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COMPUTER SIMULATION OF RADIAL TRANSPORT IN
TANDEM MIRROR MACHINES

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ABSTRACT

A code⁽¹⁾ used for simulation of classical radial transport in the 2XIIB experiment has been modified to simulate radial transport in TMX. Results have been obtained using classical transport coefficients and also using very simple trial neoclassical resonant transport⁽²⁾ coefficients.⁽³⁾ Comparison of the results obtained with solely classical transport and with both classical and neo-classical transport indicate that neoclassical transport depresses the ion density by approximately 5%. The central cell ion temperature is increased by approximately 5% by the neo-classical transport, as is the electron temperature.

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Introduction

The model used to describe the tandem mirror machine is as follows: It is assumed that in the solenoid there are two ion species. Species 1 is deuterium or a mass averaged deuterium-tritium species. Species 2 is thermalized alphas. It is assumed that the hot alphas interact with all solenoid species solely by energy transfer. In the solenoid there are 2 neutral sources of species 1; one atomic and one molecular. The plug is assumed frozen, i.e., the plug ion species is given initial radial density and temperature profiles which remain unchanged in time. The plug ions interact only with the electrons through energy transfer. The electrons are bounce averaged and interact with both the plug and solenoid ion species.

The overall electron confining potential, ϕ_e , is determined by equating the overall ion loss rates (radial and axial) to the electron loss rate (radial and axial) line by line. The ion potential, ϕ_i , is determined by the assumption of Maxwell-Boltzmann electrons free streaming along field lines.

Finally, a check is made line by line that there is a sufficient end-loss current to satisfy the DCLC stabilization condition. If there is not sufficient end-loss current, the plug ion loss rate, and hence in steady state the source, is correspondingly increased.

For the preliminary runs made, it has been assumed that there is but one ion species in the solenoid, deuterium. The frozen plug ions and the bounce-averaged electrons enter the problem as previously indicated. Also, provisions in the code for finite Larmor radius effect corrections to source terms in the density equations have been disabled.

Equations Describing the System

The ion potential, or potential difference between the center of the solenoid and the plugs, is determined by Maxwell-Boltzmann electrons:

$$\phi_i(r) = T_e(r) \ln \left(\frac{z_p n_p(r)}{z_j n_j(r)} \right), \quad (1)$$

where z_p is the charge of the plug ions having radial density profile $n_p(r)$, j runs over solenoid ion species having charge z_j and radial density profile $n_j(r)$. The label r always refers to the radius of a given flux tube at the solenoid midplane. The overall electron confining potential is determined by:

$$L_s \sum_j z_j \frac{dn_j(r)}{dr} = \frac{dN(r)}{dr}, \quad (2)$$

where L_s is the solenoid length and $N(r)$ is the density of electrons per unit flux tube. The plug ions do not appear explicitly in (2) as they are in steady state. However, the plug portion of the electron source in (2) is numerically equal to the plug ion loss.

The DCLC stabilization criterion is as follows: line by line it is required that the following inequality hold:

$$\left| \frac{2L_p}{R} z_p \left[\frac{\partial n_p}{\partial t} \right]_{EL} + L_s \left[\frac{\partial n_1}{\partial t} \right]_{EL} + 2 L_s \left[\frac{\partial n_2}{\partial t} \right]_{EL} \right| \geq L_s J_D, \quad (3)$$

$$J_D = 6.75 \cdot 10^3 \lambda \frac{v_p}{v_c} \frac{T_e^{3/2}}{E_p L_p} n_p \quad (4)$$

with T_e, E_p in $^{\circ}\text{K}$, n_p in cm^{-3} . v_p is the volume of the plug, v_c is the volume of the solenoid, λ is a constant, and R is the mirror ratio between the solenoid and the middle of the plug. The subscript EL in (3) refers to end-loss.

The plug end-loss is given by:

$$\frac{\partial n_p(r)}{\partial t} = - \frac{n_p(r)}{\tau_p(r)} \quad (5)$$

$$\tau_p(r) = \tau_{\text{DRAG}}^* \eta \lambda_n \left(\frac{E_{inj} - E_e}{E_{out} - E_e} \right) \quad (6)$$

$$(6a) E_e = 3/2 kT_e$$

$$\tau_{\text{DRAG}}^* = A (\bar{v}_e e/p)^{-1} \quad (7)$$

$K \equiv \text{Boitzmann's Constant}$

$$\frac{e/p}{\bar{v}_e} \approx \frac{1.8 \cdot 10^{-19}}{m_p} \frac{(m_e)^{1/2} (z_p)^2 n_p(r) \lambda_{ep}(r)}{T_e^{3/2}} \quad (8)$$

T_e, T_p in eV

λ_{ep} = Coulomb logarithm

n_p in cm^{-3}

$E_{\text{OUT}} \equiv$ energy at which ions are lost from the plug⁽⁴⁾

where A, η and λ are constants, and p refers to plug ions. The expression (6) is used to calculate the initial τ_p before the inequality (3) is checked. If (3) does not hold, τ_p is changed so that (3) becomes an equality.

The ion equations for the density and temperature profiles are of the general form:

$$n_p(r) = n_0 \exp(-ar^2) + n_1; \quad n_0, n_1, a \text{ constants} \quad (9a)$$

$$n_0 = 5 \cdot 10^{13}, a = 1.04 \cdot 10^{-3}, n_1 = 0 \text{ for TMX}$$

$$\frac{\partial n_i}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_i) + S_i \quad (9)$$

$$\frac{\partial T_i}{\partial t} = - \frac{\Gamma_i}{n_i} \frac{\partial T_i}{\partial r} - \frac{2}{3} \frac{T_i}{r} \frac{\partial}{\partial r} \left(r \frac{\Gamma_i}{n_i} \right) - \frac{2}{3} \frac{1}{n_i r} \frac{\partial}{\partial r} (r q_i) + \sum_L \quad (10)$$

$$\frac{(T_L - T_i)}{\tau_{iL}} + \frac{T_i S_i}{n_i} ,$$

where $i = 1, 2$ for the two ion species, and z varies over all the species, ions and electrons. The specific expressions for the fluxes and sources indicated in Equations (9) and (10) are given in Appendix I.

