UCRL-CR--105773

DE91 007155

REPORT

(Subcontract 2266703)

**ELECTROMAGNETIC PENETRATION THROUGH NARROW
SLOTS IN CONDUCTING SURFACES AND COUPLING
TO STRUCTURES ON THE SHADOW SIDE**

to

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July 1990

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ABSTRACT

Electromagnetic field penetration through a curved narrow slot in a planar conducting surface and coupling to a curved, loaded thin wire on the shadow side are determined in the time domain (TD) and the frequency domain (FD) by integral equation methods. Coupled integral equations are derived and solved numerically for the equivalent magnetic current in the slot (transverse slot voltage) and the electric current on the wire, from which the field that penetrates the slotted surface is determined. One employs a piecewise linear approximation of the unknown currents and performs equation enforcement by pulse testing. The resulting TD equations are solved by a scheme incorporating a finite-difference approximation for a second partial time derivative which allows one to solve for the unknown currents at a discrete time instant $t + 1$ in terms of the known excitation and currents calculated at a discrete time instant t and earlier. The FD equations are solved by the method of moments. One observes excellent agreement among the TD and FD computed data after the TD data are transformed via FFT.

A hybrid time-domain integral equation — finite-difference time-domain solution technique is described whereby one solves for the field which penetrates a slotted cavity-backed surface. One models the fields in the exterior region and in the slot with integral operators and models the fields inside the cavity with a discretized form of Maxwell's equations.

Narrow slots following various contours were chemically etched in thin brass sheets and an apparatus was fabricated to measure shadow-side fields, electric current on a thin wire on the shadow side, and, separately, fields inside a rectangular cavity which backed the slotted brass sheet. The experimentation was conducted

at the Lawrence Livermore National Laboratory on a frequency-domain test range (EMPEROR) employing a monocone source over a large ground plane. One observes very good agreement among the experimental and theoretical results.

ACKNOWLEDGMENTS

This work was performed under the auspices of the United States Department of Energy by Clemson University under subcontract 2266703 to the Lawrence Livermore National Laboratory. Lawrence Livermore National Laboratory is supported in this effort by contract W-7405-ENG-48 (United States Department of Energy) and by contract AFWL-87-217 (United States Air Force).

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CHAPTER I

INTRODUCTION

Development of the theory pertaining to the general problem of electromagnetic penetration through narrow slots in perfectly conducting screens is the primary purpose for the work described in this report. Equations are formulated and solutions are obtained in both the time domain and the frequency domain and the validity of these solutions is established by demonstrating close correlation between calculations and corresponding laboratory measurements. In the following sections the scope of the present investigation is outlined and pertinent related research is discussed.

Scope of Present Investigation

In this investigation, attention is focused on solutions of equations for three geometries related to finite-length narrow slots. The first geometry of interest is the narrow slot that follows an arbitrary contour in an infinite planar perfectly conducting screen. Integral equations for arbitrarily shaped finite-length narrow slots are formulated and solved in both the time domain and the frequency domain. The second geometry of interest is similar to the first but includes an arbitrarily directed thin wire scatterer on the shadow side of the screen. Again, time-domain and frequency-domain integral equations are formulated and solved. The third geometry is also similar to the first but instead of a scattering wire on the shadow side of the screen, the slotted screen is backed by a rectangular cavity. One solves this problem in the time domain with a hybrid technique that borrows from both integral equation and differential equation techniques. Each technique is employed in the region of the problem where its strengths apply.

As discussed above, integral equations play a significant role in the present work. An integral equation is an equation in which the unknown resides under an integral. The unknown in the integral equation for the narrow slot is the slot electric field or equivalent magnetic surface current. When a wire is placed on the shadow side of the slotted screen, an additional unknown, the electric surface current on the wire, appears in the slot's integral equation. Now an additional integral equation is required for the wire whose unknowns are the electric surface current on the wire and the equivalent magnetic surface current in the slot. The electromagnetic fields radiated by the slot (and the wire, if present) are calculated from knowledge of the slot's equivalent magnetic current (and the electric current on the wire) which is found by solution of the integral equation.

The general coupled slot-wire structure is illustrated in Figure 1-1, where one sees a slotted planar conducting screen illuminated by an incident field ($\mathbf{E}^{inc}, \mathbf{H}^{inc}$). The screen is considered to be vanishingly thin and of infinite extent. On the shadow side of the screen resides an arbitrarily shaped thin wire scatterer. Coupled integral equations are developed in both the time and frequency domains and the equations are solved to determine the equivalent magnetic current in the slot and the electric current on the wire from which the penetrated field is subsequently calculated. Laboratory measurements are described and the validity of the two solution techniques is established by demonstrating close correlation of calculated field values and wire currents with the corresponding laboratory measurements for a wide assortment of slot and wire geometries and excitations.

The slotted cavity-backed screen structure is illustrated in Figure 1-2, where one sees a slotted planar conducting screen which is backed by a rectangular cavity. A suitable time-domain integral representation is developed for the slot while the

volume of the cavity is divided into many cubic cells wherein Maxwell's equations in the time domain are directly enforced. Field-coupling mechanisms are defined that provide for illumination of the cavity by the slot and subsequent re-illumination of the slot by the fields that scatter from the walls of the cavity. Laboratory measurements are described and the validity of the hybrid technique is established by demonstrating close correlation of calculated field values with corresponding laboratory measurements.

A practical application of this work is in prediction of the shielding effectiveness of enclosures whose walls, by construction, contain slits and cracks. This work provides the basis for investigation of a wide range of slot geometries and also permits study of coupling to thin wires on the shadow side of the slotted surface.

Related Research Concerning Aperture Penetration and/or Radiation

Considerable work has been done on the problem of finding the properties of slots in conducting screens. Booker [1] was the first to apply Babinet's principle of optics to find the electromagnetic properties of slot antennas based on knowledge of strip and wire antennas. He recognized that Babinet's principle must be refined to account for the vector nature of electromagnetic fields. Specifically, he pointed out that thin wires or narrow strips are excited strongly by TM fields while narrow slots are excited strongly by TE fields, a distinction not made in the optics context. Booker stressed that Babinet's principle was valid only for vanishingly thin planar screens of infinite extent. He also suggested that leakage of electromagnetic fields through perforated conducting enclosures could be predicted by application of Babinet's principle.

R. W. P. King and Owyang [2] and [3] studied the distribution of the electric field in slot antennas and established correlation between measured data and

theoretical values based on analysis of cylindrical wire antennas. They also measured far-field quantities radiated by slot antennas, coupled slot antennas, and combinations of slot antennas and dipoles. They observed that the effective length of a slot depends on the thickness of the screen and indicated that the slot-strip-wire equivalence would break down in other ways if the conducting screen were not planar, vanishingly thin, and of infinite extent.

Mayes, Warren, and Wiesenmeyer [4] constructed a radiating structure which comprised a cavity-backed slot and a conical monopole operating on a large ground plane. They found that the combination of a slot and a monopole provided stable input impedance and a unidirectional radiation pattern (on the surface of the ground plane). A bandwidth ratio of up to 10:1 was achieved but they found that the efficiency of the radiator could fall to 10% at the low end of the band. Measurements of input impedance and radiation patterns of the structure are presented in their work.

Long [5,6] measured the impedance at the center of a slot antenna as a function of frequency and measured the radiated field of a combination of two coupled slots and a dipole. Long found good agreement between measurements and the far-field quantities predicted by assumed distributions of electric field in half-wavelength slots.

At this stage in the development of the theory, the ability to measure the relative distribution of electric field in an excited slot and in coupled slots had been demonstrated. However, little progress had been made toward accurate prediction of the slot electric field under other than resonant, uncoupled conditions. Also, precise prediction of near-field quantities in the neighborhood of a slot had not been demonstrated, especially for the case of coupled slots. Butler, Rahmat-Samii, and Mittra [7] review the formulation of integro-differential equations that describe the

behavior of the electric field (or equivalent surface magnetic current) in an aperture in a conducting screen of infinite extent. The formulation is based on application of the equivalence principle of electromagnetics and is generalized to allow non-planar geometries to be studied. These equations were solved numerically for the case of a slotted planar screen in order to find the slot electric field generated by specific incident excitation, and near-field quantities were calculated based on knowledge of the aperture electric field (or equivalent magnetic current). Coupling between a slot and a thin wire is also discussed based on numerical solution of coupled integro-differential equations and representative data are presented. These data show that, if one neglects the effects of coupling on the distribution of the slot electric field, significant errors can result.

Taflove and Umashankar [8] suggest a hybrid moment method – finite-difference time-domain (FDTD) method, based on Shelkunoff's third field equivalence theorem, for the solution of aperture perforated cavity problems. By this method, one covers (fills in) the aperture with conducting material and solves the exterior problem with a so called "patch code" which is a frequency-domain moment method code suitable for calculation of arbitrarily directed electric currents on conducting surfaces. The electric surface current on the shorted aperture is identified, the aperture is once again opened, and the "short-circuit" electric current that once existed on the shorted aperture is negated and forced to radiate in the now open aperture. One transforms the negated frequency-domain surface current into the time domain and subsequently excites the FDTD system of equations (that model the cavity) with this current. The authors imply that the resulting fields calculated inside the cavity by the FDTD equations are those that exist in the original problem. Whether or not the features of the problem's exterior region are adequately modeled by the

FDTD system of equations while the "short-circuit" current radiates is unclear in the authors' description of the method. In order for the fields of the interior and the exterior regions to be truly coupled, both the interior region and the exterior region must be modeled by the FDTD equations while the "short-circuit" current radiates in the aperture. Were this truly the case, there does not appear to be any advantage of this hybrid technique over a standard FDTD solution. If the interior region alone were excited by the "short-circuit" current, the influence on the internal fields by the features of the exterior region would not be completely considered.

Another significant development regarding the penetration of fields through finite-length narrow slots is Butler's [9] formulation of the finite-length narrow slot integral equation based on precise knowledge of the electric field in a narrow slot of infinite extent. Although the formulation is not based on wire concepts, a by-product of the effort is an analytical verification that the equivalent radius of a narrow slot is one quarter of its width [10].

Reed [11] describes a time-domain solution for straight narrow slots and coupled straight narrow slots which employs the known transverse variation of electric field in an infinite narrow slot. Shadow-side electric field data computed in the time domain are transformed to the frequency domain and are compared with data computed with the method-of-moments and data gathered experimentally. Good agreement is found among the time-domain computed data, the frequency-domain (MoM) computed data, and the experimental data. Shadow-side electric field behavior is explained on the basis of knowledge of the total axially directed equivalent magnetic current in the slot (or transverse slot voltage). With regard to the present effort, the time-domain solution technique mentioned above is based on a single-potential formulation which does not readily generalize to curved slots.

Fang and Wenbing [12] develop solutions for the transient fields radiated by a linear antenna. These solutions are based on the assumption (well founded) that the transient current along the antenna behaves as a traveling wave. The authors obtain the solutions directly by taking the Fourier transform of the transient current wave, finding the frequency domain vector potential and thus, by differentiation, the frequency-domain magnetic field generated by the current wave, and then recovering the time-domain transient nature of the magnetic field via inverse Fourier transform. Expressions for the radiated electric field are developed in a similar manner. The authors conclude that the transient fields radiated by a linear antenna are composed of transient wavelets that are radiated from the ends and the feedpoint of the antenna as the traveling current wave is reflected (causing acceleration and deceleration of charge) from these discontinuities in the "transmission line." Good agreement in the transient features of the radiated wave with numerically determined results is claimed. Due to the dual nature of thin wires and narrow slots, the results of this work are applicable to the solution of finite-length straight slot problems. This work also lends insight into the physical mechanisms involved in transient radiation by slots and wires.

The most recent work related to the present investigation was done by Hur and Lee [13] who describe the radiation properties of a thin strip coupled with a slotted planar conducting screen. Frequency-domain integral equations are formulated for the straight narrow slot and tilted straight strip after the fashion of Butler [14], and the coupled integral equations are solved by the method of moments. The authors show that the general form of the radiation is elliptically polarized and that, for appropriate choices of strip and slot length and strip orientation and distance from the slot, it is possible to achieve a circularly polarized wave.

Interest in the problem of field penetration through narrow slots continues as effective techniques to shield systems from electromagnetic radiation are pursued. It is hoped that the present work is a contribution to this effort in that it provides time-domain and frequency-domain integral equation formulations well suited to slots and wires of arbitrary contours, provides a new hybrid time-domain integral equation - finite-difference time-domain solution technique suitable for cavity backed slots and other related geometries, and provides a selection of associated experimental data not currently available in the literature.

Figure 1-1. Coupled Slot-Wire Structure

Figure 1-2. Slotted Cavity-Backed Screen Structure

CHAPTER II

INTEGRAL EQUATION FORMULATION FOR APERTURES AND SPECIALIZATION TO NARROW SLOTS

In this chapter are derived the equations for the unknown tangential electric field in an aperture of general shape in a conducting screen and subsequently specialize the equations to the case of a narrow slot of finite length. The formulation of the equations is aided by equivalent models based upon the equivalence principle and the jump discontinuity in tangential electric field created at a surface by the presence there of a surface magnetic current. The fields in the two regions separated by the screen are expressed as sums of the field due to the source and that due to the electric field (or equivalent magnetic current) in the aperture. They are written in terms of potential integrals over the free space Green's function ensuring that they satisfy Maxwell's equations and the radiation condition. The boundary condition on the conducting screen and continuity of tangential electric field through the aperture are satisfied, and the final integral equation follows from enforcing continuity of the tangential magnetic field along all paths through the aperture. Thus, the equations formulated here are guaranteed to produce a unique solution since they meet all requirements of the uniqueness theorem. The aperture in the conducting surface is described below and the derivation of the integral equation is presented in detail.

Equivalent Model of Aperture Perforated Screen

In Figure 2-1 is illustrated a perfectly conducting planar screen of infinite extent and of vanishing thickness in which an aperture A is cut. The aperture-perforated

screen separates two homogeneous half spaces of semi-infinite extent, each of material characterized by (μ, ϵ) . For convenience, the screen resides in the xy plane, and the upper half space ($z > 0$) is designated region a while the lower ($z < 0$) is designated region b . The aperture-perforated screen is illuminated by known sources in the two half spaces as illustrated in Figure 2-1. An electromagnetic model equivalent to the original structure and sources is developed for region a and another is developed for region b . Aided by these models, one writes expressions for the electric and magnetic field in each region and from these expressions readily arrives at an integral equation for the unknown tangential (to the xy plane) electric field $\mathbf{e}_{tan}^A$ in the aperture A .

To obtain an intermediate equivalent model one devises the structure of Figure 2-2b. The original sources are retained in the model as suggested in the figure as is the original material (μ, ϵ) in the two half spaces. Then one shorts the aperture A in the screen, i.e., fills in the hole with a vanishingly thin planar conductor, and places a surface magnetic current $\mathbf{m}$ of density $\mathbf{m} = \mathbf{e}^A \times \hat{\mathbf{z}}$ over the region- a side of the shorted aperture and places $-\mathbf{m}$ over the region- b side, where $\hat{\mathbf{z}}$ is the unit vector along the positive z axis (perpendicular to the screen). The continuous screen with the aperture shorted and the surface magnetic currents are illustrated in cross-section in Figure 2-2b.

Since the tangential electric field is zero directly on the surface of a perfect electric conductor (pec) and since a magnetic surface current causes a jump discontinuity proportional to the value of the magnetic current, the presence of $\mathbf{m}$ on the region- a side of the shorted aperture gives rise to a jump in tangential electric field from zero immediately on the surface of the short to $\mathbf{e}_{tan}^A$ at $(x, y, 0^+)$ for (x, y) in A . Similarly, $-\mathbf{m}$ serves to cause the tangential electric field to jump from zero on

the region- b side of the shorted aperture to $\mathbf{e}_{tan}^A$ at $(x, y, 0^-)$ for (x, y) in A . Thus, the limit of the tangential component of electric field in region a is $\mathbf{e}_{tan}^A$ as the point of observation (x, y, z) approaches the aperture ((x, y) in A and $z \downarrow 0^+$) and that of the tangential component in region b is also $\mathbf{e}_{tan}^A$ as (x, y, z) approaches the aperture ((x, y) in A and $z \uparrow 0^-$). Consequently, in this special limiting sense where the path never actually penetrates the vanishingly thin surface current on either side of the shorted aperture, the tangential component of electric field is continuous along any path through the aperture. (The reader should note that *continuity*, as it is used in the special limiting sense above, is not in keeping with the mathematical definition of continuity. This license is taken in the interest of brevity.)

The model of Figure 2-2b has half spaces extending to infinity and sources identical to those of the original problem. Now, since the tangential electric field is zero on the conducting screen for $(x, y) \in A$ and approaches $\mathbf{e}_{tan}^A$ in the aperture due to the presence of the two equivalent magnetic surface currents which reside over the shorted aperture, one is assured by the uniqueness theorem that the model of Figure 2-2b is electromagnetically equivalent to the original structure and sources of Figure 2-1.

To simplify the derivation of expressions for the field in the two regions of Figure 2-2b, one employs image theory to obtain the two models of Figures 2-3c and 2-4c. The model of Figure 2-3b is the same as that of Figure 2-2b for region a while the model of Figure 2-4b is the same as that of Figure 2-2b for region b . These models are repeated in Figures 2-3 and 2-4 for convenience. One notes that due to the presence of the shorted screen the coupling between regions a and b is entirely through the related magnetic currents $\mathbf{m}$ and $-\mathbf{m}$. For $z > 0$ the model of Figure 2-3c is equivalent to that of Figure 2-3b in region a and for $z < 0$ the model

of Figure 2-4c is equivalent to that of Figure 2-4b in region b . These equivalences follow from simple application of image theory.

One notes that there is no conducting screen in Figure 2-3c but that the tangential component of electric field is held to zero on the xy plane due to the presence of m and its image and the presence of the original region- a sources and their images. With its value clamped to zero on the xy plane, the tangential electric field jumps to its correct value at $z = 0^+$ for (x, y) in A due to m . Similar comments may be made about Figure 2-4c, the region b equivalent model valid for $z < 0$. The advantage of the equivalent models of Figures 2-3c and 2-4c over that of Figure 2-2b is that, in the case of the former pair, all sources reside in infinite homogeneous space allowing one to express the fields as functions of potential integrals with the free space Green's function. On the other hand, on the basis of the model of Figure 2-2b, as well as of those of Figures 2-3b and 2-4b, one must compute the fields from the sources m and (j^a, m^a) and (j^b, m^b) which radiate in the presence of the conducting screen, a computation which is more difficult than that based on the models of Figures 2-3c and 2-4c. Of course, the models of Figures 2-3c and 2-4c are nothing more than the image theory equivalents of the region a and region b half-spaces of Figure 2-2b.

With the models of Figures 2-3c and 2-4c available it is an easy matter to compose expressions for the fields in the two half spaces. In these models, (j^a, m^a) and (j^b, m^b) are known sources and $(j^{a'"}, m^{a'"})$ and $(j^{b'"}, m^{b'"})$ are their images, while the magnetic currents in these figures comprise the equivalent magnetic current and its image to form $2m$ and $-2m$ on the xy plane in A . The field due to the known source currents is determined by computing the sum of that due to (j^a, m^a) and (j^b, m^b) and that due to the images $(j^{a'"}, m^{a'"})$ and $(j^{b'"}, m^{b'"})$.

This computation can be done in a number of ways depending upon the precise nature of the sources. To write expressions for the fields in the two regions due to the magnetic current $2m$ (Figure 2-3c) or $-2m$ (Figure 2-4c), one takes advantage of the electric vector potential $\mathbf{f}$ and the magnetic scalar potential ψ , since these potentials are simply integrals of the magnetic current against the known free space Green's function. The equivalent models of Figures 2-3c and 2-4c were developed specifically to take advantage of these simple potential integrals.

Formulation of Equation for Aperture in Screen

In this section are developed the equations for the unknown equivalent magnetic current m (or $\mathbf{e}_{tan}^A \times \hat{\mathbf{z}}$). One finds it convenient to appeal to superposition and write the field in each region as the sum of that due to the known sources and that due to the presence of the aperture. This use of superposition is suggested by the equivalent models of Figures 2-3 and 2-4 from which one sees that the total field in each region can be written as the sum of that due to the known impressed sources and of that due to the equivalent magnetic current m . One observes that the field due to the impressed sources is independent of the presence of the aperture – the aperture is shorted in Figures 2-3b and 2-4b – and, further, one observes that the field due to m accounts for the presence of the aperture.

Guided by the equivalent models of Figures 2-3 and 2-4, one readily sees that the field due to the known sources can be determined immediately. This field is simply due to $(\mathbf{j}^a, m^a)$ and $(\mathbf{j}^b, m^b)$ radiating in the presence of the shorted aperture (Figures 2-3b and 2-4b) or, equivalently, that due to $(\mathbf{j}^a, m^a)$ and its image plus that due to $(\mathbf{j}^b, m^b)$ and its image, all radiating in homogeneous space (Figures 2-3c and 2-4c). Because $(\mathbf{j}^a, m^a)$ and $(\mathbf{j}^b, m^b)$ radiate in the presence of the screen with the aperture shorted (Figures 2-2b, 2-3b, and 2-4b), one gives this known field the

name *short-circuit field*, and designates it $(\mathbf{e}^{sc^a}, \mathbf{h}^{sc^a})$ in region a and $(\mathbf{e}^{sc^b}, \mathbf{h}^{sc^b})$ in region b . Clearly, it is a simple matter to determine the so-called short-circuit fields from the specified sources $(\mathbf{j}^a, \mathbf{m}^a)$ and $(\mathbf{j}^b, \mathbf{m}^b)$.

The field due to the presence of the aperture is computed from the equivalent magnetic current $\mathbf{m}$ and its image $(\mathbf{m})$ – see Figures 2-3c and 2-4c. This magnetic current or its equivalent $\mathbf{e}_{tan}^A \times \hat{\mathbf{z}}$ is the unknown of the integral equation to be formulated below. In the derivation of these integral equations, the electric and magnetic fields due to $\mathbf{m}$ and its image are written in terms of the vector operators $\mathcal{E}^A$ and $\mathcal{H}^A$, which in turn are expressed as functions of the electric vector potential $\mathbf{f}$ and the magnetic scalar potential ψ .

Specific expressions for the electric and magnetic fields $(\mathbf{e}^a, \mathbf{h}^a)$ in region a are

$$\mathbf{e}^a = \mathbf{e}^{sc^a} + \mathcal{E}^A[2\mathbf{m}], \quad z > 0 \quad (2.1a)$$

and

$$\mathbf{h}^a = \mathbf{h}^{sc^a} + \mathcal{H}^A[2\mathbf{m}], \quad z > 0 \quad (2.1b)$$

while the fields $(\mathbf{e}^b, \mathbf{h}^b)$ in region b are

$$\mathbf{e}^b = \mathbf{e}^{sc^b} + \mathcal{E}^A[-2\mathbf{m}], \quad z < 0 \quad (2.2a)$$

and

$$\mathbf{h}^b = \mathbf{h}^{sc^b} + \mathcal{H}^A[-2\mathbf{m}], \quad z < 0 \quad (2.2b)$$

where the operators are defined by

$$\mathcal{E}^A[\mathbf{m}] = -\frac{1}{\epsilon} \nabla \times \mathbf{f}(\mathbf{r}; t) \quad (2.3a)$$

and

$$\mathcal{H}^A[\mathbf{m}] = -\frac{\partial}{\partial t} \mathbf{f}(\mathbf{r}; t) - \nabla \psi(\mathbf{r}; t) \quad (2.3b)$$

in terms of the electric vector potential

$$\mathbf{f}(\mathbf{r}; t) = \epsilon \iint_A \mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS' \quad (2.4a)$$

and magnetic scalar potential

$$\psi(\mathbf{r}; t) = \frac{1}{\mu} \iint_A \mathfrak{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS'. \quad (2.4b)$$

The surface magnetic current density $\mathbf{m}$ and the surface magnetic charge density $\mathfrak{m}$ are related by the equation of continuity,

$$\nabla_s \cdot \mathbf{m}(\mathbf{r}; t) + \frac{\partial}{\partial t} \mathfrak{m}(\mathbf{r}; t) = 0, \quad (2.5a)$$

and

$$R = |\mathbf{r} - \mathbf{r}'| \quad (2.5b)$$

is the distance from $\mathbf{r}'$ locating a source point to $\mathbf{r}$ locating the point of observation, while

$$\tau = t - R/c \quad (2.5c)$$

is *delayed time* in which t is time measured from some reference instant $t = 0$ and c is the speed of light. τ is called delayed time because it stands for absolute time t less the time delay R/c from the instant a disturbance occurs at the source location $\mathbf{r}'$ to the instant it is sensed at the observation point $\mathbf{r}$, under the condition that the disturbance travels at the speed of light $c (= \frac{1}{\sqrt{\mu\epsilon}})$ in the medium.

The above expressions for electric and magnetic fields satisfy all but one condition imposed by electromagnetic theory upon them in the two regions, in the

aperture, and on the screen. This remaining condition to be satisfied is that the tangential component of magnetic field must be continuous along any path through the aperture. Consequently, one next enforces this condition:

$$\lim_{z \rightarrow 0} h_{tan}^a = \lim_{z \rightarrow 0} h_{tan}^b, \quad \mathbf{r} \in \mathcal{A}, t > 0 \quad (2.6a)$$

or simply

$$h_{tan}^a|_{z=0+} = h_{tan}^b|_{z=0-}, \quad \mathbf{r} \in \mathcal{A}, t > 0. \quad (2.6b)$$

Replacing h^a and h^b by their equivalents from Equations (2.1b) and (2.2b), one converts (2.6b) to

$$\left\{ h^{sc^a} + \mathcal{H}^A[2\mathbf{m}] \right\}_{tan} = \left\{ h^{sc^b} + \mathcal{H}^A[-2\mathbf{m}] \right\}_{tan}, \quad \mathbf{r} \in \mathcal{A}, t > 0 \quad (2.7a)$$

or

$$\left\{ \mathcal{H}^A[2\mathbf{m}] - \mathcal{H}^A[-2\mathbf{m}] \right\}_{tan} = \left\{ h^{sc^b} - h^{sc^a} \right\}_{tan}, \quad \mathbf{r} \in \mathcal{A}, t > 0 \quad (2.7b)$$

and, due to the linearity of the operator $\mathcal{H}^A$, to

$$2\mathcal{H}_{tan}^A[2\mathbf{m}] = -h_{tan}^{sc}, \quad \mathbf{r} \in \mathcal{A}, t > 0 \quad (2.7c)$$

where

$$h^{sc} = \{ h^{sc^a} - h^{sc^b} \}. \quad (2.7d)$$

Equation (2.7c) is an operator equation for the unknown magnetic current $\mathbf{m}$, with known forcing function h^{sc} .

Inspection of (2.7c), (2.3b), and (2.4) reveals that the operator equation actually depends directly upon the magnetic charge as well as upon the magnetic current. Since it is desirable to remove the direct dependency upon $\mathbf{m}$ in favor of

dependency upon the single unknown $\mathbf{m}$, one finds it expedient to differentiate both sides of (2.7c) with respect to t in order to be able to employ the relationship in Equation (2.5a) between $\mathbf{m}$ and $\mathbf{m}$:

$$2 \frac{\partial}{\partial t} \chi_{tan}^A [2\mathbf{m}] = - \frac{\partial}{\partial t} \mathbf{h}_{tan}^{*c}, \quad \mathbf{r} \in A, t > 0. \quad (2.8)$$

Making use of (2.3b) and (2.4), one readily expresses (2.8) in the more explicit form,

$$\begin{aligned} -2 \left\{ \epsilon \frac{\partial^2}{\partial t^2} \iint_A 2\mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS' + \frac{1}{\mu} \nabla \iint_A 2 \frac{\partial}{\partial t} \mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS' \right\}_{tan} \\ = - \frac{\partial}{\partial t} \mathbf{h}_{tan}^{*c}(\mathbf{r}; t), \quad \mathbf{r} \in A, t > 0, \end{aligned} \quad (2.9)$$

which can be written in terms of the single unknown $\mathbf{m}$ as

$$\begin{aligned} -2 \frac{1}{\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \iint_A 2\mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS' - \nabla \iint_A 2 \nabla' \cdot \mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS' \right\}_{tan} \\ = - \frac{\partial}{\partial t} \mathbf{h}_{tan}^{*c}(\mathbf{r}; t), \quad \mathbf{r} \in A, t > 0. \end{aligned} \quad (2.10)$$

As suggested above, the time differentiation of (2.7c) leads to (2.9) from which $\mathbf{m}$ is eliminated via (2.5a) to obtain (2.10), an integro-differential equation in the single unknown $\mathbf{m}$.

Equation (2.10) is an integro-differential equation in the unknown equivalent magnetic current $\mathbf{m}$, or $\mathbf{e}_{tan}^A \times \hat{\mathbf{z}}$, for the aperture-perforated screen separating two homogeneous half-spaces characterized by (μ, ϵ) subject to the known excitation $\mathbf{h}^{*c}$ ($= \{\mathbf{h}^{*c^a} - \mathbf{h}^{*c^b}\}$). This equation is an explicit version of Equations (2.6) and (2.7) and is simply enforcement of continuity of tangential magnetic field in the aperture.

Equation for Finite-Length Narrow Slot in a Screen

In this section attention is turned to the development of the integral equation for m in a finite-length narrow slot in a screen such as that depicted in Figure 2-5. One develops the desired integral equation by imposing simplifying approximations (peculiar to a long narrow slot in a screen) on Equation (2.10) which is valid for the aperture of general shape. These simplifying approximations are founded upon observations made about the time-harmonic electromagnetic field in an infinitely long slot that is narrow relative to the wavelength of the time-harmonic signal. Thus, the approximations hold only for those time-domain signals whose Fourier transforms have significant spectral content with wavelength that is long relative to the width of the slot.

In the case of a screen with an infinite slot that is narrow relative to the wavelength of the field of interest, it is known that the only significant component of tangential electric field in the slot is transverse to the slot axis and that this field is a result of excitation that is transverse electric to the axis. Therefore, the total excitation of the slot is taken to be the component of short-circuit magnetic field directed along the slot axis, while the transverse component of the excitation is ignored. If one assumes that a slot which is long relative to its width and narrow relative to wavelength exhibits the major electromagnetic features of a narrow slot of infinite length, at least to a high degree of approximation over the portion of the slot away from the immediate vicinity of the ends, one claims that the equivalent magnetic current m possesses only a component in the direction of the slot axis in keeping with the approximation that the slot electric field has no axial component. For the long, narrow slot ($L \gg w$) of Figure 2-5, subject to excitation whose spectrum satisfies the band-limiting condition that its shortest wavelength of significance is long relative to $2w$ as discussed above, one approximates the equivalent

magnetic current by

$$\mathbf{m}(\mathbf{r}; t) = m(x, y; t) \hat{\mathbf{y}} \quad (2.11)$$

having only a y component, whose surface divergence in (2.10) is simply

$$\nabla_s \cdot \mathbf{m}(\mathbf{r}; t) = \frac{\partial}{\partial y} m(x, y; t). \quad (2.12)$$

Since only the part of the excitation that is transverse electric to the axis of a narrow slot causes a significant slot field, one sees that the known short-circuit magnetic field of (2.10) simplifies to

$$\mathbf{h}_{tan}^{sc}(\mathbf{r}; t) = h_y^{sc}(x, y; t) \hat{\mathbf{y}}, \quad (2.13)$$

which is the forcing function for the equation specialized to the narrow slot structure of Figure 2-5. It is further known from investigations of the narrow slot of infinite length that the transverse variation of the tangential component of electric field in the slot is proportional to $[u^2 - x^2]^{-1/2}$ for excitation that is not exclusively an odd function with respect to x , a highly unlikely situation, and that the tangential component of magnetic field in the slot due to the magnetic current is essentially invariant with respect to transverse displacement, *i.e.*, with respect to x [9]. One recalls from (2.6) through (2.8) that (2.10) is a statement of equality of the time derivative of tangential magnetic field in the aperture – simplified to a slot in the present discussion – so, in view of the invariance of tangential magnetic field in the slot, he concludes that (2.10), specialized to the case of the slot of Figure 2-5, must be independent of x . In other words, the general distance

$$R(\mathbf{r}, \mathbf{r}') = \sqrt{(x - x')^2 + (y - y')^2} \quad (2.14a)$$

in (2.10) between field point (x, y) and source point (x', y') can be simplified for the case of the slot to

$$R(\mathbf{r}, \mathbf{r}') \approx \sqrt{x'^2 + (y - y')^2} \quad (2.14b)$$

in which one selects the representative field point to be $(0, y)$ on the slot axis. The rationale for this selection is simply that, since (2.10) specialized to the case of the narrow slot is independent of x , one chooses $x = 0$ as a representative value of x for $x \in (-w, w)$. Subject to the simplifications of (2.11) through (2.14) based on the narrow slot approximations, Equation (2.10), particularized to the case of the finite-length narrow slot of Figure 2-5, reduces to

$$\begin{aligned} & -\frac{1}{2\pi\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{-L^*}^{L^*} \int_{-w}^w 2m(x', y'; \tau) \frac{1}{\sqrt{x'^2 + (y - y')^2}} dx' dy' \right. \\ & \quad \left. - \frac{\partial}{\partial y} \int_{-L^*}^{L^*} \int_{-w}^w 2 \frac{\partial}{\partial y'} m(x', y'; \tau) \frac{1}{\sqrt{x'^2 + (y - y')^2}} dx' dy' \right\} \\ & = -\frac{\partial}{\partial t} h_y^{*c}(y; t), \quad y \in (-L^*, L^*), \quad t > 0. \end{aligned} \quad (2.15)$$

One notes that (2.15) is a scalar integro-differential equation for the unknown slot equivalent magnetic current m , where $\mathbf{m} = m\hat{\mathbf{y}}$. Equation (2.10) for the general aperture in a screen is a vector equation, which, when specialized to the equation for a narrow slot, becomes a vector equation with only a y component or a scalar equation (Equation (2.15)) for the y component.

Equation (2.15) can be converted to a different form which is essentially the same equation that governs the behavior of the current induced on a thin conducting wire. Because the wire equation has received so much attention, it is worthwhile to make this conversion in order to take advantage of what knowledge is available from

thin-wire theory. First, one makes explicit use of the known transverse variation of m and, second, introduces a change of variables to convert (2.15) into an equation whose form is recognized to be that for the current on a thin wire.

The magnetic current m can be written as

$$m(x, y; t) = \frac{\frac{1}{\pi}}{\sqrt{w^2 - x^2}} k(y; t) \quad (2.16)$$

which incorporates the known transverse variation and in which k is the total axial magnetic current possessing m 's variation with y along the slot axis and its dependence upon time t . To effect the variable transformation, one focuses attention on the first integral with respect to x' in (2.15),

$$I = \int_{-w}^w 2m(x', y'; \tau) \frac{1}{\sqrt{x'^2 + (y - y')^2}} dx', \quad (2.17)$$

and replaces m by the right side of (2.16) to arrive at

$$I = \int_{-w}^w 2 \frac{\frac{1}{\pi}}{\sqrt{w^2 - x'^2}} k(y'; \tau) \frac{1}{\sqrt{x'^2 + (y - y')^2}} dx', \quad (2.18)$$

and then makes the change of variables

$$x' = w \sin \frac{\alpha'}{2},$$

to obtain

$$I = 2k(y'; \tau) \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\sqrt{(y - y')^2 + w^2 \sin^2 \frac{\alpha'}{2}}} d\alpha' \quad (2.19a)$$

or

$$I = 2k(y'; \tau) k^0(y - y') \quad (2.19b)$$

where

$$k^*(y - y') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\sqrt{(y - y')^2 + w^2 \sin^2 \frac{\alpha'}{2}}} d\alpha'. \quad (2.20)$$

The second integral with respect to x' in (2.15) can be transformed by the same procedure. Making use of these transformations, one readily converts (2.15) to

$$\begin{aligned} -\frac{1}{\pi\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{-L^*}^{L^*} k(y'; \tau) k^*(y - y') dy' - \frac{\partial}{\partial y} \int_{-L^*}^{L^*} \frac{\partial}{\partial y'} k(y'; \tau) k^*(y - y') dy' \right\} \\ = -\frac{\partial}{\partial t} h_y^{*c}, \quad y \in (-L^*, L^*), t > 0 \quad (2.21) \end{aligned}$$

which is, of course, the integro-differential equation for the unknown total axial magnetic current k of the finite-length narrow slot in the screen depicted in Figure 2-5. k^* of (2.21) is called the kernel of the integral equation and is known, as is the forcing function $-\frac{\partial}{\partial t} h_y^{*c}$. One readily observes that (2.21) is the same equation as that for the thin wire with proper interpretation of terms and parameters. Note in particular that the kernel k^* is the same as the wire kernel if the slot width $2w$ is equal to one-fourth the wire radius. Obviously, the techniques for solving the wire equation can be used directly to solve (2.21). Often, for very narrow slots, the exact kernel of (2.20) can be approximated by the so-called reduced kernel or approximate kernel,

$$k^*(y - y') \approx \frac{1}{\sqrt{(y - y')^2 + (w/2)^2}}, \quad (2.22)$$

which is easier than (2.20) to compute since one need not perform the integration in α' that is required in (2.20) and also avoids the singularity that arises when $y = y'$ and $\alpha' = 0$.

