	197	300	450	424
103	4.5	3	17	780930	497	197	300	450	425
104	4.5	3	18	780930	497	197	300	450	451

TABLE 5. Continued

OBS	MIX TYPE (PSI)	BATCH #	SPECIMEN #	DATE	AGE AT TEST (DAYS)	TIME IN FOG ROOM (DAYS)	TIME IN OVEN (DAYS)	TEMPERATURE AT TEST (DEGREES F)	SPLITTING TENSILE STRENGTH (PSI)
105	3.0	3	26	770331	679	270	409	450	357
106	3.0	3	27	770331	679	270	409	450	391
107	4.5	3	2	770331	679	270	409	450	424
108	4.5	3	22	770331	679	270	409	450	407
109	3.0	4	7	770929	861	270	591	450	286
110	3.0	4	10	770929	861	270	591	450	284
111	4.5	4	13	770929	861	270	591	450	355
112	4.5	4	19	770929	861	270	591	450	347
113	3.0	5	4	780908	1203	283	920	450	261
114	3.0	5	10	780908	1203	283	920	450	324
115	4.5	5	2	780908	1203	283	920	450	341
116	4.5	5	13	780908	1203	283	920	450	368
117	3.0	8	19	790913	1580	276	1304	350	431
118	4.5	9	20	790913	1580	283	1297	350	413
119	4.5	9	28	790913	1580	283	1297	350	397

Note: The parentheses on the splitting tensile strength value of observation 35 indicates that the corresponding data point was not used in fitting equations to the data.

TABLE 6. Least Squares Estimated Coefficients From Equation (3)
for Modulus of Elasticity, Compressive Strength, and
Splitting Tensile Strength

Modulus of Elasticity

<u>Mix Type</u>	<u>Temperature (F)</u>	<u>Intercept</u>	<u>Slope</u>
3.0K	250	4.0338	-0.1854
	350	3.2623	-0.1476
	450	2.7073	-0.1622
4.5K	250	4.2464	-0.1617
	350	3.6143	-0.1795
	450	3.1196	-0.2396

Compressive Strength

<u>Mix Type</u>	<u>Temperature (F)</u>	<u>Intercept</u>	<u>Slope</u>
3.0K	250	5829.4	-126.97
	350	5943.1	-217.18
	450	5277.2	-187.81
4.5K	250	6625.4	- 67.22
	350	6738.7	-273.93
	450	6290.3	-270.83

Splitting Tensile Strength

<u>Mix Type</u>	<u>Temperature (F)</u>	<u>Intercept</u>	<u>Slope</u>
3.0K	250	525.71	-16.98
	350	503.57	-17.04
	450	451.58	-17.47
4.5K	250	551.17	-12.61
	350	560.13	-23.96
	450	493.03	-17.56

Note that S is the specified minimum initial compressive strength (in units of 1000 psi) of the concrete specimens from the PCA data base. The actual initial compressive strengths were considerably higher than the specified minimum values of 3.0 and 4.5. Therefore, in using equations (4), (5), and (6), either specified values between 3.0 and 4.5 should be substituted for S , or the equations should be rewritten to utilize "actual" values. Equations (4), (5), and (6) are nominally only valid for constant temperature histories, and for T , S , and t within the ranges of the PCA data. The extrapolative and variable-temperature use of these equations is discussed in Section 4.4.

Before making use of the fitted equations (4), (5), and (6), it is important to check (statistically) whether they adequately fit the data. Table 7 lists the residual standard deviations and within-batch standard deviations for each of the three properties. The residual standard deviations (measures of data variation not explained by the fitted equation) are significantly larger (statistically) than the within-batch standard deviations. Although there were no PCA data available to explicitly estimate the between-batch variation, it is clear from the mixture design and other information in Ref. 1 that there was considerable batch-to-batch variation (within each mixture type). Comparisons of the residual standard deviations of Table 7 to the corresponding "overall variation" values in Tables 2 and 3 show that the majority of the residual variation in the PCA data is probably typical within- and between-batch variation (including test variations and measurement uncertainties). Hence, there does not appear to be any evidence of lack-of-fit of equations (4), (5), and (6) to the data.

Although equations (4), (5), and (6) fit the data acceptably well given the inherent variability of the data, it is important to realize that there is uncertainty in the fitted equations. We characterize this uncertainty in two ways, depending on the use of the fitted equation.

1. Expected or "average" behavior. Equations (4), (5), and (6) predict the expected or "average" behavior of concrete properties at elevated temperatures. The uncertainty in "average" behavior is given statistically by confidence bands.

TABLE 7. Residual Standard Deviations and Within-Batch Standard Deviations for Modulus of Elasticity, Compressive Strength, and Splitting Tensile Strength

<u>Property</u>	<u>Residual Standard Deviation</u> (a)	<u>Within-Batch Standard Deviation</u> (b)
E	0.2181	0.1129
f_c	327.74	185.92
f_{st}	35.44	23.07

- (a) The residual standard deviation is a measure of the variability in the property data not explained by the fitted equation. Sources of variation contributing are equation lack-of-fit, within- and between-batch variations, variations in performing the tests, and measurement uncertainties.
- (b) The within-batch standard deviations were estimated using pairs or triples of property values (from tests performed on two or three specimens from the same batch) at individual mixture type x temperature x time-at-temperature combinations. Measurement uncertainties and variations in performing the tests (as well as specimen-to-specimen variation within a batch) contribute to the estimates.
2. Range of behavior. All concrete tanks will not exhibit expected or "average" behavior. The range of behavior for a given concrete property at a given set of conditions (i.e., temperature, time, mixture) will vary around the average. Statistical tolerance bands provide uncertainty estimates for this range of behavior.

Figures 4 to 21 present the PCA data, fitted equations, 95% confidence bands, and 95%/95% tolerance bands for the six combinations of mixture design and temperatures tested by PCA. Details of, and equations for, the confidence and tolerance bands are presented in Appendix A. However, a brief explanation of the concepts is given here.

- 95% confidence bands. Provided the fitted equation adequately models the property behavior, we have 95% confidence that the expected or "average" behavior falls within the upper and lower confidence bands.
- 95%/95% tolerance bands. Provided the fitted equation adequately models the property behavior, we have 95% confidence that 95% of the property values will fall between the upper and lower tolerance bands.

FIGURE 4. Modulus of Elasticity Versus Time at 250 F for 3,000 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 - - - - 95%/95% TOLERANCE BANDS

LOGE(DAYS AT TEMP + 1)

FIGURE 5. Modulus of Elasticity Versus Time at 350 F for 3,000 psi Hanford Concrete

FIGURE 6. Modulus of Elasticity Versus Time at 450 F for 3,000 psi Hanford Concrete

FIGURE 7. Modulus of Elasticity Versus Time at 250 F for 4,500 psi Hanford Concrete

FIGURE 8. Modulus of Elasticity Versus Time at 350 F for 4,500 psi Hanford Concrete

FIGURE 9. Modulus of Elasticity Versus Time at 450 F for 4,500 psi Hanford Concrete

FIGURE 10. Compressive Strength Versus Time at 250 F for 3,000 psi Hanford Concrete

◇ PCA DATA POINTS

— LEAST SQUARES FIT

... 95% CONFIDENCE BANDS

--- 95%/95% TOLERANCE BANDS

FIGURE 11. Compressive Strength Versus Time at 350 F for 3,000 psi Hanford Concrete

FIGURE 12.