The equations for the electron density per unit flux tube and the electron temperature are of the general form:

$$\frac{\partial N(r)}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_e) + S_e \quad (11)$$

$$\frac{\partial}{\partial t} (\tau_e(a) + 2/3 \langle \phi \rangle) = G \quad (12)$$

$$\begin{aligned} G = & - \frac{\Gamma_e}{N} \frac{\partial \tau_e}{\partial r} - \frac{2}{3} \frac{\tau_e}{r} \frac{\partial}{\partial r} \left(\frac{r \Gamma_e}{N} \right) - \frac{2}{3} \frac{1}{rN} \frac{\partial}{\partial r} (r q_e) + \frac{2}{3N} Q_e \\ & + \left(\frac{\tau_1 - \tau_e}{\tau_{e1}} \right) \frac{L_s n_1}{N} + \left(\frac{\tau_2 - \tau_e}{\tau_{e2}} \right) \frac{2L_s n_2}{N} + \left(\frac{\tau_p - \tau_e}{\tau_{ep}} \right) \frac{2L_p n_p z_p}{RN} \\ & + \frac{\tau_e S_5}{N} \end{aligned} \quad (13)$$

$$\langle \phi \rangle = \frac{\phi_i}{1 + \left(\frac{2L_p n_p}{L_s n_s} \frac{1}{R} \right)} \quad \phi_e > 0; \quad \langle \phi \rangle = 0 \quad \phi_e \leq 0 \quad (14)$$

$$\frac{\partial \langle \phi \rangle}{\partial t} = \frac{\partial \tau_e}{\partial t} \frac{\langle \phi \rangle}{\tau_e} + \frac{1}{n_1} \frac{\partial n_1}{\partial t} \frac{\langle \phi \rangle}{\phi_i} (\phi_i - \langle \phi \rangle - \tau_e), \quad (15)$$

where the explicit expressions for the fluxes and sources indicated in (11) and (13) are given in Appendix I. The electron density equation is not solved simultaneously with the other equations given. It is used to determine ϕ_e .

The above equations are finite differenced in the usual straightforward manner with the fluxes evaluated at the half mesh points and all

other quantities evaluated at the mesh points. Provision is made for making the equations fully implicit, fully explicit, or a mixture thereof. When finite differenced, the differential equations yield a tri-diagonal system of algebraic equations which may be written as follows:

$$-\bar{\alpha}_j^{N+1} \bar{u}_{j+1}^{N+1} + \bar{\beta}_j^{N+1} \bar{u}_j^{N+1} - \bar{\gamma}_j^{N+1} \bar{u}_{j-1}^{N+1} = \bar{\delta}_j^N \quad (16)$$

$$\bar{u}_j^N = \begin{pmatrix} n_{1j}^N \\ n_{2j}^N \\ T_{1j}^N \\ T_{2j}^N \\ T_{ej}^N \end{pmatrix}, \quad (17)$$

where j refers to the radial mesh point j and N refers to time step N . The time step and the mesh width are constants.

The tri-diagonal system is inverted using the usual tri-diagonal algorithm:

$$\bar{u}_j^{N+1} = E_j \bar{u}_{j+1}^{N+1} + F_j \quad (18)$$

$$E_j = (\bar{\beta}_j^{N+1} - \bar{\gamma}_j^{N+1} E_{j-1})^{-1} \bar{\alpha}_j^{N+1} \quad (19)$$

$$F_j = (\bar{\beta}_j^{N+1} - \bar{\gamma}_j^{N+1} E_{j-1})^{-1} (\bar{\delta}_j^N + \bar{\gamma}_j^{N+1} F_{j-1}). \quad (20)$$

The solutions obtained are not found to depend strongly upon the degree to which the finite differenced equations are made implicit or explicit, but are found to depend upon the length of the time step, Δt .

Results

Runs have been made with all diffusion coefficients set equal to zero, classical ion and classical ambipolar central cell electron diffusion, classical and neoclassical ion diffusion with classical ambipolar central cell electron diffusion, and with classical and neoclassical ion diffusion, classical ambipolar central cell electron diffusion and neoclassical plug electron diffusion. The above results are presented in Figures 1-4.

The initial profiles are Gaussian: $f(r) = f(0) \exp(-r^2/R_{\max}^2)$.

The boundary condition at the center of the plasma is zero gradient. The boundary condition at the edge is fixed gradient, where:

$$n_i(R+\Delta r) = 0.915 n_i(R-\Delta r)$$

$$T_{i,e}(R+\Delta r) = 0.915 T_{i,e}(R-\Delta r) .$$

Here R is the solenoid or central cell plasma radius, Δr is the mesh width and the numbers chosen were the smallest such that the potential solver would converge.

The central cell source strength and all other parameters, such as B_s , B_p , R_p , R , L_s , L_p , etc., were obtained from Table 1 of the TMX proposal.⁽⁶⁾

Referring to Figures 1-4 one sees that neoclassical transport depresses the central cell ion density by approximately 5% across the mesh when compared with the results obtained in which solely classical transport is present. However, the central cell ion temperature and the electron temperature are increased by approximately 5% across the mesh. Also, one notes that higher equilibrium central cell ion densities are obtained when classical and neoclassical transport are added to the problem, as compared to the results obtained with no cross-field transport. The equilibrium ion and electron temperatures are, however, decreased by about 20%. Evidently the increased cooling of the ions by the additional cross-field transport leads to better confinement, given about the same ambipolar potential. In fact, in a run with particle diffusion but no radial energy transport, the central density is lower than the no-diffusion value (see Figures 5-8). Finally, the electron confining potential, ϕ_e , is seen to be rather insensitive to the change in transport.