Equation for Curved Narrow Slot in a Screen

An aperture geometry of major interest in the present study is the curved narrow slot illustrated in Figure 2-6. The slot width $2w$ is the same all along the contour C' of the slot and it is narrow in the sense described above for the finite-length straight slot. The equation for the unknown equivalent magnetic current for the curved slot can be obtained by generalizing (2.21) for the straight slot or it can be deduced by simplification of Equation (2.10) for the aperture of general shape. Here one chooses to derive the curved-slot equation by an abbreviation of the procedure leading from (2.10) to (2.21).

The aperture A is specialized to the shape of the slot of Figure 2-6 and the simplifications adopted in the straight slot case are assumed to hold for the curved slot, subject, of course, to proper interpretation. The tangential electric field in the slot is again assumed to have only a component in the direction transverse to the slot axis, and the variation of this field with respect to displacement transverse to the slot axis is assumed to be the same as that in the straight slot. Consequently, the equivalent magnetic current $\mathbf{m}$ remains a vector but one with only a component in the axial direction at any given point on the slot axis designated C' . The positive axial direction in Figure 2-6 is denoted by the unit vector $\hat{s}$, which is in the plane of the screen, is tangential to the contour C' , and has sense of increasing arc displacement s . Because the slot is narrow, it is further assumed that only the axial component of short-circuit magnetic field gives rise to a significant slot response, so the transverse component of short-circuit magnetic field is ignored. If advantage is taken of these approximations and simplifications, one can show by simplification of (2.10), or generalization of (2.21), that the total axial equivalent magnetic current k for the curved narrow slot is governed by the integro-differential equation

$$\begin{aligned}
& -\frac{1}{\pi\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{C'} k(s'; \tau) [\hat{s}'(s') \cdot \hat{s}(s)] k^*(s, s') ds' - \frac{\partial}{\partial s} \int_{C'} \frac{\partial}{\partial s'} k(s'; \tau) k^*(s, s') ds' \right\} \\
& = -\frac{\partial}{\partial t} [\hat{s}(s) \cdot \mathbf{h}^{*o}(s; t)], \quad s \in C', t > 0. \quad (2.23)
\end{aligned}$$

The kernel in (2.23) for the curved narrow slot is by

$$k^*(s, s') = \frac{1}{\sqrt{[d(s, s')]^2 + (w/2)^2}} \quad (2.24a)$$

in which

$$d(s, s') = |d(s) - d'(s')| \quad (2.24b)$$

is the distance from a source point on C' located by d' to an observation point on C' located by d in which the two vectors are dependent upon arc displacement s measured from a common reference $s = 0$ on C' . Obviously, (2.23) is a statement that (the time derivative of) the axial component of the magnetic field of region a is equal to the axial component of the magnetic field of region b for points on the slot axis.

The scalar product $[\hat{s}'(s') \cdot \hat{s}(s)]$ in the first integral of (2.23) serves to extract the component of the magnetic field in the direction $\hat{s}$ contributed at s by the axial magnetic current $k(s'; \tau)\hat{s}'(s')$ at s' . In the second integral, $\frac{\partial}{\partial s'} k(s'; \tau)$ is the magnetic charge density (written with explicit dependence upon the (source point) variable of integration) associated with an equivalent magnetic current which has only a component in the direction of the unit vector $\hat{s}'$. This is a specialization of the first term under the second integral of (2.10). The gradient operator on the second integral of (2.10) simplifies to the derivative operator $-\frac{\partial}{\partial s'}$ on the second integral

of (2.23) because continuity of only the axial component, i.e., the $\hat{s}$ component, of the magnetic field is enforced in the slot.

Finally, the kernel $k'(s, s')$ is approximate and it is the reciprocal of the distance from the source to observation points on the axis, augmented by one-fourth of the slot width ($2w/4$) according to the expression in (2.24). In a numerical solution of (2.23) one employs the exact kernel of (2.20) in those computations for which doing so is justified (see Chapter IV).

Frequency-Domain Equations

The time-domain integral equations above can be converted to their frequency-domain counterparts by means of Fourier transformation. The Fourier transform pair employed here is

$$\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt \quad (2.25a)$$

and

$$\mathcal{F}^{-1}\{F(\omega)\} = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} d\omega \quad (2.25b)$$

and the two theorems below prove useful:

$$\mathcal{F}\left\{\frac{\partial}{\partial t}f(t)\right\} = j\omega\mathcal{F}\{f(t)\} \quad (2.26a)$$

and

$$\mathcal{F}\{f(t - t_0)\} = e^{-j\omega t_0} \mathcal{F}\{f(t)\}. \quad (2.26b)$$

One observes that, since $\tau = t - R/c$, (2.26b) implies

$$\mathcal{F}\{m(\mathbf{r}'; \tau)\} = M(\mathbf{r}')e^{-j\omega R/c} = M(\mathbf{r}')e^{-jkR} \quad (2.27)$$

where $\mathbf{M}(\mathbf{r}')$ is the transform of $\mathbf{m}(\mathbf{r}'; t)$ and in which k stands for ω/c ($= \omega\sqrt{\mu\epsilon}$).

In addition, (2.26a) and (2.27) imply

$$\mathcal{F}\left\{\frac{\partial^2}{\partial t^2}\mathbf{m}(\mathbf{r}'; \tau)\right\} = -\omega^2\mathbf{M}(\mathbf{r}')e^{-jkR}. \quad (2.28)$$

By taking the transform of both sides of (2.10) and invoking (2.27) and (2.28), one arrives at

$$\begin{aligned} -2\frac{1}{\mu}\left\{-\frac{\omega^2}{c^2}\iint_A 2\mathbf{r}'\cdot\mathbf{r}'\frac{e^{-jkR(\mathbf{r},\mathbf{r}')}}{4\pi R(\mathbf{r},\mathbf{r}')}dS' - \nabla\iint_A 2\nabla'\cdot\mathbf{M}(\mathbf{r}')\frac{e^{-jkR(\mathbf{r},\mathbf{r}')}}{4\pi R(\mathbf{r},\mathbf{r}')}dS'\right\}_{tan} \\ = -j\omega\mathbf{H}_{tan}^{sc}(\mathbf{r}), \quad \mathbf{r} \in A, \end{aligned} \quad (2.29)$$

where $\mathbf{H}^{sc}(\mathbf{r})$ is the transform of $\mathbf{h}^{sc}(\mathbf{r}, t)$. The above equation readily simplifies to

$$\begin{aligned} -j\frac{4}{k\eta}\left\{k^2\iint_A \mathbf{M}(\mathbf{r}')\frac{e^{-jkR(\mathbf{r},\mathbf{r}')}}{4\pi R(\mathbf{r},\mathbf{r}')}dS' + \nabla\iint_A \nabla'\cdot\mathbf{M}(\mathbf{r}')\frac{e^{-jkR(\mathbf{r},\mathbf{r}')}}{4\pi R(\mathbf{r},\mathbf{r}')}dS'\right\}_{tan} \\ = -\mathbf{H}_{tan}^{sc}(\mathbf{r}), \quad \mathbf{r} \in A, \end{aligned} \quad (2.30)$$

where $\eta = \sqrt{\mu/\epsilon}$.

The curved-slot equation (2.23) can be transformed and simplified by the procedure outlined above:

$$\begin{aligned} -\frac{j}{k\eta\pi}\left\{k^2\int_{C'} K(s')[\hat{\mathbf{s}}'(s')\cdot\hat{\mathbf{s}}(s)]K^*(s,s')ds' + \frac{\partial}{\partial s}\int_{C'} \frac{\partial}{\partial s'}K(s')K^*(s,s')ds'\right\} \\ = -[\hat{\mathbf{s}}(s)\cdot\mathbf{H}^{sc}(s)], \quad s \in C' \end{aligned} \quad (2.31)$$

where K is the transform of k and

$$K^*(s,s') = \frac{e^{-jk\sqrt{[d(s,s')]^2 + (w/2)^2}}}{\sqrt{[d(s,s')]^2 + (w/2)^2}}. \quad (2.32)$$

Figure 2-1. Aperture in Screen Illuminated by Sources in both Half Spaces

Figure 2-2. Aperture in Screen and its Equivalent Model

Figure 2-3. Region-a Equivalent Models (Valid for $z > 0$)

Figure 2-4. Region-b Equivalent Models (Valid for $z < 0$)

Figure 2-5. Finite-Length Straight Narrow Slot in Screen Illuminated by Sources in both Half Spaces

Figure 2-6. Finite-Length Curved Narrow Slot in Screen Illuminated by Sources in both Half Spaces

CHAPTER III

INTEGRAL EQUATION FORMULATION FOR A THIN WIRE IN THE PRESENCE OF A GROUND PLANE OR A SLOTTED PLANAR SCREEN

In this chapter, a time-domain integral equation formulation for a thin wire in the presence of a ground plane is presented. The unknown of this integral equation is the electric current on the wire. After formulation in the time domain, the integral equation is converted to its frequency-domain counterpart by means of Fourier transformation. The ground plane is then replaced with a slotted planar pec (perfect electric conductor) screen and the time-domain integral equation for the wire is specialized to accommodate the screen. This specialization involves formulation of an additional integral equation that governs field behavior in the slot and introduction of field coupling terms that account for the slot's excitation of the wire and the wire's excitation of the slot. As a result of this specialization, one obtains two equations (one for the wire and one for the slot) and two unknowns (electric current on the wire and magnetic current in the slot). The frequency-domain counterparts of the coupled slot-wire equations are then determined by Fourier transformation.

Integral Equation Formulation for a Thin Wire in the Presence of a Ground Plane

Thin-wire antennas and scatterers subject to time-harmonic excitation have been analyzed by numerous researchers in the past. Far less attention has been devoted to such structures excited by general excitation in the time domain. In this section is presented a derivation of the integral equation for the current on

a thin-wire structure in the presence of an ideal ground plane. The thin-wire assumptions and simplifications are well understood in the frequency domain and are employed here in the time-domain development which follows. This means that the equation resulting from the present analysis is valid for those wire structures and time-dependent excitations for which the wire radius is small compared to the shortest wavelength in the significant spectral content of the Fourier transform of the excitation. Subject to this restriction, the wire current is taken to be axially directed and constant over the wire circumference at any point along the wire axis. In addition, the current responds to the axial component of the electric field of the excitation but is insensitive to the component transverse to the wire axis at any given point. One also requires that the total length of wire be much greater than the wire radius and that the entire wire configuration not be a "tight coil" contained within a region whose maximum dimension is only a few wire radii.

A resistance-loaded, curved thin wire of radius a in the presence of an ideal pec screen is illustrated in Figure 3-1, together with independent sources (j^a, m^a) which give rise to excitation. The half-space to the right of the screen ($z > 0$) is filled with material characterized by (μ, ϵ) and is designated region a . The axis of the wire defines the contour C^w , and arc displacement along the axis, denoted l , is measured relative to an arbitrary reference $l = 0$ as indicated in the figure. For convenience in defining the direction of the wire and a positive sense at a point on C^w , one introduces $\hat{l}(l)$ as that unit vector tangential to the wire axis at l with positive sense taken to be along increasing l .

Time-Domain Formulation of Equations

To facilitate the formulation of the thin-wire equation, one appeals to image theory in order to remove the pec plane and thereby arrive at an electromagnetic

model equivalent to the original structure and sources in region a . The wire structure and sources to the right of the ground plane are illustrated in cross-section in Figure 3-2a, and the equivalent model is depicted in Figure 3-2b in which one sees the original sources and wire plus their images, all residing in infinite homogeneous space characterized by (μ, ϵ) .

Aided by the equivalent model, one readily constructs expressions for the electric and magnetic fields in region a ($\mathbf{e}^a, \mathbf{h}^a$):

$$\mathbf{e}^a = \mathbf{e}^{sc} + \mathcal{E}^w[\mathbf{j}] \quad (3.1a)$$

and

$$\mathbf{h}^a = \mathbf{h}^{sc} + \mathcal{H}^w[\mathbf{j}] \quad (3.1b)$$

where $\mathcal{E}^w$ and $\mathcal{H}^w$ are operators by means of which one determines electric and magnetic field from electric current $\mathbf{j}$ and where $(\mathbf{e}^{sc}, \mathbf{h}^{sc})$ is the electromagnetic field that would be produced by the sources if the loaded wire were not present. This field is called a *short circuit* field as suggested by the superscript *sc* because it is that field which exists due to the sources radiating in the presence of the perfectly conducting screen in the xy plane. The total field in region a is the sum of the short-circuit field and the field due to the wire current radiating in the presence of the ground plane. Alternatively, one can view these contributions from the image theory point of view in which the short-circuit field is the sum of that due to the original sources and that due to the images of the original sources, while $(\mathcal{E}^w, \mathcal{H}^w)$ is due to the wire current $\mathbf{j}$ and its image $\mathbf{j}_i$, with all sources and their images radiating in unbounded homogeneous space characterized by (μ, ϵ) . (See the equivalent model of Figure 3-2b.) The electric and magnetic field due to the wire

current are conveniently represented by

$$\mathcal{E}^w[j] = -\frac{\partial}{\partial t} \mathbf{a} - \nabla \phi \quad (3.2a)$$

and

$$\mathcal{H}^w[j] = \frac{1}{\mu} \nabla \times \mathbf{a} \quad (3.2b)$$

in which $\mathbf{a}$ is the magnetic vector potential and ϕ is the electric scalar potential due to the sources induced on the wire in the presence of the conducting screen.

The tangential component of the total electric field on the surface of the conducting wire must be zero at all points away from the location of the load at $l = l_\ell$. In view of the thin-wire simplifications, this boundary condition means that the *axial* component of the total electric field must be zero on the wire surface for $l \neq l_\ell$ and must equal the electric field created by the current in the load resistor at $l = l_\ell$. In equation form this boundary condition becomes

$$\hat{\mathbf{l}}(l) \cdot \mathbf{e}^a(l; t) = 2\pi a j(l; t) R \delta(l - l_\ell), \quad l \in C^w \quad (3.3a)$$

or, in view of (3.1b),

$$\hat{\mathbf{l}}(l) \cdot [\mathbf{e}^{aa} + \mathcal{E}^w[j]] = 2\pi a j(l; t) R \delta(l - l_\ell), \quad l \in C^w \quad (3.3b)$$

in which $2\pi a j(l; t)$ is the total axial current on a wire of radius a bearing surface current of density $\mathbf{j}(l; t) = j(l; t)\hat{\mathbf{l}}(l)$. The presence of the delta function accounts for reciprocal distance needed in the approximation of electric field at the load by the ratio of voltage across the load to length of load, and it suggests that the length of the load is taken to be vanishingly short.

The electric field in (3.3) is determined by (3.1a and (3.2a) in which the magnetic vector potential is

$$\begin{aligned} \mathbf{a}(l; t) = & \mu \int_{C^w} \oint j(l'; \tau) \hat{l}'(l') \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} a d\phi' dl' \\ & + \mu \int_{C_i^w} \oint j_i(l'; \tau) \hat{l}'_i(l') \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} a d\phi' dl' \end{aligned} \quad (3.4)$$

and the electric scalar potential is

$$\begin{aligned} \phi(l; t) = & \frac{1}{\epsilon} \int_{C^w} \oint q(l'; \tau) \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} a d\phi' dl' \\ & + \frac{1}{\epsilon} \int_{C_i^w} \oint q_i(l'; \tau) \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} a d\phi' dl'. \end{aligned} \quad (3.5)$$

In (3.4) and (3.5), j and q are the current and charge densities on the wire while j_i and q_i are the image current and image charge densities on the image of the wire. The axis or contour of the image wire is designated C_i^w and is defined by the following tangential unit vector:

$$\begin{aligned} \hat{l}_i(l) &= -\hat{l}(l) + 2[\hat{l}(l) \cdot \hat{z}]\hat{z} \\ &= -\hat{l}(l) \cdot [\hat{\mathbf{I}} - 2\hat{z}\hat{z}]. \end{aligned} \quad (3.6)$$

The integral $\oint a d\phi'$ in (3.4) and (3.5) is over the circumference of the wire, treated locally as a cylinder of radius a .

By invoking the assumptions of thin-wire theory, one can simplify the above equations. For example, the first integral of (3.4) is approximated as

$$\begin{aligned} & \mu \int_{C^w} 2\pi a j(l'; \tau) \hat{l}'(l') \frac{1}{2\pi} \oint \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} d\phi' dl' \\ & \approx \frac{\mu}{4\pi} \int_{C^w} i(l'; \tau) \hat{l}'(l') k^w(l, l') dl' \end{aligned} \quad (3.7a)$$

and the second as

$$\begin{aligned} & \mu \int_{C_i^w} 2\pi a j_i(l'; \tau) \hat{l}'_i(l') \frac{1}{2\pi} \oint \frac{1}{4\pi |\mathbf{r} - \mathbf{r}'|} d\phi' dl' \\ & \approx \frac{\mu}{4\pi} \int_{C^w} i(l'; \tau_i) \hat{l}'_i(l') k_i^w(l, l') dl', \end{aligned} \quad (3.7b)$$

where the kernels k^w and k_i^w in these integrals are

$$k^w(l, l') = \frac{1}{\sqrt{|\mathbf{r} - \mathbf{r}'|^2 + a^2}} \quad (3.8a)$$

and

$$k_i^w(l, l') = \frac{1}{|\mathbf{r} - \mathbf{r}' + 2[\mathbf{r}' \cdot \hat{\mathbf{z}}]\hat{\mathbf{z}}|}. \quad (3.8b)$$

The image current contribution to the vector potential $\mathbf{a}$ is given by (3.7b), which is seen to be an integral (right side) over the original wire contour C^w of the original current i with unit vector $\hat{l}'_i$ and kernel k_i^w . This is as one expects, since the image current possesses the same intensity as the original but is in a different direction and resides at a different location. The different (but related by (3.6)) direction is accounted for by $\hat{l}'_i$ and the different location is accounted for by k_i^w . Employing (3.7) in (3.4), one expresses the vector potential simply as

$$\mathbf{a}(l; t) = \frac{\mu}{4\pi} \int_{C^w} \{i(l'; \tau) \hat{l}'(l') k^w(l, l') + i(l'; \tau_i) \hat{l}'_i(l') k_i^w(l, l')\} dl'. \quad (3.9)$$

k^w of (3.8a) is the usual approximate wire kernel while k_i^w of (3.8b) is the kernel for the image wire with the wire radius ignored. Also,

$$i = 2\pi a j \quad (3.10)$$

is the total axial current and

$$\tau_i = t - \{|\mathbf{r} - \mathbf{r}' + 2[\mathbf{r}' \cdot \hat{\mathbf{z}}]\hat{\mathbf{z}}|\}/c \quad (3.11)$$

is actual time t delayed by the amount of time required for a signal to travel from a point located by $\{\mathbf{r}' - 2[\mathbf{r}' \cdot \hat{\mathbf{z}}]\hat{\mathbf{z}}\}$ on the image wire axis to a point located by $\hat{\mathbf{r}}$ on the axis of the actual wire. Similarly, the integrals that constitute the scalar potential of (3.5) can be approximated as

$$\frac{1}{\epsilon} \int_{C^w} 2\pi a q(l'; \tau) \frac{1}{2\pi} \oint \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} d\phi' dl' \approx \frac{1}{4\pi\epsilon} \int_{C^w} q(l'; \tau) k^w(l, l') dl' \quad (3.12a)$$

and

$$\frac{1}{\epsilon} \int_{C^w} 2\pi a q_i(l'; \tau) \frac{1}{2\pi} \oint \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} d\phi' dl' \approx -\frac{1}{4\pi\epsilon} \int_{C^w} q(l'; \tau_i) k_i^w(l, l') dl' \quad (3.12b)$$

in which q is the total axial charge density:

$$q = 2\pi a q. \quad (3.13)$$

The scalar potential ϕ of the wire to the right of the ground plane follows directly from (3.5) with the appropriate approximations of (3.12):

$$\phi(l; t) = \frac{1}{4\pi\epsilon} \int_{C^w} \{q(l'; \tau) k^w(l, l') - q(l'; \tau_i) k_i^w(l, l')\} dl'. \quad (3.14)$$

An integral equation for the structure of Figure 3-1 can be obtained by substituting (3.9) and (3.14) into (3.2a) and then (3.2a) into (3.3b). But in the resulting equation both i and q are unknown, so it is desirable to eliminate the charge density in order that the integral equation has only a single unknown. This one accomplishes by differentiating (3.3) to obtain

$$\frac{\partial}{\partial t} \{\hat{\mathbf{l}}(l) \cdot [\mathbf{e}^{e^*} + \mathcal{E}^w[\mathbf{j}]]\} = \frac{\partial}{\partial t} i(l; t) R \delta(l - l_t), \quad l \in C^w \quad (3.15a)$$

or, more explicitly,

$$-\frac{\partial^2}{\partial t^2}[\hat{\mathbf{l}} \cdot \mathbf{a}] - \frac{\partial}{\partial t} \frac{\partial}{\partial l} \phi - R\delta(l - l_c) \frac{\partial}{\partial t} i = -\frac{\partial}{\partial t} [\hat{\mathbf{l}} \cdot \mathbf{e}^{c*}], \quad l \in C^w \quad (3.15b)$$

and by subsequently using the continuity equation,

$$\frac{\partial}{\partial l} i(l; t) + \frac{\partial}{\partial t} q(l; t) = 0, \quad (3.16)$$

to replace $\frac{\partial}{\partial t} q$ by $-\frac{\partial}{\partial l} i$ where the former occurs in (3.15). From this procedure, one arrives at

$$\begin{aligned} & -\frac{1}{4\pi\epsilon} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{C^w} \{ i(l'; \tau) \hat{\mathbf{l}}'(l') \cdot \hat{\mathbf{l}}(l) k^w(l, l') + i(l'; \tau_i) \hat{\mathbf{l}}'_i(l') \cdot \hat{\mathbf{l}}(l) k_i^w(l, l') \} dl' \right. \\ & \quad \left. - \frac{\partial}{\partial l} \int_{C^w} \left\{ \frac{\partial}{\partial l'} i(l'; \tau) k^w(l, l') - \frac{\partial}{\partial l'} i(l'; \tau_i) k_i^w(l, l') \right\} dl' \right\} \\ & \quad - R\delta(l - l_c) \frac{\partial}{\partial t} i(l; t) \\ & = -\frac{\partial}{\partial t} [\hat{\mathbf{l}}(l) \cdot \mathbf{e}^{c*}(l; t)], \quad l \in C^w \end{aligned} \quad (3.17)$$

which is a time-domain integro-differential equation for the unknown current i induced on the curved wire to the right of the ground plane by the sources $(\mathbf{j}^a, \mathbf{m}^a)$. The use of k_i^w of (3.8b) with no account of wire radius implies that any portion of the wire which is not perpendicular to the ground plane, or nearly so, must be at least three radii to the right of it.

Extensions of the above to cover cases of thin-wire structures with multiple elements and/or with junctions follow with relative ease. Also, if the load were a small lumped inductor of inductance L , one would replace Ri by $L \frac{\partial}{\partial t} i$, or, if it were a small capacitor of capacitance C , the replacement would be by $[\int_0^t i dt]/C$. (It is assumed that the energy stored in L or C at $t = 0$ is zero.)

Frequency-Domain Integral Equation

One obtains the frequency-domain integral equation that corresponds to the time-domain equation (3.17) by means of Fourier transformation in a manner similar to that employed to obtain (2.30) and (2.31) from (2.10) and (2.23). Since the wire equation in the coupled slot-wire formulation below is a more general equation than is (3.17), transformation of the wire equation is deferred to a subsequent section.

Formulation of Equations for Wire in the Presence of Screen with Narrow Slot

In this section is presented a formulation of coupled integral equations for a loaded wire above a conducting screen perforated by a slot, with the composite structure excited by general time-dependent sources located to the left and right of the screen. The loaded wire, slotted screen, and sources are illustrated in Figure 3-3. The wire is thin and the slot narrow in the senses described above and in Chapter II. The present derivation is simply a combination of those presented in Chapter II for the slotted screen and above for a wire in the presence of a screen, with proper account taken of coupling between the two. The boundary conditions that the component of electric field tangential to the wire axis be zero and that the component of magnetic field tangential to the slot axis be continuous are again enforced.

Coupled Slot and Wire Equations

The electric and magnetic fields in region a can be expressed conveniently as

$$\mathbf{e}^a = \mathbf{e}^{ac} + \mathcal{E}^A [2\mathbf{m}] + \mathcal{E}^W [\mathbf{j}], \quad z > 0 \quad (3.18a)$$

and

$$\mathbf{h}^a = \mathbf{h}^{ac} + \mathcal{H}^A [2\mathbf{m}] + \mathcal{H}^W [\mathbf{j}], \quad z > 0 \quad (3.18b)$$

whose constituents are defined in Chapter II and above. The fields in region b are given in Equations (2-2a) and (2-2b). One requires that (3.3) hold on the wire and that $\mathbf{h}^a \cdot \hat{\mathbf{s}} = \mathbf{h}^b \cdot \hat{\mathbf{s}}$ be true in the slot. Again, in order to obtain equations for the unknown axial currents alone rather than for combinations of currents and charge densities, one differentiates both sides of the fundamental equations with respect to time t to facilitate elimination of charge densities.

Enforcement of the condition that the axial electric field be zero on the wire surface, i.e., application of (3.3) to (3.18a), yields

$$\hat{\mathbf{l}} \cdot \boldsymbol{\varepsilon}^w [\mathbf{j}] - 2\pi a j(l; t) R \delta(l - l_c) + \hat{\mathbf{l}} \cdot \boldsymbol{\varepsilon}^s [2\mathbf{m}] = -\hat{\mathbf{l}} \cdot \mathbf{e}^{ec} \quad (3.19)$$

which, when differentiated with respect to t , leads to

$$-\frac{\partial^2}{\partial t^2} [\hat{\mathbf{l}} \cdot \mathbf{a}] - \frac{\partial}{\partial l} \frac{\partial}{\partial t} \phi - R \delta(l - l_c) \frac{\partial}{\partial t} i - 2 \frac{1}{\epsilon} \hat{\mathbf{l}} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{f}] = -\frac{\partial}{\partial t} [\hat{\mathbf{l}} \cdot \mathbf{e}^{ec}], \quad l \in C^w. \quad (3.20)$$

The coupling from the slot to the wire is accounted for by $-2 \frac{1}{\epsilon} \hat{\mathbf{l}} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{f}]$ which can be written in the more explicit form revealing dependence upon the total axial magnetic current in the slot and the slot geometry:

$$\begin{aligned} -2 \frac{1}{\epsilon} \hat{\mathbf{l}} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{f}] &= -\frac{1}{2\pi} \hat{\mathbf{l}}(l) \cdot \frac{\partial}{\partial t} \left\{ \nabla \times \int_{C^s} \mathbf{k}(s'; \tau) \hat{\mathbf{s}}'(s') k^{ws}(l, s') ds' \right\} \\ &= \frac{1}{2\pi} \frac{\partial}{\partial t} \int_{C^s} \hat{\mathbf{l}}(l) \cdot \{ \hat{\mathbf{s}}'(s') \times \nabla [k(s'; \tau) k^{ws}(l, s')] \} ds' \quad (3.21a) \end{aligned}$$

where the kernel k^{ws} , given by

$$k^{ws}(l, s') = \frac{1}{|\mathbf{r}(l) - \mathbf{d}'(s')|}, \quad (3.21b)$$

is the reciprocal of the distance from a source point located by $\mathbf{d}'(s')$ on the slot axis to an observation point located by $\mathbf{r}(l)$ on the wire axis. The expansion

$$\nabla [k(s'; \tau) k^{ws}(l, s')] = k(s'; \tau) \nabla k^{ws}(l, s') - \frac{1}{c} \frac{\partial}{\partial t} k(s'; \tau) k^{ws}(l, s') \hat{\mathbf{u}}(l, s') \quad (3.22a)$$

where

$$\hat{u}(l, s') = \frac{\mathbf{r}(l) - \mathbf{d}'(s')}{|\mathbf{r}(l) - \mathbf{d}'(s')|} \quad (3.22b)$$

allows one to express (3.21a) in the more explicit form

$$\begin{aligned} -2\frac{1}{\epsilon}\hat{\mathbf{l}} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{f}] &= -\frac{1}{2\pi} \frac{\partial}{\partial t} \int_{C^w} k(s'; \tau) [\hat{\mathbf{s}}'(s') \times \hat{\mathbf{l}}(l)] \cdot \nabla k^{w*}(l, s') ds' \\ &+ \frac{1}{2\pi c} \frac{\partial^2}{\partial t^2} \int_{C^w} k(s'; \tau) \{ [\hat{\mathbf{s}}'(s') \times \hat{\mathbf{l}}(l)] \cdot \hat{\mathbf{u}}(l, s') \} k^{w*}(l, s') ds'. \end{aligned} \quad (3.23)$$

The vector potential $\mathbf{a}$ and scalar potential ϕ in (3.20) are integrals over C^w of wire current and charge, respectively, against appropriate kernels, which can be determined by procedures paralleling those leading from (3.4) and (3.5) to (3.9) and (3.14). Replacing $\mathbf{a}$ and ϕ by such integrals and $-2\frac{1}{\epsilon}\hat{\mathbf{l}} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{f}]$ by the right side of (3.23), one converts (3.20) to

$$\begin{aligned} &-\frac{1}{4\pi\epsilon} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{C^w} \{ i(l'; \tau) \hat{\mathbf{l}}'(l') \cdot \hat{\mathbf{l}}(l) k^w(l, l') + i(l'; \tau_i) \hat{\mathbf{l}}'_i(l') \cdot \hat{\mathbf{l}}(l) k_i^w(l, l') \} dl' \right. \\ &\quad \left. - \frac{\partial}{\partial t} \int_{C^w} \left\{ \frac{\partial}{\partial l'} i(l'; \tau) k^w(l, l') - \frac{\partial}{\partial l'_i} i(l'; \tau_i) k_i^w(l, l') \right\} dl' \right\} - R\delta(l - l_t) \frac{\partial}{\partial t} i(l; t) \\ &\quad - \frac{1}{2\pi} \frac{\partial}{\partial t} \int_{C^w} k(s'; \tau) [\hat{\mathbf{s}}'(s') \times \hat{\mathbf{l}}(l)] \cdot \nabla k^{w*}(l, s') ds' \\ &\quad + \frac{1}{2\pi c} \frac{\partial^2}{\partial t^2} \int_{C^w} k(s'; \tau) \{ [\hat{\mathbf{s}}'(s') \times \hat{\mathbf{l}}(l)] \cdot \hat{\mathbf{u}}(l, s') \} k^{w*}(l, s') ds' \\ &= -\frac{\partial}{\partial t} [\hat{\mathbf{l}}(l) \cdot \mathbf{e}^{*c*}(l; t)], \quad l \in C^w. \end{aligned} \quad (3.24)$$

Equation (3.24) is an integro-differential equation in the unknowns i and k which forces the axial component of the total electric field to be zero on the surface of the wire.

Continuity of the axial component of magnetic field along any path passing through the slot is assured by

$$2\hat{s} \cdot \mathcal{H}^A[2\mathbf{m}] + \hat{s} \cdot \mathcal{H}^W[\mathbf{j}] = -\hat{s} \cdot \mathbf{h}^{*c} \quad (3.25)$$

which upon differentiation with respect to time t becomes

$$-4 \frac{\partial^2}{\partial t^2} [\hat{s} \cdot \mathbf{f}] - 4 \frac{\partial}{\partial t} \frac{\partial}{\partial s} \psi + \frac{1}{\mu} \hat{s} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{a}] = -\frac{\partial}{\partial t} [\hat{s} \cdot \mathbf{h}^{*c}] \quad (3.26)$$

where $\frac{1}{\mu} \hat{s} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{a}]$ is given by

$$\begin{aligned} \frac{1}{\mu} \hat{s} \cdot \frac{\partial}{\partial t} [\nabla \times \mathbf{a}] &= \frac{1}{2\pi} \hat{s}(s) \cdot \frac{\partial}{\partial t} \left\{ \nabla \times \int_{C^w} i(l'; \tau) \hat{l}'(l') k^{*w}(s, l') dl' \right\} \\ &= \frac{1}{2\pi} \frac{\partial}{\partial t} \int_{C^w} i(l'; \tau) [\hat{l}'(l') \times \hat{s}(s)] \cdot \nabla k^{*w}(s, l') dl' \\ &\quad - \frac{1}{2\pi c} \frac{\partial^2}{\partial t^2} \int_{C^w} i(l'; \tau) \{ [\hat{l}'(l') \times \hat{s}(s)] \cdot \hat{u}(s, l') \} k^{*w}(s, l') dl' \end{aligned} \quad (3.27)$$

in which the wire-to-slot kernel is

$$k^{*w}(s, l') = \frac{1}{|\mathbf{d}(s) - \mathbf{r}'(l')|} \quad (3.28a)$$

and

$$\hat{u}(s, l') = \frac{\mathbf{d}(s) - \mathbf{r}'(l')}{|\mathbf{d}(s) - \mathbf{r}'(l')|}. \quad (3.28b)$$

Taking advantage of the experience gained above with $\mathbf{f}$ and ψ for the slot and of (3.27), one can readily convert (3.25) to

$$\begin{aligned} -\frac{1}{\pi\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{C^s} k(s'; \tau) [\hat{s}'(s') \cdot \hat{s}(s)] k^s(s, s') ds' - \frac{\partial}{\partial s} \int_{C^s} \frac{\partial}{\partial s'} k(s'; \tau) k^s(s, s') ds' \right\} \\ + \frac{1}{2\pi} \frac{\partial}{\partial t} \int_{C^w} i(l'; \tau) [\hat{l}'(l') \times \hat{s}(s)] \cdot \nabla k^{*w}(s, l') dl' \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2\pi c} \frac{\partial^2}{\partial t^2} \int_{C^w} i(l'; \tau) \{ [\hat{l}'(l') \times \hat{s}(s)] \cdot \hat{u}(s, l') \} k^{sw}(s, l') dl' \\
& = -\frac{\partial}{\partial t} [\hat{s}(s) \cdot \mathbf{h}^{sc}(s; t)], \quad s \in C^s, t > 0, \quad (3.29)
\end{aligned}$$

an integro-differential equation in i and k , which enforces continuity of axial magnetic field in the slot. Equations (3-24) and (3-29) are coupled equations which govern the behavior of the slot magnetic current k and wire electric current i subject to the excitation $\mathbf{e}^{sc}$ and $\mathbf{h}^{sc}$ that are due to the known sources $(\mathbf{j}^a, \mathbf{m}^a)$ and $(\mathbf{j}^b, \mathbf{m}^b)$.