Compressive Strength Versus Time at 450 F for 3,000 psi Hanford Concrete

FIGURE 13. Compressive Strength Versus Time at 250 F for 4,500 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 - - - 95%/95% TOLERANCE BANDS

FIGURE 14. Compressive Strength Versus Time at 350 F for 4,500 psi Hanford Concrete

◇ PCA DATA POINTS

— LEAST SQUARES FIT

... 95% CONFIDENCE BANDS

--- 95%/95% TOLERANCE BANDS

FIGURE 15. Compressive Strength Versus Time at 450 F for 4,500 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 - - - 95%/95% TOLERANCE BANDS

FIGURE 16. Splitting Tensile Strength Versus Time at 250 F for 3,000 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 --- 95%/95% TOLERANCE BANDS

FIGURE 17. Splitting Tensile Strength Versus Time at 350 F for 3,000 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 --- 95%/95% TOLERANCE BANDS

FIGURE 18. Splitting Tensile Strength Versus Time at 450 F for 3,000 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 - - - 95%/95% TOLERANCE BANDS

FIGURE 19. Splitting Tensile Strength Versus Time at 250 F for 4,500 psi Hanford Concrete

LOGE(DAYS AT TEMP + 1)

FIGURE 20.

Splitting Tensile Strength Versus Time at 350 F for 4,500 psi Hanford Concrete

FIGURE 21. Splitting Tensile Strength Versus Time at 450 F for 4,500 psi Hanford Concrete

It can be seen from Figures 4 to 21 that equations (4), (5), and (6) take the form of a straight line (for a fixed mixture type and temperature) when the property of interest is plotted against time-at-temperature on a logarithmic scale. The confidence and tolerance band limits are not straight lines, although they appear nearly so in many of the plots (see Appendix A for details). The lines and uncertainty bands in Figures 4 to 21 are extrapolated out to 50 years, with the fact that extrapolation is occurring indicated by the lack of plotted data symbols. The validity of this and other extrapolative uses of the concrete property equations and the corresponding confidence and tolerance bands is discussed in the following section.

4.4 EXTRAPOLATIVE AND VARIABLE-TEMPERATURE USE OF PROPERTY EQUATIONS

The concrete property equations (4), (5), and (6) of Section 4.3 were developed using the PCA Hanford concrete data base. The PCA data for modulus of elasticity, compressive strength, and splitting tensile strength were all taken from constant-temperature tests. Thus, strictly speaking, equations (4), (5), and (6) and the corresponding confidence and tolerance bands are directly applicable only to constant temperature situations. However, SAFE-CRACK requires property equations applicable to variable-temperature histories. Rashid and James (Ref. 17) have proposed an incremental analysis method for adapting the "constant temperature" property equations and their confidence and tolerance bands to "variable temperature" histories. This method is discussed further in Appendix B.

Because of the empirical approach used to develop equations (4), (5), (6), and their corresponding confidence and tolerance bands, it must be determined whether it is appropriate to apply them outside the range of the PCA data. The ranges for T , S , and t in the PCA data were:

T: 250 to 450 (F)
S: 3.0 to 4.5 (1000 psi)
t: 3-10 to 920-1300 (days).

The range on t varies depending on the values of T and S.

SAFE-CRACK requires equations valid for temperatures from 70 to 450 F, and for times from 0 to 18,250 days (0 to 50 years). For waste tank structural analyses, the following specific extrapolative uses of the concrete property equations may be necessary with SAFE-CRACK.

- In the range 250 F to 450 F, the property equations are assumed to be valid from 0 to 18,250 days (0 to 50 years). This involves extrapolating the property equations at both ends of the time scale within the 250 F to 450 F temperature range.
- The property equations, omitting the $\ln(t + 1)$ terms, are assumed to be valid in the range 70 F to 200 F. This corresponds to an assumption of negligible concrete degradation at temperatures below 200 F. Note that the equation for modulus of elasticity still yields decreasing property values with increasing temperatures, but the decrease is no longer related to time-at-temperature.
- In the range 200 F to 250 F, the $\ln(t + 1)$ terms in the property equations are phased in linearly via a multiplier taking the value 0 at 200 F and the value 1 at 250 F.

Before we begin our discussion of the validity of extrapolating the concrete property equations and uncertainty bands, note that validity of extrapolation is not entirely a statistical question. Statistics can address whether or not a given equation adequately models the available data. If so, then extrapolation may be allowable if the underlying theory and knowledge support the extension of the equation being used.

Extrapolating the concrete property equations on the time scale appears to be reasonable. Theory and knowledge support a general deterioration trend with time for concrete exposed to elevated temperatures, and the fitted

property equations appear to adequately fit the data. If anything, the equations may be somewhat conservative in some cases, since some of the PCA data show property deterioration leveling off after 100 days at temperature. Overall, extrapolation to 50 years appears to be reasonable given current knowledge. Extrapolation to 0 days also appears reasonable.

Justification of extrapolating concrete properties on the temperature scale must take into account the empirical approach used to develop the temperature dependence of fitted property equations (see Section 4.3). It also must take into account Anatech's deletion of the $\ln(t + 1)$ terms in the equations for 70 to 200 F and the linear phase-in of these terms for 200 to 250 F. In the following paragraphs we consider these aspects for each of the three properties (modulus of elasticity, compressive strength, and splitting tensile strength) in turn.

For modulus of elasticity, temperature only affects the intercept and not the slope of the linear relationship of modulus with $\ln(t + 1)$. Hence, the deletion or "phasing-in" of the $\ln(t + 1)$ term of equation (4) does not affect the assessment of the validity of extrapolating the equation for 70 to 250 F. The linear effect of T in the modulus of elasticity equation (4) extrapolates reasonably well down to 70 F, based on comparisons with the PCA data. Hence, we conclude that extrapolating modulus of elasticity to lower temperatures in SAFE-CRACK is reasonable.

For compressive strength and splitting tensile strength, things are more complicated due to the TA and TB terms in equations (5) and (6). TA provides for an effect of temperature above 350 F but not below, while TB provides for an effect of temperature below 350 F but not above. [These terms are a consequence of the empirical approach used to develop the temperature dependence, and the fact that the PCA data contained only three elevated values of temperature ($T = 250, 350$, and 450 F).] The assumption that the effect of temperature for 250 to 350 F also holds for 70 to 250 F is a major one, especially when terms such as TA and TB are involved. We consider this more specifically, in turn, for compressive strength and splitting tensile strength.

The property equation for compressive strength [equation (5)] contains terms involving both T_A and T_B . However, T_B shows up only in a $\ln(t + 1)$ term, which is not used (according to the Anatech implementation) for 70 to 200 F. Since T_A is zero in the temperature range 70 to 200 F, the Anatech implementation of equation (5) over this range indicates that compressive strength does not depend on temperature. This extrapolation of equation (5) is in keeping with the belief that concrete in waste storage tanks will experience little if any degradation in compressive strength at temperatures below 200 F.

The SAFE-CRACK equation (6) for splitting tensile strength contains one term involving T_A , so the splitting tensile strength does not depend on temperature under 350 F. As for compressive strength, this is in keeping with the belief that degradation should be essentially negligible for lower temperatures.

In summary, we conclude that the extrapolative use in SAFE-CRACK of the modulus of elasticity, compressive strength, and splitting tensile strength property equations (4), (5), and (6) is reasonable.

5.0 MATHEMATICAL EXPRESSIONS FOR CONCRETE CREEP STRAIN

This section discusses the SAFE-CRACK treatment of concrete creep relative to the PCA data (Appendix E of Ref. 1). Also presented are mathematical expressions for strain and creep strain fitted via least squares to the PCA data. However, the limited applicability of the expressions leads to the recommendation that alternate expressions be incorporated in SAFE-CRACK.

5.1 SAFE-CRACK'S USE OF CREEP DATA

The PCA study (Ref. 1) measured the creep strain (long time deformation under load) of Hanford concrete specimens at 250 F and 350 F. A total of six specimens (6 in. x 12 in. cylinders) were tested for about 650 days under two loadings, 500 psi (at 350 F) and 1500 psi (at 250 and 350 F). Figure 22 shows the PCA's best fit curve of creep strain data to a logarithmic equation, the type of equation often used to describe creep behavior. The equation used is of the form

$$\epsilon_{cr} = A \log_{10} (t) + B \quad (7)$$

where t is the time-at-temperature in days.