Notes on Code Eccentricities

Problems are encountered with the convergence of the potential solver if the density and/or temperature profiles vary too quickly near the mesh edge. Profiles which vary smoothly to low values at the mesh edge do not cause problems. However, profiles which are flat across most of the mesh and which then drop quickly to some small value (say 0.1 of their value along the rest of the mesh) over one or two mesh spacings at the mesh edge do not allow for convergence of the potential solver.

Also, problems are encountered if the plug ion density profile, normalized to $r/\sqrt{R}$ where r is the radial coordinate in the solenoid and R is the mirror ratio from the center of the solenoid to the center of the plug, drops off too quickly. The potential solver converges to negative values in the region where the plug ion density is too small, which leads to a myriad of internal code problems.

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Figure 1. Central Cell Ion Temperature Profile (TMX)

Figure 2. Density Profiles (TMX)

100 OK)

Figure 3. Electron Temperature Profiles (TMX)

(10⁶°K)

Figure 4. ϕ_F (Electron Confining Potential) (TMX)

Figure 5. Density Profiles (No Energy Transport) (Central Cell)

Figure 6. Ion Temperature Profiles (No Energy Transport) (Central Cell)

Figure 7. Electron Temperature Profiles (No Energy Transport)

16° OK

Figure 8. ϕ_E (No Energy Transport)

Appendix I

Ion density equations fluxes and sources:

$$\Gamma_1 = D_{11} \frac{\partial n_1}{\partial r} + D_{12} \frac{\partial n_2}{\partial r} + D_{13} \frac{\partial T_1}{\partial r} + D_{15} \frac{\partial T_e}{\partial r}$$

$$D_{11} = D_{11}^C + D_{11}^{NC} \quad C \leftrightarrow \text{Classical}$$

$$D_{12} = D_{12}^C + D_{12}^{NC} \quad NC \leftrightarrow \text{Neoclassical}$$

$$D_{13} = D_{13}^C + D_{13}^{NC}$$

$$D_{15} = D_{15}^C + D_{15}^{NC}$$

$$\begin{aligned} S_1 = & (n_1 + 2n_2) \langle n_s \sigma_{13}^e v_{e3} \rangle + n_4 \langle \sigma_{14}^e v_{e4} \rangle \\ & + n_2 \langle n_3 \sigma_{cx}^{23} v_{23} \rangle + n_4 \langle \sigma_{cx}^{24} v_{24} \rangle \\ & + n_2 \langle n_3 \sigma_{13}^2 v_{23} \rangle + n_4 \langle \sigma_{14}^2 v_{24} \rangle \\ & + n_1 \langle n_3 \sigma_{13}^1 v_{13} \rangle + n_4 \langle \sigma_{14}^1 v_{14} \rangle \\ & - n_1 (n_1 + 2n_2) \langle \sigma_{1R} v_{e1} \rangle - n_1^2 \langle \sigma_f v_1 \rangle - \left(\frac{dn_1}{dt} \right)_{\text{PAST}} \end{aligned} \quad (1)$$

$$\Gamma_2 = D_{21} \frac{\partial n_1}{\partial r} + D_{22} \frac{\partial n_2}{\partial r} + D_{24} \frac{\partial T_2}{\partial r} + D_{25} \frac{\partial T_e}{\partial r}$$

$$D_{21} = D_{21}^C + D_{21}^{NC}$$

$$D_{22} = D_{22}^C + D_{22}^{NC}$$

$$D_{24} = D_{24}^C + D_{24}^{NC}$$

$$D_{25} = D_{25}^C + D_{25}^{NC}$$

$$\begin{aligned} S_2 = & - n_2 \langle n_3 \sigma_{cx}^{23} v_{23} \rangle + n_4 \langle \sigma_{cx}^{24} v_{24} \rangle \\ & + n_1^2 \langle \sigma_f v_1 \rangle f_T - n_2 (n_1 + 2n_2) \langle \sigma_{2R} v_{2e} \rangle \\ & - \left(\frac{dn_2}{dt} \right)_{\text{PAST}} \end{aligned} \quad (2)$$

$f_T \equiv$ fraction of hot α 's from fusion which are thermalized and trapped.

The quantities in angular brackets are the relevant reaction rates, i.e.,

$\langle \sigma_{I3}^e v_{e3} \rangle \equiv$ cross section for ionization of the atomic neutral source of species 1 by the electrons times the electron-neutral relative velocity, averaged over a Maxwellian. The other terms are then read off similarly, where cx refers to charge-exchange, f refers to fusion, and R refers to recombination, when used as a subscript for the cross-sections. The Pastukhov end-loss terms are given subsequently.

Ion temperature equations heat fluxes and sources:

$$q_1 = K_{33} \frac{\partial T_1}{\partial r}$$

$$K_{33} = K_{33}^C + K_{33}^{NC}$$

$$\begin{aligned} T_1 S_3 = & (n_1 + 2n_2) (n_3 \langle \sigma_{I3}^e v_{e3} \rangle (\hat{T}_3 - T_1) + n_4 \langle \sigma_{I4}^e v_{e4} \rangle (\hat{T}_4 - T_1)) \\ & + n_1 (n_3 \langle \sigma_{cx}^{I3} v_{I3} \rangle (\hat{T}_3 - T_1) + n_4 \langle \sigma_{cx}^{I4} v_{I4} \rangle (\hat{T}_4 - T_1)) \\ & + n_3 \langle \sigma_{I3}^I v_{I3} \rangle (T_3 - T_1 - 2/3 \epsilon_{I3}) + n_4 \langle \sigma_{I4}^I v_{I4} \rangle \times (\hat{T}_4 - T_1 \\ & - 2/3 \epsilon_{I4})) + n_2 (n_3 \langle \sigma_{I3}^2 v_{23} \rangle (\hat{T}_3 - T_1) + n_4 \langle \sigma_{I4}^2 v_{24} \rangle (\hat{T}_4 - T_1) \\ & + n_3 \langle \sigma_{cx}^{23} v_{23} \rangle (\hat{T}_3 - T_1) + n_4 \langle \sigma_{cx}^{24} v_{24} \rangle (\hat{T}_4 - T_1)) - 2/3 n_1 n_4 \\ & \langle \sigma_{D4}^I v_{I4} \rangle \epsilon_{D4} - n_1^2 \langle \sigma_f v_I \rangle (\bar{\epsilon}_f - f_1 E_\alpha) - \left(\frac{dT_1}{dt} \right)_{PAST}. \quad (3) \end{aligned}$$