Frequency-Domain Integral Equations

The frequency-domain integral equations corresponding to the time-domain equations (3.24) and (3.29) can be obtained by Fourier transforming them in a manner similar to that employed to obtain (2.30) and (2.31) from (2.10) and (2.23). In preparation for taking the transforms of these equations and interpreting the results, one notes that

$$\mathcal{F}\{i(l'; \tau)\} = I(l') e^{-jk\sqrt{|r-r'|^2 + a^2}}, \quad (3.30a)$$

$$\mathcal{F}\{i(l'; \tau_i)\} = I(l') e^{-jk|r-r'| + 2|r' \cdot \hat{s}| |\hat{s}|}, \quad (3.30b)$$

and

$$\mathcal{F}\{k(s'; \tau)\} = K(s') e^{-jk|r-d'|} \quad (3.30c)$$

where $I(l')$ and $K(s')$ are the transforms of $i(l'; t)$ and $k(s'; t)$, respectively. And for convenience one defines the following frequency-domain kernels by means of the relationships below:

$$\begin{aligned}
I(l') K^w(l, l') &= \mathcal{F}\{i(l'; \tau)\} k^w(l, l') \\
&= I(l') \frac{e^{-jk\sqrt{|\mathbf{r}-\mathbf{r}'|^2+a^2}}}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2+a^2}}, \quad (3.31a)
\end{aligned}$$

$$\begin{aligned}
I(l') K_i^w(l, l') &= \mathcal{F}\{i(l'; \tau_i)\} k_i^w(l, l') \\
&= I(l') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|+2|\mathbf{r}'\cdot\hat{\mathbf{s}}|\hat{\mathbf{s}}|}}{|\mathbf{r}-\mathbf{r}'|+2|\mathbf{r}'\cdot\hat{\mathbf{z}}|\hat{\mathbf{z}}|}, \quad (3.31b)
\end{aligned}$$

and

$$\begin{aligned}
K(s') K^{w*}(l, s') &= \mathcal{F}\{k(l'; \tau)\} k^{w*}(l, s') \\
&= K(s') \frac{e^{-jk|\mathbf{r}-\mathbf{d}'|}}{|\mathbf{r}-\mathbf{d}'|}. \quad (3.31c)
\end{aligned}$$

Now one transforms (3.24) and employs the observations and definitions above to obtain

$$\begin{aligned}
&-j \frac{\eta}{4\pi k} \left\{ k^2 \int_{C^w} I(l') \{ \hat{\mathbf{l}}'(l') K^w(l, l') + \hat{\mathbf{l}}'_i(l') K_i^w(l, l') \} \cdot \hat{\mathbf{l}}(l) dl' \right. \\
&\quad \left. + \frac{d}{dl} \int_{C^w} \frac{d'}{dl'} I(l') \{ K^w(l, l') - K_i^w(l, l') \} dl' \right\} - R\delta(l-l_e) I(l) \\
&\quad + \frac{1}{j\omega} E(l) = -\hat{\mathbf{l}}(l) \cdot \mathbf{E}^{*c*}(l), \quad l \in C^w, \quad (3.32)
\end{aligned}$$

where $\mathbf{E}^{*c*}(l)$ is the transform of $\mathbf{e}^{*c*}(l; t)$ and $E(l)$ is the transform of the sum of the last two terms on the left side of (3.24):

$$\begin{aligned}
E(l) &= -j\omega \frac{1}{2\pi} \int_{C^s} K(s') [\hat{\mathbf{s}}'(s') \times \hat{\mathbf{l}}(l)] \cdot \\
&\quad \left\{ [\nabla k^{w*}(l, s') - jk\hat{\mathbf{u}}(l, s') k^{w*}(l, s')] e^{-jk|\mathbf{r}-\mathbf{d}'|} \right\} ds'. \quad (3.33)
\end{aligned}$$

Making use of the well known identity $\nabla(fg) = f\nabla g + g\nabla f$ and the fact that $\nabla(e^{-jk|r-d'|}) = -jke^{-jk|r-d'|}\hat{u}(l, s')$ one readily demonstrates that the term in the braces in (3.33) above can be reduced to $\nabla(k^{w*}(l, s')e^{-jk|r-d'|})$, which according to the relationship in (3.31c) is simply $\nabla K^{w*}(l, s')$. Thus, $E(l)$ of (3.33) becomes

$$E(l) = -j\omega \frac{1}{2\pi} \int_{C^*} K(s') [\hat{s}'(s') \times \hat{l}(l)] \cdot \nabla K^{w*}(l, s') ds' \quad (3.34)$$

and consequently (3.32) simplifies to

$$\begin{aligned} & -j \frac{\eta}{4\pi k} \left\{ k^2 \int_{C^*} I(l') \{ \hat{l}'(l') K^w(l, l') + \hat{l}'_i(l') K_i^w(l, l') \} \cdot \hat{l}(l) dl' \right. \\ & \quad \left. + \frac{d}{dl} \int_{C^*} \frac{d'}{dl'} I(l') \{ K^w(l, l') - K_i^w(l, l') \} dl' \right\} - R\delta(l - l_e) I(l) \\ & \quad - \frac{1}{2\pi} \int_{C^*} K(s') [\hat{s}'(s') \times \hat{l}(l)] \cdot \nabla K^{w*}(l, s') ds' \\ & = -\hat{l}(l) \cdot \mathbf{E}^{*c}(l), \quad l \in C^w, \end{aligned} \quad (3.35)$$

a frequency-domain integro-differential equation for the wire electric current I and slot magnetic current K of the wire/slotted-screen structure of Figure 3-3.

The companion equation (3.29) can be Fourier transformed too to obtain

$$\begin{aligned} & -j \frac{1}{\pi k \eta} \left\{ k^2 \int_{C^*} K(s') [\hat{s}'(s') \cdot \hat{s}(s)] K^*(s, s') ds' + \frac{d}{ds} \int_{C^*} \frac{d}{ds'} K(s') K^*(s, s') ds' \right\} \\ & \quad + \frac{1}{2\pi} \int_{C^*} I(l') [\hat{l}'(l') \times \hat{s}(s)] \cdot \nabla K^{*w}(s, l') dl' \\ & = -\hat{s}(s) \cdot \mathbf{H}^{*c}(s), \quad s \in C^* \end{aligned} \quad (3.36)$$

in which $\mathbf{H}^{*c}(s)$ is the transform of $\mathbf{h}^{*c}(s; t)$. The kernel K^* is defined in (2.32) and K^{*w} is given below:

$$K^w(s, l') = \frac{e^{-jk|\mathbf{d} - \mathbf{r}'|}}{|\mathbf{d} - \mathbf{r}'|}. \quad (3.37)$$

Equations (3.35) and (3.36) are coupled frequency-domain integro-differential equations for the wire electric current I and slot magnetic current K of the wire/slotted-screen structure of Figure 3-3.

Figure 3-1. Loaded Wire in the Presence of Sources in Region a

Figure 3-2. Loaded Wire in Region a and its Image-Theory Equivalent Model (Cross-Sectional View)

Figure 3-3. Loaded Wire in the Presence of a Slotted Screen and Sources in Region a

CHAPTER IV

TIME-DOMAIN INTEGRAL EQUATION

SOLUTION TECHNIQUE

In this chapter the time-domain integral equation for a planar conducting screen perforated by a finite-length slot having arbitrary contour is solved. One enforces the integral equation by pulse testing and expands the unknown equivalent magnetic current in a series of triangular expansion functions. The partial derivatives in the integral equation are approximated with central differences which lead to the field-advancement expression necessary for the "marching in time" solution. By similar steps, the field advancement expressions are developed for the more general case of a slotted screen coupled to an arbitrarily directed thin wire scatterer located on the shadow side of the screen.

With the field-advancement expressions available, one acquires the solution by specifying the incident time-domain excitation and applying the field-advancement expressions until the amplitude of the solution (magnetic current in the slot or electric current on the wire, if present) decays to an acceptably small value. In the following sections, the field-advancement expressions are developed and issues related to the numerical stability of the solution process are discussed.

Field-Advancement Formula for the Arbitrarily Directed Narrow Slot

The arbitrarily directed narrow slot in a planar perfectly conducting screen is governed by the integro-differential equation (Equation (2.23))

$$-\frac{1}{\pi\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{C'} k(s'; \tau) [\hat{s}'(s') \cdot \hat{s}(s)] k^*(s, s') ds' - \frac{\partial}{\partial s} \int_{C'} \frac{\partial}{\partial s'} k(s'; \tau) k^*(s, s') ds' \right\}$$

$$= -\frac{\partial}{\partial t} [\hat{s}(s) \cdot \mathbf{h}^{sc}(s; t)], \quad s \in C', t > 0, \quad (4.1)$$

where k is the total axially directed equivalent magnetic current, $\hat{s}$ is the unit vector in the plane of the screen which is tangential to the contour C' and has sense of increasing arc displacement s , τ is delayed time (discussed in Chapter II and defined in (2.5c)), and $\mathbf{h}^{sc}$ is the short-circuit magnetic field given by

$$\mathbf{h}^{sc} = \mathbf{h}^{sc^*} - \mathbf{h}^{sc^b}. \quad (4.2)$$

The kernel in (4.2) for the curved narrow slot is given by

$$k^*(s, s') = \frac{1}{\sqrt{[d(s, s')]^2 + (w/2)^2}} \quad (4.3)$$

in which $d(s, s')$ is the distance from the point of observation s to the source point s' . The kernel in (4.3) is approximate, as discussed in Chapter II, and it can be replaced by the "exact" kernel (Equation (2.20)) under certain favorable circumstances which are discussed subsequently.

To facilitate a numerical solution of (4.1), one approximates the curved slot as an ensemble of contiguous straight subslots. This piecewise linear approximation of the slot's contour is depicted in Figure 4-1a where one sees the slot's contour (denoted C') and points s_{p-1} , s_p , and s_{p+1} that fall on C' and which specify locations (x_p, y_p) , $p = 0, 1, 2, \dots, N + 1$. Note that the points s_{p-1} , s_p , and s_{p+1} need not be evenly spaced along C' . Thus, it is possible to specify more points where large spatial changes in slot behavior are expected. In all cases, the spacing of the points is chosen such that the distance between any two neighboring points is short compared to the shortest wavelength in the significant spectral content of the Fourier transform of the excitation.

The curved slot is approximated from s_{p-1} to s_{p+1} by two contiguous straight subslots whose lengths are given by

$$\Delta s_{p \pm \frac{1}{2}} = [(x_{p \pm \frac{1}{2}} - x_p)^2 + (y_{p \pm \frac{1}{2}} - y_p)^2]^{\frac{1}{2}} \quad (4.4)$$

and whose directions are specified by the unit vectors

$$\hat{s}_{p \pm \frac{1}{2}} = \pm \frac{(x_{p \pm \frac{1}{2}} - x_p)\hat{x} + (y_{p \pm \frac{1}{2}} - y_p)\hat{y}}{\Delta s_{p \pm \frac{1}{2}}}. \quad (4.5)$$

The unit vector in the average direction of the slot at point p is given by

$$\hat{s}_p = \frac{(x_{p+1} - x_{p-1})\hat{x} + (y_{p+1} - y_{p-1})\hat{y}}{\Delta s_p} \quad (4.6)$$

where the normalizing distance Δs_p is determined from

$$\Delta s_p = [(x_{p+1} - x_{p-1})^2 + (y_{p+1} - y_{p-1})^2]^{\frac{1}{2}}. \quad (4.7)$$

Finally, a useful quantity is the weighted vector in the average direction of the slot at point p (ΔS_p) which one determines from (4.6) and (4.7) using the relation

$$\Delta S_p = \frac{1}{2} \Delta s_p \hat{s}_p = \frac{1}{2} \Delta s_{p-\frac{1}{2}} \hat{s}_{p-\frac{1}{2}} + \frac{1}{2} \Delta s_{p+\frac{1}{2}} \hat{s}_{p+\frac{1}{2}}. \quad (4.8)$$

The significance of the direction and sense of the slot is that they are used to attribute these properties to the axial magnetic current.

Piecewise linear approximation of the slot contour by selection of the points s_p per Figure 4-1a sets the stage for initial discretization of the integral equation for the curved narrow slot (4.1). Enforcement of (4.1) by pulse testing is the first step in the discretization process. A typical testing pulse Π_p is depicted in Figure

4-1b. The testing pulse has unit amplitude and is non-zero from $p - \frac{1}{2}$ to $p + \frac{1}{2}$. For points of observation designated by the integer index m , the "pulse tested" form of (4.1) is

$$\begin{aligned}
 & -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \langle \Pi_m, \int_{C^*} k(s'; \tau) [\hat{s}'(s') \cdot \hat{s}(s)] k^*(s, s') ds' \rangle \\
 & + \frac{1}{\pi\mu} \langle \Pi_m, \frac{\partial}{\partial s} \int_{C^*} \frac{\partial}{\partial s'} k(s'; \tau) k^*(s, s') ds' \rangle \\
 & = -\frac{\partial}{\partial t} \langle \Pi_m, [\hat{s}(s) \cdot h^{*c}(s; t)] \rangle,
 \end{aligned} \tag{4.9}$$

where the inner product is given by

$$\langle \Pi_m, g \rangle = \int_{s_{m-\frac{1}{2}}}^{s_{m+\frac{1}{2}}} g(s) ds, \tag{4.10}$$

and m is an integer from 1 to N .

When the integrals suggested by (4.10) are written explicitly, (4.9) becomes

$$\begin{aligned}
 & -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{s_{m-\frac{1}{2}}}^{s_{m+\frac{1}{2}}} \int_{C^*} k(s'; \tau) [\hat{s}'(s') \cdot \hat{s}(s)] k^*(s, s') ds' ds \\
 & + \frac{1}{\pi\mu} \int_{s_{m-\frac{1}{2}}}^{s_{m+\frac{1}{2}}} \frac{\partial}{\partial s} \int_{C^*} \frac{\partial}{\partial s'} k(s'; \tau) k^*(s, s') ds' ds \\
 & = -\frac{\partial}{\partial t} \int_{s_{m-\frac{1}{2}}}^{s_{m+\frac{1}{2}}} [\hat{s}(s) \cdot h^{*c}(s; t)] ds.
 \end{aligned} \tag{4.11}$$

Making use of the geometric approximations depicted in Figure 4-1, one approximates $\hat{s}(s)$ with

$$\hat{s}(s) = \begin{cases} \hat{s}_{m-\frac{1}{2}} & , s \in (s_{m-\frac{1}{2}}, s_m) \\ \hat{s}_{m+\frac{1}{2}} & , s \in (s_m, s_{m+\frac{1}{2}}) \end{cases} \tag{4.12}$$

and writes (4.11) as

$$\begin{aligned}
 & -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[\hat{s}_{m-\frac{1}{2}} \cdot \int_{s_{m-\frac{1}{2}}}^{s_m} \int_{C^*} k(s'; \tau) \hat{s}'(s') k^*(s, s') ds' ds \right. \\
 & \quad \left. + \hat{s}_{m+\frac{1}{2}} \cdot \int_{s_m}^{s_{m+\frac{1}{2}}} \int_{C^*} k(s'; \tau) \hat{s}'(s') k^*(s, s') ds' ds \right] \\
 & + \frac{1}{\pi\mu} \int_{s_{m-\frac{1}{2}}}^{s_{m+\frac{1}{2}}} \frac{\partial}{\partial s} \int_{C^*} \frac{\partial}{\partial s'} k(s'; \tau) k^*(s, s') ds' ds \\
 & = -\frac{\partial}{\partial t} \left[\hat{s}_{m-\frac{1}{2}} \cdot \int_{s_{m-\frac{1}{2}}}^{s_m} h^{*c}(s; t) ds + \hat{s}_{m+\frac{1}{2}} \cdot \int_{s_m}^{s_{m+\frac{1}{2}}} h^{*c}(s; t) ds \right]. \quad (4.13)
 \end{aligned}$$

The kernel in (4.13) does not vary smoothly as s' approaches s . However, after one performs the integration in s' , the integrated kernel varies only slightly with s . Thus it is valid for one to approximate the integrated kernel as invariant with respect to s and choose the point of enforcement in (4.13) to be s_m (at the center of the testing pulse). After one makes this approximation, the integrations in s produce simple distance terms except in the case of the second term of (4.13) where the integration of the partial derivative with respect to s simply causes the kernel of the contour integral along C^* (w.r.t. s') to be evaluated at the endpoints of the integration in s . These steps cause (4.13) to become

$$\begin{aligned}
 & -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[\frac{1}{2} \Delta s_{m-\frac{1}{2}} \hat{s}_{m-\frac{1}{2}} \cdot \int_{C^*} k(s'; \tau_m) \hat{s}'(s') k^*(s_m, s') ds' \right. \\
 & \quad \left. + \frac{1}{2} \Delta s_{m+\frac{1}{2}} \hat{s}_{m+\frac{1}{2}} \cdot \int_{C^*} k(s'; \tau_m) \hat{s}'(s') k^*(s_m, s') ds' \right] \\
 & + \frac{1}{\pi\mu} \left[\int_{C^*} \frac{\partial}{\partial s'} k(s'; \tau_{m+\frac{1}{2}}) k^*(s_{m+\frac{1}{2}}, s') ds' \right. \\
 & \quad \left. - \int_{C^*} \frac{\partial}{\partial s'} k(s'; \tau_{m-\frac{1}{2}}) k^*(s_{m-\frac{1}{2}}, s') ds' \right]
 \end{aligned}$$

$$= -\frac{\partial}{\partial t} \left[\frac{1}{2} \Delta s_{m-\frac{1}{2}} \hat{s}_{m-\frac{1}{2}} \cdot \mathbf{h}^{ec}(s_m; t) + \frac{1}{2} \Delta s_{m+\frac{1}{2}} \hat{s}_{m+\frac{1}{2}} \cdot \mathbf{h}^{ec}(s_m; t) \right], \quad (4.14)$$

which in light of (4.8) can be simplified to

$$\begin{aligned} & -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[\Delta \mathbf{S}_m \cdot \int_{C^+} k(s'; \tau_m) \hat{s}'(s') k^*(s_m, s') ds' \right] \\ & + \frac{1}{\pi\mu} \left[\int_{C^+} \frac{\partial}{\partial s'} k(s'; \tau_{m+\frac{1}{2}}) k^*(s_{m+\frac{1}{2}}, s') ds' \right. \\ & \quad \left. - \int_{C^+} \frac{\partial}{\partial s'} k(s'; \tau_{m-\frac{1}{2}}) k^*(s_{m-\frac{1}{2}}, s') ds' \right] \\ & = -\frac{\partial}{\partial t} \left[\Delta \mathbf{S}_m \cdot \mathbf{h}^{ec}(s_m; t) \right], \end{aligned} \quad (4.15)$$

where τ takes on a subscript to account for the discrete points of observation along the arc displacement s embodied in $\mathbf{d}(s)$ in $\tau = t - \frac{1}{c} |\mathbf{d}(s) - \mathbf{d}'(s')|$.

Equation (4.15), which actually represents N pulse tested equations for N observation points s_m , $m = 1, 2, \dots, N$, is now further discretized by piecewise linear expansion of the unknown axial magnetic current $k(s'; t)$. One expands $k(s'; t)$ in a series of triangular expansion functions having peak amplitude of 1 at s_n and weighting coefficients $K(n; \tau_{mn})$ where τ_{mn} represents delayed time determined from the time at the source location, designated by the integer index n , which is delayed by the propagation time $\frac{1}{c} |\mathbf{d}_m(s_m) - \mathbf{d}'_n(s'_n)|$ from the source point s_n to the point of observation s_m . Note that the symbol τ_{mn} is subsequently re-defined to represent a discretized delayed time index when the integral equation is discretized with respect to time.

A typical triangular expansion function Δ_p is depicted in Figure 4-1c. The expansion function Δ_p is zero for $s < s_{p-1}$ and $s > s_{p+1}$ and rises linearly to unit

amplitude at s_p . Observe that, in the second term of (4.15), $k(s'; \tau)$ is differentiated with respect to s' to obtain equivalent magnetic charge. In Figure 4-1d, one sees the result of this differentiation which is a pulse doublet of charge whose amplitude is governed by the slope of Λ_p (that is, directly related to the spacing of the points s_p).

Not only is the source magnetic current's amplitude discretized by piecewise linear expansion, but also its vector direction and sense are discretized in keeping with the geometric approximations suggested by Figure 4-1a. Thus, $\hat{s}'(s')$ is now given by

$$\hat{s}'(s') = \begin{cases} \hat{s}_{n-\frac{1}{2}} & , s \in (s_{n-1}, s_n) \\ \hat{s}_{n+\frac{1}{2}} & , s \in (s_n, s_{n+1}). \end{cases} \quad (4.16)$$

Expressing the source magnetic current in a series of triangular expansion functions per Figure 4-1c, the source magnetic charge in a series of weighted pulse doublets per Figure 4-1d, and taking advantage of the geometric approximations suggested by Figure 4-1a, one writes (4.15) in the further discretized form

$$\begin{aligned} & -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[\Delta S_m \cdot \sum_{n=1}^N K(n; \tau_{mn}) \left\{ \hat{s}_{n-\frac{1}{2}} \int_{s_{n-1}}^{s_n} \Lambda_n(s') k'(s_m, s') ds' \right. \right. \\ & \quad \left. \left. + \hat{s}_{n+\frac{1}{2}} \int_{s_n}^{s_{n+1}} \Lambda_n(s') k'(s_m, s') ds' \right\} \right] \\ & + \frac{1}{\pi\mu} \sum_{n=1}^N \left\{ \frac{K(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k'(s_{m+\frac{1}{2}}, s') ds' \right. \\ & \quad \left. - \frac{K(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k'(s_{m+\frac{1}{2}}, s') ds' \right\} \end{aligned}$$

$$\begin{aligned}
& - \frac{K(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^s(s_{m-\frac{1}{2}}, s') ds' \\
& + \frac{K(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^s(s_{m-\frac{1}{2}}, s') ds' \Big\} \\
& = - \frac{\partial}{\partial t} \left[\Delta S_m \cdot h^{sc}(s_m; t) \right], \tag{4.17}
\end{aligned}$$

where the delay specified by $\tau_{m+\frac{1}{2}, n-\frac{1}{2}}$ is the propagation time from $s_{n-\frac{1}{2}}$ which is at the center of one pulse (of magnetic charge) of the pulse doublet located at s_n to $s_{m+\frac{1}{2}}$ which is the observation point determined by the integration limits on the s integration of the second term of (4.13).

Note that, owing to the near invariance of quantities over short intervals (where an interval is short with respect to the shortest wavelength in the significant spectral content of the Fourier transformed excitation), a good approximation for the integrals in the first term of (4.17) can be made if one replaces Λ_n with a pulse Π_n which is non-zero over half the subdomain (identical to the testing pulse depicted in Figure 4-1b). This very good approximation greatly simplifies evaluation of the integral in which it is applied.

The remaining variable in need of discretization is time. One discretizes time by approximating the second partial derivative with respect to time in (4.17) with a second central difference defined by

$$\frac{\partial^2}{\partial t^2} g = \frac{g(t + \Delta t) - 2g(t) + g(t - \Delta t)}{\Delta t^2}, \tag{4.18}$$

where the once continuous time variable t is now sampled at discrete times $t = \tau \Delta t$, with $\tau = 0, 1, 2, \dots, N_t$ where Δt is an appropriate time interval.

Since the incident excitation h^e is known for all time, one differentiates h^e analytically and then samples the time derivative at discrete times $t = \tau \Delta t$. Time discretization of (4.17) and approximation of Λ_n by Π_n , as discussed above, yield

$$\begin{aligned}
& -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{1}{\Delta t^2} \sum_{n=1}^N [K(n; \tau_{mn} + 1) - 2K(n; \tau_{mn}) + K(n; \tau_{mn} - 1)] \Delta S_m \\
& \cdot \left\{ \hat{s}_{n-\frac{1}{2}} \int_{s_{n-\frac{1}{2}}}^{s_n} k^e(s_m, s') ds' + \hat{s}_{n+\frac{1}{2}} \int_{s_n}^{s_{n+\frac{1}{2}}} k^e(s_m, s') ds' \right\} \\
& + \frac{1}{\pi\mu} \sum_{n=1}^N \left\{ \frac{K(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^e(s_{m+\frac{1}{2}}, s') ds' \right. \\
& \quad - \frac{K(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^e(s_{m+\frac{1}{2}}, s') ds' \\
& \quad - \frac{K(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^e(s_{m-\frac{1}{2}}, s') ds' \\
& \quad \left. + \frac{K(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^e(s_{m-\frac{1}{2}}, s') ds' \right\} \\
& = -\Delta S_m \cdot \frac{\partial}{\partial t} h^e(s_m; \tau), \tag{4.19}
\end{aligned}$$

where τ_{mn} is now explicitly re-defined as an integer index which one employs to discern the correct coefficient $K(n; \tau_{mn})$ from the discretized time history of the magnetic current at s_n :

$$\tau_{mn} = \text{int} \left\{ \frac{t}{\Delta t} - \frac{R}{c\Delta t} \right\} = \tau - \text{int} \left\{ \frac{d(s_m, s_n)}{c\Delta t} \right\} \tag{4.20}$$

where $d(s_m, s_n)$ is the distance from the center of the source subdomain s_n to the

point of observation s_m , defined in (2.24a) for non-discretized source and observation points.

The kernel in (4.19) is given by the approximation

$$k^s(s_m, s') = \frac{1}{\sqrt{[d(s_m, s')]^2 + (w/2)^2}} \quad (4.21)$$

in which $d(s_m, s')$ is the distance from the continuous source point s' to the discrete point of observation s_m . For cases where s_m and s' fall on the same straight segment of the approximated contour of the slot, one can effectively employ the exact kernel

$$k^s(s_m, s') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\sqrt{[d(s_m, s')]^2 + w^2 \sin^2 \frac{\alpha'}{2}}} d\alpha' \quad (4.22)$$

in place of the approximate (reduced) kernel in (4.21). One chooses to employ the approximation (4.21) for all other cases because (4.22) does not apply when the axis of the slot at the observation point is not common with the axis of the slot at the source point. However, whenever applicable, the exact kernel is employed to enhance solution accuracy.

Having discretized the slot's integral equation in both space and time, one develops the expression necessary for acquiring the time domain solution by "marching in time." Observe that, for $m = n$, the current coefficient $K(n; \tau_{m_n} + 1)$ at time t in (4.19) reduces to $K(m; t + 1)$ when one employs (4.20) to evaluate the integer index τ_{m_n} . All other current coefficients in (4.19) occur at time index t and earlier. Thus, it is possible for one to reorganize (4.19) such that the current coefficient at the point of observation $K(m; t + 1)$ is calculated from the known incident magnetic field and source current coefficients which are known at time index t and earlier. When one reorganizes (4.19) in the manner suggested above, he obtains

$$\begin{aligned}
K(m; t+1) = & \frac{1}{\Delta S_m \cdot \left\{ \hat{s}_{m-\frac{1}{2}} \int_{s_{m-\frac{1}{2}}}^{s_m} k^s(s_m, s') ds' + \hat{s}_{m+\frac{1}{2}} \int_{s_m}^{s_{m+\frac{1}{2}}} k^s(s_m, s') ds' \right\}} \\
& \left[- \sum_{\substack{n=1 \\ n \neq m}}^N K(n; \tau_{mn} + 1) \Delta S_m \right. \\
& \quad \cdot \left\{ \hat{s}_{n-\frac{1}{2}} \int_{s_{n-\frac{1}{2}}}^{s_n} k^s(s_m, s') ds' + \hat{s}_{n+\frac{1}{2}} \int_{s_n}^{s_{n+\frac{1}{2}}} k^s(s_m, s') ds' \right\} \\
& \quad + \sum_{n=1}^N [2K(n; \tau_{mn}) - K(n, \tau_{mn} - 1)] \Delta S_m \\
& \quad \cdot \left\{ \hat{s}_{n-\frac{1}{2}} \int_{s_{n-\frac{1}{2}}}^{s_n} k^s(s_m, s') ds' + \hat{s}_{n+\frac{1}{2}} \int_{s_n}^{s_{n+\frac{1}{2}}} k^s(s_m, s') ds' \right\} \\
& \quad + c^2 \Delta t^2 \sum_{n=1}^N \left\{ \frac{K(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^s(s_{m+\frac{1}{2}}, s') ds' \right. \\
& \quad - \frac{K(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^s(s_{m+\frac{1}{2}}, s') ds' \\
& \quad - \frac{K(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^s(s_{m-\frac{1}{2}}, s') ds' \\
& \quad \left. + \frac{K(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^s(s_{m-\frac{1}{2}}, s') ds' \right\} \\
& \quad \left. + \frac{\pi}{\epsilon} \Delta t^2 \Delta S_m \cdot \left\{ \frac{\partial}{\partial t} h^{sc^*}(s_m; t) - \frac{\partial}{\partial t} h^{sc^b}(s_m; t) \right\} \right], \tag{4.23}
\end{aligned}$$

where one employs the definition of h^{sc} found in (2.7d).

Again, since everything on the right hand side of (4.23) is known at time index t and earlier, K is known at $t+1$. It is now possible for one to establish

initial conditions (usually $K(m;0) = 0$), define incident excitation (h^{ao} and h^{ob}), select an appropriate time step duration Δt , and calculate $K(m;1)$ for each of N subdomains. When all $K(m;1)$ are known, one advances the time index and calculates all $K(m;2)$. This "marching in time" procedure continues until all energy coupled into the slot via h^{ao} has been re-radiated. Additional comments regarding the selection of the time step duration Δt are made in a subsequent section.

For practical reasons, such as limited availability of computational resources, one terminates the solution process when the amplitude of $K(m;t)$ at a reference location consistently falls below some threshold percentage of its peak amplitude. With $K(m;t)$ known for all useful time, one can calculate all field quantities of interest on either side of the slotted screen. One calculates these field quantities by weighting the triangular expansion functions for magnetic current with their respective coefficients $K(n;t)$ to generate the piecewise linear approximation of the slot's magnetic current. He then employs the integral operators $\mathcal{E}^A[m]$ and $\mathcal{H}^A[m]$ of (2.3) in the total field expressions of (2.1) which yield the desired fields.

Field Advancement Formula for an Arbitrarily
Directed Narrow Slot Coupled with an
Arbitrarily Directed Thin Wire

Integral equations for the combination of a slotted screen and a nearby thin wire are derived in two steps in Chapter III. The first step involves derivation of the integral equation for an arbitrarily directed thin wire (with optional load) that radiates in the presence of an infinite screen. The second step involves introduction of a slot in the screen and subsequent modification of the wire equation to include a field coupling term to account for the presence of the slot. Also, a new slot integral equation is written that includes a field coupling term to account for the

wire. Thus, there are two coupled integral equations (3.24) and (3.29) to be solved in the coupled slot-wire problem.

Since the integral equation for the wire in the coupled slot-wire problem (3.24) is a more general equation than that of the wire in the presence of the unslotted screen (3.17), the solution of the integral equation for the wire above an unslotted screen is not presented separately, but, instead, attention is focused on the solution of the more general coupled slot-wire problem. In almost all respects, the solution of the coupled slot-wire integral equations proceeds along the same path as that outlined above for the simpler slotted screen integral equation. One discretizes (and thus approximates) the geometry of the the slot and the wire by selecting points s_p and l_p along each contour (C^s and C^w) and then connecting the points with straight segments of slot or wire as suggested in Figure 4-1a for the case of the slot. The wire differs from the slot only in that the wire's contour is three-dimensional ($\hat{l}$ can possess a z component) whereas the slot is confined to the xy plane (the screen). One then pulse tests the integral equations for the slot and wire, choosing the point of evaluation (or observation) to be located at the center (s_m or l_m) of the testing pulse Π_m of Figure 4-1b.

There are two distinctions to be made at this point. First, the wire integral equation contains an extra term $[-R\delta(l - l_\ell)\frac{\partial}{\partial t}i]$ that does not appear in the slot equation. The purpose of this term is to account for the load resistor R which is placed at one of the points l_p . The specific point is designated by the integer subscript ℓ . When one pulse tests (3.24), the load term $\mathcal{L}$ is given by

$$\mathcal{L} = \langle \Pi_m, -R\delta(l - l_\ell)\frac{\partial}{\partial t}i(l; t) \rangle \quad (4.24)$$

which, by means of (4.10) (specialized to the notation of the wire), becomes

$$\mathcal{L} = \int_{l_{m-\frac{1}{2}}}^{l_{m+\frac{1}{2}}} -R\delta(l-l_t)\frac{\partial}{\partial t}i(l;t)dl. \quad (4.25)$$

The integral in (4.25) is evaluated by inspection as one recalls the sampling property of the delta-dirac function

$$\int_{-\infty}^{\infty} \delta(x-x')f(x)dx = f(x') \quad (4.26)$$

and applies it to (4.25) to obtain

$$\mathcal{L} = \begin{cases} -R\frac{\partial}{\partial t}i(l_t;t) & , \ell = m \\ 0 & , \ell \neq m. \end{cases} \quad (4.27)$$

Note that the slot integral equation (2.23) and the coupled slot-wire integral equation for the slot (3.29) can both be generalized to account for loads. The pulse tested load term for the case of the slot is

$$\mathcal{L}^* = \begin{cases} -G\frac{\partial}{\partial t}k(s_t;t) & , \ell = m \\ 0 & , \ell \neq m, \end{cases} \quad (4.28)$$

where the load is treated as a lumped conductance connected across the narrow dimension of the slot at the point s_t and has vanishingly short length along the contour of the slot. The magnetic current in the slot produces electric current in the load conductance that contributes to the slot's magnetic field, the continuity of which is enforced along a path through the slot by the tested integral equation. In a manner similar to that discussed in Chapter III, one can make the load either capacitive or inductive by appropriate use of differentiation (replace G by $C\frac{\partial}{\partial t}k$) or integration (replace G by $[\int_0^t kdt]/L$).

The second distinction in the derivation of the field advancement formula for the isolated slot and the derivation of the formula for the coupled slot and wire is that the coupled slot-wire equations contain new terms that account for the field coupling between the two structures. The form of the coupling terms in (3.24) and (3.29) is brief and reasonably intuitive but does not readily lend itself to discretization. To facilitate the reader's understanding of the discretization of the coupling terms and, consequently, to allow the final field advancement formulas to be written without redundant derivation, one of the coupling terms in the wire equation is reorganized and then explicitly discretized. Discretization of the second coupling term of the wire equation and both coupling terms of the slot equation follow by inspection.

The first coupling term of the wire equation in the slot-wire coupling problem is

$$\tau = -\frac{1}{2\pi} \frac{\partial}{\partial t} \int_{C'} k(s'; \tau) [\hat{s}'(s') \times \hat{l}(l)] \cdot \nabla k^{w*}(l, s') ds' \quad (4.29)$$

which, by taking advantage of vector identities, one writes as

$$\tau = \frac{1}{2\pi} \frac{\partial}{\partial t} \int_{C'} k(s'; \tau) \hat{l}(l) \cdot [\hat{s}'(s') \times \nabla k^{w*}(l, s')] ds'. \quad (4.30)$$

When (4.30) is pulse tested by steps paralleling those of the previous section, one obtains

$$\tau = \frac{1}{2\pi} \frac{\partial}{\partial t} \left\{ \Delta L_m \cdot \int_{C'} k(s'; \tau) [\hat{s}'(s') \times \nabla k^{w*}(l_m, s')] ds' \right\}. \quad (4.31)$$

Having discretized the coupled integral equations with respect to the point of observation, one now discretizes the magnetic current in the slot and electric current on the wire. He does this by expansion of k and l in two series of triangular

expansion functions in the same fashion as the expansion of k in the previous section (also see Figure 4-1c). Magnetic and electric charge m and q are determined by differentiation of Λ_n as is suggested for the slot by Figure 4-1d.

After one expands the electric current on the wire, the pulse tested load term $\mathcal{L}$ (4.27) of the wire equation becomes

$$\mathcal{L} = \begin{cases} -R \frac{\partial}{\partial t} J(l_\ell; t) & , \ell = m \\ 0 & , \ell \neq m, \end{cases} \quad (4.32)$$

where J is the ℓ th coefficient of the series of triangular expansion functions for the electric current on the wire. The pulse tested coupling term τ (4.31) of the wire equation becomes (recall that Λ_n is approximated by Π_n for vector potential terms)

$$\tau = \frac{1}{2\pi} \frac{\partial}{\partial t} \left[\Delta L_m \cdot \sum_{n=1}^{N^s} K(n; \tau_{mn}) \left\{ \hat{s}_{n-\frac{1}{2}} \times \int_{s_{n-\frac{1}{2}}}^{s_n} \nabla k^{ws}(l_m, s') ds' \right. \right. \\ \left. \left. \hat{s}_{n+\frac{1}{2}} \times \int_{s_n}^{s_{n+\frac{1}{2}}} \nabla k^{ws}(l_m, s') ds' \right\} \right]. \quad (4.33)$$

Observe that the wire-excited-by-slot kernel k^{ws} of (3.21b) can be explicitly written

$$k^{ws}(l_m, s') = \frac{1}{[(x_m - x')^2 + (y_m - y')^2 + z_m^2]^{\frac{1}{2}}} \quad (4.34)$$

where z' is zero for source locations in the plane of the slotted screen. The gradient of (4.34) is easily found to be

$$\nabla k^{ws}(l_m, s') = - \frac{(x_m - x')\hat{x} + (y_m - y')\hat{y} + z_m\hat{z}}{[(x_m - x')^2 + (y_m - y')^2 + z_m^2]^{\frac{3}{2}}} \quad (4.35)$$

or by the definition of the unit vector $\hat{u}(l, s')$ which lies in the direction from the source location s' to the point of observation l (defined in (3.22b)),

$$\nabla k^{w*}(l_m, s') = -\hat{u}(l, s') [k^{w*}(l_m, s')]^2. \quad (4.36)$$

When one substitutes (4.36) into (4.33), he obtains the pulse tested triangle (approximated by pulse) expanded coupling term

$$\tau = -\frac{1}{2\pi} \frac{\partial}{\partial t} \left[\Delta L_m \cdot \sum_{n=1}^{N^s} K(n; \tau_{mn}) \left\{ [\hat{s}_{n-\frac{1}{2}} \times \hat{u}] \int_{s_{n-\frac{1}{2}}}^{s_n} [k^{w*}(l_m, s')]^2 ds' \right. \right. \\ \left. \left. [\hat{s}_{n+\frac{1}{2}} \times \hat{u}] \int_{s_n}^{s_{n+\frac{1}{2}}} [k^{w*}(l_m, s')]^2 ds' \right\} \right]. \quad (4.37)$$

To complete the discretization of the wire and slot equations for the coupled slot-wire problem, one approximates the partial derivatives with respect to time with finite differences. All second partial derivatives are approximated by second central differences, almost all first partial derivatives are approximated by first central differences, but the first partial derivative in the load term $\mathcal{L}$ is treated differently.

