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TWO PHASE POTENTIAL FLOW

Progress Report

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Abstract

New results for the flow of a dispersion of particles in an inviscid irrotational flow are reported. Equations of motion for an isotropic assembly have been derived and applied to several example problems. Theorems have been derived relating the macroscopic (averaged) properties of flows composed of unit cells. The effective conductivity of a suspension has been obtained in new ways, using the method of images, and related to forces exerted by a fluid on particles when there is relative motion.

Summary

The objective is to develop theorems for two-phase potential flow that are analogous to those existing for single-phase flow.

During the previous year, macroscopic equations of motion for each phase were developed; they are compatible with Geurst's formulations. They have now been applied to specific situations such as the motion of a porous sphere in an unbounded fluid [1].

A comprehensive treatment of the average properties of "unit cells" has led to several theorems for one-dimensional duct flows and three-dimensional continua [2].

Specific results have been derived by solving Laplace's equation numerically for rows of spheres of any diameter and any spacing in a general round pipe [3]. The result is a set of graphs and correlations that can be used to solve practical problems using the theorems in [2]. Numerical checks of equations in [2] have also been successfully made.

The classical results of Maxwell [4] and Rayleigh [5] for the conductivity of a suspension have been derived in new ways using the method of images [6].

New theorems have been derived, relating the dipole moment of internal sources to the particle "impulse" and "added mass" [7]. These parameters are being computed for various structures of an assembly of particles [7]; they can be related to forces and "pressures" exerted on the particles. We are also developing new theorems regarding "averaging" in discontinuous media and the levels of approximation involved.

Predictions of the propagation speed of compressibility waves have been made [11].

Theoretical predictions have been checked by measuring the natural frequency of oscillation of arrays of particles and discs suspended in a pipe [3,7].

Approach

The approach has been mostly theoretical, based on fundamental laws of fluid mechanics. Numerical methods have been used to solve specific examples and to check theorems. Experiments have also been performed to test and check predictions.

Results

Last year [8] we reported a new derivation on Geurst's equations based on the averaged Bernoulli equation for fluid flowing around an isotropic suspension. This work has now been put on a firmer basis by proving that it is compatible with both the "resistivity" and "polarization" approaches in the literature [1,9]. It has also been illustrated, and some details confirmed, by application to specific examples including the motion of a porous sphere or cylinder relative to a large expanse of fluid. All the results are predictable from one function, the "exertia," which depends only on the void fraction in an isotropic suspension.

A comprehensive paper on dispersed flows with a "unit cell" structure [2] has extended earlier work [10] in which it was shown how the dimensionless resistivity, β , could be used to determine several properties of a dispersion of particles in a potential fluid flow. Once the geometry of a unit cell is defined, β can be determined by solving Laplace's equation. Other properties can then be deduced by straightforward mathematics.

The new features of the present work included:

- Extension to three-dimensional cells in which β becomes a tensor.
- A new derivation of the equation of motion of a uniform array and the related "two-fluid model" equations.

- Evaluation of the “impulse” $\int \phi ds'$ for particles and its relation to momentum and kinetic energy density.
- Evaluation of the “impulse function” for stationary and moving cells.
- Derivation of the averaged Bernoulli equation for the fluid for stationary and moving cells.
- Prediction of the mean pressure on the wall of the one-dimensional cells and in the particle.
- Derivation of the energy conservation equation for steady flow with one-dimensional cells and Bernoulli’s equation for phase 2 which makes it possible to solve for the local conditions.
- A proof that the kinetic energy density is minimized in one-dimensional flow from a single stagnation pressure.
- Various results concerning the existence of solutions, forces at interfaces, time-varying macroscopic potentials, etc.

Unit cells of a simple structure are particularly suitable for the generation of numerical solutions to the microscopic flow field that can be used to check these results and to predict the behavior of specific systems (e.g. spheres in a pipe). Experimental confirmation is also possible by constructing situations, such as small amplitude oscillations or rapid accelerations, for which potential flow provides a good approximation.

It would be interesting to see how these results compare with those involving less orderly, or random, structures that might be analyzed using ensemble averaging. There are also questions about how far a continuum flow can be represented by unit cells that elongate in the direction of extension rather than maintaining an isotropic character.

The above results have been tested, confirmed and illustrated both by numerical and by experimental methods. Since all of the predictions can be made from knowledge of the resistivity, β , it has been derived by solving Laplace's equation for strings of spheres and discs in pipes in terms of general dimensionless variables. β can then be inserted into the macroscopic equations of motion to predict the natural frequency of arrays of spheres and discs suspended on springs in a pipe, with various end conditions (such as an "open" or "closed" end). A comprehensive set of results is being obtained and will be reported in the Ph.D. theses of X. Cai and C. Luo [3,7].

For isotropic arrays of spheres or cylinders there exist predictions for β as a function of void fraction, dating back to classical works of Maxwell [4] and Rayleigh [5]. We have found new ways to derive these results for several particle geometries and boundary conditions, using the method of images. One by-product of this work has been to justify, to higher order, a (fortunate) assumption earlier made by Maxwell. A further result has been the prediction of edge and end effects for assemblies of spheres arranged as disc or rod-like slabs with their axes parallel to a uniform external flow [6].

At the very basic level, several theorems have been derived. One relates the dipole moment of internal sources to the "impulse," or $\int \phi ds$ over the particles. This leads to several general results involving "added mass" and forces exerted by (or on) particles and is related to the stability of various arrays in the presence of relative motion.

We have been led to re-examine the basics of "averaging" in discontinuous media containing discrete particles. Though this work is incomplete, it indicates that successive levels of approximation can be achieved in terms of Taylor series containing new properties that can be defined at the microscopic level.

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PERSONNEL

Professor Graham Wallis, Principal Investigator, has devoted approximately 15% of the period January-June, 1991 and 3/4 of a summer month to this research. He was able to spend more time on these topics during August-December, 1990 under a Royal Society Visiting Fellowship at University College, London.

During the year, Simon Harvey completed his M.S. thesis and two Ph.D. students (Xiaolong Cai and Chao Luo) were supported by the contract.

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