Superimposed on the graph of Figure 22 is a plot of the creep predicted for 1500 psi at 350 F by the exponential equation used in SAFE-CRACK which is of the form (simplified):

$$\epsilon_{cr} = C (1 - e^{-r \cdot t}) + D \cdot t \quad (8)$$

The SAFE-CRACK equation predicts the creep correctly at about five days but between 50 and 1000 days, the prediction is significantly higher than the creep as measured by PCA. Figures 23, 24, and 25 show the difference between the PCA extrapolation of the data (the first 600 days of which are actual) and the SAFE-CRACK predictions out to 50 years (18,250 days). The reason that the SAFE-CRACK equation over-predicts the creep strain beyond about

FIGURE 22. Best Fit of Creep Strain Data to Logarithmic Equation,
 $\epsilon_{cr} = A \cdot \log_{10} (\text{TIME}) + B$

FIGURE 23. SAFE CRACK Creep Exponential Equation Versus PCA Cylinder Data
(Logarithmic equation, best-fit Curve) for 500 psi Stress, 350 F

FIGURE 24. SAFE-CRACK Creep Exponential Equation Versus PCA Cylinder Data (Logarithmic Equation, Best-Fit Curve) for 1,500 psi Stress, 250 F)

FIGURE 25. SAFE-CRACK Creep Exponential Equation Versus PCA Cylinder Data
(Logarithmic Equation, Best-fit Curve) for 1,500 psi Stress,
350 F

100 days is that this creep equation was formulated when the available Hanford concrete creep data were from the first approximately 70 days of elevated temperature creep testing by PCA.⁽⁸⁾

For use in analyzing Hanford waste tanks, the SAFE-CRACK equation should be modified to more closely model the actual creep data from Hanford concrete. This topic and creep equation modifications for SAFE-CRACK are discussed in the following section.

5.2 FITTED EQUATIONS, CONFIDENCE BANDS, AND TOLERANCE BANDS FOR STRAIN AND CREEP

The creep data of Ref. 1, as displayed in Figure 22, was computed from measured strain data via

$$\epsilon_{cr} = a_0 - \epsilon \quad (9)$$

where ϵ_{cr} = computed creep strain at t days (millionths),

a_0 = measured strain at zero days, after load applied and test temperature reached (millionths),

ϵ = measured strain at t days (millionths).

Note that equation (9) merely corrects the total strain for the effects of elastic strain and thermal expansion.

The PCA study measured strain for two specimens for each of the three load/temperature combinations considered (1500 psi/250 F, 500 psi/350 F, and 1500 psi/350 F). The measured a_0 values for the two specimens of a combination varied considerably, especially for the 1500 psi/350 F combination. This variation contributes substantially to the creep uncertainty, and so it was decided to model both strain and creep. In this way, if so desired, the fitted equation for strain can be combined with an assumed or computed a_0 to obtain creep (thereby avoiding the large uncertainty imposed by the variation of the PCA a_0 values).

The basic equation form

$$\epsilon = a + b \log_e (t + 1) \quad (10)$$

was fitted to the PCA data^(a) for each of the three load/temperature combinations, with the least-squares estimated coefficients summarized in Table 8. Note that equation (10) is essentially the same as equation (7), with $t + 1$ substituted for t and the natural log substituted for the common log. These changes were made to maintain similarities with the fitted property equations of Chapter 5.

Statistical comparisons of the slopes (the b column) in Table 8 show that they are not significantly different. However, the dependence of the intercept on the test conditions is obvious. To include load and temperature in the model of equation (10), it is necessary to assume a linear function for each, since there were only three combinations of two temperatures (250 F, 350 F) and two loads (500 psi, 1500 psi) in the PCA study. The resulting fitted equation is

$$\hat{\epsilon} = 3064.152 - 0.863 L - 6.152 T - 127.526 \log_e (t + 1) \quad (11)$$

Because six specimens from the same batch were used for the two tests at each of the three test condition combinations, only within-batch variation (in addition to variation in the test itself and in measurements) is present in the data. The two tests at each combination provide an estimate of the overall experimental variation that can be used to statistically test the fitted model (Eq. 11) for lack-of-fit (see Ref. 11, Section 1.5). There was not a significant lack-of-fit (i.e., the model appears to adequately fit the data).

(a) All of the data in Appendix E of Ref. 1 was used, with the exception of the zero-day data (labeled "6 hr. heat to test temperature").

TABLE 8. Unweighted Least Squares Estimates of Slope and Intercept of Equation (10) for Strain

Test Conditions			
Load (psi)	Temperature (F)	a	b
1500	250	230.48	-127.27
500	350	447.16	-118.45
1500	350	-344.46	-138.22

where $\hat{\epsilon}$ = predicted strain (millionths)
 L = applied load or stress (psi)
 T = temperature of test (F)
 t = time at temperature (days).

The fitted equation (11) should only be applied at or very close to the three load/temperature combinations for which it was developed. The possibility was noted by PCA⁽¹⁾ that creep strain of Hanford concrete may be greater at temperatures from ambient to 150 F than at 250 F to 450 F. A prediction equation based on wider ranges of temperature and load is not possible based solely on the PCA data. The SAFE-CRACK creep equations modified using a larger base of data are recommended.

A prediction equation for creep strain valid only for the three test condition combinations is given by

$$\hat{\epsilon}_{cr} = \bar{a}_0 - \hat{\epsilon} \quad (12)$$

where $\hat{\epsilon}_{cr}$ = predicted creep strain at t days (millionths),
 $\bar{a}_0$ = average strain reading at zero days (after load applied and test temperature reached). The average is over the two specimens at the specified test combination (millionths),
 $\hat{\epsilon}$ = predicted strain at t days, from equation (11) (millionths).

The values of $\bar{a}_0$ are given in the following chart.

Test Conditions		Individual Specimen Values		Avg.
Load (psi)	Temperature (F)	a_{01}	a_{02}	$\bar{a}_0$
500	350	700	550	625
1500	250	490	260	375
1500	350	-230	860	315

The substantial variation in the a_0 values for individual specimens at a given test condition is apparent, and hence a dependence of $\bar{a}_0$ on load and temperature was not determined.

Figures 26 to 31 present the PCA data, ^(a) fitted equations, 95% confidence bands, and 95%/95% tolerance bands on strain and creep strain for the three load and temperature test conditions. Although we present strain and creep strain prediction equations [equations (11) and (12)] and formulas for confidence bands [equations (A.10) and (A.12)] and tolerance bands [equations (A.17) and (A.18)] in Appendix A, we do not recommend their use in SAFE-CRACK. The type of creep equations previously used in SAFE-CRACK modified to include a wider data base are recommended. Rashid and James⁽¹⁷⁾ have developed equations of this type for use in SAFE-CRACK.

(a) The "data" for creep plotted in Figures 29 to 31 are the individual specimen strain values adjusted by the average $\bar{a}_0$ values. Plotted this way, the data does not reflect the variability in the individual a_0 values. The plots were done in this fashion in keeping with the procedure used to obtain the fitted creep equations.

◇ PCA DATA POINTS

— LEAST SQUARES FIT

····· 95% CONFIDENCE BANDS

--- 95%/85% TOLERANCE BANDS

FIGURE 26. Strain Versus Time at 350 F and 500 psi for 4,500 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 - - - 95%/95% TOLERANCE BANDS

FIGURE 27. Strain Versus Time at 250 F and 1,500 psi for 4,500 psi Hanford Concrete

◇ PCA DATA POINTS
 — LEAST SQUARES FIT
 95% CONFIDENCE BANDS
 --- 95%/95% TOLERANCE BANDS

FIGURE 28. Strain Versus Time at 350 F and 1,500 psi for 4,500 psi Hanford Concrete

FIGURE 29. Creep Strain Versus Time at 350 F and 500 psi for 4,500 psi Hanford Concrete

FIGURE 30. Creep Strain Versus Time at 250 F and 1,500 psi for 4,500 psi Hanford Concrete

FIGURE 31. Creep Strain Versus Time at 350 F and 1,500 psi for 4,500 psi Hanford Concrete

6.0 EVALUATION OF EFFECTS OF THERMAL CYCLING ON MODULUS OF ELASTICITY FOR HANFORD CONCRETE

In this chapter, we first evaluate the SAFE-CRACK approach to quantifying the effects of thermal cycling on modulus of elasticity.^(a) We also consider the PCA data which cycled temperatures to 350 F for 17 cycles of length 14 days, and for 18 cycles of length 28 days.