$$q_2 = K_{44} \frac{\partial T_2}{\partial r}$$

$$K_{44} = K_{44}^C + K_{44}^{NC}$$

$$\begin{aligned} T_2 S_4 = & - 2/3 \epsilon_{I3} n_2 n_3 \langle \sigma_{I3}^2 v_{23} \rangle - 2/3 \epsilon_{I4} n_2 n_4 \langle \sigma_{I4}^2 v_{24} \rangle \\ & - 2/3 \epsilon_{D4} n_2 n_4 \langle \sigma_{D4}^2 v_{24} \rangle - \left(\frac{dT_2}{dt} \right)_{PAST} + n_1^2 \langle \sigma_f v_I \rangle f_2 E_\alpha \quad (4) \end{aligned}$$

where: $\bar{\epsilon}_f$ $\equiv$ mean energy of fusing D-T

$\bar{\epsilon}_\alpha$ $\equiv$ mean energy of hot α

f_1 $\equiv$ fraction of energy transferred to D-T by hot α and it drags down

$\epsilon_{I3,4}$ $\equiv$ energy required to ionize 3,4

ϵ_{D4} $\equiv$ energy required to dissociate 4

f_t $\equiv$ fraction of hot alphas which are thermalized and trapped

f_2 $\equiv$ fraction of energy which the hot alphas supply to thermalized alphas as they drag down.

$\hat{T}_3, \hat{T}_4$ are the ion injection energies from the neutral sources.

$$\left(\frac{dn_i}{dt}\right)_{\text{PAST}} \equiv \frac{4}{\sqrt{\pi}} \frac{Z_i n_i}{T_i} \frac{T_i}{\phi_i Z_i} \exp\left(-\frac{Z_i \phi_i}{T_i}\right) \frac{I(T_i/Z_i \phi_i)}{G(RZ_i)} \quad (5)$$

$i=1,2$

$$\tau_i^{-1} = 4\pi e^4 \sum_j \frac{Z_j^2 Z_i^2 n_j \lambda_{ij}}{m_i m_j v_i^3}$$

$$v_i = \sqrt{\frac{2kT_i}{m_i}}$$

for m in AMU and T in keV

$$\lambda_{ij} \equiv \lambda_n \{1.6162 \cdot 10^{15} \frac{m_i m_j}{m_i + m_j} \sqrt{\frac{3/2 T}{n_e}} \times \max\left(\sqrt{\frac{3/2 T_i}{m_i}}, \sqrt{\frac{3/2 T_j}{m_j}}\right)\}$$

$$Z_i \equiv \frac{\sum_j Z_j^2 n_j \lambda_{ij}}{2 \sum_j (m_i/m_j) Z_j n_j \lambda_{ij}}$$

$$I(x) = (1+x/2)/(1+x^2/4)$$

The above expression for $I(x)$ is an approximate expression used in reference 4.

$$G(x) = (2x+1)/2x \ln(4x+2) \quad (5)$$

Note: for ϕ_i , i is not a running subscript. z_i refers to charge of ion species i .

$$\left(\frac{dT_i}{dt}\right)_{\text{PAST}} = \frac{2/3 z_i \phi_i}{T_i} \left(\frac{dn_i}{dt}\right)_{\text{PAST}} \frac{1}{n_i} \quad (6)$$

$$I\left(\frac{z_i \phi_i}{T_i}\right)$$

For the classical diffusion coefficients:

let j label the coefficient of $\frac{\partial n_j}{\partial r}$

let T_j label the coefficient of $\frac{\partial T_j}{\partial r}$

$$\text{Then: } r_j^C = \sum_k D_{jk}^C \frac{\partial n_k}{\partial r} + \sum_k D_{jT_k}^C \frac{\partial T_k}{\partial r} \quad k = 1, 2, e$$

$$D_{jj}^C = -\frac{\beta}{(n_1 + 2n_2)} (n_j T_e + (n_1 + 2n_2) T_j) \quad j = 1, 2$$

$$D_{ji}^C = \frac{-\beta n_j}{(n_1 + 2n_2)} T_e \quad \beta \equiv \frac{1}{m_e \Omega_e^2 \tau_{ee}}$$

$$D_{jT_j}^C = -\beta n_j \quad \Omega_e \equiv \frac{eB}{m_e c}$$

$$D_{jT_k}^C = 1/2 \beta n_j = D_{jT_e}^C \quad (7)$$

$$K_{33}^C = -2\beta_1 T_1 n_1$$

$$K_{44}^C = -2\beta_2 T_2 n_2$$

$$\beta_j \equiv \frac{1}{m_j \Omega_j^2 \tau_{ij}} \quad \Omega_j = \frac{z_j e B}{m_j c}$$

$$\tau_{jk} = \frac{3}{8\sqrt{2\pi}} \frac{m_j m_k}{\lambda_{jk} e^4 n_k} \left(\frac{kT_j}{m_j} + \frac{kT_k}{m_k} \right)^{3/2} \frac{1}{z_j^2 z_k^2} \quad (8)$$

For the neoclassical transport one adopts order of magnitude estimates for plateau resonance transport such as those in reference 3:

$$D_{11}^{NC} \approx \sqrt{2/\pi} (T_1/m_1)^{3/2} \left(\frac{m_1 c}{e B_s}\right)^2 \frac{R^2}{L_{TR} L_s}$$

$$D_{22}^{NC} \approx \sqrt{2/\pi} (T_2/m_2)^{3/2} \left(\frac{m_2 c}{2e B_s}\right)^2 \frac{R^2}{L_{TR} L_s}$$