The time derivative in the pulse tested load term $\mathcal{L}$ of (4.32) must be carefully discretized. A simple central difference approximation is unsatisfactory since it requires knowledge of J at $\tau = t + 1$ when $\ell = m = n$ at time t . Although an algebraic expression can subsequently be written for J at $\tau = t + 1$ in terms of J at $\tau = t$ and earlier (based on a central difference), such an expression is found to be numerically unstable. On the other hand, if the time derivative in (4.32) is approximated by the three-point rule (the first derivative of a second order polynomial approximation of g evaluated at t)

$$\frac{\partial}{\partial t} g(t) = \frac{3g(t) - 4g(t - \Delta t) + g(t - 2\Delta t)}{2\Delta t}, \quad (4.38)$$

no future knowledge of J is required at time index t and the resulting field advancement formula is numerically stable. For reference, an even more precise approximation, known as the five-point rule, is available,

$$\frac{\partial}{\partial t}g(t) = \frac{25g(t) - 48g(t - 1\Delta t) + 36g(t - 2\Delta t) - 16g(t - 3\Delta t) + 3g(t - 4\Delta t)}{12\Delta t}, \quad (4.39)$$

but is not employed here in order to limit the complexity of the resulting field advancement formula. The time derivatives in the coupling terms of (3.24) and (3.29) are approximated by central differences and no numerical stability problems occur as long as the wire is separated from the slot by at least $c\Delta t$, a generally practical limitation.

Field Advancement Formula for the Wire in the Coupled Slot-Wire Problem

Now that the integral equation for the wire is fully discretized, it is possible for one to write the field advancement formula for electric current on the wire by isolating a term containing J at $\tau = t + 1$ and then writing an expression by which $J(m; t + 1)$ is determined from knowledge of J at time index t and earlier. The field advancement formula for $J(m; t + 1)$ on the wire is

$$J(n; t + 1) = \frac{1}{\Delta L_m \cdot \left\{ \hat{1}_{m-\frac{1}{2}} \int_{l_{m-\frac{1}{2}}}^{l_m} k^w(l_m, l') dl' + \hat{1}_{m+\frac{1}{2}} \int_{l_m}^{l_{m+\frac{1}{2}}} k^w(l_m, l') dl' \right\}} \left[- \sum_{\substack{n=1 \\ n \neq m}}^{N^w} J(n; \tau_{mn} + 1) \Delta L_m \cdot \left\{ \hat{1}_{n-\frac{1}{2}} \int_{l_{n-\frac{1}{2}}}^{l_n} k^w(l_m, l') dl' + \hat{1}_{n+\frac{1}{2}} \int_{l_n}^{l_{n+\frac{1}{2}}} k^w(l_m, l') dl' \right\} \right]$$

$$\begin{aligned}
& + \sum_{n=1}^{N^w} J(n; \tau_{m n}^i + 1) \Delta \mathbf{L}_m \\
& \cdot \left\{ \hat{\mathbf{l}}_{n-\frac{1}{2}} \cdot [\hat{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}] \int_{l_{n-\frac{1}{2}}}^{l_n} k_i^w(l_m, l') dl' + \hat{\mathbf{l}}_{n+\frac{1}{2}} \cdot [\hat{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}] \int_{l_n}^{l_{n+\frac{1}{2}}} k_i^w(l_m, l') dl' \right\} \\
& + \sum_{n=1}^{N^w} [2J(n; \tau_{m n}) - J(n, \tau_{m n} - 1)] \Delta \mathbf{L}_m \\
& \cdot \left\{ \hat{\mathbf{l}}_{n-\frac{1}{2}} \int_{l_{n-\frac{1}{2}}}^{l_n} k^w(l_m, l') dl' + \hat{\mathbf{l}}_{n+\frac{1}{2}} \int_{l_n}^{l_{n+\frac{1}{2}}} k^w(l_m, l') dl' \right\} \\
& - \sum_{n=1}^{N^w} [2J(n; \tau_{m n}^i) - J(n, \tau_{m n}^i - 1)] \Delta \mathbf{L}_m \\
& \cdot \left\{ \hat{\mathbf{l}}_{n-\frac{1}{2}} \cdot [\hat{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}] \int_{l_{n-\frac{1}{2}}}^{l_n} k_i^w(l_m, l') dl' + \hat{\mathbf{l}}_{n+\frac{1}{2}} \cdot [\hat{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}] \int_{l_n}^{l_{n+\frac{1}{2}}} k_i^w(l_m, l') dl' \right\} \\
& + c^2 \Delta t^2 \sum_{n=1}^N \left\{ \frac{J(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}})}{\Delta l_{n-\frac{1}{2}}} \int_{l_{n-1}}^{l_n} k^w(l_{m+\frac{1}{2}}, l') dl' \right. \\
& - \frac{J(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}})}{\Delta l_{n+\frac{1}{2}}} \int_{l_n}^{l_{n+1}} k^w(l_{m+\frac{1}{2}}, l') dl' \\
& - \frac{J(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}})}{\Delta l_{n-\frac{1}{2}}} \int_{l_{n-1}}^{l_n} k^w(l_{m-\frac{1}{2}}, l') dl' \\
& + \frac{J(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}})}{\Delta l_{n+\frac{1}{2}}} \int_{l_n}^{l_{n+1}} k^w(l_{m-\frac{1}{2}}, l') dl' \\
& - \frac{J(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}}^i)}{\Delta l_{n-\frac{1}{2}}} \int_{l_{n-1}}^{l_n} k_i^w(l_{m+\frac{1}{2}}, l') dl' \\
& \left. + \frac{J(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}}^i)}{\Delta l_{n+\frac{1}{2}}} \int_{l_n}^{l_{n+1}} k_i^w(l_{m+\frac{1}{2}}, l') dl' \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{J(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}}^i)}{\Delta l_{n-\frac{1}{2}}} \int_{l_{n-1}}^{l_n} k_i^w(l_{m-\frac{1}{2}}, l') dl' \\
& - \frac{J(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}}^i)}{\Delta l_{n+\frac{1}{2}}} \int_{l_n}^{l_{n+1}} k_i^w(l_{m-\frac{1}{2}}, l') dl' \Big\} \\
& - \frac{2\pi}{\mu} \Delta t \delta_{mn} R \Big\{ 3J(m, t) - 4J(m, t-1) + J(m, t-2) \Big\} \\
& - \frac{\Delta t}{\mu} \Delta L_m \cdot \sum_{n=1}^{N^s} [K(n; \tau_{mn} + 1) - K(n; \tau_{mn} - 1)] \\
& \Big\{ [\hat{s}_{n-\frac{1}{2}} \times \hat{u}_{mn}^{ws}] \int_{s_{n-\frac{1}{2}}}^{s_n} [k^{ws}(l_m, s')]^2 ds' + [\hat{s}_{n+\frac{1}{2}} \times \hat{u}_{mn}^{ws}] \int_{s_n}^{s_{n+\frac{1}{2}}} [k^{ws}(l_m, s')]^2 ds' \Big\} \\
& - 2\epsilon c \Delta L_m \cdot \sum_{n=1}^{N^s} [K(n; \tau_{mn} + 1) - 2K(n; \tau_{mn}) + K(n; \tau_{mn} - 1)] \\
& \Big\{ [\hat{s}_{n-\frac{1}{2}} \times \hat{u}_{mn}^{ws}] \int_{s_{n-\frac{1}{2}}}^{s_n} k^{ws}(l_m, s') ds' + [\hat{s}_{n+\frac{1}{2}} \times \hat{u}_{mn}^{ws}] \int_{s_n}^{s_{n+\frac{1}{2}}} k^{ws}(l_m, s') ds' \Big\} \\
& + \frac{4\pi}{\mu} \Delta t^2 \Delta L_m \cdot \frac{\partial}{\partial t} e^{scs}(l_m; t) \Big], \tag{4.40}
\end{aligned}$$

where δ_{mn} is the Kronecker delta defined by

$$\delta_{mn} = \begin{cases} 1 & , m = n \\ 0 & , m \neq n, \end{cases} \tag{4.41}$$

the superscript i in τ_{mn}^i suggests that the propagation delay is calculated from the source point on the wire's image to the observation point on the actual wire, and

$\hat{u}_{mn}^{ws}$ is the discretized form of (3.28b) given by

$$\hat{u}_{mn}^{ws} = \frac{r(l_m) - d'(s_n)}{|r(l_m) - d'(s_n)|}. \quad (4.42)$$

Field Advancement Formula for the Slot
in the Coupled Slot-Wire Problem

As was true in the case of the wire, once the integral equation for the slot is fully discretized, it is possible for one to write the field advancement formula for magnetic current in the slot by isolating a term containing K at $\tau = t + 1$ and then writing an expression by which $K(m; t + 1)$ is determined from knowledge of K at time index t and earlier. The field advancement formula for $K(m; t + 1)$ on the wire is

$$\begin{aligned} K(n; t + 1) = & \frac{1}{\Delta S_m \cdot \left\{ \hat{s}_{m-\frac{1}{2}} \int_{s_{m-\frac{1}{2}}}^{s_m} k^s(s_m, s') ds' + \hat{s}_{m+\frac{1}{2}} \int_{s_m}^{s_{m+\frac{1}{2}}} k^s(s_m, s') ds' \right\}} \\ & \left[- \sum_{\substack{n=1 \\ n \neq m}}^{N^s} K(n; \tau_{mn} + 1) \Delta S_m \right. \\ & \cdot \left\{ \hat{s}_{n-\frac{1}{2}} \int_{s_{n-\frac{1}{2}}}^{s_n} k^s(s_m, s') ds' + \hat{s}_{n+\frac{1}{2}} \int_{s_n}^{s_{n+\frac{1}{2}}} k^s(s_m, s') ds' \right\} \\ & + \sum_{n=1}^{N^s} [2K(n; \tau_{mn}) - K(n, \tau_{mn} - 1)] \Delta S_m \\ & \cdot \left\{ \hat{s}_{n-\frac{1}{2}} \int_{s_{n-\frac{1}{2}}}^{s_n} k^s(s_m, s') ds' + \hat{s}_{n+\frac{1}{2}} \int_{s_n}^{s_{n+\frac{1}{2}}} k^s(s_m, s') ds' \right\} \\ & + c^2 \Delta t^2 \sum_{n=1}^{N^s} \left\{ \frac{K(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-\frac{1}{2}}}^{s_n} k^s(s_{m+\frac{1}{2}}, s') ds' \right. \end{aligned}$$

$$\begin{aligned}
& - \frac{K(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^s(s_{m+\frac{1}{2}}, s') ds' \\
& - \frac{K(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^s(s_{m-\frac{1}{2}}, s') ds' \\
& + \frac{K(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^s(s_{m-\frac{1}{2}}, s') ds' \Big\} \\
& + \frac{\Delta t}{4\epsilon} \Delta S_m \cdot \sum_{n=1}^{N^w} [J(n; \tau_{mn} + 1) - J(n; \tau_{mn} - 1)] \\
& \left\{ [\hat{\mathbf{l}}_{n-\frac{1}{2}} \times \hat{\mathbf{u}}_{mn}^{sw}] \int_{l_{n-\frac{1}{2}}}^{l_n} [k^{sw}(s_m, l')]^2 dl' + [\hat{\mathbf{l}}_{n+\frac{1}{2}} \times \hat{\mathbf{u}}_{mn}^{sw}] \int_{l_n}^{l_{n+\frac{1}{2}}} [k^{sw}(s_m, l')]^2 dl' \right\} \\
& + \frac{\mu c}{2} \Delta S_m \cdot \sum_{n=1}^{N^w} [J(n; \tau_{mn} + 1) - 2J(n; \tau_{mn}) + J(n; \tau_{mn} - 1)] \\
& \left\{ [\hat{\mathbf{l}}_{n-\frac{1}{2}} \times \hat{\mathbf{u}}_{mn}^{sw}] \int_{l_{n-\frac{1}{2}}}^{l_n} k^{sw}(s_m, l') dl' + [\hat{\mathbf{l}}_{n+\frac{1}{2}} \times \hat{\mathbf{u}}_{mn}^{sw}] \int_{l_n}^{l_{n+\frac{1}{2}}} k^{sw}(s_m, l') dl' \right\} \\
& + \frac{\pi}{\epsilon} \Delta t^2 \Delta S_m \cdot \left\{ \frac{\partial}{\partial t} h^{ca}(s_m; t) - \frac{\partial}{\partial t} h^{cb}(s_m; t) \right\}, \tag{4.43}
\end{aligned}$$

where $\hat{\mathbf{u}}_{mn}^{sw}$ is the discretized form of (3.28b) given by

$$\hat{\mathbf{u}}_{mn}^{sw} = \frac{\mathbf{d}(s_m) - \mathbf{r}'(l_n)}{|\mathbf{d}(s_m) - \mathbf{r}'(l_n)|}. \tag{4.44}$$

Solution of Coupled Slot-Wire Problem

Equations (4.40) and (4.43) provide one the means to calculate $J(m, t + 1)$ and $K(m, t + 1)$ from knowledge of J and K at time index t and earlier. Thus, it is possible for one to establish initial conditions (usually $J(m, 0) = K(m, 0) = 0$),

define incident excitation (h^{ac} , h^{cb} , and e^{ac}), select an appropriate time step interval Δt , and calculate all $J(m, 1)$ and $K(m, 1)$. When all $J(m, 1)$ and $K(m, 1)$ are known, one advances the time index and calculates all $J(m, 2)$ and $K(m, 2)$. Again, this "marching in time" procedure continues until all energy induced in the slot-wire structure by the incident excitation h^{ac} , h^{cb} , and e^{ac} has been re-radiated.

As in the case for the simple slotted screen, one terminates the solution process when the amplitude of J or K at a reference location consistently falls below some threshold percentage of its peak amplitude. With J and K known for all useful time, one calculates all field quantities of interest by forming the piecewise linear approximation of $i(l, t)$ and $k(l, t)$ from J and K and employing the integral operators $\mathcal{E}^A[m]$ and $\mathcal{K}^A[m]$ of (2.3) and $\mathcal{E}^w[j]$ and $\mathcal{K}^w[j]$ of (3.2) in the total field expressions of (3.18). Of course, one can take advantage of the approximations that lead to the field advancement formulas in order to write efficient expressions for the desired fields.

Stability of the Time-Domain Solution Technique

The destabilizing effect of careless approximation of the differentiation of $i(l, t)$ of (4.25), when one includes the load resistor R in the field advancement formula (4.40), is discussed above. There are other causes of instability, including careless selection of the time step duration Δt , which is limited by the Courant condition

$$\Delta t \leq \Delta t_{max} = \frac{\delta_{min}}{c}, \quad (4.45)$$

where δ_{min} is the shortest distance between a discretized point of observation (s_m , l_m or $s_m - \frac{1}{2}$, $l_m - \frac{1}{2}$) and the center of the current or charge expansion function that

excites it (s_n, l_n or $s_{n-\frac{1}{2}}, l_{n-\frac{1}{2}}$). One must preview the discretized geometry of the problem in order to determine δ_{min} prior to the start of the solution process in order to guarantee that (4.45) is satisfied. Choices of Δt that violate (4.45) cause the solution process to rapidly become unstable since one is, in effect, attempting to excite a point of observation with a field that is not given sufficient time to propagate at the speed of light to the point of observation. One might be tempted to violate (4.45) since the total solution is acquired in fewer time steps as the time step duration Δt grows, thus saving computation time. Reduced computation time is a compelling reason to maximize Δt , but one must satisfy (4.45) to avoid instability.

In simple problems where the slot follows a straight contour and the subdomains are all of equal length, the field advancement formulas reduce to a more compact form that can be derived from a simple one-potential integral equation (see Appendix C). In this very special case, it is found that the choice $\Delta t = \Delta t_{max}$ provides a stable and time efficient solution that avoids instability for thousands of time steps.

In cases where non-uniform subdomain lengths are specified or where the slot or wire contour is not straight, it is sometimes necessary to select a smaller value for the duration of the time step Δt than is theoretically allowed by (4.45). This is done to maintain a stable solution after a large number of time steps. Long solution times are often necessary when tightly coupled slots and wires exhibit resonant responses that are only gradually damped by re-radiation.

In order to investigate the impact of Δt on the stability of the solution process, when non-uniform subdomains are employed or when the slot or wire contour is curved, a series of simple slot problems, depicted in Figure 4-2, are solved. Simple

slot geometries are chosen since, other than the resonant behavior of tightly coupled slots and wires, all significant stability effects can be observed in the case of a simple slotted screen. Conclusions drawn from the data of these simple slot geometries apply equally well to the wire problem and to the coupled slot-wire problem.

The slots depicted in Figure 4-2 are excited by a normally incident plane wave having Gaussian time history given by

$$h_y^{inc} = e^{-(t-t_0)^2 A^2}, \quad (4.46)$$

where the short-circuit magnetic field in region b ($z < 0$) is

$$h^{sc} = 2h_y^{inc} \hat{y}, \quad (4.47)$$

in which t_0 is chosen such that the amplitude of h_y^{inc} is less than 0.001 prior to the start of the solution and A^2 is chosen such that the amplitude of h_y^{inc} is greater than 0.1 for a duration of 0.5 ns. A typical excitation pulse appears in Figure 4-3 for A^2 chosen as discussed above for the slots of Figure 4-2. h_y^{inc} exceeds 0.1 for at least 20 time steps (when $\Delta t = \Delta t_{max}$) and more when $\Delta t < \Delta t_{max}$. Thus, the general shape of the Gaussian pulse is well preserved by the temporal discretization process.

Stability Effects Related to the Choice of Non-Uniform Subdomain Lengths

In this section, acquisition of the time-domain solution is described for a straight slot with uniform subdomain length and for a straight slot with non-uniform subdomain length. These slots are depicted in Figures 4-2a and 4-2b where one should observe the dimensions of each slot and the relative sizes of the subdomains. Observe that the subdomains in Figure 4-2a are of uniform length while the ratio

of maximum to minimum subdomain lengths in Figure 4-2b is almost 3:1. In both cases, the slot is 10 cm long, 1 mm wide, y-directed, and comprises 10 segments.

In Figure 4-4, one sees the time-domain magnetic current in subdomain 5 (the center subdomain) of each of the two slots (one set of data is presented for the slot of Figure 4-2a and two sets of data are presented for the slot of Figure 4-2b). The data for the case of uniform subdomains (slot of Figure 4-2a) are clearly stable for all observed time when $\Delta t = \Delta t_{max}$. However, when $\Delta t = \Delta t_{max}$ for the non-uniform subdomain length case of Figure 4-2b, the solution is clearly unstable at 5 ns. When the time step is reduced to $0.33 \Delta t_{max}$, the solution remains stable until roughly $t = 7$ ns, but at the cost of more than three times the number of time steps used in the $\Delta t = \Delta t_{max}$ case.

Several comments are now in order. First, it is observed that solution stability can be improved by reducing the duration of Δt . This is generally found to be true, although the improvement in this case is not as dramatic as in the case presented in the following section. The less than dramatic improvement in stability in the present case has its roots in the poor choice of the subdomains in Figure 4-2b. The spatial distribution of magnetic current in the slot is such that its maximum gradient (maximum magnetic charge in the slot) occurs at the ends of the slot. With the subdomains chosen as they are in Figure 4-2b, the spatial variation of the magnetic current, and thus the magnetic charge, is very poorly approximated by the spatial discretization, and this leads to the observed instability.

When the subdomains of Figure 4-2b are rearranged in the fashion of Figure 4-2c, the magnetic current observed in Figure 4-5 results. In this figure one observes that, for $\Delta t = \Delta t_{max}$, the solutions for the slots depicted in Figures 4-2a and 4-2c are stable for long periods. Obviously, when the small subdomains are placed

at the ends of the slot, one obtains greater stability than when they are placed at the middle of the slot since the spatial variation of the magnetic current is better approximated. However, the choice of small subdomains restricts Δt_{max} and, consequently, more time steps are required to advance the solution to the same point in time. In Figure 4-5, one observes that for 1000 time steps the solution for the uniform subdomain case (of Figure 4-2a) is advanced further than that of the non-uniform subdomain case (of Figure 4-2c). Thus, one must carefully balance the potential benefits of using non-uniform subdomain lengths against the related computational costs.

Stability Effects Related to Curved Contours

In this section, acquisition of the time domain solution is described for a curved slot with uniform subdomain length in order to further investigate the effect of the time step duration on stability. The geometry of the curved slot is depicted in Figure 4-2d where one observes that the slot's contour has a radius of 3 cm and that the slot comprises 10 segments or 9 unknowns.

In Figure 4-6, one observes the time-domain magnetic current in subdomain 5 at the axial midpoint of the curved slot. For the case of $\Delta t = \Delta t_{max}$, the solution grows unstable at about 2.5 ns. When the time step duration is reduced to 0.8 Δt_{max} , the solution remains stable until about 9 ns, a considerable improvement since only 25% more time steps are required to reach the same point in time. When the time step duration is further reduced to 0.7 Δt_{max} , the solution remains stable for all time observed in this exercise.

No precise theory is presently available to explain the source of the instability in the case presented above or to explain the cure that one obtains by reducing the duration of the time step. It is suspected that smaller time steps allow finer

resolution in the selection of the correct current coefficient after the discretized delayed time index $\tau_{m,n}$ is calculated, and, thus, that small discretization errors which could lead to instability are reduced. Stable solutions for annular contours are possible even after more than 8000 time steps when one chooses the time step duration Δt appropriately.

One concludes that stable time-domain solutions for slotted screens, loaded wires in the presence of a screen, and coupled slotted screens and wires are possible if one avails himself of the techniques developed in this chapter. With a suitable choice of subdomain length and placement and with a sensible choice of Δt , one can successfully continue the "marching in time" process for many thousands of time steps and thereby progressively acquire the time-domain solution.

Figure 4-1. Approximated Geometry of Curved Slot and Associated Testing and Expansion Functions

Figure 4-2. Simple Slot Geometries Suitable for Investigation of Time-Domain Solution Stability

Figure 4-3. Gaussian Pulse Time History of h_v^{inc} for Stability Investigation

Figure 4-4. Time History of Equivalent Magnetic Current at the Center of the 10 cm by 1 mm Straight Slots of Figures 4-2a and 4-2b

Figure 4-5. Time History of Equivalent Magnetic Current at the Center of the 10 cm by 1 mm Straight Slots of Figures 4-2a and 4-2c

Figure 4-6. Time History of Equivalent Magnetic Current at the Center of the 9.4 cm by 1 mm Curved Slot of Figure 4-2d

CHAPTER V

FREQUENCY-DOMAIN INTEGRAL EQUATION

SOLUTION TECHNIQUE

In this chapter one finds a brief description of the standard frequency-domain solution technique that is employed to solve the Fourier transformed integral equation for the slot (2.31) and the transformed integral equations for the wire (3.35) and the slot (3.36) in a coupled wire-slot structure. The solution technique briefly outlined in this chapter is a method of moments solution based on work originally done by Harrington [15] and subsequently refined by Wilton and Butler [16] and others. The method of equation enforcement is discussed as is the representation chosen for the unknown in the integral equation. Ultimately, a matrix equation is obtained whose elements have specific physical interpretation and whose solution is obtained by standard techniques of linear algebra. Finally, a means by which one can easily accommodate junctions in slots and wires is discussed and a simple test case is explored to establish the validity of the technique.

Frequency-Domain Solution of the Slot Equation

The frequency domain integral equation for the curved slot (2.31), which is determined from the time-domain integral equation (2.23) by Fourier transformation, is solved by a standard method of moments technique. Many aspects of this technique closely parallel the time-domain solution technique that is described in detail in Chapter IV.

The slot's geometry is approximated by the same piecewise linear approximation that is suggested by Figure 4-1a. That is, all points, vectors, unit vectors,

and distances are defined as they are in Chapter IV. The frequency-domain integral equation is enforced by pulse testing, where the testing pulses Π_m are depicted in Figure 4-1b. The unknown axially directed equivalent magnetic current K is expanded in a series of triangular basis functions A_n , which are depicted in Figure 4-1c. The spatial derivative of the triangular basis function provides the pulse-doublet expansion function for magnetic charge that is depicted in Figure 4-1d. Indeed, after these approximations, the resulting frequency-domain integral equation is the same as that which one would obtain by Fourier transformation of (4.17) which is the spatially discretized time-domain slot equation just prior to temporal discretization.

After the frequency-domain equation is spatially discretized in a manner identical to the discretization of the time-domain equation, the method of moments technique and the "marching in time" technique of Chapter IV diverge. In the Fourier transformed complement to Equation (4.17), the coefficient of the expansion function K_n is complex and the frequency-domain kernel ($K'(s_m, s')$ in Equation 2.32) carries the phase delay information that is inherent in the delayed time variable τ of (2.23). In the method of moments technique, one, in essence, factors out the complex coefficients K_n from the left hand side of the Fourier transformed version of (4.17) and forms a system of equations which comprises M pulse tested equations each of which is written in terms of N unknown complex expansion function coefficients. The solution of this system of equations is then obtained by standard techniques of linear algebra.

The moment method process is now described in a slightly different way that may shed more light on the physical meaning of the individual elements in the system of equations. The pulse tested and triangle expanded version of the transformed

frequency domain slot integral equation (2.31) is written as the matrix equation

$$(-Y)(K) = (-H^c) \quad (5.1)$$

where the individual elements Y_{mn} of the square matrix (Y) represent the pulse-tested magnetic field which is caused in subdomain m by the magnetic current basis function that resides in subdomain n . As is suggested by the symbol, these elements have the units of admittance, i.e., Siemens. The column matrix (H^c) contains the tested tangential component of the short-circuit magnetic field which is due to the complex excitation in each subdomain and which has the units of electric current. The unknown (K) is a column matrix whose elements are the basis function coefficients which have the units of voltage. Thus, in a sense, the system of equations is an electromagnetic generalization of Ohm's law ($YV=I$).

For a given point of observation s_m in the slot, the m th equation inherent in (5.1) is a statement that the tested magnetic field observed at s_m which is due to contributions from N triangular expansion functions (appropriately weighted by the coefficients K_n) is exactly equal to the pulse-tested short-circuit magnetic field that exists at the point of observation and which is due to the incident excitation. The system of equations is an explicit statement that the coefficients K_n must adjust themselves such that the pulse tested magnetic field observed at s_m , which is radiated by the N weighted expansion functions, must precisely equal the pulse tested short-circuit magnetic field at s_m at all M locations simultaneously. Since the equations are linearly independent (a result of the choice of linearly independent expansion functions and due to properties of operator equations) one can determine the current coefficients (K) by using a linear equation solution technique such as matrix inversion or Gaussian elimination.

The elements of (Y) are only valid at a single frequency for a given geometry. In order to determine the frequency response of a slot, the slot equation must be solved at each of many individual frequencies in the range of interest. This is in contrast to the time domain case where all available frequency information is contained within the time record and can easily be recovered by use of the fast Fourier transform (FFT).

For the case of multiple subslots, (5.1) is still valid. However, the notation associated with the elements of each matrix is adjusted to accommodate the additional complexity of the geometry. For coupled subslots, an element of the square matrix (Y) is Y_{mn}^{uv} which represents the tested magnetic field generated in subdomain m of subslot u by the magnetic current basis function in subdomain n of subslot v . An element of the column matrix (K) is K_n^v which represents the coefficient of the unknown magnetic current in subdomain n of subslot v . Finally, an element of the column matrix (H^{sc}) is H_m^{sc} which represents the tested tangential component of the short-circuit magnetic field in subdomain m of subslot u .

Frequency-Domain Solution of the Coupled Wire and Slot Equations

The notation given above for coupled subslots is also helpful when it is adapted to accommodate coupled slots and wires. Following the path indicated above, one approximates the geometry of the slot and the wire in the manner suggested by Figure 4-1a. He then pulse tests the frequency-domain integral equations for the slot (3.36) and the wire (3.35) and subsequently expands the unknown equivalent magnetic current K in the slot and electric current I on the wire in the same manner that leads from (4.1) to (4.17) in the time-domain case. This process yields a system of equations which comprises $M = M^s + M^w$ equations written in terms of

$N = N^s + N^w$ unknowns, where $M = N$ is the total number of subdomains in the problem, s indicates subdomains located in the slot, and w indicates subdomains on the wire.

Following the lead of the previous section, the pulse tested and triangle expanded versions of the Fourier transformed coupled slot (3.36) and wire (3.35) integral equations are written as the matrix equation

$$\begin{pmatrix} -Y_{mn}^{ss} & C_{mn}^{sw} \\ C_{mn}^{ws} & -Z_{mn}^{ww} \end{pmatrix} \begin{pmatrix} K_n^s \\ I_n^w \end{pmatrix} = \begin{pmatrix} -H_m^{scs} \\ -E_m^{scw} \end{pmatrix} \quad (5.2)$$

where the individual elements of the square submatrix Y_{mn}^{ss} represent the pulse-tested magnetic field which is caused in subdomain m of the slot by the magnetic current basis function that resides in subdomain n of the slot. Again, these elements have the units of admittance.

The elements of C_{mn}^{sw} represent the pulse-tested magnetic field in subdomain m of the slot that is radiated by the electric current basis function of subdomain n on the wire. Conversely, C_{mn}^{ws} represents the pulse tested electric field which is observed in subdomain m of the wire and is generated by the magnetic current basis function n in the slot. As is implied by the symbols, C_{mn}^{sw} and C_{mn}^{ws} account for coupling between the slot and the wire.

Z_{mn}^{ww} represents the pulse tested electric field in subdomain m of the wire that is due to the electric current basis function n of the wire. As is implied by the symbol, these elements have the units of impedance. The submatrices K_n^s and I_n^w represent the unknown coefficients of the magnetic and electric current expansion functions in the slot at s_n and on the wire at l_n , while H_m^{scs} and E_m^{scw} represent the short-circuit magnetic field that illuminates the slot at s_m and the short-circuit electric field that illuminates the wire at l_m .

One clearly sees that the slot and the wire are coupled when he inspects (5.2). In order to satisfy (5.2), the current coefficients K_n^s and I_n^w must adjust themselves in such a manner that the total pulse-tested magnetic field observed at s_m in the slot, which is due to all K_n^s and I_n^w , must precisely equal the pulse-tested short-circuit magnetic field at s_m . The current coefficients K_n^s and I_n^w must also adjust themselves in such a manner that the total pulse-tested electric field observed at l_m on the wire, which is also due to all K_n^s and I_n^w , must precisely equal the pulse-tested short-circuit electric field at s_m . Again, one solves the system of equations represented by (5.2) by standard techniques of linear algebra to obtain the unknown complex coefficients K_n^s and I_n^w .

Junctions in Slots and Wires

A junction in a slot or wire structure is easily accommodated by the method of moments solution technique. Two methods one can use to implement a junction capability in a moment method code are discussed, an algorithm by which one detects junctions is described, a simple test problem is solved by means of the second method, and the numerical solution is compared with experimental data to check the validity of the junction model and the overall solution technique.

Standard Junction Model

An expedient (and commonly employed) approach one uses to model the junction involves placing N half-triangle expansion functions on each of the N slot or wire "ends" at the junction (which adds N unknowns to the problem), pulse testing the integral equation at the junction (which provides $N - 1$ additional linearly independent pulse-tested equations), and supplying one more equation that explicitly enforces Kirchoff's current law at the junction (i.e., forces the currents directed into

the junction to sum to zero). This approach is entirely valid, but the equation that enforces the current law disturbs the properties and interpretation of the matrix (Y) for a slot or (Z) for a wire, since most of the new elements in (Y) or (Z) are zero and those that are non-zero no longer carry the physical significance of pulse-tested magnetic field or pulse-tested electric field (as discussed above).

Improved Junction Model

If one employs a different, but simple and intuitively satisfying approach to enforce Kirchoff's current law at the junction, the properties of the resulting (Y) or (Z) matrix are preserved and he need not add a supplementary equation to enforce the current law. As discussed above, pulse testing at the junction of N slots or wires produces $N - 1$ new linearly independent equations. What one desires is a way to represent the unknown current at the junction so that only $N - 1$ additional unknowns are created. This one easily accomplishes, in an intuitively satisfying way, by selecting one of the "ends" at the junction to be the "source" for current and the other $N - 1$ ends at the junction to be "sinks." One then introduces $N - 1$ whole triangular expansion functions such that the rising half of Λ_n rests on the source slot or wire's end and the falling half of each Λ_n rests on only one of the $N - 1$ sink slot or wire ends.

Thus, by construction, the total current entering the junction from the source (the sum of the $N - 1$ rising halves of the triangles) precisely equals the total current that exits the junction via the $N - 1$ sinks, and there is no need for an additional equation to explicitly enforce the current law. Moreover, all elements of the matrix (Y) for a slot or (Z) for a wire continue to carry the physical significance of either pulse-tested magnetic field or electric field and the properties of the matrix are preserved since one avoids a new row of elements almost all of which are zero.

Automatic Junction Handling Algorithm

If one has a moment method code that accommodates multiple coupled (but not joined) slots or wires, an expedient approach to defining the necessary additional expansion functions required for a junction is to define $N - 1$ "jumpers" that all overlap the end segment of the source slot or wire and subsequently overlap one end segment of each of the $N - 1$ sinks. Each jumper bears one of the $N - 1$ required expansion functions and introduces one of the $N - 1$ unknown current coefficients into the system of equations for the slot or wire. The moment method code then automatically assigns a testing pulse to each of the jumpers (since it cannot distinguish them from any other very short slot or wire), and the required $N - 1$ additional linearly independent equations are produced. It is important to understand that the function of the jumper is only to carry the new expansion function and stimulate production of another pulse-tested equation. The width of the slot or the radius of the wire is not modified at the junction by the presence of the jumper.

It is a simple matter to incorporate an algorithm into the code that examines the geometry of the problem, identifies common endpoints of slots or wires, and generates the required jumpers which lead to the additional unknowns and pulse tested equations that are required to model the junction. Such an algorithm identifies the "start" and "end" of each slot or wire and creates a table of starts and ends that are initially unflagged. For each of $n = 1, 2, \dots, N$ starts and $m = 1, 2, \dots, N$ ends, which may or may not be part of a junction, the following series of checks and responses is performed.

First, one checks whether the start of slot or wire n resides at the same location as the unflagged start of any "other" slot or wire m ($n \neq m$). If so, a jumper between the two is generated and the m th slot or wire start is flagged as "used." He then

checks whether the n th start resides at the same location as the unflagged end of any slot or wire m including its own end ($n = m$). If so, a jumper is generated to go between the n th start and the m th end and the m th end is flagged as used.

Now, the n th unflagged end is checked against the unflagged start of any slot or wire m , including its own start ($n = m$). If an unflagged start is found to reside at the same location, a jumper is generated and the m th start is flagged. Finally, the n th unflagged end is checked for proximity to the unflagged end of any other slot or wire m ($n \neq m$). If two different unflagged ends reside at the same location, a jumper is generated and the m th end is flagged. These checks are performed N^2 times in a double loop indexed in n and m . This algorithm successfully identifies and accommodates junctions in a wide variety of narrow slot and thin wire geometries.

Validation of the Improved Junction Technique

One finds it a simple task to verify the efficacy of the jumper generation algorithm by observing its performance when it is challenged by a series of difficult geometries. However, the validity of the simplified junction model, once it is incorporated in a moment method code, is more difficult to assess. In an effort to validate the simpler and more intuitively satisfying junction model, an experiment was devised in which a 15 cm by 1 mm slot was cut in a 30 cm by 30 cm sheet and the input impedance of the slot was measured at its axial center from 100 MHz to 4 GHz. A discussion of the experimental apparatus and the measurement techniques involved in this experiment appears in Appendix B. Figure B-1 depicts the manner in which the slot was excited and Figure B-2 depicts the evolution of the slot as extensions were made to it by removal of brass from the 30 cm by 30 cm sheet. As one observes in Figure B-2, the extension process ultimately produces a slot that forks at both ends and thus contains two junctions. One understands that there are

limitations on accuracy caused by the screen size and feed scheme. These limitations are discussed in Appendix B. An impedance measurement was performed after each extension was fabricated, and the measured data are compared, below, with data computed by a moment method code that is based on the concepts discussed above and which incorporates the simplified junction model and the automatic junction identification and jumper generation functions described above.

In Figure 5-1, one sees the input impedance at the axial center of the simple 15 cm by 1 mm slot. Notice the excellent agreement of computed and measured data below 2 GHz and the generally good agreement above. The slot obviously experiences a strong half-wave resonant response at about 1 GHz, but does not respond significantly at the full-wave resonant frequency near 2 GHz. Since the slot is fed at its axial center, the excitation is an even function of displacement from the feedpoint at the center of the slot and the resulting response of the slot's magnetic current must therefore be an even function of displacement. Even-function excitation discriminates against the odd-function full wave resonance at 2 GHz, and all that remains at that frequency is the very small non-resonant even-function response of the slot. The resonance near 3 GHz also produces an even-function distribution of magnetic current and is clearly evident in Figure 5-1.