6.1 SAFE-CRACK TREATMENT OF THERMAL CYCLING

SAFE-CRACK utilizes a common mathematical expression for modulus of elasticity for both constant and non-constant temperature histories. The expression is of the form

$$\hat{E} = a - bT - c \log_e (t + 1) \quad (13)$$

where $\hat{E}$ = predicted modulus of elasticity (psi)
T = temperature (F)
t = time at temperature (days).

For non-constant temperature histories, an incremental form of (13) is given in Ref. 8 as

$$\Delta E_n = -b \Delta T_n - c \Delta t_n \exp [(E_n - a + b T_n)/c] \quad (14)$$

where ΔE_n = the change in modulus due to changes in temperature and time during the n^{th} time step (psi)
 ΔT_n = the change in temperature during the n^{th} time step (F)
 Δt_n = the incremental time during the n^{th} step (days)
 E_n = the modulus of elasticity prior to the n^{th} time step (psi)
 T_n = the temperature prior to the n^{th} time step (F).

(a) The effects of thermal cycling on other concrete properties are also of interest, but were not included within the scope of work covered by this document. Therefore, only modulus of elasticity is considered.

Values for a, b, and c^(a) from Ref. 8 are

$$\begin{aligned} a &= 6.0 \times 10^6 \\ b &= \begin{cases} 7.0 \times 10^3 & \text{for positive } \Delta T \\ 3.5 \times 10^3 & \text{for negative } \Delta T \end{cases} \\ c &= 1.95 \times 10^5 \end{aligned}$$

The reduced value of b for negative ΔT provides for partial recovery of the modulus with a temperature decrease. SAFE-CRACK apparently also has the option of suppressing recovery upon cooling if it is deemed appropriate to do so.

The fitted equation (4) for modulus of elasticity presented in Section 4.3 has a slightly different form than (13), but it is presumed that SAFE-CRACK could easily be modified to accept the new equation. However, the important question of what to do about recovery remains. This is part of a larger question concerning whether it is appropriate to apply an equation with temperature dependence based on constant temperature tests to non-constant temperature applications. We attempt to shed some light on these questions in the following two sections.

6.2 PCA CYCLIC TEMPERATURE TESTS

PCA performed two sets of cyclic temperature tests, where the concrete specimens went through a number of temperature cycles. The two sets involved cycle lengths of 14 and 28 days. Both sets had the same maximum temperature of 350 F, and both had the same rate of heating and cooling (140 F/day). The modulus of elasticity was measured after cooling the specimens to 70 F. Figure 32 illustrates the two cycle configurations.

(a) The coefficients are labeled E_0 , E_1 , and E_2 in reference 8, but this notation conflicts with the E_n notation also used in reference 8. The change was made here to avoid confusion.

FIGURE 32. Fourteen- and Twenty-Eight-Day Temperature Cycles in PCA Study

The modulus of elasticity results for the PCA cycle temperature tests are given in Appendix G of Ref. 1. Figures 33 and 34 plot the PCA cyclic and constant temperature modulus of elasticity data averages^(a) on the same plots, with "time at maximum temperature" used as the x-axis. Since the cyclic temperature tests attained a maximum temperature of 350 F, it is most reasonable to compare the 14- and 28-day cyclic data to the 350 F constant temperature data. It is seen in Figures 33 and 34 that the 14- and 28-day cyclic temperature modulus values are significantly larger than the 350 F constant temperature values. However, this may be due to the fact that specimens from different batches were used for the cyclic tests than were used for the constant temperature tests. The cyclic temperature batches had significantly lower water/cement ratios and other differences.

Figures 35 and 36 show the cyclic and constant temperature data replotted with modulus of elasticity expressed as a fraction of its time-zero value. These plots show the 28-day cycle data agreeing very well with the

(a) The plotted modulus of elasticity values are averages of two specimen values for the cyclic temperature tests, and of three specimen values for the constant temperature tests. Missing data or data deleted as potential outliers are exceptions to this in a few cases.

FIGURE 33. Modulus of Elasticity for 3,000 psi Hanford Concrete from Cyclic and Constant Temperature Tests

FIGURE 34. Modulus of Elasticity for 4,500 psi Hanford Concrete from Cyclic and Constant Temperature Tests

FIGURE 35. Modulus of Elasticity (as a Fraction of Time Zero Value) for 3,000 psi Hanford Concrete from Cyclic and Constant Temperature Tests

FIGURE 36. Modulus of Elasticity (as a Fraction of Time Zero Value) for 4,500 psi
 Hanford Concrete from Cyclic and Constant Temperature Tests

350 F constant temperature data. However, these plots also have some complicating factors. Recall that the constant temperature tests used specimens from various batches, and that time-zero measurements of modulus (and the other properties) were not made for each batch by PCA. However, such measurements were made as part of the ambient temperature (70 F) time-in-fog-room tests. Two batches each of the 3000 and 4500 psi mix types had cure times similar to those for the batches used in the constant temperature tests. Results from these specimens were averaged (for each of the mix types) to yield "time-zero" modulus values which were used to form the modulus ratios for the 250, 350, and 450 F data. While the cyclic data did have time-zero modulus values, they are only based on one or two specimens. In summary, the starting values of modulus are uncertain for both the cyclic and constant temperature data, making the ratioing of Figures 35 and 36 open to question.

Another complicating factor in comparing cyclic and constant temperature test data is that modulus measurements on the constant temperature specimens were made immediately after removal from test temperature, whereas the cyclic temperature specimens were measured after the end of a two-day cooling ramp to 70 F.

6.3 FINAL COMMENTS ON CONSTANT VERSUS NON-CONSTANT TEMPERATURE HISTORIES

Questions were raised in Section 6.1 about how to appropriately apply the fitted modulus equation (4), which was developed from constant temperature data, to non-constant temperature situations. Figures 33 and 34 suggest that there was little difference in 14- and 28-day cycle lengths, but that cycling provided some recovery of modulus. On the other hand, Figures 35 and 36 suggest that there is a difference between 14- and 28-day cycle lengths, but that there was not any recovery on the 28-day cycle tests. Obviously, the complicating factors in the PCA data affect the conclusions reached in evaluating it.

Our conclusion concerning the PCA data, based on the complicating factors already mentioned and combined with the limited nature of the cyclic data (only one maximum temperature), is that they are insufficient to be of

definitive use in answering the questions of interest. There are additional investigations that could be performed with the PCA data that would shed some light on the matter, but would not completely answer the questions. Such investigations were beyond the scope of work.

Changes in chemistry and physical changes in the concrete do not support a phenomenon of partial recovery of properties on cooling. The loss of water is the major factor in the degradation of concrete properties at elevated temperatures. Since there is no recovery of water upon cooling (unless subjected to soaking), there is no basis for assuming recovery. Marechal (Ref. 15) found that losses in modulus due to drying or heating are irrecoverable, even in the 100-200 F range and that the variation of the modulus with temperature was very slight compared to the variation because of loss of water (change of state). Further, restoring the lost water by exposing once-heated specimens to 98% humidity over several months did not result in recovery of compressive strength lost by heating. Also, Refs. 1 and 7 indicate substantial microcracking resulting from heating. The cracking along the paste-aggregate interface would not be expected to re-bond to the aggregate upon cooling and cracks in the paste itself could not be expected to experience significant autogenous healing unless the concrete were in the early stages of hydration, as in the first 90 days.

7.0 MITIGATING FACTORS IN ELEVATED TEMPERATURE CONCRETE DESIGN

There are several mitigating factors in the design of structures for elevated temperatures. One is that concrete heated from one side only is less affected than concrete heated from all sides (Ref. 2). Thus the temperature effects on the tank structures in the field will not be as severe as those for the cylinders heated from all sides in the PCA testing program.