T in ergs

$R \equiv$ radius of solenoidal plasma in cm

$L_{TR} \equiv$ length of quadrupole transition region in cm

$L_s \equiv$ length of solenoid in cm

$B_s \equiv$ solenoid magnetic field strength in Gauss

or any other set of consistent units.

$$\text{All other } D_{ij}^{NC} = 0 \quad (9)$$

$$K_{33}^{NC} = D_{11}^{NC} n_1, \quad K_{31}^{NC} = -\frac{5}{2} T_1 D_{11}^{NC}$$

$$K_{44}^{NC} = D_{22}^{NC} n_2, \quad K_{42}^{NC} = -\frac{5}{2} T_2 D_{22}^{NC} \quad (10)$$

Electron density eqn. sources and fluxes:

$$\Gamma_e = D_{51} \frac{\partial n_1}{\partial r} + D_{52} \frac{\partial n_2}{\partial r} + D_{53} \frac{\partial T_1}{\partial r} + D_{54} \frac{\partial T_2}{\partial r} + D_{55} \frac{\partial T_e}{\partial r}$$

$$S_e = \frac{2Z_p n_p L_p}{\tau_p R} + (n_1 + 2n_2) L_s (n_3 \langle \sigma_{13}^e v_{e3} \rangle + n_4 \langle \sigma_{14}^e v_{e4} \rangle$$

$$- \langle \sigma_{2R} v_{1e} \rangle n_1 - 2 \langle \sigma_{2R} v_{2e} \rangle n_2)$$

$$+ n_1 L_s (n_3 \langle \sigma_{13}^1 v_{13} \rangle + n_4 \langle \sigma_{14}^1 v_{14} \rangle)$$

$$+ n_2 L_s (n_3 \langle \sigma_{13}^2 v_{23} \rangle + n_4 \langle \sigma_{14}^2 v_{24} \rangle)$$

$$- \frac{2Z_p n_p L_p}{R} \langle \sigma_{PR} v_{ep} \rangle - (dN/dt)_{PAST} \quad (11)$$

$$(dN/dt)_{\text{PAST}} = \frac{2L_p n_p z_p}{R \langle \tau \rangle}$$

$$\langle \tau \rangle = \frac{\tau_e}{4} \sqrt{\pi} \frac{2n(4\langle Z_e \rangle R_p + 2)}{1 \left(\frac{\tau_e}{\phi_m} \right)} \frac{(2R_p \langle Z_e \rangle + 1)}{2R_p \langle Z_e \rangle} \phi_m / \tau_e$$

$$\times \exp(\phi_m / \tau_e) \frac{1}{\langle Z_e \rangle}$$

$$\tau_e^{-1} = \frac{4\pi e^4 z_p n_p \lambda_{\text{eep}}}{m_e^2 v_e^3 \langle \epsilon_e \rangle} \quad v_e = \sqrt{\frac{2kT_e}{m_e}}$$

$$\langle \epsilon_e \rangle \equiv 1 + \frac{L_s}{2L_p} \frac{\lambda_{\text{ees}}}{\lambda_{\text{eep}}} \frac{(n_1 + 2n_2)}{z_p n_p} (\phi_m / \phi_e)$$

$$\phi_m = \phi_i + \phi_e$$

s $\leftrightarrow$ solenoid

p $\leftrightarrow$ plug

$$\langle Z_e \rangle \equiv \frac{Z_{\text{ep}} + Z_{\text{es}} R(n_1 + 2n_2) / (z_p n_p) (\phi_m / \phi_e) (L_s / 2L_p) (\lambda_{\text{ees}} / \lambda_{\text{eep}})}{1 + \frac{(n_1 + 2n_2)}{z_p n_p} \left(\frac{\phi_m}{\phi_e} \right) \frac{L_s}{2L_p} \frac{\lambda_{\text{ees}}}{\lambda_{\text{eep}}}}$$

$$Z_{\text{es},p} = 1/2 \left(1 + \frac{1}{n_{\text{es},p}} \sum_{\text{solenoid ions}} \left(\frac{n_i z_i^2 \lambda_{\text{ei}}}{\lambda_{\text{ees},p}} \right) \right) \quad (12)$$

$$\tau_e S_5 = (n_1 + 2n_2) L_s \langle n_3 \sigma_{I3}^e v_{e3} \rangle (\hat{T}_{e3} - T_e - 2/3 \epsilon_{I3} + 2/3 (\phi_i - \langle \phi \rangle))$$

$$+ n_4 \langle \sigma_{I4}^e v_{e4} \rangle (\hat{T}_{e4} - T_e - 2/3 \epsilon_{I4} + 2/3 (\phi_i - \langle \phi \rangle))$$

$$- 2/3 n_4 \epsilon_{D4} \langle \sigma_{D4}^e v_{e4} \rangle + n_s^* f(Z, T_e)_s + L_s f_{BS} (n_1 + 2n_2)_p^2$$

$$T_e^{1/2} + \frac{2L_p}{R} \beta_{8p} (z_p n_p)^2 T_e^{1/2} + \frac{2n_p^* z_p n_p}{R} L_p f(Z, T_e)_p$$

