Ion Acoustic Double Layers^{*}

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It is found that, by means of numerical simulations, an ion acoustic instability ($v_d < v_{te}$) results in the formation of double layers in a sufficiently long system. Anomalous resistivity generated by the ion acoustic instability causes the buildup of a dc potential which in turn accelerates electrons further to enhance the original instability leading to the formation of double layers. A necessary condition required for generating a double layer is given.

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Anomalous resistivity and double layers have long been important subjects in both laboratory and space plasmas.¹⁻⁵ Because of the highly nonlinear aspect of double layers, existing theories deal only with the steady-state solutions.^{1,2} In order for double layers to be a reality, however, one must prove that they can be reached from a realistic initial condition. Computer simulation is the most powerful tool to study this dynamical process. The necessary condition for the formation of double layers obtained from the previous simulations,^{3,4} the steady-state theory,¹ and laboratory experiments^{5,6} is that the electron drift velocity exceeds the electron thermal velocity ($v_d > v_{te}$), or double layers are a result of two-stream (Buneman) instability.

There are, however, observations⁷ which support the existence of double layers (electrostatic shocks) along the auroral field lines where the electron drift speed is much less than the electron thermal speed. The low-intensity field-aligned current may be able to excite electrostatic ion cyclotron waves or ion acoustic waves, but cannot excite two-stream instability.⁸

In this Letter we show that, contrary to the earlier conclusion, double layers can be formed even if the electron drift speed is less than the electron thermal speed. This is proved by means of extensive one-dimensional particle simulations to be discussed. The underlying physical picture for this new process is the following: Suppose the system under consideration is sufficiently long. Then, no matter how small, the anomalous resistivity may be caused by an ion acoustic instability, and the

dc potential buildup associated with the resistivity across the system becomes large enough to further accelerate electrons and to detrapp trapped electrons. The acceleration in turn should enhance the original instability so that the anomalous resistivity and dc potential are further enhanced by a bootstrap action. Therefore, a new stage of the instability will take place which would not happen in a short system where little effective acceleration occurs.

In order to study this process in detail, we have performed one-dimensional particle simulations with the system length $L = 1024\lambda_D$, $512\lambda_D$, and $256\lambda_D$, and $128\lambda_D$ where λ_D is the initial electron Debye length which is equal to computational mesh size. Initially the density is uniform and the electron velocity distribution is assumed to be a drifting Maxwellian. $T_e/T_i = 20$, $M_i/M_e = 100$, and v_d (electron drift speed) $= 0.6v_{te}$. A smaller drift speed at $v_d = 0.3v_{te}$ and $L = 1024\lambda_D$ is also tried with the other parameters remaining the same. Periodic boundary conditions are used throughout the calculations. No external potential is applied across the system.

Figure 1 shows the results of $L = 1024\lambda_D$ and $v_d = 0.6v_{te}$ for the electron and ion phase space distributions, Fig. 1 (a) and (b), the electron and ion density profiles in real space, (c) and (d), and the potential profile, (e), at $t\omega_{pe} = 960$ when the localized structure (double layer) has fully developed. In Fig. 1 (e), a large potential jump comparable to the thermal energy, $e\Delta\phi/T_e \approx 1$, can be seen across the double layer. Along with the potential jump, large density discontinuities, $\delta n/n_0 \sim 1$, are generated for both electrons and ions. Furthermore, electron

and ion densities are considerably compressed in the upstream region of the electron flow and are rarified in the downstream region. In a narrow transition layer where a strong negative potential spike exists, similar large spikes in both the electron and ion densities are also found. The phase space plots shown in Fig. 1 (a) and (b) indicate that ions are considerably decelerated in the potential double layer while electrons are accelerated. From these observations it is clear that a double layer can be formed through the ion acoustic instability.

It is interesting to observe a conspicuous leading oscillatory structure in the downstream region of the potential profile.⁹ The wavelength of the structure roughly corresponds to that of the most unstable ion acoustic wave ($k\lambda_D \approx 0.2$).

Shown in Fig. 2 are the time evolution of the wave energy, (a), and the electron current, (b), for the same run. In the early stage of wave evolution ($t\omega_{pe} = 0 \sim 500$) the total field energy increases almost linearly and the current relaxes slowly at roughly a constant rate, which indicates the generation of a small but finite anomalous resistivity associated with the ion acoustic instability. The magnitude of this resistivity, η_a , is given by $\eta_a/\eta_0 \approx 5 \times 10^{-5}$ where $\eta_0 = 4\pi/\omega_{pe}$. A drastic reduction of the current, hence, a sudden enhancement of the anomalous resistivity, takes place at about $t\omega_{pe} \approx 600$. The enhanced anomalous resistivity becomes more than ten times larger than the 'acoustic' resistivity, $\eta_a/\eta_0 \approx 6 \times 10^{-4}$. At this stage, an explosive increase of the wave energy takes place resulting in the formation of the double layer by a bootstrap action. The drastic changes suddenly cease at $t\omega_{pe} \approx 1000$

and the anomalous resistivity is completely shut off and the resistivity drops to almost zero less than the classical value.