In Figure 5-2, one observes the input impedance after one extension to the slot has been fabricated. Note that the total slot is now longer and, as a result, the half-wave resonance occurs at a lower frequency than before the extension was fabricated. Also observe that the full-wave resonance that was discriminated against when the slot was fed at its center is clearly evident at about 1.5 GHz now that the total slot is no longer center fed and there is a significant odd-function component of slot excitation. Again, observe the good agreement among the computed and measured data.

Data for the first slot junction appear in Figure 5-3. The excellent agreement observed among the computed and measured data suggest that the junction in the slot has been well modeled. The half-wave resonance occurs at a still lower frequency since the slot is now electrically longer than in the previous case, and the full-wave resonance is still evident due to the odd-function excitation that results from the location of the feedpoint.

When the third slot extension is fabricated in the sheet, the response observed in Figure 5-4 occurs. Observe that the structure is significantly balanced by the presence of the third extension. Now the excitation point is closer to the geometric center of the structure and thus the excitation is almost completely an even function of displacement from the geometric center of the structure. Thus, it is not a surprise that the full-wave resonance at about 1.3 GHz is almost completely squelched. Agreement among the computed and measured data is excellent with respect to the shape of the response and the frequency accuracy of the resonant peaks. However, agreement of the amplitude of the measured and computed input impedance has deteriorated somewhat. It is believed that the computed data are more accurate and that the measured data suffer from error that results from the large magnitude of the measured impedance at the resonant peaks which is perturbed by the presence of the feedline that carries the excitation to the slot. A thorough discussion of these errors is found in Appendix B.

Finally, the data that pertain to the fully extended slot appear in Figure 5-5. There are two slot junctions in this structure, and the input impedance is very high at about 2 GHz where the structure attains its second even-function resonance. Observe the generally good agreement between the computed and measured data for measured impedances below about 600 Ω . For impedances above 600 Ω , a significant discrepancy is observed that is most likely due to "unaccounted for"

loading of the slot by the feed line and scattering from the overall feedline structure which is encouraged by the transition from the unbalanced feedline to the balanced radiating structure. Also observe that the full-wave resonance at about 1.3 GHz is now completely squelched since the excitation is once again strictly an even function of displacement from the feedpoint.

This simple experiment establishes the basic validity of the method of moments solution technique and the simplified junction model presented above. In the next chapter, a method of moments code is employed to predict the fields that penetrate a slotted screen and subsequently couple to a loaded thin-wire scatterer on the shadow side. There, also, one observes good agreement among computed and measured data which provides additional assurance of the validity of this frequency domain technique.

Figure 5-1. Impedance at the Center of a 15 cm by 1 mm Slot with no Extensions

Figure 5-2. Impedance at the Center of a 15 cm by 1 mm Slot with 1 Extension

Figure 5-3. Impedance at the Center of a 15 cm by 1 mm Slot with 2 Extensions

Figure 5-4. Impedance at the Center of a 15 cm by 1 mm Slot with 3 Extensions

Figure 5-5. Impedance at the Center of a 15 cm by 1 mm Slot with 4 Extensions

CHAPTER VI

DISCUSSION OF SLOT AND COUPLED SLOT-WIRE DATA

In this chapter, data generated by numerical solutions of integral equations and data gathered by laboratory experimentation are presented, compared, and discussed. The time-domain response of magnetic current in narrow slots, electric current on nearby coupled thin wires, and the electric field at a location roughly midway between the slot and the wire are given and the data are discussed. Data generated in both the time and frequency domains are compared in the frequency domain for a screen that is perforated by a narrow slot having annular or rectangular contour, by ensembles of coupled narrow slots having generally annular or generally rectangular contour, and by a narrow slot having straight, annular, or rectangular contour that is coupled with a straight or bent thin wire located on the shadow side of the slotted screen. Also, electric field data on the shadow side of a slotted screen and electric current data at the center of a nearby coupled thin wire are compared with measured data in an effort to validate the time-domain and frequency-domain solution techniques. Electric field data collected on the shadow side of a screen perforated by a straight narrow slot or an ensemble of straight colinear slots are not considered here since such slots are thoroughly treated elsewhere [11]. Throughout this chapter, key features of the data are discussed in order to illustrate significant theoretical concepts and to draw attention to dominant physical effects.

The essential geometry pertinent to each figure appears in a small sketch at the top of the figure. Specific details regarding the angle of incidence of the excitation or the distance and angle of the sensor are also marked on the figure. The frequency-

domain form of the incident excitation is given by

$$H^{inc} = H_0^{inc} (\sin \theta^i) e^{jk_y \cos \theta^i} \quad (6.1)$$

where the incident angle θ^i is defined in Figure A-11 such that excitation at $\theta^i = 90^\circ$ is normally incident on the slotted screen. Sensor positioning for both the computed data and the measured data also appears in Figure A-11 of Appendix A where thorough descriptions of the experimental apparatus and the measurement techniques are provided.

One additional comment is in order. Almost all data determined by time-domain methods are actually presented in the frequency domain to facilitate comparison with the frequency-domain computed data and the measured data. The time-domain data are transformed to the frequency domain by means of the Fast Fourier Transform (FFT). These data are called time-domain computed data or just time-domain data in order that one may distinguish between the data computed in the time domain and those computed in the frequency domain.

Time-Domain Data for Slotted Screens Coupled with Nearby Thin Wires

In Figure 6-1, one sees the time-domain magnetic current (or transverse slot voltage) at the center of a 15 cm by 1 mm straight slot that is coupled to a 15 cm straight wire of radius 0.4 mm. This current results from a normally incident plane wave pulse having Gaussian time history (a typical Gaussian pulse is seen in Figure 4-3). The magnetic field of the plane wave is y-directed and the Gaussian pulse is positive for all time. The peak amplitude of the incident magnetic field is $1 \frac{A}{m}$ and the width factor of the Gaussian pulse is fixed so that the pulse is sampled 50 times by the time domain solution technique while its amplitude is greater than or equal

to $0.1 \frac{\lambda}{m}$. The slot is divided into 50 subdomains as is the wire. The time step is fixed at 20% of Δt_{max} (2 ps for this geometry) and the solution process continues for 8192 time steps. The same excitation is used to excite the structures observed in Figures 6-2 through 6-11 although for the structures in Figures 6-7, 6-10, and 6-11, the plane wave is obliquely incident with $\theta^i = 60^\circ$ per Figure A-11. Thus, the incident magnetic field is no longer exclusively y-directed but still lies in the yz plane.

Several details of the time-domain response of the magnetic current observed in Figure 6-1 are worthy of comment. First, one observes that the y-directed magnetic current in the slot is positive during the early stages of excitation. This is as expected, since if the slot were very wide (as a large window), one would expect the sense of the early penetrating field to be the same as that of the incident excitation, and, furthermore, one would not expect the sense of the field to change sign at some arbitrary slot width between "very wide" and "very narrow."

The early time history of the magnetic current has a sharply peaked response that has significant high frequency content, but later the response is dominated by the half-wave resonance of the slot and wire at about 1 GHz which has a period of about 1 ns as observed in Figure 6-1. The magnetic current does not decay in a purely exponential fashion, but nearly linearly between 3 ns and 7ns. This enhancement of the current is due to the presence of the wire and is further discussed below.

For the same structure and excitation as above, the time history of the x-directed electric current at the center of the thin wire appears in Figure 6-2. Comments similar to those made above can be made about the sense and dominant frequency of the response. Note that the response does not start at time zero, but rather is delayed by the propagation time from the slot to the wire. Also, the

buildup and subsequent decay of the current on the wire is noteworthy, since the linear decay observed in the response of the magnetic current in Figure 6-1 between 3 ns and 7 ns is clearly due to re-excitation of the slot by the field created by the wire's electric current.

In Figure 6-3, one observes the time history of the x-directed electric field at sensor location $d' = 5$ cm $\theta' = 0^\circ$ per Figure A-11. This location is marked on the geometry sketch in Figure 6-3 by the dot labeled E. Notice that the initial response is positive which is expected since the incident electric field is x-directed. Also notice the steady response between 3 ns and 7 ns that is due to the sharing of energy between the slot and the wire. This sharing of energy by the slot and the wire is discussed further below.

In Figure 6-4, one sees the time history of the magnetic current at subdomain number 13 (of 50) in an annular slot that is loosely coupled to a 15 cm loaded wire whose center is located on the z-axis 15 cm from the slotted screen. Observe that the subdomain numbers are identified on the figure. The response of the current with the wire absent is also presented for comparison. The magnetic current in the slot is seen to be only moderately affected by the presence of the wire during the early time history, but when the load resistor is set to 0Ω , the late-time response of the magnetic current is noticeably enhanced by the field energy that is shared by the slot and the wire.

When the wire is moved closer to the slot, its effect on the magnetic current in the slot (in the same subdomain) is significantly enhanced as is seen in Figure 6-5. Notice that the presence of the wire not only increases the intensity of the magnetic current, it also significantly shifts its "phase" relative to its "phase" in the wire's absence. At 8 ns, the "phase" response with the wire present lags the response with the wire absent by almost 90° . The 100Ω resistor significantly reduces the impact of

the wire's presence on the magnetic current's amplitude, but does not significantly affect the "phase" lag due to the wire.

In Figure 6-6 one observes the magnetic current in the same subdomain identified above, but with the wire located only 3.75 cm from the slotted screen. The impact on the magnetic current due to the presence of the wire is strongly evident during most of the observed time response. The coupling between the slot and the wire is so strong that, when the 100Ω resistor is included in the wire, it actually serves to damp the response of the slot's magnetic current. This damping is clearly seen between 4 ns and 6 ns. At 8 ns, the phase of the magnetic current's response lags behind that which would exist in the absence of the wire by 180° and again does not appear to be greatly affected by the magnitude of the load resistor.

In Figure 6-7, one observes the magnetic current in the same subdomain as above, but with the excitation obliquely incident at $\theta^i = 60^\circ$. The center of the wire is located on the z-axis at $z = 7.5$ cm. Significant features of this data are that the 100Ω resistor again serves to damp the response of the magnetic current between 3 ns and 6 ns and that, even though the incident magnetic field is reduced by the factor 0.866 at the point of observation due to the angle of incidence, the overall response of the magnetic current has essentially the same intensity as it does in the case of normal incidence. It is reasonable to suggest that for the spectral content of the Gaussian pulse used, the slot responds strongly to the odd-function (with respect to displacement from subdomain 13) component of excitation that is introduced by the obliquely incident wave. One sees from (6.1) (the frequency-domain expression that defines the incident excitation) that when $\theta^i = 90^\circ$, the excitation is an even function of displacement (constant) along the y-axis, but, when $\theta^i = 60^\circ$, the excitation can be partitioned into both an even-function component

of displacement along y and an odd-function component. This even-function - odd-function partitioning is more difficult to visualize in the time domain because the variation along the y -axis for obliquely incident excitation is a variation of a time delay rather than a variation in phase as it is in the frequency-domain. The distribution of the magnetic current in the annular slot and its relationship to the nature of the incident excitation are discussed below.

The instantaneous amplitude distribution of magnetic current in the annular slot discussed above appears in Figures 6-8 and 6-9 at 3.6 ns and 4.6 ns, respectively, for a normally incident plane wave. Notice that the current in subdomains 26 through 50 is the mirror image of the current in subdomains 1 through 25 when one takes into account the fact that the slot's contour is y -directed at subdomain 13 while it is oppositely directed in subdomain 38. That the slot is y -directed at subdomain 13 is a consequence of the choice that arc displacement along the slot be measured in the counter-clockwise sense when the screen is viewed from the shadow side. Thus, the vector direction of the slot and, consequently, the sign of the instantaneous amplitude of the current in subdomain 38 is opposite that observed in subdomain 13.

Behavior consistent with a plane of symmetry in the yz plane is to be expected since the incident excitation, as it is defined in (6.1), does not vary with displacement in the x -direction. Thus, subdomain 38 sees the same excitation and environment as does subdomain 13 and, hence, must have the same response. Observe that the wire is positioned so that it does not disturb this symmetry. Notice also that the response of the magnetic current is an even function of displacement from subdomains 12 and 13 and subdomains 37 and 38 which lie adjacent to the x -axis. This is a direct result of normal incidence ($\theta_i = 90^\circ$ per Figure A-11) in that there is no odd-function component in the distribution of the incident magnetic field as one moves

away from the x-axis in the plane of the slotted screen. Lack of an odd-function component of the incident magnetic field guarantees that there is no odd-function component in the current's response as the point of observation is moved away from the x-axis.

When the angle of incidence is changed to $\theta^i = 60^\circ$, there is a significant odd-function component in the incident excitation with displacement along the y-axis in the plane of the slotted screen (see Figure A-11 for geometric details). Thus, one expects to see an odd function component in the magnetic current as the point of observation is displaced from the x-axis. However, since the symmetry with respect to displacement along the x-axis remains, the mirror-image response seen above is still expected. The data that results from oblique incidence of the excitation appears in Figures 6-10 and 6-11 at 3.6 ns and 4.6 ns respectively. Observe that the predictions made above indeed hold true. One observes that the magnetic current is "bunched-up" in the vicinity of subdomains 1 and 50 which are the subdomains most distant from the source.

The wire, too, obviously plays a role in the observed response. This is clear from the data in the two figures, since the magnetic current, whose distribution is highly skewed when the wire is present, is not significantly perturbed from its response to normally incident excitation when the wire is absent.

Knowledge of the magnetic current in the slot and the electric current on the wire (if present) are the key to prediction and explanation of the field that penetrates the slotted screen. When one partitions the incident excitation into even-function and odd-function components and understands that the structure exhibits only even-function response to even-function excitation and odd-function response to odd-function excitation, he has a powerful tool with which to predict the the behavior of the penetrated field.

Electric Field on the Shadow Side of the Slotted Screen

In this section, electric field data observed on the shadow side of a planar screen perforated by a curved or bent slot are discussed. In all cases, these data are normalized by the incident electric field that exists at the origin of the coordinate system (see Figure A-11) in the absence of the slotted screen. The data are easily divided into two groups: data obtained in the absence of a coupled thin wire scatterer and data obtained in the presence of a thin wire scatterer. The data are presented in the natural order mentioned above and significant features of the data are explained. The reader is reminded that the time-domain (TD) data presented below are actually frequency-domain data but are computed by means of time-domain techniques.

Electric Field with Wire Absent

In Figure 6-12, one sees the magnitude of the normalized x-directed electric field observed at $d' = 5$ cm and $\theta' = 0^\circ$ (per the geometry of Figure A-11) on the shadow side of a screen perforated by a slot with annular contour that is excited by a normally incident plane wave which varies harmonically in time. Observe the excellent agreement of the data obtained from the two solution techniques. The computed data also compare quite favorably with measured data.

Three significant features stand out in the data. First, the slot resonates strongly at about 1 GHz. Due to the symmetry of the structure and the nature of the excitation, it is known that the magnetic current in the slot goes to zero at the points where the slot passes through the yz plane (such behavior is clearly evident in the distribution of magnetic current observed in the time-domain in Figures 6-8 through 6-11). Thus, the length that determines the first half-wave resonant frequency is half the circumference of the annular slot, or about 15.7 cm. This slot

length is consistent with the frequency of the observed resonance in that a 15.7 cm slot is 0.5λ long at 0.995 GHz.

The second observation is that, at the half-wave resonant frequency of the slot, the intensity of the observed electric field on the shadow side of the slotted screen actually exceeds the intensity of the electric field that would exist at the point of observation if the screen were removed. This observation is significant in light of the fact that the point of observation appears to be well shielded by the screen and the penetrated field cannot directly illuminate the sensor in a ray-optics sense. Thus, a "field enhancement" effect occurs in the vicinity of the slot that can only be explained on the basis of the resonant storage of energy in the magnetic current of the slot.

The third observation is that the electric field strength at the point of observation is essentially constant over a remarkably wide range of frequencies. No simple explanation of this effect is immediately available, but this effect has great significance when viewed from a shielding effectiveness perspective. That is, significant energy penetrates through a narrow annular slot over a very broad range of frequencies at and above its half-wave resonant frequency. Thus, one should expect shielded enclosures that contain narrow slits or cracks that follow a generally annular contour to be essentially transparent to the penetrating field at frequencies at or above the first half-wave slot resonance. The broadband effect observed above appears to be position sensitive, since when the point of observation is moved off the z-axis the intensity of the electric field decays with increasing frequency. These data appear in Figure 6-13. When the point of observation is moved off the z-axis, it is likely that significant phase cancellation occurs as the frequency rises.

In Figure 6-14, one sees the electric field on the z-axis at $z = 5$ cm for the case of obliquely incident excitation. Again the response decays with increasing

frequency due to phase cancellation. In this case the phase variation is due to the odd-function response of the magnetic current in the slot rather than a variation of the distance from the slot to the point of observation. The effect is essentially the same as was observed in Figure 6-13 and discussed above when the point of observation was moved away from the z-axis. It is worth noting that even though there is a significant odd-function component in the distribution of magnetic current in the slot, one still does not see any sign of a full-wave slot resonance at about 2 GHz. The full-wave resonance does occur (as is seen below) but its effect is not evident at the point of observation since the contribution of the odd-function magnetic current to electric field on the z-axis must be zero due to symmetry.

In Figure 6-15, one sees the result of both obliquely incident excitation and offset sensor location. Notice that the full wave resonance at about 2 GHz is clearly evident with the sensor moved away from the z-axis. Also notice that the broadband response seen in Figure 6-12 has returned. The return of the broadband response leads one to speculate that the narrow annular slot actually appears as an open circular window to the incident wave, but only in the direction of the wave's propagation.

In Figures 6-16 through 6-19 one sees the electric field that penetrates an ensemble of coupled subslots that follow an annular contour. Two sensor angles and two angles of incidence are shown. Observe that breaking up the single annular slot into smaller subslots by bridging the slot with small conducting "shorts" raises the halfwave resonant frequency as expected (since the subslots are shorter than the original slot), but does not significantly alter the broadband response observed above for the single annular slot. Observe the excellent agreement of the time-domain and frequency domain data and their very good agreement with the experimental data for this challenging geometry.

In Figure 6-20, one observes the electric field on the shadow side of a screen perforated by a narrow slot having rectangular contour. The response is similar to that of the annular slot in that the half-wave resonance occurs at about 1 GHz (half the circumference is 16 cm), and the electric field on the z-axis at $z = 5$ cm is roughly constant over a very broad range of frequencies. The frequency response at this location is not as flat as in the case of the annular slot, but one guesses that this lack of flatness is due to the effects of field scattering from the sharp bends in the slot's contour.

In Figures 6-21 through 6-23 one sees the electric field on the shadow side of the same structure, but the field is observed at a different point for normal incidence, and then the structure is illuminated by obliquely incident excitation. The general shape of the electric field parallels that of the annular slot in Figures 6-17 through 6-19.

The data that appear in Figures 6-24 through 6-27 pertain to an ensemble of narrow slots that follow a rectangular contour. Again, this data is similar to that obtained from the ensemble of slots that follow an annular contour. In Figure 6-24, one observes that the first half-wave resonance occurs at about 2.1 GHz which is consistent with the length of the 7 cm long subslot that occupies 6 cm of the top portion and 1 cm of the right hand portion of the rectangular contour when the screen is viewed from the shadow side.

Several general comments are in order concerning the data in this section. First, there is consistently excellent agreement among the time-domain and frequency-domain computed data. Also, there is very good agreement among the computed data and the measured data. Thus one concludes that the integral equations and the solution techniques used to compute electric field data for curved narrow slots and coupled curved narrow slots are accurate.

Electric Field with Wire Present

In Figure 6-28, one observes the normalized x-directed electric field on the z-axis at $z = 5\text{ cm}$ that results from electromagnetic penetration through a straight narrow slot and coupling to a nearby straight thin wire. The slot is 15 cm by 1 mm and the wire is 15 cm long and has a radius of 0.4 mm. The center of the wire is located on the z-axis at $z = 7.5\text{ cm}$. Pertinent geometric features appear in the sketch that accompanies the graph and the sensor angle θ' and the angle of incidence θ^i are defined in Figure A-11. Observe the good agreement among the data computed in the time and frequency domains and the good agreement among the computed data and the measured data.

The half-wave resonance of the coupled straight slot and straight wire occurs at about 1 GHz as one expects for a 15 cm slot and a 15 cm wire. Since only even-function excitation exists for normal incidence, no full-wave resonant response is evident. When the point of observation is moved off the z-axis, the electric field observed in Figure 6-29 results. As is seen in much of the data described above, electric field of significant intensity exists on the shadow side of the slotted screen over a broad range of frequencies.

The data that appear in Figures 6-30 and 6-31 result from obliquely incident excitation of the same coupled straight slot and straight wire that are discussed above. Again, good agreement exists among the computed data and among the computed and measured data. Observe in Figure 6-30 that no special response is seen at about 2 GHz where one might expect to see a strong full-wave resonant response due to the odd-function excitation of the slot by the obliquely incident excitation. For points of observation on the z-axis, the x-directed electric field radiated by the odd-function component of magnetic current in the slot must be zero due to the symmetry of the structure. However, when the point of observation

is moved away from the z-axis, the full-wave resonance at about 2 GHz is clearly evident in the data that appear in Figure 6-31.

In Figures 6-32 through 6-35, one observes the electric field at two locations on the shadow side of the slotted screen for two angles of incidence for a straight slot coupled with a bent wire. No outstanding differences exist between these data and the data for the case of the straight wire, so no additional comments are made. Again good agreement exists among the computed data and among the computed and measured data.

When the straight slot in the structure described above is replaced with a slot that follows an annular contour, the electric field data that appear in Figures 6-36 through 6-39 result. In most respects, the data in these figures do not differ greatly from the data in Figures 6-12 through 6-15 for the annular slot in the absence of the thin wire scatterer. However, upon closer examination, one observes small details in the frequency response of the electric field when the wire is present that do not appear in the wire's absence. These small deviations are clearly related to the behavior of the 15 cm wire. In Figure 6-36, one sees a distinct response at about 0.9 GHz that does not appear in Figure 6-12 when the wire is absent. Since the half-wave resonance of the wire and the slot occur at slightly different frequencies, it is possible for one to observe two distinct resonant responses in the vicinity of 1 GHz. The smaller response is due to the wire while the larger response is due to the slot. The wire's half-wave resonant response is visible in the other three figures as well. In Figure 6-38, one observes that the wire not only contributes a response at about 0.9 GHz, but also in the neighborhood of 3 GHz where the wire experiences a resonance when its length is 1.5λ .

When the annular slot in the structure described above is replaced with a slot that follows a rectangular contour, the data that appear in Figures 6-40 through

6-43 result. No significant new features appear in this data that are not discussed above for other structures. Good agreement is seen among the time-domain and frequency-domain computed data and generally good agreement is seen among the computed data and the measured data.

Several comments are in order concerning the data in this section. First, there is consistently excellent agreement among the time-domain and frequency-domain computed data. Also, there is good agreement among the computed data and the measured data. Thus one concludes that the integral equations, solution techniques, and electric field formulas that are used to compute electric field data for coupled slot-wire structures are accurate. Also, plausibility is afforded the measurement techniques.

Electric Current in a Loaded Thin Wire on the Shadow Side of a Slotted Screen

While good correlation among computed electric field data and measured electric field data provides a high degree of confidence in the validity of the integral equation formulations and the related solution techniques, a more fundamental validation results when one establishes a correlation of computed and measured currents. This observation follows from the fact that the unknowns in the slot and wire equations are magnetic current and electric current, while the electric field data observed above result from a secondary calculation involving integration of those currents. So, even though the computed and measured electric field data observed above are in good agreement, it is possible that the magnetic and electric current solutions could be somewhat in error and that this error could be masked by the integrations one performs to obtain electric field.

The input impedance measurements described in Chapter V and the subsequent correlation that is established among the measured data and data computed in the

frequency-domain by means of a moment method code provide evidence that the magnetic current in the slot is accurately modeled by the slot integral equation. Thus the moment method technique was validated by these measurements. In order to provide evidence that the electric current in a loaded wire that resides on the shadow side of a slotted screen is accurately modeled by the integral equation for a wire in a coupled slot-wire structure, a series of electric current measurements were performed at the base of a monopole on the shadow side of a slotted screen. Again, details of these measurements and the associated issues of calibration and data normalization appear in Appendix A.

In Figure 6-44, one observes the normalized x-directed electric current on a loaded straight wire that resides on the shadow side of a 15 cm by 1 mm straight slot that is excited by a normally incident plane wave. The wire's center is located on the z-axis at $z = 7.5$ cm and the wire is loaded at its center with a 100Ω resistor. Observe the excellent agreement among the time-domain computed data, the frequency-domain computed data, and the measured data. The real and imaginary parts of the current appear in Figures 6-45 and 6-46 where one also observes excellent agreement. Thus one concludes that not only is the magnitude of the electric current modeled well by the time-domain and frequency-domain equations and solution techniques, but also the phase of the current with respect to the phase of the excitation at the origin of the coordinate system (see Figure A-11 and related discussion of data normalization in Appendix A) is accurate.

Notice also that almost all of the energy that couples into the wire does so at about 1 GHz. This is also observed in Figure 6-47 when the excitation is obliquely incident. Two mechanisms are at work here. First, the field that penetrates the slot is strongest in the neighborhood of 1 GHz which is the half-wave resonance of the slot, and, second, the thin wire responds most vigorously to excitation at its

half-wave resonant frequency which also occurs at about 1 GHz. Thus, neglecting the effects of coupling, one can predict that the dominant electric current response on the wire will occur at about 1 GHz, virtually to the exclusion of all other responses. One makes this prediction by multiplying the frequency response of the penetrated electric field in the absence of the wire by the frequency response of current on the wire due to electric field excitation. Such a two-step process is not generally satisfactory when one wants to predict the precise amplitude and phase of the electric current in a tightly coupled problem, so, in such problems, the fully coupled integral equation must be solved as is done here. Now that the frequency response of the electric current at the center of the wire is known, it is clear why so little impact is observed in the electric field data above 4 GHz when the wire is located on the shadow side of the slotted screen. So little impact is observed because so little energy is coupled into the wire at those frequencies.

In Figures 6-48 through 6-51 one observes the electric current on a loaded straight wire that resides on the shadow side of a slotted screen perforated by narrow slots that follow both annular and rectangular contours. The slotted screen is excited by both normally incident and obliquely incident plane waves. Again, very good agreement exists among the computed data and the measured data. The same observations are valid concerning the data for these structures as are valid concerning the data for the structure discussed above.

A few comments are in order concerning the data in this section. First, there is consistently excellent agreement among the time-domain and frequency-domain computed data. Also, there is very good agreement among the computed data and the measured data. Thus one concludes that the integral equations and the solution techniques used to compute electric current data on the loaded wire of a coupled slot-wire structure are indeed valid.

Figure 6-1. Time History of Magnetic Current at the Center of a Straight Slot Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-2. Time History of Electric Current at the Center of a Straight Wire Coupled with a Straight Slot (Gaussian Pulse Excitation)

Figure 6-3. Time History of Electric Field in the Neighborhood of a Straight Slot Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-4. Time History of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-5. Time History of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-6. Time History of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-7. Time History of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-8. Instantaneous Amplitude Distribution of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-9. Instantaneous Amplitude Distribution of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-10. Instantaneous Amplitude Distribution of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-11. Instantaneous Amplitude Distribution of Magnetic Current in a Slot with Annular Contour Coupled with a Straight Wire (Gaussian Pulse Excitation)

Figure 6-12. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-13. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-14. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-15. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-16. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-17. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-18. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-19. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen.

Figure 6-20. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-21. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-22. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-23. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-24. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-25. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-26. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-27. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen

Figure 6-28. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-29. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-30. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-31. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-32. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-33. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-34. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-35. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-36. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-37. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-38. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-39. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-40. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-41. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-42. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-43. Frequency Response of the Normalized Magnitude of the Electric Field on the Shadow Side of a Slotted Screen in the Presence of a Wire Scatterer

Figure 6-44. Frequency Response of the Normalized Magnitude of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

Figure 6-45. Frequency Response of the Normalized Real Component of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

Figure 6-46. Frequency Response of the Normalized Imaginary Component of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

Figure 6-47. Frequency Response of the Normalized Magnitude of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

Figure 6-48. Frequency Response of the Normalized Magnitude of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

Figure 6-49. Frequency Response of the Normalized Magnitude of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

Figure 6-50. Frequency Response of the Normalized Magnitude of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

Figure 6-51. Frequency Response of the Normalized Magnitude of the Electric Current at the Center of a Loaded Wire Located on the Shadow Side of a Slotted Screen

CHAPTER VII

HYBRID TIME-DOMAIN INTEGRAL EQUATION - FINITE-DIFFERENCE TIME-DOMAIN SOLUTION TECHNIQUE

When electromagnetic energy penetrates a conducting surface through an aperture into an enclosure which has conducting walls, it is often difficult or inefficient to model the behavior within the cavity (and the cavity's influence on the aperture fields) by application of integral equation techniques alone. This observation is especially true when the cavity contains regions of differing electromagnetic properties, contains irregularly shaped scatterers, and/or has an irregularly shaped boundary. On the other hand, even though differential equation techniques model field behavior within cavities more readily than do integral equation techniques, they are less effective or efficient when confronted with large regions of free space and surface-oriented geometries such as narrow slots and thin wires.

A hybrid method based on both integral equation techniques and differential equation techniques is described in this chapter. In this method, integral equation techniques are applied in regions where they are appropriate and effective such as in a slot next to a large region of free space. Differential equation techniques are applied where they are more effective such as within a cavity. Much attention is given to suitable integral equation techniques in the preceding chapters. It is now appropriate to describe a differential equation technique that is well suited to finite-volume problems such as the cavities mentioned above.

A differential equation technique known as the finite-difference time-domain (FDTD) technique [17] has been successfully employed to determine the fields within

cavities such as those described above. To solve such a cavity problem, the cavity's volume is subdivided into many cells (typically cubical and uniformly sized) and Maxwell's equations are explicitly enforced on a cell-by-cell basis. To facilitate this process, the partial derivatives in space and time of Maxwell's equations are approximated by finite differences. Since time is now discretized, it is possible to write equations for field values at a particular point in time in terms of field values calculated at earlier times. Thus, given appropriate initial conditions, the solution can be progressively acquired by "marching in time."

Since the volume of the cavity is treated on a cell-by-cell basis, it is possible for one to independently specify the material properties of each cell. Thus one can account for free-space, inhomogeneous media, scatterers, and boundaries by specifying appropriate permittivity, permeability, and conductivity for each cell. However, a potential difficulty with this cell-by-cell treatment is that feature resolution is limited by cell size. For this reason, thin wire scatterers and narrow slots cannot be accurately modeled if their smallest dimension is significantly less than a cell width.

The specific problem addressed in this chapter is the penetration of electromagnetic energy through a *narrow* slot in an infinite planar conducting screen which is backed by a rectangular cavity (see Figure 7-1). This problem has features which suggest solution by means of a hybrid technique wherein fields in the slot and the exterior region are modeled by integral operators while fields inside the cavity are modeled by the finite-difference time-domain technique.

Time-domain integral operator treatment of the slot and the exterior region allows precise definition of the slot's dimensions and implicit satisfaction of the radiation condition via the free-space Green's function. Finite-difference time-domain treatment of the cavity provides a straightforward means to satisfy the boundary conditions on the cavity's interior walls and also provides for the possibility of

inhomogeneous media and scatterers within the cavity. If the cavity-backed slotted screen problem were solved entirely by the FDTD method, two obvious difficulties would arise. The first difficulty involves the cell size. One would either reduce the cell size until it was compatible with the width of the slot or attempt to employ a suitable equivalent slot capacitance model in order to model the slot while maintaining the original cell size. Neither choice is appealing when one is confronted with a very narrow slot. The reduced cell size would likely lead to an unacceptably large computational burden and equivalent slot capacitance models may not be sufficiently accurate for very narrow slots.

The second difficulty involves the large free-space region exterior to the cavity. Since one finds it impossible to compute fields on a grid with infinite dimensions, the grid must be truncated in the region outside the cavity and an appropriate absorbing boundary condition must be applied at the prematurely truncated boundary in order that fields radiated by the slot not scatter from the truncated boundary. The hybrid time-domain integral equation - finite-difference time-domain (TDIE-FDTD) solution technique provides an alternative that avoids these problems.

At the heart of the hybrid time-domain integral equation-finite difference time-domain solution technique is the mechanism by which the two solution techniques share field data. A successful approach to integrating the two techniques involves excitation of the cavity (the FDTD system) directly by the electric field in the slot and re-excitation of the slot (modeled with integral operators) by the magnetic field scattered from the walls of the cavity.

Implementation of the slot to cavity field coupling depends on a formulation of the FDTD equations which employs the integral form rather than the differential form of Maxwell's equations. Excitation of the slot by the cavity is accomplished as

one identifies the component of the total "short-circuit" magnetic field on the cavity side of the slotted screen that is due to scattering from the cavity's walls (versus that due to direct radiation from the slot) and then excites the cavity side of the slot with this scattered short-circuit magnetic field. A more precise distinction is made between the desired short-circuit magnetic field and the field directly radiated by the slot in a subsequent section.

The following sections contain a more detailed description of the hybrid TDIE-FDTD solution technique and its application to the solution of the problem of electromagnetic penetration through a finite-length narrow slot in an infinite planar screen backed by a finite-dimensioned rectangular cavity. A brief discussion of the formulation of the integral operator is presented (more detailed information is available in Chapter II). Also, detailed descriptions are given of two formulations of the FDTD equations and application of these equations to the solution of cavity problems. Details of the integration of the TDIE and FDTD models are discussed, and the validity of the hybrid technique is established by demonstration of good agreement among data calculated by the hybrid technique and data collected by laboratory experimentation. Several items of general interest concerning the problem of electromagnetic penetration through a slotted screen into a rectangular cavity are also discussed.

Time-Domain Integral Representation of Fields

Formulation of the time-domain integral operator that governs field behavior in the slot parallels that given in detail in Chapter II. Pertinent highlights of the formulation are reproduced below.

In the usual manner, the slot is shorted or filled in with conducting material and an equivalent magnetic surface current is applied to both surfaces of the now shorted

slot such that the original slot electric field (at this point unknown) is restored just outside the surface of the magnetic current. The unknown magnetic current is of equal magnitude but opposite sign on the two sides of the shorted aperture. Thus, by construction, the continuity of the tangential component of the slot electric field is enforced. Enforcement of the continuity of the tangential component of the slot magnetic field is the basis of the integral operator expression.

The magnetic field in region a ($z > 0$) is given by

$$\mathbf{h}^a = \mathbf{h}^{so^a} + \mathcal{H}^A [2\mathbf{m}] \quad (7.1)$$

and the magnetic field in region b ($z < 0$) is given by

$$\mathbf{h}^b = \mathbf{h}^{so^b} + \mathcal{H}^A [-2\mathbf{m}] \quad (7.2)$$

where

$$\mathcal{H}^A [\mathbf{m}] = -\frac{\partial}{\partial t} \mathbf{f}(\mathbf{r}; t) - \nabla \psi(\mathbf{r}; t), \quad (7.3)$$

$$\mathbf{f}(\mathbf{r}; t) = \epsilon \iint_A \mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS', \quad (7.4)$$

$$\psi(\mathbf{r}; t) = \frac{1}{\mu} \iint_A m(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS', \quad (7.5)$$

$$\nabla_s \cdot \mathbf{m}(\mathbf{r}; t) + \frac{\partial}{\partial t} m(\mathbf{r}; t) = 0, \quad \mathbf{r} \in A, \quad (7.6)$$

$$\tau = t - R/c, \quad (7.7)$$

and

$$R = |\mathbf{r} - \mathbf{r}'|. \quad (7.8)$$

$\mathbf{h}^{s^a}$ and $\mathbf{h}^{s^b}$ are the magnetic fields that exist in regions a and b with the slot shorted which are due to sources other than those resulting in direct illumination by the equivalent magnetic current ($\mathbf{m}$) and magnetic charge ($\mathbf{m}$) in the slot. These fields commonly result from external impressed sources. However, the short-circuit field in a region could also be due to fields originally generated by magnetic current in the slot which are subsequently scattered by an object or boundary in the vicinity of the slot. This concept is developed further in the discussion of the integration mechanism by which field data are shared by the time-domain integral operator model and the FDTD model.

The operator $\mathcal{H}^A[\mathbf{m}]$ represents the magnetic field in either region that is radiated by the equivalent magnetic current $\mathbf{m}$. Note that $\mathcal{H}^A[\mathbf{m}]$ does not account for any subsequent scattering of this radiated field. Fields scattered by nearby objects or boundaries must be accounted for by the $\mathbf{h}^{s^a}$ terms of (7.1) and (7.2) if such objects or boundaries exist.