Second, concrete stressed to about 1000 psi (Ref. 4), or to $0.4 f_c$ (Ref. 1), and tested hot shows higher compressive strengths for exposures up to 500 F and above than concrete: a) heated at zero stress and tested hot, or b) heated at zero stress and tested at room temperature. The differences are significant (plots are shown in Ref. 1) and the net result will be that the concrete in the tanks, which are under load during heating, will not show as great a reduction in strength as the test cylinders that were not loaded during heating.

A third mitigating factor, which should be taken into account in the analysis, is that the tanks will not actually be at an elevated temperature for the full 50 years of their projected service life. Therefore, the degradation of properties will not be as great as indicated by the end point on a 50-year extrapolation plot. This can be taken into account by using temperature histories simulating actual or expected operating conditions.

8.0 RECOMMENDATIONS

The following recommendations and conclusions are made.

1. SAFE-CRACK should be modified to utilize the fitted equations for modulus of elasticity, compressive strength, and splitting tensile strength presented in Section 4.3. The equations for modulus and compressive strength are of a form similar to that previously used by SAFE-CRACK. The equation for splitting strength (which has a form similar to the equations for the other two properties) should replace the previous SAFE-CRACK method of computing splitting tensile strength as 10% of compressive strength.
2. SAFE-CRACK should be modified to make use of the 95% confidence and 95%/95% tolerance bands for modulus of elasticity, compressive strength, and splitting tensile strength (as presented in Appendices A and C). As indicated in specific recommendations below, SAFE-CRACK might best implement this and the previous recommendation by letting the user choose between the fitted equation, the 95% lower confidence band, and the 95%/95% lower tolerance band.
3. The fitted equations, confidence bands, and tolerance bands for modulus of elasticity, compressive strength, and splitting tensile strength should be used in the following ways:
 - The least squares fitted equations should be used to predict structural deflections for comparison with field surveillance measurements.
 - The lower 95% confidence bands should be used for "design by analysis" with the SAFE-CRACK code.
 - The lower 95%/95% tolerance bands should be used to perform a "worst case" design by analysis to determine if any below average tanks might be subject to premature failure. If so, needed modifications could be made to the basic tank design.

- The lower 95%/95% tolerance bands should also be used for determining suitable times for detailed scheduled surveillance of the tanks in service.
4. The fitted equations, confidence bands, and tolerance bands presented for modulus of elasticity, compressive strength, and splitting tensile strength were developed from PCA test data having the following variable ranges:

Temperature	250 to 450 F
Nominal initial strength	3000 to 4500 psi
Time at temperature	3-10 to 920-1300 days.

However, SAFE-CRACK requires equations valid from 70 to 450 F and from 0 to 50 years. Anatech's proposed (see Section 4.4) extrapolative use of the modulus of elasticity, compressive strength, and splitting tensile strength expressions for these wider temperature and time ranges appears to be reasonable based on our limited investigations. According to phone conversations with Y. R. Rashid of Anatech, he also has investigated the validity of the extrapolations and judged them reasonable.

5. The modulus of elasticity, compressive strength, and splitting tensile strength property equations and corresponding confidence bands presented herein were developed from the PCA constant-temperature test data. The incremental analysis approach proposed by Anatech for adapting the constant-temperature property equations and uncertainty bands to variable-temperature situations is valid provided the inherent assumptions (see Appendix B) are judged reasonable.
6. The previous version of SAFE-CRACK had the option of allowing recovery of concrete properties with cooling. The PCA constant-temperature data give no information about the effects of cooling, while the cyclic temperature data give only limited information. What information the cyclic data do give suggests that there is little or no recovery on cooling. Physical and chemical changes do not support property recovery on cooling, and Marechal's study (Ref. 15) also indicates that modulus

losses from drying and heating are irrecoverable. Hence, we recommend that the SAFE-CRACK code neglect partial recovery of the concrete properties on cooling. This includes recovery from short-term (few days) heating because the tanks will not experience short-term heating.

7. Further efforts are required to develop a model (and the corresponding uncertainty bands) for Hanford concrete creep which is appropriate for use in SAFE-CRACK and adequately explains the PCA Hanford creep data. The previous SAFE-CRACK creep equation (Ref. 8) over-predicts creep behavior relative to the PCA Hanford concrete data. While expressions for strain and creep strain fitted to the PCA data are presented in Section 5.2 and Appendix A, the expressions are inadequate (due to insufficient PCA data) for the needs of SAFE-CRACK. We recommend that equations of both logarithmic form (as we present) and exponential form (as SAFE-CRACK presently uses) be considered in light of the code's needs, the PCA data, and other appropriate concrete creep data. Because of their expertise in this area, Anatech is recommended to perform this additional evaluation and model development. (Anatech has implemented this recommendation in Ref. 17.)
8. The modifications to SAFE-CRACK should be verified as capable of predicting the properties as intended by running the code after the modifications. Other controls should be implemented by the Hanford-area code custodian to assure that the proper version is being used and maintained without undesired modifications.

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APPENDIX A

CONFIDENCE AND TOLERANCE BANDS

APPENDIX A

CONFIDENCE AND TOLERANCE BANDS

A.1 GENERAL FORMULAS

We begin the appendix by presenting the general formulas for two-sided confidence and tolerance bands associated with a least-squares-fitted multiple regression equation. Equations (4), (5), (6), (11), and (12) are such equations.

Let

$$y(\underline{x}) = \underline{x}' \underline{\beta} = \beta_0 + \beta_1 x_1 + \dots + \beta_{p-1} x_{p-1} \quad (\text{A.1})$$

be an equation with unknown parameter vector $\underline{\beta} = (\beta_0, \beta_1, \dots, \beta_{p-1})$ and vector of predictor variables $\underline{x} = (1, x_1, x_2, \dots, x_{p-1})$. Given a data set of n observations assumed to follow the form (A.1), we write the matrix equation

$$\underline{y} = \underline{X} \underline{\beta}$$

where $\underline{y}$ is an $n \times 1$ vector, $\underline{X}$ an $n \times p$ matrix and $\underline{\beta}$ a $p \times 1$ vector. The least squares estimator of $\underline{\beta}$ is then

$$\hat{\underline{\beta}} = (\underline{X}'\underline{X})^{-1} \underline{X}' \underline{y}$$

The fitted least squares equation is then

$$\hat{y}(\underline{x}) = \underline{x}' \hat{\underline{\beta}}$$

which has variance (considering it a predictor of expected behavior at $\underline{x}$)

$$\text{Var} [\hat{y}(\underline{x})] = \underline{x}' (\underline{X}'\underline{X})^{-1} \underline{x} \hat{\sigma}^2$$

where $\hat{\sigma}$ is an estimate of the experimental error standard deviation. The residual standard deviation from the least squares fit is used for $\hat{\sigma}$ provided the equation form (A.1) adequately fits the data.

A 95% confidence band for the form (A.1) is

$$\hat{y}(\underline{x}) \pm \sqrt{p F_{.95}(p, n-p) \underline{x}' (X'X)^{-1} \underline{x} \hat{\sigma}^2} \quad , \quad (A.2)$$

where $F_{.95}(p, n-p)$ is the upper 95th percentile point of an F-distribution with p and $n-p$ degrees of freedom. Confidence bands of the type (A.2) are of the Working-Hotelling type (see Ref. 13, page 200). The interpretation of this confidence band is that it contains the expected behavior of the response y (provided the equation form (A.1) adequately fits the data) with 95% confidence for all vectors x (for which (A.1) is adequate).

A 95%/95% tolerance band for the form (A.1) is

$$\hat{y}(\underline{x}) \pm \hat{\sigma} \left\{ \left[p F_{.975}(p, n-p) \underline{x}' (X'X)^{-1} \underline{x} \right]^{1/2} + Z_{.975} \left(\frac{n-p}{\chi^2_{.025}(n-p)} \right)^{1/2} \right\} \quad (A.3)$$

where $F_{.975}(p, n-p)$ = the upper 97.5 percentile of an F-distribution with p and $n-p$ degrees of freedom,

$Z_{.975}$ = the upper 97.5 percentile of the standard normal (mean 0, variance 1) distribution,

$\chi^2_{.025}(n-p)$ = the lower 2.5 percentile of the chi-square distribution with $n-p$ degrees of freedom.