When the system length is shortened, the strength of double layers is reduced. Shown in Fig. 3 are the ion density and the potential profiles for $L = 512\lambda_D$ and $v_d = 0.6v_{te}$ at $t\omega_{pe} = 1432$ when a double layer is at its peak. By comparing the present result with that of the previous case, double layer structures are very similar to each other except for the potential jump $\Delta\phi$ across the double layer which is now reduced to about one-half of the previous value, i.e., $e\Delta\phi/T_e \approx 0.5$. The "enhanced" anomalous resistivity reached, however, is almost the same value as that for $L = 1024\lambda_D$, i.e., $\eta_a/\eta_0 \approx 6 \times 10^{-4}$. When the system is further reduced to $L = 256\lambda_D$, it is found that the formation of double layers is marginal and the jump in potential is small. This indicates that $L = 256\lambda_D$ must be the marginal system length L_c for the case of $v_d = 0.6v_{te}$.

When the system length is reduced to $L = 128\lambda_D$, no double layers are formed consistent with the previous simulations.^{3,4} Figure 4 shows the time evolutions of the wave energy, (a), and the current, (b). In the period of exponential growth ($t\omega_{pe} = 0 \sim 800$) nearly constant slowing down of the current is observed, which indicates the existence of the anomalous resistivity associated with the ion acoustic instability. The resistivity is nearly equal to that of the previous case, i.e., $\eta_a/\eta_0 \approx 5 \times 10^{-5}$.

No indication of "enhanced" anomalous resistivity or a 'burst' of ion acoustic waves is observed in this case and hence, no double layers. After $t\omega_{pe} \approx 1000$, the wave growth stopped and correspondingly, the anomalous resistivity disappeared and the classical resistivity η_c due to particle collision was observed. The classical resistivity turns out to be $\eta_c/\eta_0 \approx 10^{-5}$ for this case.¹⁰ From these results we may conclude that double layers are generated when the system is long enough and they become stronger as the system length becomes longer.

Let us consider a physical mechanism underlying the formation of double layers in the regime of the ion acoustic instability. The dc potential $\Delta\phi$ across the system may be given by $\Delta\phi = L\eta J$ where η is the resistivity generated by the ion acoustic waves and J is the current density. During the phase of acoustic instability $\eta_a/\eta_0 \approx 5 \times 10^{-5}$ and J is approximately given by its initial value, i.e., J_0 where $J_0 = n_0 e v_d$ (n_0 is the average electron density), and hence, the potential buildup across the system is $e\phi_c/T_e \approx 3 \times 10^{-5} L/\lambda_D$. The critical length L_c for the formation of double layers for $v_d = 0.6 v_{te}$ was found roughly $L_c \approx 256 \lambda_D$ through our simulations, so that the critical potential ϕ_c turns out roughly $e\phi_c/T_e \approx 10^{-2}$. Note that this value is smaller by two orders of magnitude than that given by a steady-state theory.³ Now associated with the dc potential is the acceleration of the electrons. Collective acceleration will

take place if the dc potential generated by the anomalous resistivity becomes larger than the wave potential associated with the ion acoustic instability. The energy gain for the electrons from the dc potential becomes larger than the potential energy of trapped particles and a collective acceleration should take place to further enhance the original ion acoustic instability. At this stage the instability is bootstrapped which leads to the formation of double layers. Therefore, we conclude that a necessary condition for ion acoustic double layers may be given by:

$$L\eta_a J > \phi_w \quad (1)$$

where ϕ_w is the wave potential associated with the acoustic instability.

The condition given by (1) assumed that the anomalous resistivity associated with the acoustic instability persists for a long time. We have seen in Fig. 4, however, that the anomalous resistivity reduces to much smaller classical resistivity after the acoustic instability saturates and the potential buildup will be much smaller. For such a case, a sufficient acceleration of the electrons must take place before the ion acoustic instability saturates. This latter condition gives $T_c \equiv L_c/v_d < T_s$ where T_s is the time for the ion acoustic instability to saturate. In fact, this requirement is satisfied for the simulations using $L = 1024\lambda_D$ and $512\lambda_D$.

To check this point we have performed another simulation with $v_d = 0.3v_{te}$ and $1024\lambda_D$. The result has shown the presence of acoustic anomalous resistivity ($\eta_a/\eta_0 = 3.8 \times 10^{-5}$) until $t\omega_{pe} \approx 750$

but after the saturation of the instability, it reduced to the classical value ($\eta_c/\eta_o = 1.3 \times 10^{-5}$) and no double layers were formed. Using $v_d = 0.3v_{te}$ and $L = 1024\lambda_D$ we find $T_c \approx 3300\omega_{pe}^{-1}$. At this time, the acoustic instability has saturated and no more anomalous resistivity is present and therefore $T_c < T_s$ is not satisfied.

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Fig. 1. Results of simulations using $L = 1024\lambda_D$ and $v_d/v_{te} = 0.6$ for the electron phase space (a), ion phase space (b), electron density profile (c), ion density profile (d) and the potential profile (e), at $t\omega_{pe} = 960$ (averaged over one plasma period). $n_0\lambda_D = 100$.

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Fig. 2. Time evolution of the wave energy (a) and the electric current (b) for the same run as in Fig. 1 (plotted are the values averaged during five plasma periods). $L = 1024\lambda_D$ and $v_A/v_{te} = 0.6$. Note the wave energy associated with the acoustic instability saturates at $t = 500\omega_{pe}^{-1}$ beyond which it grows explosively with a much faster growth rate.

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 Fig. 3. Results of simulations using $L = 512\lambda_D$ and $v_d/v_{te} = 0.6$ for the ion density profile (a) and the potential profile (b), at $\omega_{pe} = 1432. \tau_{e} n_0 \lambda_D = 100$.

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 Fig. 4. Time evolution of the wave energy (a) and the electric current (b) for $L = 128\lambda_D$, $v_d/v_{te} = 0.6$, and $n_0\lambda_D = 100$.