Alternately, additional terms may be added to (7.1) and (7.2) to account for scattering from nearby objects as is done in the case of a slot coupled to a nearby wire in Chapter III. Note also that the $2\mathbf{m}$ (and $-2\mathbf{m}$ in region b) argument of the operator in (7.1) (and (7.2)) result from invocation of image theory as one removes the shorted screen in order to make use of the free-space Green's function in (7.4) and (7.5). Hence, $\mathcal{H}^A[-2\mathbf{m}]$ does account for the field radiated by $-2\mathbf{m}$ which scatters off the region- b side of the infinite perfectly conducting screen.

Once again, explicit enforcement of the continuity of the tangential magnetic field in the slot produces the desired integral operator expression. In its simplest form, this equation is

$$\mathbf{h}_{tan}^a = \mathbf{h}_{tan}^b, \quad \mathbf{r} \in A, t > 0, \quad (7.9)$$

which, after differentiation with respect to time and algebraic manipulation, becomes

$$\begin{aligned} & -2 \frac{1}{\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \iint_A 2\mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS' - \nabla \iint_A 2\nabla' \cdot \mathbf{m}(\mathbf{r}'; \tau) \frac{1}{4\pi R(\mathbf{r}, \mathbf{r}')} dS' \right\}_{tan} \\ & = -\frac{\partial}{\partial t} \left\{ \mathbf{h}^{ac^a} - \mathbf{h}^{ac^b} \right\}_{tan} \quad \mathbf{r} \in A, t > 0. \end{aligned} \quad (7.10)$$

Further specialization to the case of a straight narrow y-directed slot whose transverse electric field takes on the known even-function distribution of an infinitely long narrow slot yields (see Chapter II)

$$\begin{aligned} & -\frac{1}{\pi\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{-L^*}^{L^*} k(y'; \tau) k^*(y - y') dy' - \frac{\partial}{\partial y} \int_{-L^*}^{L^*} \frac{\partial}{\partial y'} k(y'; \tau) k^*(y - y') dy' \right\} \\ & = -\frac{\partial}{\partial t} \left\{ h_y^{ac^a} - h_y^{ac^b} \right\} \quad y \in (-L^*, L^*), t > 0, \end{aligned} \quad (7.11)$$

where $k(y; t)$ is the total axial equivalent magnetic current (or transverse slot voltage), $-L^*$ and L^* denote the end points of the slot, and the kernel of the free space Green's function is given by

$$k^*(y - y') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\sqrt{(y - y')^2 + w^2 \sin^2 \frac{\alpha'}{2}}} d\alpha' \quad (7.12)$$

where $2w$ is the width of the narrow slot.

To facilitate a numerical solution of (7.11), one divides the slot into N equal-length subdomains, expands the equivalent magnetic current into N triangular basis functions, and enforces the resulting equations by pulse testing. Expansion of the current in a series of basis functions allows one to move the amplitude coefficient of the basis function outside the potential integral. With the coefficient removed from the integral, it is possible for one to formulate an expression in which a particular current coefficient is found in terms of other, previously calculated current coefficients.

The triangular basis functions that are part of the integrand for vector potential are approximated by pulses that are centered in each subdomain, but are non-zero only over half of each subdomain (the center portion of the subdomain). This very good approximation makes the integration process more efficient. Additional details of the overall discretization process are found in Chapter IV.

The choice of a straight y -directed slot divided into equal-length subdomains allows considerable simplification of the spatially discretized slot equation in Chapter IV (Equation (4.17)). The simplified equation evaluated at point y_m is

$$\begin{aligned}
 & -\frac{\partial^2}{\partial t^2} \left\{ \Delta y \frac{\epsilon}{2\pi} \sum_{n=1}^N K(n, \tau) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m - y') dy' \right\} \\
 & + \frac{1}{\mu 2\pi} \frac{1}{\Delta y} \sum_{n=1}^N \left\{ K(n, \tau) \int_{y_{n-1}}^{y_n} k^s(y_{m+\frac{1}{2}} - y') dy' - K(n, \tau) \int_{y_n}^{y_{n+1}} k^s(y_{m+\frac{1}{2}} - y') dy' \right. \\
 & \left. - K(n, \tau) \int_{y_{n-1}}^{y_n} k^s(y_{m-\frac{1}{2}} - y') dy' + K(n, \tau) \int_{y_n}^{y_{n+1}} k^s(y_{m-\frac{1}{2}} - y') dy' \right\} \\
 & = -\frac{\partial}{\partial t} \frac{\Delta y}{2} \left\{ h_v^{s^*}(y_m) - h_v^{s^b}(y_m) \right\}. \tag{7.13}
 \end{aligned}$$

One approximates the second derivative with respect to time in (7.13) with a second

central difference in order to provide temporal discretization and ultimately to provide the expression necessary for "marching in time." Now, at each point of evaluation y_m , the time-discretized form of (7.13) provides a formula with which one can predict the current at point y_m and time index $t + 1$ based on currents calculated at time t and earlier:

$$\begin{aligned}
 K(m, t + 1) = & \frac{1}{\int_{y_{m-\frac{1}{2}}}^{y_{m+\frac{1}{2}}} k^s(y_m - y') dy'} \left[- \sum_{\substack{n=1 \\ n \neq m}}^N K(n, \tau + \Delta\tau) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m - y') dy' \right. \\
 & + \sum_{n=1}^N [2K(n, \tau) - K(n, \tau - \Delta\tau)] \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m - y') dy' \\
 & + \frac{1}{\mu\epsilon} \frac{\Delta t^2}{\Delta y^2} \sum_{n=1}^N \left\{ K(n, \tau) \int_{y_{n-1}}^{y_n} k^s(y_{m+\frac{1}{2}} - y') dy' - K(n, \tau) \int_{y_n}^{y_{n+1}} k^s(y_{m+\frac{1}{2}} - y') dy' \right. \\
 & \left. - K(n, \tau) \int_{y_{n-1}}^{y_n} k^s(y_{m-\frac{1}{2}} - y') dy' + K(n, \tau) \int_{y_n}^{y_{n+1}} k^s(y_{m-\frac{1}{2}} - y') dy' \right\} \\
 & \left. + \frac{\pi}{\epsilon} \Delta t^2 \frac{\partial}{\partial t} \left\{ h_y^{sca}(y_m, t) - h_y^{scb}(y_m, t) \right\} \right]. \quad (7.14)
 \end{aligned}$$

Again, since everything on the right hand side of (7.14) is known at time $t\Delta t$ or earlier, K is known at time $(t + 1)\Delta t$. It is now possible for one to establish initial conditions (usually $K(m, 0) = 0$), define incident excitation (h_y^{sca} and h_y^{scb}), and calculate $K(m, 1)$ for each of N subdomains. When all $K(m, 1)$ are known, one advances the time index and calculates all $K(m, 2)$. This "marching in time" procedure continues until all energy coupled into the slot via h_y^{sca} and h_y^{scb} has been re-radiated.

For practical reasons, one can terminate the solution process when the amplitude of $K(m, t)$ at a reference location consistently falls below some threshold percentage of its peak amplitude. With $K(m, t)$ known for all useful time, one can calculate all field quantities of interest on either side of the slotted screen.

Finite-Difference Time-Domain Field Representation

Formulation of the field representation in the finite-difference time-domain solution technique proceeds directly from Maxwell's equations in the time-domain. The FDTD field representation is first formulated from the differential form of Maxwell's equations. The field representation is then formulated from the integral form of Maxwell's equations.

FDTD Formulation from the Differential Form of Maxwell's Equations

Given in differential form, Maxwell's equations are

$$\nabla \times \mathbf{e} = -\mu \frac{\partial}{\partial t} \mathbf{h} - \mathbf{m} \quad (7.15)$$

$$\nabla \times \mathbf{h} = \epsilon \frac{\partial}{\partial t} \mathbf{e} + \sigma \mathbf{e} + \mathbf{j} \quad (7.16)$$

where

$$\nabla \cdot \epsilon \mathbf{e} = \mathbf{q} \quad (7.17)$$

and

$$\nabla \cdot \mu \mathbf{h} = \mathbf{m}, \quad (7.18)$$

where $\mathbf{m}$ is vector magnetic current, $\mathbf{j}$ is vector electric current, $\mathbf{q}$ is electric charge density, and $\mathbf{m}$ is magnetic charge density. Equations (7.15) and (7.16) can be

reduced to the following set of scalar equations when one employs a rectangular coordinate system:

$$\frac{\partial}{\partial t} h_x = \frac{1}{\mu} \left(\frac{\partial}{\partial z} e_y - \frac{\partial}{\partial y} e_z \right) - \frac{1}{\mu} m_x \quad (7.19)$$

$$\frac{\partial}{\partial t} h_y = \frac{1}{\mu} \left(\frac{\partial}{\partial x} e_z - \frac{\partial}{\partial z} e_x \right) - \frac{1}{\mu} m_y \quad (7.20)$$

$$\frac{\partial}{\partial t} h_z = \frac{1}{\mu} \left(\frac{\partial}{\partial y} e_x - \frac{\partial}{\partial x} e_y \right) - \frac{1}{\mu} m_z \quad (7.21)$$

$$\frac{\partial}{\partial t} e_x = \frac{1}{\epsilon} \left(\frac{\partial}{\partial y} h_z - \frac{\partial}{\partial z} h_y \right) - \frac{\sigma}{\epsilon} e_x - \frac{1}{\epsilon} j_x \quad (7.22)$$

$$\frac{\partial}{\partial t} e_y = \frac{1}{\epsilon} \left(\frac{\partial}{\partial z} h_x - \frac{\partial}{\partial x} h_z \right) - \frac{\sigma}{\epsilon} e_y - \frac{1}{\epsilon} j_y \quad (7.23)$$

$$\frac{\partial}{\partial t} e_z = \frac{1}{\epsilon} \left(\frac{\partial}{\partial x} h_y - \frac{\partial}{\partial y} h_x \right) - \frac{\sigma}{\epsilon} e_z - \frac{1}{\epsilon} j_z \quad (7.24)$$

Instead of solving Equations (7.19) through (7.24) over a continuum of points, one seeks to discretize space and time in order that a solution can be achieved with a finite number of calculations. To this end, one approximates scalar Equations (7.19) through (7.24) with two-point central differences which approximate the partial derivatives in space and time with second-order accuracy. To facilitate these approximations, one discretizes three-dimensional space into equally spaced points which lie on a grid which is aligned with a rectangular coordinate system. Uniform point spacing and specification of a rectangular coordinate system are not required for successful application of this technique. However, these choices are compatible

with a wide range of problems, lead to simple equations, and are widely used in practice.

Observe the positions of the various field components on a unit cell of the FDTD grid (see Figure 7-2). If the three-dimensional problem space is so discretized, it is a straightforward matter to assign appropriate permeability, permittivity, and conductivity to each field component and thus specify the material properties of the media, scatterers, and the walls of the cavity. Material properties are often assigned on a cell-by-cell basis rather than on a component-by-component basis for convenience. This discretization of the volume of the problem space is often referred to as "gridding-up" the problem.

A grid point is identified by $(i, j, k) = (i\Delta x, j\Delta y, k\Delta z)$ where, for uniformly spaced grid points, $\Delta x = \Delta y = \Delta z$. Now one may specify a function of space and time by $\mathcal{F}^n(i, j, k) = \mathcal{F}(k\Delta x, j\Delta y, k\Delta z, n\Delta t)$ where Δt is the duration of one discrete time step. When the field components of Maxwell's equations are positioned on the grid as suggested by Figure 7-2, one may write the following finite-difference expressions for h_x and e_x (for $\sigma = 0$ and in the absence of current sources j and m):

$$\begin{aligned}
 h_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) &= h_x^{n-\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) \\
 &\quad - \frac{\Delta t}{\mu \Delta x} \left[e_x^n(i, j + 1, k + \frac{1}{2}) - e_x^n(i, j, k + \frac{1}{2}) \right. \\
 &\quad \left. - e_y^n(i, j + \frac{1}{2}, k + 1) + e_y^n(i, j + \frac{1}{2}, k) \right] \quad (7.25)
 \end{aligned}$$

and

$$e_x^{n+1}(i + \frac{1}{2}, j, k) = e_x^n(i + \frac{1}{2}, j, k)$$

$$\begin{aligned}
& + \frac{\Delta t}{\epsilon \Delta x} \left[h_x^{n+\frac{1}{2}} \left(i + \frac{1}{2}, j + \frac{1}{2}, k \right) - h_x^{n+\frac{1}{2}} \left(i + \frac{1}{2}, j - \frac{1}{2}, k \right) \right. \\
& \left. - h_y^{n+\frac{1}{2}} \left(i + \frac{1}{2}, j, k + \frac{1}{2} \right) + h_y^{n+\frac{1}{2}} \left(i + \frac{1}{2}, j, k - \frac{1}{2} \right) \right]. \quad (7.26)
\end{aligned}$$

h_x and e_x are displaced in time by half a time step because this choice offers considerable convenience when one calculates the discretized time derivatives. In Equation (7.25), which is the discretized form of (7.19), the partial derivative with respect to time of magnetic field is centered at time $n\Delta t$. Since the discretized electric field values in (7.25) are displaced from h_x by half a spatial step in the grid, it is convenient for one to make the temporal offset in the approximation for the partial derivative with respect to time equal to half a time step. By making this choice one implies that magnetic field values are known at times $(n + \frac{1}{2})\Delta t$ which is entirely consistent with the discretization of (7.22) in (7.26) where the partial derivative with respect to time of electric field is centered at $(n + \frac{1}{2})\Delta t$.

By making these choices, one defines a consistent system of scalar equations wherein both the electric field and the magnetic field are advanced once per time step but where advancement of electric field depends directly on previous advancement of magnetic field and vice-versa. One may derive discretized equations for $e_y, e_z, h_y,$ and h_z in like fashion.

FDTD Formulation from the Integral Form of Maxwell's Equations

Maxwell's equations are also written in integral form. It is now shown that one may derive the finite-difference approximation from the integral form of Maxwell's equations. Maxwell's equations in integral form are:

$$\oint_C \mathbf{e} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_S \mu \mathbf{h} \cdot d\mathbf{s} - \iint_S \mathbf{m} \cdot d\mathbf{s} \quad (7.27)$$

and

$$\oint_C \mathbf{h} \cdot d\mathbf{l} = \frac{\partial}{\partial t} \iint_S \epsilon \mathbf{e} \cdot d\mathbf{s} + \iint_S \sigma \mathbf{e} \cdot d\mathbf{s} + \iint_S \mathbf{j} \cdot d\mathbf{s} \quad (7.28)$$

where

$$\oiint_S \mu \mathbf{h} \cdot d\mathbf{s} = \iiint_V \mathbf{m} dv, \quad (7.29)$$

$$\oiint_S \epsilon \mathbf{e} \cdot d\mathbf{s} = \iiint_V \mathbf{q} dv, \quad (7.30)$$

where C is the closed contour which defines the surface S . Review the relative field positions in Figure 7-2. If one assumes the magnetic field to be constant over each face of the unit cube and the electric field to be constant along each edge, discretized expressions for the scalar components of $\mathbf{h}$ and $\mathbf{e}$ ($\sigma = 0$ and sources $\mathbf{j}$ and $\mathbf{m}$ absent) can be derived in the following manner. Equation 7.27, as it relates to the x-component of $\mathbf{h}$, is

$$\frac{\partial}{\partial t} \iint_S \mu (h_x \hat{\mathbf{x}}) \cdot (\hat{\mathbf{x}} d\mathbf{s}) = - \oint_C \mathbf{e} \cdot d\mathbf{l} \quad (7.31)$$

which is spatially discretized as

$$\frac{\partial}{\partial t} \mu h_x(i, j + \frac{1}{2}, k + \frac{1}{2}; t) \iint_S d\mathbf{s} = - \left[e_x(i, j + 1, k + \frac{1}{2}; t) \int_{k \Delta x}^{(k+1) \Delta x} dl \right]$$

$$\begin{aligned}
& + e_y(i, j + \frac{1}{2}, k + 1; t) \int_{(j+1)\Delta x}^{j\Delta x} dl + e_x(i, j, k + \frac{1}{2}; t) \int_{(k+1)\Delta x}^{k\Delta x} dl \\
& + e_y(i, j + \frac{1}{2}, k; t) \int_{j\Delta x}^{(j+1)\Delta x} dl \Big]. \tag{7.32}
\end{aligned}$$

Observe that

$$\iint_s ds = \Delta x^2 \tag{7.33}$$

and that each of the line integrals has length Δx . Thus (7.32) becomes

$$\begin{aligned}
\frac{\partial}{\partial t} \mu h_x(i, j + \frac{1}{2}, k + \frac{1}{2}; t) \Delta x^2 = & -\Delta x \left[e_x(i, j + 1, k + \frac{1}{2}; t) - e_x(i, j, k + \frac{1}{2}; t) \right. \\
& \left. - e_y(i, j + \frac{1}{2}, k + 1; t) + e_y(i, j + \frac{1}{2}, k; t) \right]. \tag{7.34}
\end{aligned}$$

Temporal discretization of (7.34) by means of a centered difference yields:

$$\begin{aligned}
& \left[\frac{\mu h_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) - \mu h_x^{n-\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2})}{\Delta t} \right] \Delta x^2 = \\
& -\Delta x \left[e_x^n(i, j + 1, k + \frac{1}{2}) - e_x^n(i, j, k + \frac{1}{2}) \right. \\
& \left. - e_y^n(i, j + \frac{1}{2}, k + 1) + e_y^n(i, j + \frac{1}{2}, k) \right], \tag{7.35}
\end{aligned}$$

which, when solved algebraically for $h_x^{n+\frac{1}{2}}$, yields

$$\begin{aligned}
h_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) = & h_x^{n-\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) \\
& - \frac{\Delta t}{\mu \Delta x} \left[e_x^n(i, j + 1, k + \frac{1}{2}) - e_x^n(i, j, k + \frac{1}{2}) \right.
\end{aligned}$$

$$-e_y^n(i, j + \frac{1}{2}, k + 1) + e_y^n(i, j + \frac{1}{2}, k)]. \quad (7.36)$$

One may derive equations for the remaining components of $\mathbf{h}$ and $\mathbf{e}$ in similar fashion. Note that Equation (7.36) is identical to Equation (7.25). The advantage of the derivation which follows from the integral form of Maxwell's equations is that the resulting equations can be interpreted in the integral sense of Faraday's law and Ampere's law. Specific application of this integral interpretation is made in the section where coupling between the slot and the cavity is discussed.

Treatment of Finite Conductivity in the FDTD Technique

To this point, one has assumed the conductivity of the media to be zero. Obviously this is not the general case. One may wish to specify a very high conductivity in cells wherein there exists a conducting wall or scatterer. While it is possible for one to explicitly force the appropriate components of $\mathbf{e}$ and $\mathbf{h}$ to be zero in cells which contain a perfectly conducting material, this approach does not easily lead to efficient, general-purpose computer codes and does not allow for conducting materials. To this end, finite-difference equations which accommodate non-zero conductivity are now derived.

Equation (7.22) is reproduced below as a starting point in the discussion of how finite conductivity is accommodated in the finite-difference equations.

$$\nabla \times \mathbf{h} = \frac{\partial}{\partial t} \epsilon \mathbf{e} + \sigma \mathbf{e} + \mathbf{j}. \quad (7.37)$$

Equation (7.37) is now temporally discretized, reorganized, and the electric current source $\mathbf{j}$ is removed (for convenience). After these steps, (7.37) becomes:

$$\frac{\mathbf{e}^{n+1} - \mathbf{e}^n}{\Delta t} = \frac{1}{\epsilon} (\nabla \times \mathbf{h}^{n+\frac{1}{2}}) - \frac{\sigma}{\epsilon} \mathbf{e}^{n+\frac{1}{2}}. \quad (7.38)$$

There is a problem with Equation (7.38). That is, one knows $\mathbf{e}$ only at integer values of time. Thus we have no direct knowledge of $\mathbf{e}$ at time $(n + \frac{1}{2})\Delta t$. One may circumvent this problem by forming the average of $\mathbf{e}$ at time $(n + 1)\Delta t$ and $n\Delta t$. The average is found to be centered at time $(n + \frac{1}{2})\Delta t$. Thus (7.38) becomes

$$\frac{\mathbf{e}^{n+1} - \mathbf{e}^n}{\Delta t} = \frac{1}{\epsilon}(\nabla \times \mathbf{h}^{n+\frac{1}{2}}) - \frac{\sigma}{\epsilon} \left(\frac{\mathbf{e}^{n+1} + \mathbf{e}^n}{2} \right) \quad (7.39)$$

which, after some algebraic manipulation, becomes

$$\mathbf{e}^{n+1} = \frac{\left[1 - \frac{\sigma \Delta t}{2\epsilon}\right]}{\left[1 + \frac{\sigma \Delta t}{2\epsilon}\right]} \mathbf{e}^n + \frac{\frac{\Delta t}{\epsilon}}{\left[1 + \frac{\sigma \Delta t}{2\epsilon}\right]} \nabla \times \mathbf{h}^{n+\frac{1}{2}}. \quad (7.40)$$

When one spatially discretizes (7.40) for the x -component of $\mathbf{e}^{n+1}$, the following expression results:

$$\begin{aligned} e_x^{n+1}\left(i + \frac{1}{2}, j, k\right) &= \frac{\left[1 - \frac{\sigma \Delta t}{2\epsilon}\right]}{\left[1 + \frac{\sigma \Delta t}{2\epsilon}\right]} e_x^n\left(i + \frac{1}{2}, j, k\right) \\ &+ \frac{\frac{\Delta t}{\epsilon \Delta x}}{\left[1 + \frac{\sigma \Delta t}{2\epsilon}\right]} \left[h_x^{n+\frac{1}{2}}\left(i + \frac{1}{2}, j + \frac{1}{2}, k\right) - h_x^{n+\frac{1}{2}}\left(i + \frac{1}{2}, j - \frac{1}{2}, k\right) \right. \\ &\quad \left. - h_y^{n+\frac{1}{2}}\left(i + \frac{1}{2}, j, k + \frac{1}{2}\right) + h_y^{n+\frac{1}{2}}\left(i + \frac{1}{2}, j, k - \frac{1}{2}\right) \right]. \end{aligned} \quad (7.41)$$

One may derive expressions for the remaining components of $\mathbf{e}$ in similar fashion. As a quick check, observe that (7.41) becomes (7.26) as $\sigma \rightarrow 0$. Also, observe that as conductivity grows, (7.41) becomes

$$e_x^{n+1}\left(i + \frac{1}{2}, j, k\right) \underset{\sigma \rightarrow \infty}{=} -e_x^n\left(i + \frac{1}{2}, j, k\right). \quad (7.42)$$

If $e_x(i + \frac{1}{2}, j, k)$ were tangential to a perfectly conducting surface, its initial value would naturally be zero. Given an initial value of zero, subsequent time advancement by means of (7.42) would continue to produce zero tangential electric field. Thus (7.41) predicts field behavior consistent with expectations for the natural extremes of σ .

As a practical note, it is prudent for one to specify σ independently for each component of $\mathbf{e}$ in a cell. This generality allows one to avoid the problem of inadvertently forcing the normal component of $\mathbf{e}$ to zero on a conducting surface.

Treatment of Boundary Conditions in the FDTD Technique

Now that field advancement formulas of the form of (7.25) for magnetic field and (7.41) for electric field are available, it is well to discuss several ancillary subjects related to the practical application of the finite-difference time-domain technique. First is the matter of boundary conditions. Since it is not possible for one to "grid up" or discretize all of free space, it is necessary that he truncate the grid in an appropriate manner.

For the case of a free-space scattering problem, it is common for one to enforce an absorbing boundary condition at the extremes of the necessarily finite grid. This boundary condition usually involves a localized approximation of a global "one-way" wave equation. Such a "one-way" wave equation is formulated in terms of a Fourier transform of the incident wave and involves enforcement of the condition that the incident wave is precisely annihilated at the boundary, squelching a potential reflected or scattered wave.

When one considers these equations in the transform domain, they have the potential to provide perfect absorption of the incident wave. However, the price for perfect absorption is that one must enforce the equation in a global sense. This

"global" enforcement requires one to have knowledge of all field values throughout the grid at all times, a huge overhead in memory for stored data.

It is not practical for one to enforce boundary conditions in such a "global" sense. Rather, what one desires is an expression that can be enforced in a "local" sense at the boundary. That is, what one desires is an expression that only requires knowledge of field values very close to the point of enforcement (spatially) and very close (temporally) to the current time step.

Highly absorbing "local" approximations have been made to a "global" perfectly absorbing boundary condition [18]. These approximations involve algebraic manipulations and suitable approximations in the transform domain, subsequent inverse-transformation into the time domain, and approximation of the resulting partial derivatives in space and time with finite-differences in order to incorporate the now localized boundary condition into the FDTD solution technique.

For the problem under consideration in this chapter (slotted, cavity-backed infinite planar screen), the free-space side of the screen, the screen, and the slot are modeled by integral operators, thus eliminating the need to apply absorbing boundary conditions. The cavity side of the slotted screen involves the natural boundary condition on the cavity walls that the tangential component of electric field must be zero. One easily enforces this condition. One initially sets the appropriate tangential components of $\mathbf{e}$ to zero and subsequently neglects them in the field-advancement process so that they always remain zero. However, one incorporates these field components (with zero value) into the field-advancement process for neighboring cells within the grid.

This approach to handling the boundary conditions on the cavity walls is very efficient for the case of a rectangular cavity whose walls are aligned with the

rectangular coordinate system. One handles almost all other geometries more efficiently by specifying high conductivity in the field-advancement formula when the electric field components that are tangential to the conducting surface are advanced.

Excitation of the FDTD Grid

Another topic of importance is the introduction of excitation into the FDTD system so that it is properly "distributed" throughout the grid. In almost all cases, the structure under investigation is unexcited prior to the start of the FDTD solution process and field components in the neighborhood of scatterers are initialized to zero. One commonly employs two methods to excite the FDTD grid in typical scattering problems. In one method, one forces the electric and magnetic field components at the boundary of the grid to take on values consistent with an incoming plane wave which has the desired time history (usually some sort of a pulse). When one enforces the field-advancement formulas for $\mathbf{e}$ and $\mathbf{h}$ in each cell at each time step, the incident wave will propagate through the grid toward the scatterer.

In an alternate but similar approach, one initializes the field components of the grid such that they take on values consistent with a plane wave pulse that has already propagated to within a few cells of the scatterer. When one starts the field-advancement process, the incident wave proceeds to propagate toward the scatterer. It is also possible to excite the grid by means of one or more current sources $\mathbf{j}$ or $\mathbf{m}$. One can locate these sources in any cell of the grid and can give them any desired time history.

In the case of the cavity under consideration in this chapter, one may excite the grid by including the slot electric field (calculated as the solution of the time-domain integral equation for the slot) in the Faraday's law line integral for cells adjacent to the slot (see Equations (7.27) through (7.31) and accompanying discussion).

Of course, since the slot width is much smaller than the dimension of the cell, one must appropriately scale the contribution of the slot electric field in order that the correct "average electric field" value be used in the general FDTD field advancement formula (see Equations (7.32) through (7.36)). The details of this excitation mechanism are discussed more thoroughly in the section which deals with integration of the time-domain integral representation of fields for the slot and the FDTD representation of fields in the cavity.

Selection of the Duration of a Time Step

Another topic worthy of discussion is selection of the duration of a time step. The duration of a time step is limited by the Courant condition

$$c_{max} \Delta t < \frac{1}{\left[\left(\frac{1}{\Delta x} \right)^2 + \left(\frac{1}{\Delta y} \right)^2 + \left(\frac{1}{\Delta z} \right)^2 \right]^{\frac{1}{2}}} \quad (7.43)$$

which one enforces to ensure the numerical stability of the field-advancement process. c_{max} is the highest propagation velocity in the grid and will not exceed the speed of light. It is possible for c_{max} to be less than the speed of light in vacuum if the entire grid is composed of materials whose constitutive parameters differ from μ_0 and ϵ_0 .

It is generally advantageous for one to give the time step the longest possible duration in order that the total solution be acquired after the fewest possible calculations. If the duration of a time step were cut in half, the discretization error for the partial derivatives with respect to time would be reduced, but the solution process would then require twice the number of calculations.

The error reduction realized as one increases the temporal resolution (by specifying smaller time steps) is usually very small when compared to the overall error

generated by insufficient spatial resolution. Thus, one usually specifies the duration of the time step to be close to the maximum allowed by (7.43), and allocates any additional computational resources to the definition of a higher resolution grid (which indirectly improves temporal resolution via (7.43)). For a three-dimensional grid composed of cubic cells, (7.43) becomes

$$c_{max} \Delta t < 0.577 \Delta x. \quad (7.44)$$

It is common for one to let $c_{max} = c_0$ (the speed of light in free space) and to specify the time step by

$$\Delta t = \frac{0.5 \Delta x}{c_0} \quad (7.45)$$

where

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \quad (7.46)$$

since this condition is stable for any choice of materials and permits simple calculation of propagation times over specified distances in free space. The time step duration in the hybrid techniques is specified by (7.45) for convenience.

Observation of Field Components in the FDTD Grid

It is not generally practical to retain all data regarding $\mathbf{e}$ and $\mathbf{h}$ at each time step (a simple $26 \times 21 \times 17$ cell problem run for 8,000 time steps would require 1.8 gigabytes of disk storage if all field components were stored in single precision). Thus it is necessary for one to decide those field components at specific locations of interest that are to be recorded as the solution is progressively acquired by "marching in time."

Since one calculates $\mathbf{e}$ and $\mathbf{h}$ directly by means of the field-advancement formulas, no special additional effort is required to calculate field quantities at a location of interest as is the case in the time-domain integral equation technique. Of course, a corresponding disadvantage in the FDTD technique is that one must choose specific points of observation prior to the start of the solution process.

Hybridization Details (Slot-Cavity Field Coupling)

At the heart of the hybrid technique is the mechanism by which the time-domain integral equation technique and the finite-difference time-domain technique are integrated. Field coupling between the slotted screen model where fields are represented by potential integrals and the cavity model where fields are represented by FDTD equations is discussed in detail in the next two sections.

Excitation of the FDTD System (Cavity) by the TDIE Model (Slot)

As mentioned earlier, the FDTD system (cavity) is excited by the slot electric field. Since the slot in the screen is narrow with respect to its length and also narrow with respect to a wavelength at the highest frequency component of interest, it is known that the slot electric field is directed transverse to the slot axis and thus that the resulting equivalent magnetic current will be axially directed.

For excitation that is predominately an even function of the transverse dimension of the slot (this is true in virtually all practical cases) the equivalent magnetic current has known transverse variation (see Equation (2.16)). Thus, when the current is integrated across the transverse dimension of the slot, one obtains the total axially directed equivalent magnetic current or the voltage from $-w$ to w across the slot. Since the voltage across the slot is known (as the solution of the time-domain integral equation), and since the tangential electric field on the surface of the screen

is zero, the line integral of e_x from $i\Delta x$ to $(i+1)\Delta x$ yields only the negative of the slot voltage (see Figure 7-3 for details of FDTD cell immediately adjacent to the slot in the screen). Thus, when the time derivative of h_y is predicted based on the line integral of e_x and e_z around the edges of the cell of Figure 7-3, via Faraday's law, the contribution to the integral due to the existence of the slot is exactly the total axially directed equivalent magnetic current in the slot (or the transverse slot voltage). In order for the slot's contribution to the integral to be properly accounted for by the general FDTD field-advancement formulas, one must convert it to an average electric field along the entire edge of the cell in which it exists. This is easily accomplished if one divides the slot voltage by Δx . Now, instead of the zero contribution to the field-advancement formula due to zero tangential electric field along the edge of the conducting screen, the proper contribution which accounts for the presence of the slot is made as one substitutes this average electric field along the cell edge into the field-advancement formula.

To check the accuracy of this method of FDTD grid excitation by narrow slots, a FDTD grid with 1 cm cells was excited by the early time history of magnetic current in a y -directed 15 cm $\times$ 1 mm slot (same geometry as Figure 7-1). The slot was excited by a Gaussian pulse. e_x was recorded at locations $x = 0$ cm, $y = 0$ cm, $z = 5$ cm and $z = 1$ cm and this data is compared with e_x (at the same locations) that was calculated by integral equation techniques. In Figure 7-4 one sees the data recorded at $z = 5$ cm. Good agreement exists among these data in the early time history (prior to reflections from the cavity's walls) which suggests the validity of this approach to grid excitation.

The data collected at $z = 1$ cm appear in Figure 7-5. The agreement among the data is not as good when one moves the point of observation closer to the slot.

Observe that the FDTD data is roughly 14 percent higher than the integral equation data. Several reasons are suggested for this discrepancy. First, it is known that the electric field intensity decays very rapidly as the point of observation is moved from locations inside the slot to locations somewhat distant from the slot. Indeed, for a vanishingly thin planar screen, the slot electric field (represented by equivalent magnetic current) is unbounded as the point of observation approaches the edge of the slot (see Equation (2.16)). Thus rapid changes in electric field intensity with spatial displacement must be modeled by the FDTD grid whose spatial resolution is limited to 1 cm in this case.

Along a similar vein, when one derives the FDTD field advancement formulas from the integral form of Maxwell's equations, one can see that the field component predicted by the field-advancement formula is an average value over a surface the size of a face of a FDTD grid cell. For a spatially decaying field, it is not unreasonable for one to expect that the average value of the field will exceed the field's value at the center of the region over which the average is calculated when the gradient of the field is large. Thus the observation that the FDTD prediction of e_x at $z = 1$ cm is higher than that predicted by integral equation techniques may be reasonably explained.

Other potential sources of error include second-order errors in the finite-difference approximations of the temporal and spatial partial derivatives and inadequate modeling of wave propagation along the slotted wall of the cavity. Such errors could occur as the FDTD solution process predicts electric field at $y = 0$, $z = 1$ cm that results from slot electric field that exists several cells away, say at $y = 5$ cm, $z = 0$. Even though precise agreement in electric field predictions is unlikely when the point of observation is chosen to be close to the slot, the good agreement at $z = 5$ cm observed in Figure 7-4 suggests that the proper electric

field energy is nevertheless injected into the grid and that electric field observations made several cells away from the slot are acceptably accurate. Thus one may conclude that a satisfactory mechanism has been found to couple energy from the time-domain integral equation model (slot) into the FDTD grid (cavity). Next one focuses attention on the problem of coupling energy back from the cavity to the slot.

Excitation of the TDIE Model (Slot)
by the FDTD System (Cavity)

As indicated above, the time-domain integral model (slot) is re-excited by the FDTD model (cavity) by means of the short-circuit magnetic field (h_y^{sc} of Equation (7.13)) that exists on the cavity side of the slotted screen. It is important to understand that the short-circuit magnetic field of interest is only that portion of the total short-circuit magnetic field (on the cavity side of the slotted screen) that is due to fields that have been scattered at least once from the cavity's walls (other than the shorted screen). Scattering from the shorted screen is included after the fields have scattered from at least one other wall.

Magnetic fields generated by direct radiation of the equivalent magnetic current in the slot (or the transverse slot electric field), which radiates in the presence of the shorted (infinite) screen, are explicitly excluded from the short-circuit magnetic field h_y^{sc} . These directly radiated fields must first be scattered by the cavity's walls (other than the shorted screen) before they can contribute to the short circuit magnetic field of interest.

The magnetic field that exists next to the cavity side of the slotted screen (within the cavity in the limit as $z \downarrow 0$) already exists in the presence of a shorted screen and thus is the "total" short-circuit magnetic field. That is, one models the

slotted region as an unbroken screen (or cavity wall) which has equivalent magnetic surface current "painted" on it such that the jump in electric field through this surface magnetic current precisely restores the original tangential slot electric field on the cavity side of the surface current. It is this "restored" electric field due to the surface magnetic current that contributes to the Faraday's law integral that excites the FDTD grid as discussed above. However, the magnetic field next to the unbroken screen (one cavity wall) comprises both direct radiation from the magnetic current in the slot as well as fields scattered from the cavity's other walls. It is necessary for one to isolate the component of this field that is due only to scattering from the cavity walls.

One can isolate the scattered component of the total field by calculating the field that is directly radiated by the equivalent magnetic current and subsequently subtracting this directly radiated component from the "total" short-circuit field. What remains is the short-circuit magnetic field (h_y^{sc} of Equation (7.13)) which is due only to fields which have scattered at least once from a cavity wall other than the infinite screen.