The interpretation of (A.3) is that we have 95% confidence that 95% of the property values y (over all vectors x) will fall within the tolerance bands (provided the equation from (A.1) adequately fits the data). Equation (A.3) is an extension of equation (30) on page 124 of Ref. 14.

A.2 95% CONFIDENCE BANDS FOR VARIOUS PROPERTIES

Modulus of Elasticity (10^6 psi)

$$\hat{E} \pm \sqrt{0.4606 \underline{x}' (X'X)^{-1} \underline{x}} \quad (A.4)$$

where $\underline{x}' = [1 \quad S \quad T \quad \log_e(t + 1)]$, $\hat{E}$ is given by (4), and

$$(X'X)^{-1} = \begin{bmatrix} 3.0226E-1 & -3.5888E-2 & -3.1970E-4 & -1.0128E-2 \\ -3.5888E-2 & 1.0323E-2 & -4.6499E-6 & -2.1691E-4 \\ -3.1970E-4 & -4.6499E-6 & 8.9565E-7 & 3.2632E-6 \\ -1.0128E-2 & -2.1691E-4 & 3.2632E-6 & 2.0813E-3 \end{bmatrix} \quad (A.5)$$

Compressive Strength (psi)

$$\hat{f}_c = \sqrt{1,240,608.6 \underline{x}' (X'X)^{-1} \underline{x}} \quad (A.6)$$

where $\underline{x}' = [1 \quad S \quad TA \quad \log_e(t + 1) \quad TB * \log_e(t + 1)]$, $\hat{f}_c$ is given by (5), and

$$(X'X)^{-1} = \begin{bmatrix} 3.6018E-1 & -6.3199E-2 & -2.7301E-4 & -1.9907E-2 & -3.3229E-5 \\ -6.3199E-2 & 1.6849E-2 & -1.7591E-5 & 1.1130E-4 & -6.8534E-7 \\ -2.7301E-4 & -1.7591E-5 & 5.3708E-6 & 1.1623E-5 & 5.1847E-7 \\ -1.9907E-2 & 1.1130E-4 & 1.1623E-5 & 4.0012E-3 & -3.4940E-6 \\ -3.3229E-5 & -6.8534E-7 & 5.1847E-7 & -3.4940E-6 & 2.1704E-7 \end{bmatrix} \quad (A.7)$$

Splitting Tensile Strength (psi)

$$\hat{f}_{st} = \sqrt{12,310.848 \underline{x}' (X'X)^{-1} \underline{x}} \quad (A.8)$$

where $\underline{x}' = [1 \quad S \quad TA \quad \log_e(t + 1)]$, $\hat{f}_{st}$ is given by (6), and

$$(X'X)^{-1} = \begin{bmatrix} 2.8935E-1 & -5.6444E-2 & -1.5953E-4 & -1.3687E-2 \\ -5.6444E-2 & 1.5066E-2 & -2.0168E-8 & -1.1318E-5 \\ -1.5953E-4 & -2.0168E-8 & 3.6334E-6 & 5.2182E-6 \\ -1.3587E-2 & -1.1318E-5 & 5.2182E-6 & 2.9283E-3 \end{bmatrix} \quad (A.9)$$

Strain (Millionths)

$$\hat{E} = \sqrt{72,881.307 \underline{x}' (X'X)^{-1} \underline{x}} \quad (A.10)$$

where $\underline{x}' = [1 \quad L \quad T \quad \log_e(t + 1)]$, $\hat{\epsilon}$ is given by (11), and

$$(X'X)^{-1} = \begin{bmatrix} 7.1038E-1 & -1.2771E-4 & -1.6669E-3 & -9.6386E-3 \\ -1.2771E-4 & 4.4272E-8 & 2.5022E-7 & -3.4731E-7 \\ -1.6669E-3 & 2.5022E-7 & 4.4243E-6 & -1.8999E-6 \\ -9.6386E-3 & -3.4731E-7 & -1.8999E-6 & 2.9464E-3 \end{bmatrix} \quad (A.11)$$

Creep Strain (Millionths)

$$\hat{\epsilon}_{cr} = \sqrt{72,881.307 \underline{x}'(X'X)^{-1} \underline{x} + 9.76 V} \quad (A.12)$$

where $\underline{x}' = [1 \quad L \quad T \quad \log_e(t + 1)]$, $\hat{\epsilon}_{cr}$ is given by (12), $(X'X)^{-1}$ is given by (A.11), and

$$V = \begin{cases} 5,625 & \text{when } L = 500, T = 350 \\ 13,225 & L = 1500, T = 250 \\ 297,025 & L = 1500, T = 350 \end{cases} \quad (A.13)$$

Note that V is merely the (standard error)² of $\bar{a}_0^{(a)}$ in (12). If an uncertain a_0 is obtained by other means, it's squared standard error should be substituted for V .

A.3 95%/95% TOLERANCE BANDS FOR VARIOUS PROPERTIES

Modulus of Elasticity (10^6 psi)

$$\hat{E} = 0.2181 \left[\sqrt{11.44 \underline{x}'(X'X)^{-1} \underline{x} + 2.19} \right] \quad (A.14)$$

where $\underline{x}' = [1 \quad S \quad T \quad \log_e(t + 1)]$, $\hat{E}$ is given by (4), and $(X'X)^{-1}$ is given by (A.5).

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- (a) Because $\bar{a}_0$ was obtained by averaging two a_0 values, the term standard error is used to differentiate the uncertainty of the average value from the uncertainty of individual values. If the uncertainty of a single a_0 is obtained in a different way, there may be no need to differentiate between standard error and standard deviation.

Compressive Strength (psi)

$$\hat{f}_c = 327.74 \left[\sqrt{13.5 \tilde{x}' (X'X)^{-1} \tilde{x}} + 2.27 \right] \quad (A.15)$$

where $\tilde{x}' = [1 \quad S \quad TA \quad \log_e(t+1) \quad TB \cdot \log_e(t+1)]$, $\hat{f}_c$ is given by (5), and $(X'X)^{-1}$ is given by (A.7).

Splitting Tensile Strength (psi)

$$\hat{f}_{st} = 35.44 \left[\sqrt{11.56 \tilde{x}' (X'X)^{-1} \tilde{x}} + 2.25 \right] \quad (A.16)$$

where $\tilde{x}' = [1 \quad S \quad TA \quad \log_e(t+1)]$, $\hat{f}_{st}$ is given by (6), and $(X'X)^{-1}$ is given by (A.9).

Strain (Millionths)

$$\hat{\epsilon} = 86.41 \left[\sqrt{11.52 \tilde{x}' (X'X)^{-1} \tilde{x}} + 2.22 \right] \quad (A.17)$$

where $\tilde{x}' = [1 \quad L \quad T \quad \log_e(t+1)]$, $\hat{\epsilon}$ is given by (11), and $(X'X)^{-1}$ is given by (A.11).

Creep Strain (Millionths)

$$\hat{\epsilon}_{cr} = 86.41 \sqrt{11.52 \tilde{x}' (X'X)^{-1} \tilde{x} + 0.00154 V + 2.22} \quad (A.18)$$

where $\tilde{x}' = [1 \quad L \quad T \quad \log_e(t+1)]$, $\hat{\epsilon}_{cr}$ is given by (12), $(X'X)^{-1}$ is given by (A.11) and V is given by (A.13) for the three PCA temperature x load combinations. If elastic strain and thermal expansion are adjusted using an uncertain a_0 obtained by other means (than from PCA data), then the squared standard deviation of a_0 should be substituted for V above.