$$\begin{aligned}
& + n_1 L_s (n_3 \langle \sigma_{13}^1 v_{13} \rangle (\hat{T}_{e3} - T_e + 2/3 (\phi_i - \langle \phi \rangle))) \\
& \quad + n_4 \langle \sigma_{14}^1 v_{14} \rangle (\hat{T}_{e4} - T_e + 2/3 (\phi_i - \langle \phi \rangle))) \\
& + n_2 L_s (n_3 \langle \sigma_{13}^2 v_{23} \rangle (\hat{T}_{e3} - T_e + 2/3 (\phi_i - \langle \phi \rangle))) \\
& \quad + n_4 \langle \sigma_{14}^2 v_{24} \rangle (\hat{T}_{ev} - T_e + 2/3 (\phi_i - \langle \phi \rangle))) \\
& + n_1^2 \langle \sigma_f v_i \rangle f_e E_\alpha L_s + \frac{2L_p^2 n_p}{R_p} (\hat{T}_{ep} - T_e - 2/3 \langle \phi \rangle) \\
& - \left(\frac{dT_e}{dt} \right)_{\text{PAST}} \quad n_{p,s} f(Z, T_e)_{p,s} \equiv \text{Lorentz ionization source in plug,} \\
& \quad \text{solenoid}
\end{aligned}$$

$$\beta_{\text{BP},s} \equiv \text{Bremstrahlung coefficient} \quad (13)$$

where $\hat{T}_{e3} \equiv$ injection energy of electrons from atomic neutral source

$\hat{T}_{e4} \equiv$ injection energy of electrons from molecular neutral source

$f_e \equiv$ fraction of energy transferred to the electrons as the hot
alphas drag down

$R_p \equiv$ plug mirror ratio

$$\left(\frac{dT_e}{dt} \right)_{\text{PAST}} = 4/3 \frac{L_p^2 n_p}{R_p} \frac{1}{\langle \tau \rangle} \left(\frac{\phi_m}{T_e} - \langle \phi \rangle \right) \quad (14)$$

$$D_{51} = D_{51}^C + D_{51}^{NC}$$

$$D_{52} = D_{52}^C + D_{52}^{NC}$$

$$D_{53} = D_{53}^C$$

$$D_{54} = D_{54}^C$$

$$D_{55} = D_{55}^C \quad (15)$$

$$D_{51}^{NC} = \frac{L_p z_p}{R} \left(\frac{\partial n_p}{\partial r} / \frac{\partial n_1}{\partial r} \right) D_{ep}^* \{2\} \quad (16)$$

$$D_{52}^{NC} = \frac{L_p z_p}{R} \left(\frac{\partial n_p}{\partial r} / \frac{\partial n_2}{\partial r} \right) D_{ep}^* \quad (17)$$

$$D_{ep}^* = -1/4 \left(\frac{R^p}{TR_p} \right)^2 T_e \left(\left(T_e + \frac{\phi_i}{(R_p-1)} \right)^{-1} - \left(T_e + \frac{\phi_m}{(R_p-1)} \right)^{-1} \right)^2 \\ \times (R^p)^2 v_{eep} \quad (18)$$

where the factor of 2 in curly brackets in (16) is included for $n_2 = 0$.

The electron neoclassical transport is due to the non-omnigenity of the drift surfaces of the plug electrons resulting from asymmetry of the axial ambipolar potential profile. Order of magnitude estimates for the transport are adopted from Reference 5.

$R^p \equiv$ plug plasma radius

$$v_{eep} = 4/3 \sqrt{\frac{\pi}{m_e}} \frac{\lambda_{ee} e^4 z_p n_p}{(kT_e)^{3/2}}$$

$$D_{51}^C = L_s (D_{11}^C + D_{21}^C)$$

$$D_{52}^C = L_s (D_{22}^C + D_{12}^C)$$

$$D_{53}^C = L_s D_{13}^C$$

ambipolar classical radial transport

$$D_{54}^C = L_s D_{24}^C$$

$$D_{55}^C = L_s (D_{15}^C + D_{25}^C) \quad (19)$$

$$\eta_e = K_{51} \frac{\partial n_1}{\partial r} + K_{52} \frac{\partial n_2}{\partial r} + K_{53} \frac{\partial T_1}{\partial r} + K_{54} \frac{\partial T_2}{\partial r} + K_{55} \frac{\partial T_e}{\partial r}$$

$$K_{5j} = K_{5j}^C + K_{5j}^{NC} \quad j = 1, 5$$

$$K_{5j}^C = 3/2 \beta T_e (T_e + T_j) L_s \quad j = 1, 2$$

$$K_{5j}^C = 3/2 \beta T_e n_j L_s \quad j = 3, 4$$

$$K_{55} = -3.16 \beta T_e n_e L_s$$

$$K_{5j}^{NC} = 0 \quad j = 2, 3, 4 \quad K_{51}^{NC} = -\frac{5}{2} T_e D_{ep}^* 2 L_p 2 p \left(\frac{\partial n_p / \partial r}{\partial n_i / \partial r} \right)$$

$$K_{55}^{NC} = 2 D_{ep}^* \frac{z_p n_p L_p}{R}$$

$$Q_E^C = N \cdot Q^C \cdot N^T$$

$$N \equiv (\partial n_1 / \partial r, \partial n_2 / \partial r, \partial T_1 / \partial r, \partial T_2 / \partial r, \partial T_e / \partial r)$$

$$Q_{jj}^C = - (T_j^2 / n_j + \frac{T_e (T_e + 2T_j)}{n_e}) \beta \quad j \neq e$$

$$Q_{jk}^C = - (T_e / n_e (T_j + T_k + T_e)) \beta \quad j, k \neq e$$

$$Q_{jj}^C T_j = - (T_j + n_j / n_e T_e) \beta \quad j \neq e$$

$$Q_{jk}^C T_k = - \left(\frac{T_e n_k}{n_e} \right) \beta \quad j, k \neq e$$

$$Q_{jj}^C T_e = 1/2 \beta (T_j + T_e) \quad j \neq e$$

$$Q_{T_j j}^C = - Q_{jj}^C T_j \quad j \neq e$$

$$Q_{T_j k}^C = \frac{\beta T_e n_j}{n_e} \quad j, k \neq e$$

$$Q_{T_j T_j}^C = n_j \beta \quad j \neq e$$

$$Q_{T_j T_e}^C = - 1/2 \beta n_j \quad j \neq e$$

$$Q_{Tej}^C = \beta(T_e - T_j) \quad j \neq e$$

$$Q_{TeTj}^C = \beta n_j \quad j \neq e$$

$$Q_{TeTe}^C = -1/2 \beta n_e \quad (20)$$

$$Q_{51}^{NC} = 2/3 \left(-\frac{D_{ep} n_L p^z p}{R} \frac{\partial \phi_m}{\partial r} \left(\frac{\partial n_p / \partial r}{\partial n_1 / \partial r} \right) \right) 1/N \{2\}$$