Since a discrepancy appears when electric field values located close to the slot's equivalent magnetic current are observed in the FDTD grid, one is not surprised to observe a similar discrepancy in the magnetic field values. Indeed, as one may observe in Figure 7-6, the early-time FDTD prediction of the first time derivative of h_y at $x = 0$ cm, $y = 0$ cm, $z = 0.5$ cm is approximately 8 percent higher than that calculated by integral equation methods when the cavity of Figure 7-1 is excited by the equivalent magnetic current of the 15 cm by 1 mm slot. Late-time discrepancies in Figure 7-6 are due to scattering from the cavity's walls. Thus, since one desires to subtract the directly radiated field (calculated by integral equation techniques) from

the total short-circuit field that exists on the cavity side of the slot, it is obvious that one must modify the integral equation prediction in order that he subtract the correct amount of directly radiated field. The appropriate correction factor for this case is 1.08. However, when the spatial resolution is doubled so that the cell size is reduced from 1 cm, to 0.5 cm, the necessary correction factor becomes roughly 1.05.

The errors that drive the need for this correction factor are essentially the same as those discussed earlier for the electric field case, but a few additional comments are in order. First, no theory exists that can predict the correction factor prior to execution of the hybrid code. So, at this point, the correction is done by trial and error (usually one or two trials are sufficient). It is believed that the relative width of the slot with respect to the size of the FDTD cell is the significant variable and that one can derive a simple empirical formula to predict the correction factor based on this ratio.

The correction factor appears to affect only the amplitude of the solution. If the 8% error discussed above were not corrected, the amplitude of the time-domain solution would be in error by 8%. No increased tendency toward instability is observed in the solution process when the error is left uncorrected.

The error that exists when one observes the magnetic field value at $z = 0.5$ cm (8%), is smaller than the error that exists when one observes the electric field value farther away from the slot at $z = 1.0$ cm (14%). This discrepancy is easily explained in that the magnetic field is bounded in the neighborhood of the slot while the electric field is not. Thus the gradient of the magnetic field and the resulting error (when the FDTD field-advancement formula calculates the average value of the field rather than a value at a precise location) is smaller than in the electric field case. Indeed, the ratio of the slot magnetic field to the magnetic field at $z = 0.5$

cm is found to be about 2.2:1 while the ratio of the average slot electric field to the electric field at $z = 1.0$ cm is found to be about 2.6:1, a larger ratio. Thus it is not surprising that the electric field value at $z = 1.0$ cm suffers greater error.

A final comment regards whether or not the remaining short-circuit magnetic field (after one removes the directly radiated magnetic field) should be corrected by some factor related to the slot width-FDTD cell size ratio. Such correction should not be necessary since the remaining short-circuit magnetic field consists of waves that have been scattered by the cavity's walls and have propagated through significant distances in free space prior to reaching the point of observation. Thus, one expects no significant gradients in the scattered field at the point of observation since the point of observation is well removed from the original concentrated source (the slot) by virtue of the "round-trip" distance traveled by the wave.

Once the time derivative of the short-circuit magnetic field ($\frac{\partial}{\partial t} h_y^{sc}$ of Equation (7.13)) has been accurately determined, one delays this data by one time step (to account for the propagation time from the middle of the cell to the cell wall) and subsequently substitutes it into Equation (7.13). This delay allows one to accurately time-advance the equivalent magnetic current in the slot at a particular slot location.

Based on the discussion above, one may conclude that a satisfactory mechanism is now available to couple the fields of the FDTD system (cavity) and the time-domain integral equation model (slot). In the next section, one's attention is focused on the application of the hybrid technique to the solution of the slotted, cavity-backed infinite planar screen problem whose geometry is seen in Figure 7-1. Geometric details of the problem are discussed, field response to plane-wave excitation is reported, and the resulting cavity-field data are compared with experimental data (via FFT) in order to establish the plausibility of the hybrid technique.

Application and Validation of the Hybrid TDIE - FDTD Technique

The problem of electromagnetic penetration through an aperture into a cavity is well suited to solution by the hybrid TDIE-FDTD technique discussed in this chapter. The specific problem under consideration is penetration through a finite-length narrow slot in an infinite, vanishingly thin planar conducting screen which is backed by a rectangular cavity (see Figure 7-1). Specific details regarding the geometry of the problem and the application of the TDIE and FDTD techniques to the slot and the cavity are discussed in the next section.

Details of the Slotted Cavity-Backed Screen Problem

The slot is y -directed, 15 cm long and 1 mm wide while the cavity is 26 cm tall, 21 cm wide, and 17 cm deep and has perfectly conducting walls. The cavity is positioned on the shadow side of the screen such that the edges of the slot are equally spaced from the walls of the cavity. The slotted screen is excited by a plane wave having a Gaussian-pulse time history and the excitation is incident on the surface of the screen at an angle θ' measured from the y -axis in the yz plane. The incident electric field is x -directed while the incident magnetic field lies in the yz plane. Electric field is sensed within the cavity at locations in the yz plane defined by the distance from the origin d' and the angle θ' which is measured from the z -axis toward the negative y -axis.

The slot and the exterior region are modeled by the integral equation techniques described previously in this chapter. The slot is divided into 14 subdomains and the yet unknown total axially-directed equivalent magnetic current is expanded into 14 triangular basis functions. As indicated earlier, equation enforcement is done by means of pulse testing. A diagram of the slot, expansion functions for current and charge (spatial derivative of current), and the testing pulses appears in Figure 7-7.

The interior of the cavity is modeled by the FDTD technique described previously in this chapter. Cubical cells are employed whose dimension is 1.0 cm. Thus the grid contains $26 \times 21 \times 17 = 9282$ cells. The relative orientation of the grid cells of the cavity with respect to the expansion functions of the slot is such that the location of appropriate values of e_x in the grid corresponds with the location of the midpoints of the respective magnetic current expansion functions in the slot. A diagram of the relative alignment of the grid cells and the slot is found in Figure 7-8.

Observe that the slot is not precisely centered with respect to the top and bottom of the grid. This spatial error results from the relative location of e_x in the FDTD grid cell which makes it impossible for one to center the slot if an even number of cells is chosen in the x-direction. If one were to choose an odd number of cells to facilitate vertical centering of the slot, then the cell dimension would be such that the length of the slot and the width of the cavity could not be accurately modeled if they were an integer multiple of 1.0cm.

For the problem presently under consideration, one must carefully specify the length of the slot and the dimensions of the cavity in order that the resonant responses in the cavity will occur at the correct frequencies. Thus, it is reasonable for one to choose that the spatial error be assigned to the vertical centering of the slot rather than to the dimensions of the cavity. One may expect that some error will exist in the calculated fields due to this spatial discrepancy.

Such spatial errors are unavoidable when one uses the FDTD technique. Increased spatial resolution can reduce the impact of these errors, but they can never be completely avoided. Also, increased spatial resolution comes at a high price. If the size of a grid cell is cut in half, 8 times the original number of cells are required for one to model a particular structure and, since the duration of a time step is

proportional to the cell size, twice as many time steps are necessary to advance the solution to the same absolute time. Thus, doubling spatial resolution increases the computational burden by a factor of 16.

Initiation of the Hybrid Solution Technique

As the first step in the initiation of the hybrid solution technique, one initializes to zero the magnetic current in the slot and all field values in the cavity prior to the first time step. Then one illuminates the slotted screen with the incident excitation and starts the time stepping (field-advancement) process. One chooses the width parameter of the Gaussian pulse such that the pulse's amplitude exceeds 10% of its peak amplitude for about 20 time steps, and one continues the solution process until the magnetic current in the slot and field components in the cavity decay to some acceptably small value (typically after many thousands of time steps).

While one progressively acquires the solution by "marching in time," he permanently stores field values at several predetermined points of observation in a data file. One subsequently analyzes the recorded data in its native time-domain form or transforms it into the frequency domain via FFT in order to facilitate comparison with frequency domain data gathered experimentally.

Time-Domain Response of the Slot Magnetic Current

A graph of the time-domain equivalent magnetic current (or transverse slot voltage) near the center (subdomain 7) of the cavity-backed slot for normally incident excitation ($\theta^i = 90^\circ$) appears in Figure 7-9. For reference, the equivalent magnetic current in the absence of the cavity is also included in the graph. Notice that the presence of the cavity has a dramatic impact on the duration of the magnetic current in the slot. The peak amplitude of the current response is unaffected since

the peak occurs before the field that penetrates the slot can re-illuminate the slot via scattering from the cavity walls.

The large periodic response in the first 10 ns is due to the half-wave resonance of the slot at roughly 1 GHz. Notice that the period of the response is about 1 ns. The slot's half-wave resonant response decays quickly when the cavity is absent but is strongly reinforced when the cavity is present. After the half-wave slot resonance decays, the cavity continues to resonate at its natural frequencies (modes) for solution times beyond 15 ns.

The short-circuit field that re-illuminates the slot from the cavity side produces a magnetic current in the slot that persists beyond the 50 ns that one observes in Figure 7-9 (the solution process was terminated after $8192\Delta t = 134$ ns). The magnetic current due to the cavity modes persists longer than that due to the half-wave slot resonance because the slot re-radiates energy more efficiently at its half-wave resonant frequency than it does at the frequencies of the cavity modes.

If one allows the field-advancement process to continue for extended periods of time (more than $12,000\Delta t = 200$ ns), the solution gradually becomes unstable. This instability appears to be due to cavity modes which are excited by small numerical errors and by small inaccuracies in the field-advancement formulas of both the integral equation model and the FDTD model. These unintentionally excited modes occur at sufficiently high frequencies that they are not effectively damped by re-radiation by the slot's integral equation model.

Time-Domain Response of the Cavity Electric Field

A graph of the time-domain, x-directed electric field inside the cavity near the location specified by $d^* = 5$ cm, $\theta^* = 0^\circ$ appears in Figure 7-10. The response in the absence of the cavity (predicted by integral equation techniques) is included for

reference. Both responses result from excitation of the slot by a normally incident plane wave.

The point of observation in the FDTD grid is actually located at $x = 0$ cm, $y = -0.5$ cm, $z = 5$ cm rather than at the intended $x = 0$ cm, $y = 0$ cm, $z = 5$ cm since this is the location of e_x in the grid that is closest to the desired location. Thus the point of observation has a spatial positioning error of 0.5 cm in the negative y -direction. (This positioning error is unrelated to the imprecise vertical centering of the slot in the cavity that was mentioned above.) Again, arbitrary points of observation in the space modeled by the FDTD grid can only be specified with accuracy on the order of a cell size, in this case 1 cm.

As is observed in the time-domain equivalent magnetic current, the half-wave resonant response of the slot dominates the first 15 ns of the response of e_x at the point of observation. After 15 ns, most of the energy associated with the half-wave resonance of the slot has been re-radiated by the slot and what remains is the response due to the cavity modes. This remaining response is relatively much more slowly damped by slot re-radiation.

Frequency Domain Response of the Cavity Electric Field

Time-domain electric field data were computed at two points of observation in the cavity for two angles of incident excitation. The computed data were subsequently transformed into the frequency domain (via FFT) to make possible a direct comparison of the computed data with data collected experimentally on a frequency-domain test range.

The x -directed component of the cavity electric field (e_x) was computed near points of observation specified by sensor locations: sensor distance $d' = 5$ cm and sensor angles $\theta' = 0^\circ$ and $\theta' = 30^\circ$. The excitation was incident at angles

$\theta^i = 90^\circ$ and $\theta^i = 60^\circ$. Details of the experimental apparatus and related measurement techniques can be found in Appendix B. In all cases the frequency-domain electric field data are normalized by the incident electric field at the origin of the coordinate system (at the center of the slot) in the absence of the slotted screen and the cavity.

Field Response for Normally Incident Excitation ($\theta^i = 90^\circ$)

The frequency-domain response of the magnitude of e_z at $d^o = 5$ cm, $\theta^o = 0^\circ$ with $\theta^i = 90^\circ$ is seen in Figure 7-11. Note the generally good agreement between computation and measurement. The half-wave resonance of the slot is clearly evident at 0.95 GHz. Observe that the response at this frequency exceeds the magnitude of the incident excitation by a factor of four. Related experimentation with the cavity absent reveals a less than 2:1 ratio of observed field to incident field. Thus the presence of the cavity enhances the resonant response of the slot even though no cavity modes are excited at the slot's half-wave resonant frequency.

It is also observed that many cavity modes are excited. However, the excited modes are only a subset of the modes that are theoretically possible below 3 GHz. The hybrid technique correctly predicts which modes are excited and predicts their relative amplitude with reasonable accuracy. It is significant to point out that the amplitude of a particular mode is determined by many factors, including the slot's response to excitation at the modal frequency, the related ability of the slot to re-radiate at that frequency, and the mode's field structure which affects the degree of coupling between energy in the mode and energy in the slot.

The real part of the complex frequency-domain response observed in Figure 7-11 appears in Figure 7-12 while the imaginary part appears in Figure 7-13. One can clearly see that the relative phase of each modal response is accurately predicted

by the hybrid technique (after the data are transformed into the frequency domain via FFT).

The frequency-domain response of the magnitude of e_z at $d' = 5$ cm, $\theta' = 30^\circ$ with $\theta^i = 90^\circ$ is seen in Figure 7-14. Again good agreement is seen between computation and measurement. For this data, the point of observation is deliberately offset from $y = 0$ and the relative amplitude of the various modal responses is consequently affected. This change in relative mode amplitude is faithfully reproduced by the hybrid technique. Another point of interest is the small modal response observed just above 2.5 GHz. This mode does not appear in Figure 7-11 for the case that the point of observation is centered in the cavity. It is likely that the structure of the mode in question is such that a relative null exists at $d' = 5$ cm, $\theta' = 0^\circ$, but not at $d' = 5$ cm, $\theta' = 30^\circ$.

Field Response for Obliquely Incident Excitation ($\theta^i = 60^\circ$)

One expects that the response of the magnitude of e_z in the cavity will be significantly affected when the angle of incidence is changed from $\theta^i = 90^\circ$ to $\theta^i = 60^\circ$. The equivalent magnetic current in the slot changes from being exclusively an even function of position in the slot to being a combination of even and odd functions of position. Thus, it is expected that additional modes having odd-function field distribution with respect to $y = 0$ will be excited that were not excited by the normally incident excitation. Another expected effect is that the overall amplitude of the response will be reduced by the factor $\cos \theta^i = 0.866$ since this factor is the component of the incident magnetic field that is tangential to the axis of the slot for this oblique angle of incidence ($\theta^i = 60^\circ$). The data of Figure 7-15 appear at first glance to only partially confirm these expectations. Indeed, even though the

overall response at $d' = 5$ cm, $\theta' = 0^\circ$ is noticeably attenuated when compared to the response for normal incidence ($\theta' = 90^\circ$) observed in Figure 7-11, the expected additional modes are not clearly evident. There are some different (but small) mode responses at 1.7 GHz, 2.25 GHz, and 2.3 GHz, but these small responses are not supported by the experimental data.

There is a reasonable explanation for these seemingly inconsistent results. First, even though new modes are excited by the odd function equivalent magnetic current in the slot, such modes must have field structure such that a null exists in the xz plane since the odd function equivalent magnetic current (or slot electric field) that excites these modes is zero at $y = 0$ in the xz plane. That these new modes are not observed at $d' = 5$ cm, $\theta' = 0^\circ$ ($y = 0$) is clearly evident in the experimental data, but not so clearly evident in the computed data where several new (but small) responses are observed.

The new responses observed in the computed data result from the fact that the point of observation in the FDTD grid is actually located at $y = -0.5$ cm rather than $y = 0$ on the xz plane due to FDTD gridding compromises such as those discussed in previous sections. That is, there is no e_x field component exactly at $d' = 5$ cm, $\theta' = 0^\circ$ ($x = 0$ cm, $y = 0$ cm, $z = 5$ cm), so one must choose the e_x field component that exists closest to the desired location and be aware that errors may result from this spatial imprecision.

When the field response in the neighborhood of a point of observation is large, small errors in position do not dramatically affect the observation, but, when the field value is a null at a specific location, any spatial deviation from that location is readily evident in the observed response. To verify that this observation applies to the erroneous new responses in the computed data, the problem was re-solved

with twice the spatial resolution and a graph of the results appears in Figure 7-16. Observe that the spurious responses at 1.7 GHz and 2.25 GHz are now essentially absent and that the response at 2.3 GHz is somewhat reduced. Additional improvement is expected if one were to further increase the spatial resolution of the grid.

When one moves the point of observation away from the xy plane for $\theta^i = 60^\circ$, the new modes expected for the case of oblique incidence become clearly evident. For $d^i = 5$ cm, $\theta^i = 30^\circ$, several new modal responses are seen in Figure 7-17 that are not evident in Figure 7-15 for $\theta^i = 0^\circ$. These new responses are observed near 1.7 GHz, 1.8 GHz, 1.95 GHz, 2.25 GHz, and 2.95 GHz. Again, the overall response at this point of observation is reduced by $\cos \theta^i = 0.866$ with respect to the case of normal incidence since the incident magnetic field is again not tangential to the axis of the slot.

General Observations Concerning the Hybrid Technique

Several general observations concerning the hybrid technique are in order. First, the good agreement among the computed data and those gathered experimentally establishes the validity of the hybrid TDIE-FDTD solution technique when is is applied to suitable problems in electromagnetics. The errors that result from spatial discretization (which limit the ability to precisely specify locations and dimensions of interest) are somewhat predictable and are controllable within the limits of available computational resources. The hybrid technique should extend readily to the case of slots with arbitrary contour although late-time instability could be more of a problem for curved slots than it is for straight slots. Finally, the hybrid technique is well suited to a class of problems that is not as efficiently solved by integral or differential techniques alone.

Figure 7-1. Geometry Associated with Slotted Cavity-Backed Infinite Planar Screen

Figure 7-2. Positions of Field Components on a Unit Cell of the FDTD Grid within the Cavity

Figure 7-3. Cross-Section of FDTD Cell adjacent to Slot in Infinite Planar Screen

Figure 7-4. FDTD Prediction of e_z in Cavity at $d' = 5.0$ cm, $\theta' = 0^\circ$ versus TDIE Prediction of e_z at Same Location in Free Space

Figure 7-5. FDTD Prediction of e_x in Cavity at $d' = 1.0$ cm, $\theta' = 0^\circ$ versus TDIE Prediction of e_x at Same Location in Free Space

Figure 7-6. FDTD Prediction of $\frac{\partial}{\partial t} h_y$ in Cavity at $d^* = 0.5$ cm, $\theta^* = 0^\circ$ versus TDIE Prediction of $\frac{\partial}{\partial t} h_y$ at Same Location in Free Space

Current Expansion Functions

Charge Expansion Functions

Testing Functions

Figure 7-7. Spatially Discretized Slot and Related Expansion and Testing Functions

Figure 7-8. Location of Current Expansion Functions with Respect to Edges of Cells in the FDTD Grid

Figure 7-9. Time History of Equivalent Magnetic Current near the Center of the Slot with and without the Cavity Present

Figure 7-10. Time History of e_x near $d' = 5.0$ cm, $\theta' = 0^\circ$ with and without the Cavity Present

Figure 7-11. Magnitude of the Normalized Electric Field at $d' = 5.0$ cm, $\theta' = 0^\circ$ on the Shadow Side of a Slotted Screen Illuminated by a Plane Wave Incident at $\theta' = 90^\circ$ (Grid Cell Size = 1.0 cm)

Figure 7-12. Real Component of the Normalized Electric Field at $d' = 5.0$ cm, $\theta' = 0^\circ$ on the Shadow Side of a Slotted Screen Illuminated by a Plane Wave Incident at $\theta_i = 90^\circ$ (Grid Cell Size = 1.0 cm)

Figure 7-13. Imaginary Component of the Normalized Electric Field at $d' = 5.0$ cm, $\theta' = 0^\circ$ on the Shadow Side of a Slotted Screen Illuminated by a Plane Wave Incident at $\theta_i = 90^\circ$ (Grid Cell Size = 1.0 cm)

Figure 7-14. Magnitude of the Normalized Electric Field at $d' = 5.0$ cm, $\theta' = 30^\circ$ on the Shadow Side of a Slotted Screen Illuminated by a Plane Wave Incident at $\theta_i = 90^\circ$ (Grid Cell Size = 1.0 cm)

Figure 7-15. Magnitude of the Normalized Electric Field at $d' = 5.0$ cm, $\theta' = 0^\circ$ on the Shadow Side of a Slotted Screen Illuminated by a Plane Wave Incident at $\theta_i = 60^\circ$ (Grid Cell Size = 1.0 cm)

Figure 7-16. Magnitude of the Normalized Electric Field at $d^* = 5.0$ cm, $\theta^* = 0^\circ$ on the Shadow Side of a Slotted Screen Illuminated by a Plane Wave Incident at $\theta^* = 60^\circ$ (Grid Cell Size = 0.5 cm)

Figure 7-17. Magnitude of the Normalized Electric Field at $d' = 5.0$ cm, $\theta' = 30^\circ$ on the Shadow Side of a Slotted Screen Illuminated by a Plane Wave Incident at $\theta_i = 60^\circ$ (Grid Cell Size = 1.0 cm)

CHAPTER VIII

OBSERVATIONS AND CONCLUSIONS

The data discussed in the previous chapters provide the basis for several observations and conclusions, the first of which is the observation that solution of the curved narrow slot integral equation leads to valid prediction of the fields that penetrate the slot. This observation holds equally for solutions based on time domain analyses as well as solutions based on frequency domain analyses. It is also observed that the equation formulations in the time and frequency domains and the related solution techniques can be extended to account for a thin wire scatterer on the shadow side of the slotted screen while still providing excellent agreement between computed and measured fields. A final observation is that the hybrid time-domain integral equation - finite-difference time-domain solution technique accurately predicts the field inside a cavity which backs a slotted planar conducting screen. The hybrid solution technique combines the strengths of both integral equation techniques and differential equation techniques in the regions of the problem where those strengths apply while avoiding the most obvious weaknesses of the two techniques.

The first conclusion is that any effort to explain the nature of the field that penetrates through a narrow slot in a planar conducting screen will achieve little success if the distribution of equivalent magnetic current (or slot electric field) remains unknown. Explanation of virtually all of the significant features of the computed and measured data follows directly from properties of the distribution of magnetic current in the slot. When a thin wire scatterer is placed on the shadow side of the slotted screen, not only must the distribution of magnetic current in the slot be

known, but also one must know the distribution of the electric surface current on the wire in order to explain or predict the penetrating field. Moreover, complete understanding of the nature of the penetrating field is enhanced by considering the incident excitation to be composed of two components, one that is an even function of position and the other that is an odd function of position. Even-function excitation generates only even-function magnetic current while odd-function excitation generates only odd-function current. The field response is simply the sum of the contributions made by each component. This even-function - odd-function dichotomy is especially relevant in the case of a rectangular cavity which backs a slotted planar screen, where the cavity modes either exist or do not exist based on whether or not a mode's field structure is compatible with the even or odd distribution of the equivalent magnetic current in the slot.

Finally, a significant practical conclusion is reached: an enclosure that has seams containing narrow slits or cracks may actually provide less shielding effectiveness than use of no shielding at all. It is observed that electric field measured on the shadow side of slotted screens at frequencies where the slot length is close to $\frac{\lambda}{2}$ very often exceeds the strength of the incident electric field in the absence of the slotted screen. When the point of observation lies within a cavity that backs the slotted screen, this "shielded" to non-shielded field strength ratio is observed to exceed 4:1. Thus, if slits or cracks are unavoidable, they should be made very short with respect to $\frac{\lambda}{2}$ at the highest frequency of the anticipated threat, since slots that are very short with respect to $\frac{\lambda}{2}$ at a given frequency do not pass significant energy.

APPENDICES

Appendix A

Measurement of Shadow-Side Fields and Currents: Experimental Apparatus and Techniques

A wide assortment of laboratory measurements were made in order to establish a body of experimental data which would provide the basis for validation of the solutions of the time-domain integral equation, frequency-domain integral equation, and hybrid TDIE-FDTD technique. This appendix comprises descriptions of the measurement system, data normalization, calibration details, experimental apparatus, and the various experimental configurations that were measured.

Measurement System and Data Normalization

The measurements described in this appendix were made with the EMPEROR (EMP Engineering Research Omnidirectional Radiator) inverted monocone facility and the BEAMS (Broadband Electromagnetic Analysis Measurement System) instrumentation package at Lawrence Livermore National Laboratory. The capabilities of this facility are documented in the literature [19]. The primary measurement instruments were a Hewlett Packard model 8510A vector network analyzer and a model 8515A S-parameter test set.

Most of the measurements described here were made from 500 MHz to 10.5 GHz in frequency steps of 5 MHz and the resulting data were smoothed by means of a 12 point (60 MHz) arithmetic moving average. However, when the slotted screen was backed by a cavity, the frequency range was limited to 3.0 GHz and the data were not smoothed. When measurements are made beyond 3.0 GHz in the presence of a cavity, the numerous excited cavity modes tend to obscure the data and do not contribute significant additional information. For this reason, measurements inside the cavity were made from 0.5 GHz to 3.0 GHz instead of 0.5 GHz to 10.5

GHz. The data measured in the presence of the cavity walls were not smoothed in order that the fine structure of the cavity modes be preserved. The presence of the cavity during such measurements blocks most sources of external interference so that the resulting signal-to-noise ratio is virtually the same as that achieved when the moving average is applied to the non-cavity measurements.

Two types of measurements were made. First, as is described in a previous report, the magnitude and phase of the electric field were detected by a Prodyne D sensor. All electric field values so measured were normalized by the value of the incident electric field at the origin of the coordinate system (see Figure A-1) in the absence of the conducting screen and the thin-wire scatterer or cavity. Thus the data are presented in the form $\frac{E_{\text{sensor}}}{E_{\text{inc}}}$. Again, the error caused by locating the D sensor electrically close to the slotted screen at the lower end of the measurement frequency range is estimated to be small and is ignored. A block diagram of the experimental configuration for electric field measurements (with the cavity absent) appears in Figure A-2.

The second type of measurement involved sensing the current at the base of a monopole located on the shadow side of the slotted screen. The monopole was inserted into the opening of the center conductor of a small female-to-female 3.5 mm coaxial microwave connector. The connector was screwed into a threaded hole in a brass baseplate and protruded through an access hole in the ground plane when the baseplate was placed on the ground plane's surface. A flexible coaxial cable was connected from the HP-8515A S-parameter test set (located under the ground plane) to the protruding end of the microwave connector. A block diagram of the experimental configuration for monopole current measurements appears in Figure A-3.

The S-parameter test set provides data in the form of the ratio of the voltage at the base of the monopole divided by the voltage at the feedpoint of the monocone.

Note that

$$S_{21} = \frac{V_{monopole}}{V_{monocone}} \quad (A.1)$$

under the condition that both the monopole and the monocone are terminated in 50Ω . This condition is satisfied by the 50Ω input impedances of the ports of the HP-8515A S-parameter test set.

It is desired to transform the data so that it represents the current at the base of the monopole normalized by the incident magnetic field ($\frac{I_{monopole}}{H_{inc}}$) at the origin of the coordinate system defined in Figure A-1 (in the absence of the conducting screen). This useful transformation is possible due to the well known properties of biconical radiators. It can be shown [20] that the magnetic field of a biconical radiator is related to the driving current at the radiator's feedpoint by the following expression:

$$H(r) = 2I_{bicone} \frac{e^{-jkr}}{4\pi r}, \quad (A.2)$$

where H is the azimuthal component of magnetic field (ϕ component in a usual cylindrical coordinate system with the bicone axis z -directed), I_{bicone} is the driving current at the feed point of the bicone, and r is the distance from the feedpoint of the bicone to the point on the azimuthal plane (xy plane in the usual cylindrical coordinate system mentioned above) at which H is determined. It can also be shown that the input impedance of an ideal infinite biconical structure depends only on the half-angle of the cones and is independent of frequency. Thus, the feed current

of the bicone is given by

$$I_{bicone} = \frac{V_{bicone}}{Z_{bicone}}, \quad (A.3)$$

where V_{bicone} is the driving voltage at the feedpoint and Z_{bicone} is the frequency-invariant input impedance of the bicone.

By application of image theory, one knows that an infinite monocone whose apex rests on the surface of a ground plane has electrical behavior identical to that of the infinite bicone if the driving current at the feedpoint is the same. Of course the radiated field exists only on the monocone side of the ground plane. Thus, equation (A.2) applies equally well to the monocone over a ground plane as it does to the bicone:

$$H(r) = 2I_{monocone} \frac{e^{-jkr}}{4\pi r}, \quad (A.4)$$

where H and r are defined as they are in (A.2) and $I_{monocone}$ is the driving current at the feedpoint of the monocone. The input impedance of the monocone is also frequency-invariant and is determined by the half-angle of the cone. The monocone input impedance must be one-half that of the bicone for a given conical half-angle in order that the same driving current exist with only one-half the driving voltage (the other one-half of the driving voltage exists in the image in the ground plane when one compares the monocone to the bicone by application of image theory). Thus the driving current at the feedpoint of the monocone is given by

$$I_{monocone} = \frac{V_{monocone}}{Z_{monocone}}, \quad (A.5)$$

where $V_{monocone}$ is the driving voltage at the feedpoint of the monocone over the ground plane and $Z_{monocone}$ is the frequency-invariant input impedance which is

half that of the bicone. By substituting (A.5) into (A.4) one obtains an equation that relates the radiated magnetic field to the driving voltage of the monocone over a ground plane and the monocone's input impedance:

$$H(r) = 2 \frac{V_{monocone}}{Z_{monocone}} \frac{e^{-jkr}}{4\pi r}. \quad (A.6)$$

The EMPEROR facility was designed such that its input impedance is given by

$$Z_{monocone} = \frac{V_{monocone}}{I_{monocone}} = 50\Omega. \quad (A.7)$$

Related investigations have established that the measured input impedance of the EMPEROR facility does not significantly deviate from 50Ω over the frequency range presently of interest. Thus one determines the ratio of the driving voltage at the feedpoint of the EMPEROR monocone to the radiated magnetic field (a ratio that will be useful subsequently) by substituting (A.7) into (A.6) and rearranging terms. These steps yield the desired ratio:

$$\frac{V_{monocone}}{H^{inc}} = 50 \cdot 2\pi r e^{jkr}, \quad (A.8)$$

where the symbol $H(r)$ of (A.6) is replaced by the new symbol H^{inc} since the magnetic field radiated by the monocone is incident upon the surface of the slotted conducting screen during measurements.

Current at the base of the monopole is sensed by the S-parameter test set via a 50Ω coaxial cable which is terminated in its characteristic impedance, the input impedance of the HP-8515A S-parameter test set, which is 50Ω . As a result, the ratio of voltage to current at the base of the monopole is constrained to 50:1 since

the monopole is terminated by a 50Ω impedance (which is assumed to be non-reactive). Thus, one determines the current at the base of the monopole in terms of the voltage measured at the base of the monopole by application of Ohm's law:

$$\frac{I_{monopole}}{V_{monopole}} = \frac{1}{50}. \quad (A.9)$$

One finds the desired normalized form for the data $(\frac{I_{monopole}}{H^{inc}})$ by employing the simple identity:

$$\frac{I_{monopole}}{H^{inc}} = \frac{I_{monopole}}{V_{monopole}} \frac{V_{monopole}}{V_{monocone}} \frac{V_{monocone}}{H^{inc}} \quad (A.10)$$

in which one employs equations (A.1), (A.8), and (A.9) to arrive at

$$\frac{I_{monopole}}{H^{inc}} = S_{21} 2\pi r e^{jk(r+\delta)}, \quad (A.11)$$

where S_{21} is the measured data, r is the distance from feedpoint of the biconical radiator to the origin of the coordinate system (defined in Figure A-1), and δ is a small phase adjustment to compensate for electrical delay not accounted for in the network analyzer's S_{21} calibration procedure. Such unaccounted for electrical delay could result from the electrical length of the hardware used to adapt the feedpoint of the monocone to a standard microwave connector.

For the measurements reported here, $r = 3.0$ m and $\delta = 0.0537$ m. r was determined by physical measurement while δ was found empirically as the necessary correction to produce constant phase response from a vanishingly short monopole (located 3.0 m from the radiator) over the frequency range 500 MHz to 10.5 GHz. In the following section the steps involved in the empirical determination of δ are described.

Calibration Details

As discussed previously, the raw S_{21} data must be transformed via equation (A.11) so that one may obtain the desired normalized form $\frac{I_{monopole}}{H^{inc}}$. It was anticipated that some correction factor δ would be required, but the precise value was unknown. In order to determine the proper value for δ , the current at the base of a 7.515 cm monopole was measured since the behavior of such a simple structure is well understood and an accurate prediction of the current can be made with a frequency-domain method-of-moments code. It is understood that calibration of a measurement system by comparison with a numerical technique is a weak foundation for subsequent validation of a second numerical technique based on measurements made with that calibration. However, a starting point was needed and the moment-method solution for current induced on a thin wire by an incident plane wave is widely accepted as accurate.

The monopole and its image in the ground plane were modeled with the moment-method code. The unknown current was expanded in a series of 101 triangular expansion functions and the 100 Ω load (50 Ω load and its image) was included during pulse testing. The wire was excited by a normally incident plane wave with x-directed electric field amplitude $1 \frac{V}{m}$. The solution was repeated every 0.02 GHz from .1 GHz to 10.0 GHz.

The measured and computed data for the magnitude of the current at the base of the loaded 7.515 cm monopole appear in Figure A-4. The measured data are normalized by means of (A.11) with $r = 3.0m$ and $\delta = 0.0m$ and are then divided by η_0 , where

$$\eta_0 = \frac{E^{inc}}{H^{inc}} = 377\Omega, \quad (A.12)$$

in order to normalize with respect to the incident electric field rather than the incident magnetic field (to facilitate comparison with the computed data that were so normalized). One observes good agreement in the frequency and amplitude of the first resonance and general agreement of the data over the entire frequency range. The good agreement observed among the measured and computed magnitude data inspires confidence in the validity of the magnitude portion of the data normalization formula (A.11).

The measured and computed phase data for the current at the base of a loaded 7.515 cm monopole appear in Figure A-5. Again the measured data are normalized by means of (A.11) with $r = 3.0m$ and $\delta = 0.0m$. With no phase compensation ($\delta = 0$), the measured phase data do not agree with the data calculated by the method-of-moments code. Indeed, the measured data indicate that zero phase occurs at roughly 0.75 GHz while the magnitude data observed in Figure A-4 and the moment-method phase data of Figure A-5 indicate that the first resonance occurs at about 0.92 GHz. The measured phase data are obviously in error. Observation of the measured data in Figure A-5 leads one to conclude that a positive phase correction ($\delta > 0$) is required in order to establish agreement between measured phase and computed phase at the monopole's first resonance.

A correction factor commonly used to adjust phase data collected with the EMPEROR facility is $\delta = 0.1025m$. The measured data were re-normalized with $\delta = 0.1025m$ and the results appear in Figure A-6. Agreement between measured and computed phase is still poor. Indeed, in the region of the first monopole resonance, the measured phase never falls to zero – an obvious error. One concludes that the measured data have been over-corrected and that the correct value of δ lies between $0.0m$ and $0.1025m$. Trial values of δ in the vicinity of $0.05m$ were employed

in (A.11) until good agreement was observed between the measured and computed phase data.

In Figure A-7 one sees good agreement between the measured and computed phase data for the choice $\delta = 0.0537m$. The agreement is very good below 3 GHz and generally good above. Thus, it is likely that $\delta = 0.0537m$ is the proper correction factor. However, it is well to verify this choice via means other than numerical computation. To this end, the 7.515 cm monopole was removed from the microwave connector in the ground plane and the S_{21} measurement was repeated with what was now a vanishingly short monopole. The current on such a short structure is known to be purely imaginary and thus the current's phase should be a constant 90° from low frequencies to well beyond 10.5 GHz, the limit of our present interest.

The phase data for the vanishingly short monopole with $\delta = 0.0m$ appear in Figure A-8. Positive phase correction is obviously necessary since the measured phase data falls behind the theoretical value by 645° or 3.583π radians as the frequency reaches 10.0 GHz. To make up this phase deficit, one solves the equation

$$k\delta = 3.583\pi \quad (A.13)$$

for delta at 10.0 GHz and finds that $\delta = 0.0537m$.

The phase data for the vanishingly short monopole with the commonly employed correction $\delta = 0.1025m$ appear in Figure A-9. The phase correction must be reduced since the measured phase data advance beyond the theoretical value of 90° by 585° or 3.250π radians as the frequency reaches 10.0 GHz. To squelch this phase advance, one solves the equation

$$k\Delta = 3.250\pi \quad (A.14)$$

to find the excess phase correction at 10 GHz which is $\Delta = 0.0488m$. The desired phase correction δ is given by

$$\delta = .1025 - \Delta, \quad (A.15)$$

which yields the same correction $\delta = 0.0537m$ that was determined above.

The phase data for the vanishingly short monopole with the proper phase correction $\delta = 0.0537m$ appear in Figure A-10. Observe that the phase response is constant to within about $\pm 10^\circ$ from 0.5 GHz to 10.5 GHz and less than $\pm 5^\circ$ over substantial portions of that range. This corrected phase data inspires additional confidence that the field radiated by the EMPEROR facility is well characterized by (A.11) with $\delta = 0.0537$ over the frequency range of interest.