APPENDIX B

ADAPTING CONCRETE PROPERTY EQUATIONS BASED ON
CONSTANT TEMPERATURE PCA TESTS TO VARIABLE TEMPERATURE HISTORIES

APPENDIX B

ADAPTING CONCRETE PROPERTY EQUATIONS BASED ON CONSTANT TEMPERATURE PCA TESTS TO VARIABLE TEMPERATURE HISTORIES

The modulus of elasticity, compressive strength, and splitting tensile strength equations (equations (4), (5), and (6) of Section 4.3) and the corresponding confidence and tolerance bands of Appendix A are based on the constant-temperature PCA data (Refs. 1 and 9). Hence, equations (4), (5), and (6) are directly applicable only for constant-temperature histories. However, the SAFE-CRACK code must be able to handle variable-temperature histories. The Anatech "incremental analysis" approach to adapting the "constant temperature" property equations and uncertainty bands for "variable temperature" situations was outlined in Ref. 17 and discussed by PNL in Ref. 18. The main findings of Ref. 18 are summarized below.

- Anatech's incremental analysis method involves differentiating the property equations with respect to time and then rewriting the equations in incremental form. The key element of the method is the calculation of a "history parameter" that (suitably re-parameterized) gives the time at the current increment's temperature necessary to yield the attained value (at that point in the time-temperature history) of the property.
- A method equivalent to the Anatech incremental analysis method has been formulated and is useful for understanding the Anatech method. A step-by-step description and graphical examples are given in Ref. 18.
- The Anatech incremental analysis method (and the equivalent method outlined in Ref. 18) has two main assumptions. First, it assumes that the property degradation during time increment n does not depend on the previous time-temperature history other than through the value of the property before increment n . Second, it assumes that no recovery or additional degradation occurs with an instantaneous change in temperature. Provided the assumptions inherent in the Anatech incremental analysis method are reasonable, it is a viable way to predict concrete properties for variable-temperature histories given constant-temperature equations.

- In the solution for t (time) in eq. (6) of Ref. 17, terms that are a function of t remain on both sides of the equation because of the quadratic nature of the equations involved. When using this approach to solve for the material properties, SAFE-CRACK iterates the solution for each time step several times in order to obtain the correct values at time t . Use of the linearized confidence and tolerance bands in Appendix C would simplify the solution and reduce or eliminate the iterative calculations required to compensate for this problem.
- The Anatech approach to uncertainty is to apply the incremental analysis method to the confidence and tolerance bands associated with the fitted property equations. This approach does not have a probabilistic interpretation. That is, applying the incremental analysis method to "constant temperature" 95% confidence and 95%/95% tolerance bands does not yield "variable temperature" 95% confidence and 95%/95% tolerance bands. The Anatech approach to uncertainty does have an intuitive interpretation to the extent that the 95% confidence and 95%/95% tolerance bands are "extreme cases" for expected and individual behavior, respectively. This "extreme cases" interpretation is still appropriate following the application of the incremental analysis method to the "constant temperature" 95% confidence and 95%/95% tolerance bands.
- Given that the Anatech incremental analysis method [when applied to the fitted property equations (4), (5), and (6)] produces concrete property predictions for variable-temperature histories, a reasonable goal would be to obtain uncertainty estimates for such predictions. However, as noted in the previous paragraph, the Anatech approach to uncertainty does not do this. Reference 18 presents a framework (based on the equivalent method presented therein) for calculating the variance of variable-temperature property predictions, from which interpretable probability statements could be made. Unfortunately, an analytic solution is very difficult, if not impossible. A Monte Carlo solution is probably the best option, although it may be infeasible to implement in SAFE-CRACK.

For additional details on the above findings, refer to Refs. 17 and 18.

APPENDIX C

LINEARIZING THE CONFIDENCE AND TOLERANCE BANDS FOR MODULUS OF
ELASTICITY, COMPRESSIVE STRENGTH, AND SPLITTING TENSILE STRENGTH

APPENDIX C

LINEARIZING THE CONFIDENCE AND TOLERANCE BANDS FOR MODULUS OF ELASTICITY, COMPRESSIVE STRENGTH, AND SPLITTING TENSILE STRENGTH

As noted in Appendix B, the use of the modulus of elasticity, compressive strength, and splitting tensile strength equations (equations (4), (5), and (6) of Section 4.3) and the corresponding confidence and tolerance bands of Appendix A is complicated by their "constant temperature" foundation and the need in SAFE-CRACK to address "variable temperature" histories. The incremental analysis method used by Anatech (see Ref. 17) to adapt the equations to variable-temperature histories is relatively straightforward for the fitted "linear" property equations (4), (5), and (6), but is more complicated for the hyperbolic-shaped confidence and tolerance bands.

Before continuing, the use of the term "linear" when referring to equations (4), (5), and (6) should be explained. For specified values of S (mix type) and T (temperature), the three property equations are linear functions of $\ln(t + 1)$. Note that when S and T are considered as variables and equations (4), (5), and (6) as fourth-dimensional response surfaces, only equation (4) is linear with respect to S , T , and $\ln(t + 1)$. However, it is the linearity with respect to $\ln(t + 1)$ in all three equations that we refer to.

Based on discussions with Anatech, Kaiser, and Westinghouse, it was decided to "linearize" the confidence and tolerance bands for modulus of elasticity, compressive strength, and splitting tensile strength (henceforth referred to as "concrete properties"). This decision was made to simplify the computations performed in SAFE-CRACK, and does not reflect on the merits of the hyperbolic Working-Hotelling bands presented in Appendix A. (The Working-Hotelling type bands are the most widely used and easily computed of the available options.)

While statistical methods exist for computing linear uncertainty bands, they are not easy to apply. Also, as noted in Appendix B, the Anatech incremental analysis method applied to "constant temperature" confidence and tolerance bands does not yield "variable temperature" confidence and tolerance bands with comparable probabilistic interpretations. For both these reasons, it was decided (in consultation with Kaiser and Westinghouse) to use an easier to compute and somewhat conservative method to linearize the bands. This method is described below.

An easy-to-apply linearization method was suggested by the nearly-linear appearance of the confidence and tolerance bands in Figures 4 to 21. These plots suggested that linear bands could be constructed by "connecting the end points" of the hyperbolic bands. Although this will introduce some conservatism in the resulting linear bands, the visual appearance is that the conservatism will not be excessive. To investigate this assumption further and to provide information for linearizing the hyperbolic bands, the widths of the bands (both confidence and tolerance) were computed for several combinations of S, T, and t. The results are given in Tables C.1, C.2, and C.3.

Tables C.1, C.2, and C.3 are each separated into two parts, one for the 95% confidence bands and one for the 95%/95% tolerance bands. The widths of the bands are given as plus/minus values (corresponding to the plus/minus portions of the formulas in Appendix A). Widths are given for several values of time (days in oven) for each of eight mix type-temperature combinations. Note that the tables contain band widths for values of t and T outside of the range of data used to develop the property equations. This is because of the way the equations are used in the new version of SAFE-CRACK (as presented in Section 4.4 and repeated here for convenience).

- In the range 250 to 450 F, the property equations are assumed to be valid from 0 to 18,250 days (0 to 50 years). The PCA data used to develop the equations ranged from 3-10 days to 920-1300 days. Hence, the equations will be extrapolated at both ends of the time scale within the 250 to 450 F temperature range.

- The property equations, omitting the $\ln(t + 1)$ terms, are assumed to be valid in the range 70 to 200 F. This corresponds to an assumption of negligible concrete degradation at temperatures below 200 F. Note that the equations still yield decreasing property values with increasing temperatures, but the decrease is no longer related to time-at-temperature.
- In the range 200 to 250 F, the $\ln(t + 1)$ terms in the property equations are phased in linearly via a multiplier taking the value 0 at 200 F and the value 1 at 250 F.

The above assumptions and uses of the property equations are due to Anatech, and are mentioned here to explain why it is necessary to consider the widths of the uncertainty bands at extrapolated conditions. Phone conversations with Y. R. Rashid of Anatech indicate that they have evaluated the extrapolative use of the property equations relative to other data and have determined such use to be reasonable.