$$Q_{52}^{NC} = 2/3 \left(-\frac{D_{ep} n_L p^z p}{R} \frac{\partial \phi_m}{\partial r} \left(\frac{\partial n_p / \partial r}{\partial n_2 / \partial r} \right) \right) 1/N$$

$$Q_{5j}^{NC} = 0 \quad j = 3, 4, 5. \quad (21)$$

Appendix 11

Notes on the Method Used to Solve for ϕ_e

The method used to solve for the electron confining potential is a fixed point method of the form:

$$\phi_e^{k+1} = f(\phi_e^k), \quad (1)$$

where k labels the iteration. The function $f(\phi_e^k)$ has the form:

$$f(\phi_e^k) = -\phi_i - T_e \ln \{ R / (2Z_p n_p L_p \psi) (X - L_s (\partial n_i / \partial t + 2 \partial n_2 / \partial t)) \}$$

where: $\frac{\partial N}{\partial t} = X - \frac{2Z_p n_p L_p}{R} \psi \exp(-\frac{\phi_m}{T_e}) \psi^{-1} - \frac{1}{4} \frac{e^{\sqrt{\pi}} \ln(4 <Z_e> R + 2)}{I(\frac{T_e}{\phi_m})}$

$$\frac{(2R <Z_e> + 1)}{2R_p <Z_e>} \left(\frac{\phi_m}{T_e} \right) <Z_e> \cdot \quad (2)$$

This method is used in conjunction with the convergence criterion:

$$| \exp((\phi_i + \phi_e^k)/T_e) - \exp((\phi_i + \phi_e^{k+1})/T_e) | \leq 1 \cdot 10^{-4}.$$

The method, when it converges to a non-negative ϕ_e is found to converge within 1-6 iterations. The potential solver convergence depends critically upon the magnitudes of the radial gradients which enter the problem and its failure is usually the first indication of something amiss in the initial profiles chosen or of other internal code problems.

User's Notes

The required input data are read into the code in subroutine INITIAL. The various code variables and the format in which they are read, in the order in which they are read, are:

- (1) IDENTFI (A10)
 - (2) DTR (5E16.6)
 - (3) RH01, RH02, RH03, RH04 (5E16.6)
 - (4) RH05 (5E16.6)
 - (5) NPRINT, NPL0T, ITIMFLG (1615)
 - (6) NSTOP, NRSTRT (1615)
 - (7) TAU0, TO, EL, BP (5E16.6)
 - (8) BS (5E16.6)
 - (9) SIG0, SIG1, SIG2, SIG3 (5E16.6)
 - (10) EN0, ENA, ENS, ENNO (5E16.6)
 - (11) V3, V4, V8 (5E16.6)
 - (12) T3, T4, T13HAT, T14, HAT (5E16.6)
 - (13) TBHAT, TE3HAT, TE4HAT, TEBHAT, TEBLHAT (5E16.6)
 - (14) EPS0, EPS13, EPS14, EPS1B (5E16.6)
 - (15) BETAB (5E16.6)
 - (16) PL0TMIN (1-5) (5E16.6)
 - (17) PL0TMIN (6-7) (5E16.6)
 - (18) PL0TMAX (1-5) (5E16.6)
 - (19) PL0TMAX (6-7) (5E16.6)
 - (20) CCCC, ETA0, EPSIFB, EALPHB (5E16.6)
 - (21) F100, F200, FE00 (5E16.6)
 - (22) LSSS, LPPP, ZPPP, ALBDAD (5E16.6)
 - (23) EINJ, EPP, PMPP, AAA (5E16.6)
 - (24) ITYES, ITMAX, NCHKM (1615)
 - (25) FT00, AN0, AN01, AN02 (5E16.6)
 - (26) RMAX, DB, RMIRP (5E16.6)
 - (27) UJX (1-5) (5E16.6)
 - (28) UJY(1-3) (5E16.6)
 - (28a) ANCD, AD11, AD22, AD51, AD52
 - (28b) AAK51, SAMB
 - (28c) ENDPT, ENDPT1
 - (28d) DR0PP, AHLF, ALTR, ARPP
 - (29) NAMELIST/SIGVE/
- SI3EVE3, SI4EVE4, SI31V13, SI32V23, SI41V14, SI42V24, SCX13V13, SCX23V23, SCX14V14, SCX24V24, SD41V14, SD42V24, SPRVEP, S1RVE1, S2RVE2, SFUV1, DC0N, CL0SN, CL0ST, STEEP, RRI, RR2, LIMAX, STP2, RR3, RR4, S51, S52, S53, S54, S55, EFAL, EPS, SD43V34, SD4EVE4.
- (30) NAMELIST /SWITCH1/
- SWD11, SWD12, SWD13, SWD15, SWD21, SWD22, SWD24, SWD25, SWK31, SWK32, SWK33, SWK35, SWK41, SWK42, SWK44, SWK45, SWK51, SWK52, SWK53, SWK54, SWK55, SWV31, SWV32, SWV33, SWV35, SWV41, SWV42, SWV44, SWV45, SWV51, SWV52, SWV54, SWV55, SWW11, SWW12, SWW21, SWW22, SWW32, SWW33, SWW34, SWW35, SWW41, SWW43, SWW44, SWW45, SWW52, SWW55, SW51, SW52, SW53, SW54, SW55, SWS0URLE, SWL055, SWW61, SWW62, SWW63, SWW64, SWW65, SWP1, SWP2, SWW31, SWD51, SHD52, SWD53, SWD54, SHD55

where:

- (1) Identifying label for the run
- (2) Real Time step in seconds
- (3), (4) Implicit-explicit parameters, 0 = fully explicit, 1 = fully implicit
- (5) Number of time steps between prints and plots of variables, ITIMFLG=0
- (6) Number of time steps to be taken in total, NRSTRT=0
- (7) Normalizing time, temperature and length in seconds, K, and cm.
Plug magnetic field strength.
- (8) Central cell magnetic field strength
- (9) Normalizing cross sections in cm^2
- (10) Normalizing densities in cm^{-3}
- (11) Neutral and beam velocities, consistent with assumed energies, in cm s^{-1}
- (12) Neutral injection temperatures in K
- (13) Temperatures of electrons produced by ionization and charge exchange upon the neutral species in K
- (14) Dissociation and ionization energies of neutral species in K
- (15) Central cell Bremsstrahlung coefficient
- (16)-(19) Minimum and maximum expected values of outputted plasma quantities used by the plotting routine
- (20) Constants used in calculation of τ_p ; $\bar{e}_f$, E_2
- (21) f_1, f_2, f_e
- (22) L_s, L_p, Z_p, λ
- (23) Plug ion injection energy, plug ion energy (in K), plug ion mass in g, A (used in calculation of τ_p)
- (24) Iteration switch for numerical integrator (subroutine ITERATE), maximum number of iterations, maximum number of iterations for potential solver (subroutine POTENT)
- (25) f_T , ANO, ANO1, ANO2; note:

$$n_p(r) = \text{ANO} \exp(-\text{ANO1 } r^2) + \text{ANO2}$$

- (26) Maximum plasma radius in solenoid (in cm), neutral beam half-width (not used), plug mirror ratio
- (27) $n_1(r=0)$, $n_2(r=0)$, $T_1(r=0)$, $T_2(r=0)$, $T_e(r=0)$ in cm^{-3} and K. Density profiles are Gaussian and temperature profiles are gaussian. The profiles are initialized in subroutine XINIT.
- (28) Neutral species values of $r=0$
- (28a-d) Switches for the various types of diffusion used, fixed gradient params., initial profile switch, plasma half-length in plug, quadrupole transition length, plug plasma radius.
- (29) Thermal averaged reaction rates in $\text{cm}^3 \text{s}^{-1}$
- (30) Switches for terms in the finite differenced equations as explained below. 0 = not used, 1 = used.

One notes that the density equations may be written in the following form:

$$\frac{\partial n_i}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (G) + W_{11} n_1 + W_{12} n_2 + W_{13} T_1 + W_{14} T_2 + W_{15} T_e + S_1$$

$$G = r \Gamma_1 \quad (1)$$

The temperature equations may be written in the general form:

$$\begin{aligned} \frac{\partial T_1}{\partial t} = & \frac{T_1}{n_1 r} \frac{\partial}{\partial r} (G) - \frac{1}{n_1 r} \frac{\partial P}{\partial r} + V_{31} \frac{\partial n_1}{\partial r} + V_{32} \frac{\partial n_2}{\partial r} + V_{33} \frac{\partial T_1}{\partial r} \\ & + V_{32} \frac{\partial T_2}{\partial r} + V_{35} \frac{\partial T_e}{\partial r} + W_{31} n_1 + W_{32} n_2 + W_{33} T_1 + W_{34} T_2 \\ & + W_{35} T_e + S_3 \end{aligned}$$

$$P = r (2/3 q_1 + 5/3 T_1 r_1)$$

$$K_{31} = 5/3 T_1 D_{11}$$

$$K_{32} = 5/3 T_1 D_{12}$$

$$K_{33} = 2/3 K_{33} + 5/3 T_1 D_{13}$$

$$K_{34} = 0$$

$$K_{35} = 5/3 T_1 D_{15} \quad (2)$$

with analogous forms for the other species. The coefficients indicated above are stored in arrays in the code as follows:

<u>Array</u>	<u>Variable</u>
TCØG1(1)	D11
(2)	D12
(3)	D13
(4)	D15
(5)	D21
(6)	D22
(7)	D24
(8)	D25
(9)	K31
(10)	K32
(11)	K33
(12)	K35
(13)	K41
(14)	K42
(15)	K44
(16)	K45
(17)	K51
(18)	K52
(19)	K53
(20)	K54
(21)	K55

<u>Array</u>	<u>Variable</u>
(35)	D51
(36)	D52
(37)	D53
(38)	D54
(39)	D55

<u>Array</u>	<u>Variable</u>
TCØF1 (1)	V31
(2)	V32
(5)	V41
(6)	V42
(7)	D24
(8)	D25
(9)	V51
(10)	V52
(11)	V53
(12)	V54
(13)	V55
(14)	W11
(15)	W12
(16)	W21
(17)	W22
(18)	W32
(19)	W33
(20)	W34
(21)	W35
(22)	W41
(23)	W43
(24)	W44
(25)	W45
(26)	W52
(27)	W55
(28)	S1
(29)	S2
(30)	S3
(31)	S4
(32)	S5
(36)	W53
(37)	W54
(38)	W51
(50)	W61
(51)	W62
(52)	W63
(53)	W64
(54)	W65
(56)	W31

The switches in /SWITCHI/ are then used to include or delete the above terms in the calculation.

The plasma ion and electron densities and temperatures are stored in the u(# of eqns., # of mesh pts.) array as indicated. The neutral quantities are stored in the uu(# of neutral mesh points, # of neutral eqns.) array. The neutral routine is not presently being used. The coefficients

stored in TCØG1 and TCØF1 are calculated in subroutine CØEF and CØGNEØ.

$$\begin{aligned}u(1,r) &= N_1(r) \\u(2,r) &= n_2(r) \\u(3,r) &= T_1(r) \\u(4,r) &= T_2(r) \\u(5,r) &= T_e(r)\end{aligned}$$

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