At this point it is well to discuss a possible source of confusion when one applies the normalization formula (A.11). When one computes the current on a monopole which is x-directed, per the coordinate system defined in Figure 1, one usually defines positive current to be x-directed. Along a similar vein, one also assumes that the incident electric field is x-directed. However, when one makes measurements using the EMPEROR facility, positive voltage at the base of an x-directed monopole corresponds to negative x-directed current and positive voltage at the base of the x-directed monocone corresponds to negative x-directed current and thus negative x-directed radiated electric field. Thus, while (A.11) is accurate as written, one must exercise caution when relating the sense of the voltage to the sense of the current at the base of the monopole. Likewise, caution is necessary when the sense of the voltage is related to the sense of the current at the base of the monocone since the current at the base of the monocone determines the sense of the radiated field.

Experimental Apparatus

The positioning of the slotted screen, D sensor, and thin wire scatterer is illustrated in Figure A-11. The slotted screen comprised several components. The first was a 1.22 m by 3.335 m sheet of aluminum 1.6 mm thick. The sheet of aluminum had a 14 cm by 24 cm section removed from the center of one long edge and was mounted to a light wooden frame and secured to the ground plane so that the opening was next to the ground plane. The opening was filled by a 15 cm by 26 cm insert which extended into a 30 cm by 30 cm by 3 mm brass baseplate which formed part of the ground plane. The baseplate contained a narrow slit into which the insert was placed and clamped. The insert was fashioned from a 0.25 mm thick strip of brass shim stock which had been chemically etched to produce a slot with precise dimensions. A typical insert and the brass baseplate are illustrated in Figure A-12. The brass baseplate contained several holes to allow access to sensors whose connections led to the instrumentation which was located below the ground plane. The various components of the slotted screen were secured to each other and to the ground plane with copper tape coated with perforated adhesive. The mechanical attachment of the screen to the ground plane was fortified by small angle brackets, screws, and nuts.

Since cavity backed slotted screen electric field measurements were required, it was necessary to fabricate a suitable cavity-like extension to the measurement apparatus. To this end, four walls of a 13 cm by 17 cm by 21 cm cavity were fabricated from a 1.6 mm thick aluminum sheet. These walls were attached to each other with copper tape to form a four-wall cavity assembly (see Figure A-13). Before the cavity-backed slot data were collected, the four-wall assembly was positioned so that the slotted insert and the brass baseplate became the fifth and sixth walls of the cavity. The four-wall cavity assembly was then secured to the slotted

insert and the brass baseplate with copper tape. Figure A-14 depicts the evolution of the measurement apparatus as one progresses from electric field measurements on the shadow side of the slotted screen with the $\dot{D}$ sensor (Figure A-14(a)) to current measurements at the base of a monopole (Figure A-14(b)) to electric field measurements within the rectangular cavity (Figure A-14(c)).

Finally, it was necessary to simulate off-normal incidence of the exciting field. Since it is impractical to move the permanently fixed monocone, it was decided that the slotted screen would be moved instead. In order to simulate off-normal incidence, the large aluminum sheet (containing the slotted insert) and the brass baseplate were untaped, disconnected from the ground plane, rotated clockwise 30 degrees, reattached to the ground plane, and retaped. The "rotated screen" structure corresponding to that of Figure A-14(a) is depicted in Figure A-14(d). Thus θ° of Figure A-11 was changed from 90° to 60° while the origin of the coordinate system remained at the same point 3.0m from the feedpoint of the monocone.

Data Collection

A variety of measurements were made on the experimental apparatus. These measurements involved ten separate thin brass inserts in which there were slots chemically etched in three configurations. Two inserts contained straight slots, four inserts contained slots following an annular contour, and four inserts contained slots following a rectangular contour. The differences among inserts within a configuration consisted of the presence (or absence) and location of "shorts" along the slot's contour. "Shorts" are small sections along a slot's contour where the brass is not etched away. Thus a slot with a "short" is actually a pair of coupled subslots which follow a common contour. The slots were all 0.5 mm wide which caused the straight slots to have an effective width of 1.0 mm (slot plus its image in the ground plane).

The length of the slots prior to the inclusion of "shorts" was made to be roughly 15 cm so that the slots would resonate at approximately 1 GHz. (Precise dimensions for each slot are clearly marked on each figure.)

Each of the ten inserts was subjected to both normally incident ($\theta^i = 90^\circ$) and off-normally incident ($\theta^i = 60^\circ$) excitation (see Figure A-11 for pertinent geometric details). Electric field was measured at two locations on the shadow side of the slotted screen either in the presence or absence of a thin wire scatterer or in the presence or absence of the 13 cm by 17 cm by 21 cm cavity. The sensor was located at $d' = 5$ cm with two angular displacements ($\theta' = 0^\circ$ and $\theta' = 30^\circ$).

The optional thin wire scatterer (radius = 0.4 mm) was located at each of four locations on the shadow side of the slotted screen. The wire was located at two distances from the origin ($d^w = 3.75$ cm and $d^w = 7.5$ cm) and at two angular displacements ($\theta^w = 0^\circ$ and $\theta^w = 30^\circ$). For several cases, the thin wire scatterer was bent at a 45° angle with respect to the ground plane. The angle opened either in the $\hat{z}$ or $-\hat{y}$ direction.

For many measurements, the electric field sensor was removed and the input of the S-parameter test set was connected to the base of a thin wire scatterer via a connector which was screwed into a threaded hole in the brass baseplate. Thus the wire became an extension of the center conductor of a coaxial cable whose outer conductor terminated at the surface of the brass baseplate.

Not all possible combinations of sensor and scatterer location, slot configuration, cavity presence or absence, and angle of incidence were measured. A balance between measurement economy and reasonable redundancy of measurements was sought while an effort was made to provide sufficient variety to adequately validate the three numerical solution techniques.

Figure A-1. Coordinate System Associated with a Slotted Screen and a Thin Wire Scatterer on the Shadow Side

Figure A-2. Block Diagram of Instrumentation for Electric Field Measurements

Figure A-3. Block Diagram of Instrumentation for Monopole Current Measurements

Figure A-4. Magnitude of Normalized Electric Current at the Base of a 7.515 cm Monopole Terminated with a 50Ω Load

Figure A-5. Phase of Normalized Electric Current at the Base of a 7.515 cm Monopole Terminated with a 50Ω Load

Figure A-6. Phase of Normalized Electric Current at the Base of a 7.515 cm Monopole Terminated with a 50Ω Load

Figure A-7. Phase of Normalized Electric Current at the Base of a 7.515 cm Monopole Terminated with a 50Ω Load

Figure A-8. Phase of Normalized Electric Current at the Base of a Vanishingly Short Monopole Terminated with a 50Ω Load

Figure A-9. Phase of Normalized Electric Current at the Base of a Vanishingly Short Monopole Terminated with a 50Ω Load

Figure A-10. Phase of Normalized Electric Current at the Base of a Vanishingly Short Monopole Terminated with a 50Ω Load

Figure A-11. Pertinent Geometric Features of the Monocone, Screen, Thin Wire Scatterer, and Electric Field Sensor

Figure A-12. Slotted Insert and Brass Baseplate

Figure A-13. Configuration of Four-Wall Cavity Assembly

Figure A-14. Evolution of Measurement Apparatus to Accommodate various Measurements

Appendix B

Measurement of Slot Input Impedances: Experimental Apparatus and Techniques

A series of laboratory measurements was performed in order to establish a body of experimental data which provides the basis for validation of the junction theory for the frequency domain integral equation for slots with arbitrary contour. It was decided that slot input impedance would be not only a sensitive indicator of junction modeling performance, but also would be simple to measure. Slot input impedance is sensitive to the basic slot model since accurate impedance prediction implies accurate knowledge of the distribution of the total axially directed equivalent magnetic current (or transverse slot voltage) at a point (the driving point) along the slot.

To provide a suitable apparatus on which to perform slot input impedance measurements, a 30 cm by 30 cm sheet of 0.001 inch thick brass was perforated to fabricate a 15 cm by 1 mm slot (see Figure B-1). A short (3 cm) segment of 0.085 inch diameter 50 Ω semi-rigid coaxial cable was "connected across" the slot at the slot's axial center and was terminated with a 3.5 mm microwave connector. The coax outer conductor was soldered to the sheet on one side of the slot and the center conductor was soldered to the sheet on the opposite side (see insert in Figure B-1). The complex reflection coefficient was measured at the point where this feeder cable was connected to the slot. The measurements were made with an HP-8510A microwave network analyzer and an HP-8515A S-parameter test set. All measurements were made from 0.1GHz to 4.1 GHz in frequency steps of 10 MHz. The resulting S-parameter (S_{11}) data were transformed to impedance by means of the standard microwave circuit formula

$$Z = Z_0 \frac{1 + S_{11}}{1 - S_{11}}, \quad (B.1)$$

where $Z_0 = 50\Omega$.

The network analyzer was calibrated at the end of a 4 foot flexible coaxial cable. The calibration was performed with the HP-85052A 3.5 mm calibration kit. Thus, the calibration plane of reference was initially located at the end of the 4 foot cable. When the 4 foot cable was attached to the 3.5 mm connector at the end of the short (3 cm) feeder cable, some compensation was necessary to account for the additional electrical length of the feeder cable. The port extension facility of the HP-8510A network analyzer provided a means to accomplish this compensation. A 3 cm section of 0.085 inch 50Ω coaxial cable was fabricated to be identical to the 3 cm feeder cable that was attached to the slot. This second 3 cm cable was terminated with a short circuit and attached to the 4 foot cable which was attached to the HP-8515A test set. The port extension parameter of the HP-8510A was adjusted until the response displayed by the network analyzer closely approximated that of a perfect short circuit. The 4 foot cable was then disconnected from the shorted cable and connected to the feeder cable which was attached to the slot in the brass sheet.

In order to measure the behavior of slots that contain junctions, e.g., a slot that forks into two or more slots, the single straight slot in the 30 cm by 30 cm sheet was extended so that each of its two ends forked into separate slots. The slot was extended by the progressive removal of 4 cm by 1 mm sections that were cut such that one end of each extension coincided with an end of the original slot (see Figure B-2). Four extensions were made in a natural progression in order to provide several intermediate cases for comparison with computed data and also to

provide insight into the physical mechanisms at work in the structure. The input impedance at the center of the original straight slot was measured first and then, after each 4 cm by 1 mm slot extension was cut from the sheet, the impedance was again measured at the same feedpoint.

Several sources of measurement error are anticipated. First, the transition from the feeder cable to the slot has the potential to cause error since the fields in the neighborhood of the truncated cable deviate from those that would result from an ideal "delta-gap" magnetic field generator. Another source of error is the presence of the feeder cable and the 4 foot cable connected to the HP-8510A. Fields radiated by the slot are scattered by these cables and subsequently re-illuminate the slot. Since the feeder cable is located flush with the surface of the brass sheet, this effect is minimized but not eliminated. The finite dimensions of the brass sheet also contribute error to the measurements since waves propagating along the surface of the sheet are diffracted by the edges of the sheet and re-illuminate the slot.

For a fixed absolute error in the measured reflection coefficient S_{11} (due to one or more of the sources described above), measured impedances of 50Ω suffer less error than measured impedances of $2,000\Omega$. This is readily seen from equation (B.1) where a reflection coefficient of 0.00 that is in error by 0.01 suffers an impedance error of about 2% (with respect to the unperturbed impedance of 50Ω) while a reflection coefficient of 0.95 that is in error by 0.01 suffers a relative impedance error of about 24%. Indeed, if equation (B.1) is differentiated with respect to the reflection coefficient (S_{11}), one obtains

$$\frac{dZ}{dS_{11}} = Z_0 \frac{2}{(1 - S_{11})^2} \quad (B.2)$$

which predicts that very small errors in S_{11} lead to large errors in the calculated

impedance as the reflection coefficient S_{11} approaches 1. Thus, more error is expected when the measured slot input impedance is high with respect to the 50Ω characteristic impedance of the measurement system than is expected when the measured impedance is close to 50Ω .

Not only is the measurement error sensitive to the magnitude of the measured reflection coefficient, it is also sensitive to the frequency of the measurement. More error is expected at 4.1 GHz than is expected at 1 GHz since the discontinuities at the feedpoint are expected to increase as the cable dimensions grow in wavelength with increasing frequency. Also, the feed coax and the measuring instruments form a more efficient radiator of "unaccounted for" energy at higher frequencies. Some "unaccounted for" radiated energy could result from the transition from the unbalanced feedline to the balanced radiating structure.

Figure B-1. Center-Fed 15 cm by 1 mm Slot Cut in a 30 cm by 30 cm by 0.001 inch Brass Sheet

Figure B-2. Progression of Slot Extensions in Brass Sheet

Appendix C

Equivalence of Discretized Dual-Potential and Single-Potential Integral Equations for Straight Slots with Uniform Subdomain Length

The narrow slot integral equation is derived in Chapter II in what is commonly known as the "dual-potential" form. That is, the magnetic field in each half-space is written in terms of electric vector potential and scalar magnetic potential as well as the short-circuit magnetic field in each half-space that exists due to impressed sources. The resulting integral equation (Equation (2.23)) is

$$\begin{aligned}
 & -\frac{1}{\pi\mu} \left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_C k(s'; \tau) [\hat{s}'(s') \cdot \hat{s}(s)] k^*(s, s') ds' \right. \\
 & \quad \left. - \frac{\partial}{\partial s} \int_C \frac{\partial}{\partial s'} k(s'; \tau) k^*(s, s') ds' \right\} \\
 & = -\frac{\partial}{\partial t} [\hat{s}(s) \cdot \mathbf{h}^{sc}(s; t)], \tag{C.1}
 \end{aligned}$$

where all symbols are defined in Chapter II. An alternate formulation of the narrow slot integral equation is possible. The alternate formulation has the same dual-potential roots, but, by means of the Lorentz gauge, one expresses the magnetic scalar potential in terms of the electric vector potential and writes the general vector slot integral equation as

$$\left[-\frac{\partial^2}{\partial t^2} \mathbf{f} + \nabla \nabla \cdot \mathbf{f} \right]_{tan} = -\frac{1}{2} \frac{\partial}{\partial t} \mathbf{h}_{tan}^{sc} \tag{C.2}$$

where $\mathbf{f}$ is the electric vector potential.

For the case of a curved narrow slot, one finds the dual-potential formulation convenient to implement since the spatial derivatives inherent in the scalar

potential term reduce to partial derivatives along the axial direction of the slot which are readily discretized. On the other hand, the derivatives inherent in the gradient-divergence term of the single-potential formulation prove difficult to accurately discretize in the case of curved narrow slots. However, if the narrow slot is straight and y-directed, the general vector slot equation (C.2) reduces to the simple form

$$-\frac{\partial^2}{\partial t^2} f_v + \frac{1}{\mu\epsilon} \frac{\partial^2}{\partial y^2} f_v = -\frac{1}{2} \frac{\partial}{\partial t} h_v^{*c} \quad (C.3)$$

which one readily discretizes by dividing the straight narrow slot into equal-length subdomains and approximating the second partial derivatives in space and time with second central differences. This discretization leads to a simple and efficient "marching in time" solution technique which yields accurate results [11].

The dual-potential formulation of Equation (C.1) appears somewhat unwieldy after it is discretized by the steps outlined in Chapter IV. The spatially discretized form of (C.1) is

$$\begin{aligned} & -\frac{1}{\pi\mu} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[\Delta S_m \cdot \sum_{n=1}^N K(n; \tau_{mn}) \left\{ \hat{s}_{n-\frac{1}{2}} \int_{s_{n-1}}^{s_n} \Lambda_n(s') k^*(s_m, s') ds' \right. \right. \\ & \quad \left. \left. + \hat{s}_{n+\frac{1}{2}} \int_{s_n}^{s_{n+1}} \Lambda_n(s') k^*(s_m, s') ds' \right\} \right] \\ & + \frac{1}{\pi\mu} \sum_{n=1}^N \left\{ \frac{K(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^*(s_{m+\frac{1}{2}}, s') ds' \right. \\ & \quad - \frac{K(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k^*(s_{m+\frac{1}{2}}, s') ds' \\ & \quad \left. - \frac{K(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}})}{\Delta s_{n-\frac{1}{2}}} \int_{s_{n-1}}^{s_n} k^*(s_{m-\frac{1}{2}}, s') ds' \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{K(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}})}{\Delta s_{n+\frac{1}{2}}} \int_{s_n}^{s_{n+1}} k'(s_{m-\frac{1}{2}}, s') ds' \Big\} \\
& = -\frac{\partial}{\partial t} \left[\Delta S_m \cdot \mathbf{h}^{ec}(s_m; t) \right], \tag{C.4}
\end{aligned}$$

where all symbols are defined either in Chapter II or Chapter IV. After observing remarkable similarity in the data produced by a "marching in time" code based on the dual-potential equation (C.4) and a "marching in time" code based on the single-potential equation (C.3) for the case of straight slots that were sub-divided into uniform-length subdomains, one determines that the field advancement formulas employed by the two codes are identical for this special case. What follows is a demonstration of the equivalence of the two discretized formulations.

For the case of a y-directed straight narrow slot which is divided into uniform subdomains, the following simplifications can be made:

$$\hat{s}_{n \pm \frac{1}{2}} = \hat{y}, \tag{C.5}$$

$$\Delta s_{n \pm \frac{1}{2}} = \Delta y, \tag{C.6}$$

and

$$\Delta S_m = \Delta y \hat{y}. \tag{C.7}$$

When one substitutes (C.5) through (C.7) into (C.4), employs the definition $c^2 = \frac{1}{\mu\epsilon}$, and approximates the triangular expansion function Λ_n in the vector potential term with a pulse Π_n , he obtains

$$-\frac{\epsilon}{\pi} \Delta y \frac{\partial^2}{\partial t^2} \sum_{n=1}^N K(n; \tau_{m,n}) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k'(y_m, y') dy'$$

$$\begin{aligned}
& + \frac{1}{\pi\mu} \frac{1}{\Delta y} \sum_{n=1}^N \left\{ K(n; \tau_{m+\frac{1}{2}, n-\frac{1}{2}}) \int_{y_{n-1}}^{y_n} k^s(y_{m+\frac{1}{2}}, y') dy' \right. \\
& \quad - K(n; \tau_{m+\frac{1}{2}, n+\frac{1}{2}}) \int_{y_n}^{y_{n+1}} k^s(y_{m+\frac{1}{2}}, y') dy' \\
& \quad - K(n; \tau_{m-\frac{1}{2}, n-\frac{1}{2}}) \int_{y_{n-1}}^{y_n} k^s(y_{m-\frac{1}{2}}, y') dy' \\
& \quad \left. + K(n; \tau_{m-\frac{1}{2}, n+\frac{1}{2}}) \int_{y_n}^{y_{n+1}} k^s(y_{m-\frac{1}{2}}, y') dy' \right\} \\
& = -\Delta y \frac{\partial}{\partial t} h_y^s(y_m; t). \tag{C.8}
\end{aligned}$$

A simple sketch of the geometry of a straight narrow slot divided into equal-length subdomains appears in Figure C-1a. If one makes $m = n = 3$, the second integral in (C.8) becomes

$$\int_{y_{n-1}}^{y_n} k^s(y_{m+\frac{1}{2}}, y') dy' = \int_{y_2}^{y_3} k^s(y_{\frac{7}{2}}, y') dy', \tag{C.9}$$

where the integration geometry is indicated pictorially in Figure C-1b. Now, guided by Figures C-1b and C-1c, observe that the integral in (C.9) is also equal to another integral

$$\int_{y_2}^{y_3} k^s(y_{\frac{7}{2}}, y') dy' = \int_{y_{\frac{5}{2}}}^{y_{\frac{7}{2}}} k^s(y_4, y') dy' = \int_{y_{2+\frac{1}{2}}}^{y_{3+\frac{1}{2}}} k^s(y_{3+\frac{1}{2}}, y') dy', \tag{C.10}$$

by virtue of the fact that all subdomains have the same length and that the kernel is sensitive only to the distance from the source to the point of observation and the slot width, not to the absolute locations in the slot. By this argument, the result obtained in (C.10) generalizes to

$$\int_{y_{n-\frac{1}{2}}}^{y_n} k^s(y_{m+\frac{1}{2}}, y') dy' = \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_{m+1}, y') dy'. \quad (C.11)$$

The remaining integrations in the second summation in (C.8) are treated in like manner. Thus, it follows that

$$\int_{y_n}^{y_{n+1}} k^s(y_{m-\frac{1}{2}}, y') dy' = \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_{m-1}, y') dy' \quad (C.12)$$

and that

$$\int_{y_n}^{y_{n+1}} k^s(y_{m+\frac{1}{2}}, y') dy' = \int_{y_{n-1}}^{y_n} k^s(y_{m-\frac{1}{2}}, y') dy' = \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m, y') dy'. \quad (C.13)$$

Observe that for $m = n$, the time delay $\tau_{m+\frac{1}{2}, n-\frac{1}{2}}$ in the second integral of (C.8) reduces to $t - \frac{\Delta y}{c}$ since the distance that the signal must propagate is measured from the middle of the integration range to the point of observation. After the integration limits and point of observation are "shifted" by $\frac{\Delta y}{2}$ to obtain (C.10) from (C.9), the time delay is given by $\tau_{m+1, n}$ which also reduces to $t - \frac{\Delta y}{c}$ for $m = n$. Thus the time delay is preserved in the change of variables that one implicitly makes when "shifting" the integration limits and the point of observation as is done above.

It is important to reiterate that the simplifications and substitutions suggested above by (C.11) through (C.13) are made possible only because the subdomains are of uniform length and because the slot is straight. When one makes these substitutions in (C.8) he obtains

$$\begin{aligned}
& -\frac{\epsilon}{2\pi} \frac{\partial^2}{\partial t^2} \sum_{n=1}^N K(n; \tau_{m,n}) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m, y') dy' \\
& + \frac{1}{\mu\epsilon} \frac{\epsilon}{2\pi} \frac{1}{(\Delta y)^2} \sum_{n=1}^N \left\{ K(n; \tau_{m+1,n}) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_{m+1}, y') dy' \right. \\
& \quad - 2K(n; \tau_{m,n}) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m, y') dy' \\
& \quad \left. + K(n; \tau_{m-1,n}) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_{m-1}, y') dy' \right\} \\
& = -\frac{1}{2} \frac{\partial}{\partial t} h_y^{s,c}(y_m; t), \tag{C.14}
\end{aligned}$$

which is a spatially discretized form of the single-potential integral equation for a straight y-directed narrow slot. To recognize that this is indeed the case, one observes, in the second summation of (C.14), a second central difference approximation to the second partial derivative with respect to y of the current coefficient and the integrated kernel. Thus (C.14) is recognized to be an approximation of

$$\begin{aligned}
& -\frac{\epsilon}{2\pi} \frac{\partial^2}{\partial t^2} \sum_{n=1}^N K(n; \tau_{m,n}) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m, y') dy' \\
& + \frac{1}{\mu\epsilon} \frac{\epsilon}{2\pi} \frac{\partial^2}{\partial y^2} \sum_{n=1}^N K(n; \tau_{m,n}) \int_{y_{n-\frac{1}{2}}}^{y_{n+\frac{1}{2}}} k^s(y_m, y') dy' \\
& = -\frac{1}{2} \frac{\partial}{\partial t} h_y^{s,c}(y_m; t), \tag{C.15}
\end{aligned}$$

which is simply a spatially discretized version of the single-potential straight narrow slot equation (Equation (C.3)).

(a)

(b)

(c)

Figure C-1. Geometry of a Straight Narrow Slot with Uniform-Length Subdomains and Pictorial Representation of Kernel Integrations

Appendix D

Alternate Formulation of Time-Domain Potentials for 2-D Problems

The usual formulation of time-domain potentials which is commonly employed for 3-d problems is unsuitable for 2-d problems because of difficulties one encounters when attempting the infinite integration in the z direction. An alternate formulation for 3-d problems readily reduces to a suitable 2-d form.

In order to find the 2-d scalar Green's function for a line source, the alternate 3-d scalar Green's function for a point source is found as the solution of the following scalar wave equation:

$$\nabla^2 g - \frac{1}{c^2} \left(\frac{\partial^2}{\partial t^2} g \right) = -\delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \quad (D.1)$$

where g is assumed to be continuous through the source. Equation (D.1), in spherical coordinates, is:

$$\frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} g = -\delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \quad (D.2)$$

where

$$R = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}. \quad (D.3)$$

The following solution form is proposed:

$$g = \frac{h\left[\frac{R}{c} - (t - t')\right]}{R} + \frac{k\left[\frac{R}{c} + (t - t')\right]}{R} \quad (D.4)$$

where h and k are any functions.

First, one verifies that (D.4) satisfies the homogeneous form of (D.2). Details are presented for the first term of (D.4) ($g^- = \frac{h}{R}$), the second term ($g^+ = \frac{k}{R}$) where $g = g^- + g^+$ follows.

$$\frac{\partial}{\partial R} g^- = \frac{\frac{R}{c} h' \left[\frac{R}{c} - (t - t') \right] - h \left[\frac{R}{c} - (t - t') \right]}{R^2} \quad (D.5)$$

$$\frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g^- = \frac{\partial}{\partial R} \left\{ \frac{R}{c} h' \left[\frac{R}{c} - (t - t') \right] - h \left[\frac{R}{c} - (t - t') \right] \right\} \quad (D.6)$$

$$\begin{aligned} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g^- &= \frac{1}{c} \left\{ h' \left[\frac{R}{c} - (t - t') \right] + \frac{R}{c} h'' \left[\frac{R}{c} - (t - t') \right] \right\} \\ &\quad - \frac{1}{c} h' \left[\frac{R}{c} - (t - t') \right] \end{aligned} \quad (D.7)$$

$$\frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g^- = \frac{R}{c^2} h'' \left[\frac{R}{c} - (t - t') \right] \quad (D.8)$$

and

$$\frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g^- = \frac{h'' \left[\frac{R}{c} - (t - t') \right]}{Rc^2}. \quad (D.9)$$

So, for $g = g^- + g^+$,

$$\frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g = \frac{h'' \left[\frac{R}{c} - (t - t') \right]}{Rc^2} + \frac{k'' \left[\frac{R}{c} + (t - t') \right]}{Rc^2}. \quad (D.10)$$

The second term in (D.10) follows directly from (D.9) since none of the derivatives leading to (D.9) is performed with respect to t or t' .

Now,

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} g^- = \frac{h'' \left[\frac{R}{c} - (t - t') \right]}{c^2 R} \quad (D.11)$$

and, for $g = g^- + g^+$,

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} g = \frac{h'' \left[\frac{R}{c} - (t - t') \right]}{c^2 R} + \frac{k'' \left[\frac{R}{c} + (t - t') \right]}{c^2 R} \quad (D.12)$$

since two differentiations of $(t - t')$ with respect to t' produce no sign change. Thus,

$$\begin{aligned} \frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} g &= \frac{h'' \left[\frac{R}{c} - (t - t') \right]}{R c^2} + \frac{k'' \left[\frac{R}{c} + (t - t') \right]}{R c^2} \\ &- \frac{h'' \left[\frac{R}{c} - (t - t') \right]}{c^2 R} - \frac{k'' \left[\frac{R}{c} + (t - t') \right]}{c^2 R} = 0. \end{aligned} \quad (D.13)$$

Hence, the proposed solution (D.4) satisfies the homogeneous form of (D.2). Now it is necessary to specify the functions h and k .

Since we insist that effects of the source not be felt until after the source is activated, $k \left[\frac{R}{c} + (t - t') \right]$ is an unacceptable solution and, even though it satisfies the homogeneous form of (D.2), it is discarded ($\Rightarrow k \left[\frac{R}{c} + (t - t') \right] = 0$) because it is not causal. Now, a small sphere of radius R is placed about the point r' and (D.2) is integrated over this volume. The volume of the sphere is reduced to zero in order to obtain a limiting form for $g = h \left[\frac{R}{c} - (t - t') \right]$.

For the first term of (D.2),

$$\iiint_v \frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g \, dv = \iiint_v \frac{1}{R^2} \frac{\partial}{\partial R} R^2 \left(\frac{\partial}{\partial R} g \right) R^2 \sin \theta \, dR d\theta d\phi \quad (D.14)$$

$$\iiint_v \frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g \, dv = \int_0^{2\pi} \int_0^\pi R^2 \frac{\partial}{\partial R} g \sin \theta \, d\theta d\phi \quad (D.15)$$

$$\iiint_v \frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} g \, dv = 4\pi R^2 \frac{\partial}{\partial R} g. \quad (D.16)$$

For the second term of (D.2),

$$\iiint_v \frac{1}{c^2} \frac{\partial^2}{\partial t^2} g \, dv = \frac{4\pi R^3}{3c^2} \frac{\partial^2}{\partial t^2} g. \quad (D.17)$$

And, for the source,

$$\iiint_v -\delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \, dv = -\delta(t - t'). \quad (D.18)$$

Thus, equation (D.2) becomes

$$4\pi R^2 \frac{\partial}{\partial R} g - \frac{4\pi R^3}{3} \frac{\partial^2}{\partial t^2} g = -\delta(t - t') \quad (D.19)$$

or

$$\frac{\partial}{\partial R} g - \frac{R}{3} \frac{\partial^2}{\partial t^2} g = \frac{-\delta(t - t')}{4\pi R^2}. \quad (D.20)$$

Now, take the limit as $R \rightarrow 0$. Keep in mind that g is continuous through the source. Thus, one infers that $\frac{\partial}{\partial R} g$ must support the discontinuous behavior implied by $\delta(\mathbf{r} - \mathbf{r}')$ in (D.2). These steps yield

$$\frac{\partial}{\partial R} g \xrightarrow{R \rightarrow 0} \frac{-\delta(t - t')}{4\pi R^2}. \quad (D.21)$$

And, after integration with respect to R ,

$$g \xrightarrow{R \rightarrow 0} \frac{\delta(t - t')}{4\pi R}, \quad (D.22)$$

where the constant of integration must be zero to satisfy the original differential equation whose solution must maintain the form in (D.4). Thus $g = h[\frac{R}{c} - (t - t')]$ must be given by

$$g = \frac{1}{4\pi R} \delta\left[\frac{R}{c} - (t - t')\right] \quad (D.23)$$

or

$$g = \frac{1}{4\pi R} \delta\left[(t - t') - \frac{R}{c}\right]. \quad (D.24)$$

Equation (D.24) is a 3-d scalar Green's function for a point source in space and time.

Now, to find a 2-d Green's function, one integrates (D.24) with respect to z' from $-\infty$ to $+\infty$:

$$g_{2d} = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{\delta(\frac{R}{c} - \tau)}{R} dz', \quad (D.25)$$

where

$$\tau = t - t' \quad (D.26)$$

and

$$R = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}. \quad (D.27)$$

Let

$$\zeta = (z' - z) \quad (D.28)$$

and

$$P = [(x - x')^2 + (y - y')^2]^{1/2} \quad (D.29)$$

such that

$$R^2 = P^2 + \zeta^2. \quad (D.30)$$

Now Equation (D.25) is written as

$$g_{2d} = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{\delta\left[\frac{R}{\sigma} - \tau\right]}{R} d\zeta \quad (D.31)$$

since

$$\frac{d\zeta}{dz'} = 1. \quad (D.32)$$

Note that

$$\zeta = [R^2 - P^2]^{1/2} \quad (D.33)$$

so

$$\frac{d\zeta}{dR} = \frac{R}{\zeta}. \quad (D.34)$$

Equation (D.34) allows the integral in (D.31) to be written as

$$g_{2d} = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{\delta\left[\frac{R}{\sigma} - \tau\right]}{R} \frac{R}{\zeta} dR \quad (D.35)$$

or

$$g_{2d} = \frac{2}{4\pi} \int_0^{\infty} \frac{\delta\left[\frac{R}{c} - \tau\right]}{\sqrt{R^2 - P^2}} dR. \quad (D.36)$$

The sampling property of the delta function implies that (D.36) can be written as

$$g_{2d} = \frac{2}{4\pi} \frac{1}{\sqrt{c^2 \tau^2 - P^2}} \int_0^{\infty} \delta\left[\frac{R}{c} - \tau\right] dR, \quad (D.37)$$

since the square root in (D.36) is sampled when

$$\frac{R}{c} - \tau = 0 \quad (D.38)$$

which implies that, at the moment of sampling,

$$R = c\tau. \quad (D.39)$$

By definition,

$$\int_0^{\infty} \delta\left[\frac{R}{c} - \tau\right] dR = c. \quad (D.40)$$

Thus, the 2-d scalar Green's function for a line source in space and time is given by:

$$g_{2d} = \begin{cases} \frac{c}{2\pi} \frac{1}{\sqrt{c^2 \tau^2 - P^2}} & , c\tau > P \\ 0 & , c\tau < P \end{cases} \quad (D.41)$$

or

$$g_{2d} = \begin{cases} \frac{c}{2\pi} \frac{1}{\sqrt{c^2(t-t')^2 - P^2}} & , c(t-t') > P \\ 0 & , c(t-t') < P. \end{cases} \quad (D.42)$$

The expression $c(t-t') > P$ must be satisfied in order to honor the causality condition that is invoked when the second solution of (D.2) found in (D.4) is discarded (due to a physical argument).

Now equations (D.24) and (D.42) are specialized to the case of electric vector potential generated by elementary magnetic current sources. For the 3-d case, let $\mathbf{m}$ be a point source (dipole) in space and time given by

$$\mathbf{m} = \delta(\mathbf{r} - \mathbf{r}')\delta(t - t')\hat{\mathbf{z}}, \quad (D.43)$$

and let f_z satisfy the 3-d scalar wave equation

$$\nabla^2 f_z - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} f_z = -\epsilon_0 \delta(\mathbf{r} - \mathbf{r}')\delta(t - t'). \quad (D.44)$$

The solution f_z may be written by inspection of equations (D.2) and (D.24):

$$f_{z, \text{point}} = \epsilon_0 \frac{1}{4\pi R} \delta\left[(t - t') - \frac{R}{c}\right]. \quad (D.45)$$

For the 2-d case of a magnetic line source, (D.45) is integrated from $z' = -\infty$ to $z' = \infty$. The 2-d vector potential may be written by inspection of equation (D.42):

$$f_{z, \text{line}} = \begin{cases} \frac{\epsilon_0 c}{2\pi} \frac{1}{\sqrt{c^2(t-t')^2 - P^2}} & , c(t-t') > P \\ 0 & , c(t-t') < P. \end{cases} \quad (D.46)$$

In order to develop an expression for f_z due to an assemblage of line sources on a cylindrical surface, the 3-d Green's function for f_z (D.45) is used in the following integral:

$$f_z(x, y, t) = \int_{\Gamma} \int_{t'=-\infty}^{\infty} \int_{z'=-\infty}^{\infty} \epsilon_0 \frac{m_z(\mathbf{r}'; t')}{4\pi R} \delta\left[(t - t') - \frac{R}{c}\right] dz' dt' dl', \quad (D.47)$$

where f_z is found in the $z = 0$ plane, the conversion to 2-d is explicitly performed by the z' integration, and the integration with respect to time is also explicitly performed (recall that the magnetic dipole defined in (D.43) is a point source in both space and time). Observe that

$$\int_{z'=-\infty}^{\infty} \epsilon_0 \frac{m_z(\mathbf{r}'; t')}{4\pi R} \delta\left[(t - t') - \frac{R}{c}\right] dz' = \begin{cases} \frac{\epsilon_0 c}{2\pi} \frac{m_z(x', y'; t')}{\sqrt{c^2(t - t')^2 - P^2}} & , c(t - t') > P \\ 0 & , c(t - t') < P \end{cases} \quad (D.48)$$

which follows directly from (D.46). Thus (D.47) becomes

$$f_z(x, y, t) = \int_{\Gamma} \int_{t'=-\infty}^{\infty} \frac{\epsilon_0 c}{2\pi} \frac{m_z(x', y'; t')}{\sqrt{c^2(t - t')^2 - P^2}} dt' dl'. \quad (D.49)$$

No sources are active until $t' > 0$, and (D.48) is zero after $t' > t - \frac{P}{c}$, so the integration limits in (D.49) are modified such that the 2-d vector potential is given by:

$$f_z(x, y, t) = \frac{\epsilon_0 c}{2\pi} \int_{\Gamma} \int_0^{t - \frac{P}{c}} \frac{m_z(x', y'; t')}{\sqrt{c^2(t - t')^2 - P^2}} dt' dl'. \quad (D.50)$$

Note that the 2-d vector potential given by (D.50) requires integration in time as well as integration along the contour defined by Γ . Thus, time-domain integral equation codes based on this formulation are different in basic structure from codes based on the usual 3-d formulation in which no temporal integration is required. That is, for each subdomain of the problem, a running summation (continuous time integral) is required in order to keep track of the history of contributions made by all subdomains to the potential in the subdomain of observation.

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02 / 21 / 91