Returning attention to Tables C.1, C.2, and C.3, widths are given for $t = 0$, t_s , and 18,250 days, where t_s is the number of days for which the width is smallest (usually around 100-150 days). The comparison of widths for $t = t_s$ to widths for $t = 0$ and $t = 18,250$ shows the extent of conservatism that will be introduced by linearizing the hyperbolic bands in a "connect the end points" fashion.

Note in Tables C.1 and C.3 that the $t = 0$ and $t = 18,250$ widths are very close for 250 to 450 F. There are noticeable differences between the $t = 0$ and $t = 18,250$ widths in Table C.2, but they are still relatively close. Further, in Tables C.1 and C.3 there is very little difference in widths over the six (S, T) combinations with $T = 250, 350, 450$ and $S = 3.0, 4.5$. Again, there are some differences over these six (S, T) combinations in Table C.2, but the widths are still relatively close. Finally, we compare the widths for $T = 70$ at $t = 0$ (an extrapolation on both the T and t scales) to the other values. In Tables C.2 and C.3 the widths are similar to the other values, while in Table C.1 the widths are somewhat larger (but not terribly so) than the other values. As a gauge to interpreting the widths and

differences in widths in Table C.1, compare the width of 0.252×10^6 at $T = 70$ and $S = 3.0$ to predicted values of modulus of elasticity that range from 4.0×10^6 to 1.4×10^6 for various combinations of t , T , and S .

The preceding observations suggest that over the ($250 \leq T \leq 450$, $3.0 \leq S \leq 4.5$, $0 \leq t \leq 18,250$) region, linearization could consist of the simple choice of a common width. This is an appealing result, since it yields uncertainty bands parallel to the fitted property equations and does not involve a dependency on T , S , or t . Table C.4 contains our recommendations for the common widths for the uncertainty bands. These recommendations are merely the largest widths (suitably rounded) given in Tables C.1, C.2, and C.3. Choosing the largest values provides for maintaining the specified 95% confidence levels relative to the conditions for which the fitted equations have the most uncertainty, while incorporating conservatism for other conditions.

Overall, the linear bands recommended in Table C.4 surely provide more than the nominal 95% confidence (confidence bands) and nominal 95% confidence of 95% coverage (tolerance bands). Computing the exact levels would involve considerable effort, and has not been performed. The need for doing so is questionable, however, when the confidence levels so obtained would only apply to constant temperature situations. As noted earlier and as discussed in Appendix B, the Anatech incremental analysis method does not allow for direct applicability of the "constant temperature" confidence levels to "variable temperature" applications.

TABLE C.1. Widths of 95% Confidence Bands and 95%/95% Tolerance Bands for Modulus of Elasticity(a)

Mix Type (S, 1000 psi)	Temperature (T, deg F)	Widths of 95% Confidence Bands (10 ⁶ psi)		
		t = 0 days	t = ts days(b)	t = 18,250 days
3.0	70	* .252	* .252	* .252(c)
4.5	70	* .258	* .258	* .258(c)
3.0	250	* .178	* .100	* .184
4.5	250	* .185	* .103	* .183
3.0	350	* .161	* .073	* .176
4.5	350	* .166	* .074	* .172
3.0	450	* .168	* .094	* .190
4.5	450	* .170	* .092	* .185

Mix Type (S, 1000 psi)	Temperature (T, deg F)	Widths of 95%/95% Tolerance Bands (10 ⁶ psi)		
		t = 0 days	t = ts days(b)	t = 18,250 days
3.0	70	* .751	* .751	* .751(c)
4.5	70	* .759	* .759	* .759(c)
3.0	250	* .672	* .586	* .678
4.5	250	* .678	* .590	* .676
3.0	350	* .652	* .557	* .669
4.5	350	* .658	* .559	* .655
3.0	450	* .660	* .580	* .685
4.5	450	* .663	* .578	* .679

(a) The widths contained in this table result from the * portions of Eq. (A.6) for the confidence bands and Eq. (A.14) for the tolerance bands.

(b) ts = time for which the width is smallest, usually in the 100-150 days range.

(c) Width does not change with t when $T \leq 200$.

TABLE C.2. Widths of 95% Confidence Bands and 95%/95% Tolerance Bands For Compressive Strength(a)

Mix Type (S, 1000 psi)	Temperature (T, deg F)	Widths of 95% Confidence Bands (psi)		
		t = 0 days	t = ts days(b)	t = 18,250 days
3.0	70	± 406	± 406	± 406(c)
4.5	70	± 406	± 406	± 406(c)
3.0	250	± 406	± 209	± 506
4.5	250	± 406	± 211	± 507
3.0	350	± 406	± 214	± 407
4.5	350	± 406	± 219	± 412
3.0	450	± 388	± 213	± 424
4.5	450	± 379	± 201	± 421

Mix Type (S, 1000 psi)	Temperature (T, deg F)	Widths of 95%/95% Tolerance Bands (psi)		
		t = 0 days	t = ts days(b)	t = 18,250 days
3.0	70	± 1182	± 1182	± 1182(c)
4.5	70	± 1182	± 1182	± 1182(c)
3.0	250	± 1182	± 970	± 1291
4.5	250	± 1182	± 972	± 1292
3.0	350	± 1182	± 975	± 1184
4.5	350	± 1182	± 980	± 1190
3.0	450	± 1163	± 974	± 1203
4.5	450	± 1154	± 962	± 1199

(a) The widths contained in this table result from the ± portions of Eq. (A.7) for the confidence bands and Eq. (A.15) for the tolerance bands.

(b) ts = time for which the width is smallest, usually in the 100-150 days range.

(c) Width does not change with t when $T \leq 200$.

TABLE C.3. Widths of 95% Confidence Bands and 95%/95% Tolerance Bands For Splitting Tensile Strength(a)

Mix Type (S, 1000 psi)	Temperature (T, deg F)	Widths of 95% Confidence Bands (psi)		
		<u>t = 0</u> days	<u>t = ts</u> days(b)	<u>t = 18,250</u> days
3.0	70	± 32.6	± 32.6	± 32.6(c)
4.5	70	± 32.6	± 32.6	± 32.6(c)
3.0	250	± 32.6	± 16.5	± 34.9
4.5	250	± 32.6	± 16.5	± 34.9
3.0	350	± 32.6	± 16.5	± 34.9
4.5	350	± 32.6	± 16.5	± 34.9
3.0	450	± 33.4	± 19.6	± 37.4
4.5	450	± 33.4	± 19.6	± 37.4

Mix Type (S, 1000 psi)	Temperature (T, deg F)	Widths of 95%/95% Tolerance Bands (psi)		
		<u>t = 0</u> days	<u>t = ts</u> days(b)	<u>t = 18,250</u> days
3.0	70	± 115	± 115	± 115(c)
4.5	70	± 115	± 115	± 115(c)
3.0	250	± 115	± 98	± 118
4.5	250	± 115	± 98	± 118
3.0	350	± 115	± 98	± 118
4.5	350	± 115	± 98	± 118
3.0	450	± 116	± 101	± 120
4.5	450	± 116	± 101	± 120

(a) The widths contained in this table result from the ± portions of Eq. (A.8) for the confidence bands and Eq. (A.16) for the tolerance bands.

(b) ts = time for which the width is smallest, usually in the 100-150 days range.

(c) Width does not change with t when $T \leq 200$.

TABLE C.4. Widths of "Linearized" 95% Confidence and 95%/95% Tolerance Bands

<u>Concrete Property</u>	<u>Confidence Band Width</u>	<u>Tolerance Band Width</u>
Modulus of elasticity (10 ⁶ psi)	± 0.26	± 0.76
Compressive strength (psi)	± 507	± 1292
Splitting tensile strength (psi)	± 37	± 120

Note: "Linearized" uncertainty bands are formed by adding and subtracting the appropriate widths from above to the fitted property equations (4), (5), and (6). Note that the resulting bands modify equations (4), (5), and (6) by adjusting only the constant terms.

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