by J. Schivell

Methods are discussed for reconstructing spatial distributions from two types of projection -- in a direction tangential to a torus and in a cross-sectional plane. For a tangential projection the reconstruction is unambiguous. For cross-sectional projections one usually does not have enough information for an unambiguous reconstruction, unless strong simplifying assumptions are made. A method is given for fitting a spatial distribution to such projections. Tests of both methods are discussed. The fitting method for cross-sectional projections is compared with a Fourier-series method.

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Methods for Reconstructing Spatial Distributions of Radiation from Plasmas

INTRODUCTION

In the detection of radiation from plasmas, the measurements are often made for a series of directions across the plasma. In bolometry either an array of detectors or a tilting scanner is used, while in spectroscopy a rotating mirror is used to make a vertical scan thru the plasma. Each measurement is the line integral of radiation from a chord thru the plasma. If the distribution of radiation is cylindrically symmetric, then the Abel inversion is used to obtain the radial distribution. However, in many cases the distribution is manifestly not cylindrically symmetric. In spectroscopy, for example, the raw data is sometimes not consistent with any cylindrically symmetric distribution [1]. In bolometry the data typically shows approximate top-bottom symmetry, but experience on T-10 [2] shows in-out asymmetry, raising the possibility that data taken looking radially inward could be interpreted incorrectly. Furthermore, in machines such as PDX, one expects a non-circular cross-section, plus the divertor regions, which implies great difficulties in determining the spatial distribution of radiation.

In analogy with computed tomography in the medical field [3], ideas have been put forth to make measurements on plasmas in several cross-sectional projections. With suitable combinations of the number of projections with weaker assumptions of symmetry, 2-dimensional distributions can be obtained [4,5].

However, these procedures appear to be not very satisfactory, because of the assumptions which are still required and because of the number of projections (hence, number of ports) which are required (as many as 4 or more).

One purpose of this report is to point out that a single view in the tangential direction (tangential to the major circumference, or "long view") appears to be a very effective way of obtaining a 2-dimensional distribution. Compared with views in the radial direction, it has the advantage of requiring no assumptions about the 2-dimensional distribution. On the other hand, it requires the assumption of toroidal symmetry over that part of torus which is viewed. Typically 120 degrees of the torus would be visible, so that a position far from the limiter could readily be chosen.

Since the line intensity along any chord is most sensitive to the radiation from the cross section at the point of tangency of the line of sight, the raw data is already a good rough indication of the 2-dimensional distribution, as has been seen with a visible-light "TV" system. The reconstruction method discussed here can be used to determine quantitatively the 2-dimensional distribution. For bolometry a pinhole camera combined with an image detector could be employed to measure the 2-dimensional projection; for spectroscopy a mechanical scanning optical system, such as, for example, two rotating mirrors, could measure the projection.

The other purpose of this report is to examine two types of approach to the problem of spatial reconstruction from views in the cross-sectional direction. These remain interesting in many situations, of course, because even partial toroidal symmetry may not exist or a 2-dimensional tangential projection may not be practical.

RECONSTRUCTION FROM A TANGENTIAL VIEW

Method: (Figs. 1 and 2)

If one had available a vertical array of ports thru each of which he could obtain a projection of a horizontal slice of the plasma, then the reconstruction could be done for each slice separately by simple Abel inversion. However, for the more likely case in which only one port is available, it is necessary to form a 2-dimensional image and to unfold the spatial distribution from it.

Consider the torus to be divided into a set of rings. The rings are formed by revolving the rectangular grid in Fig. 1 around the axis of the torus. The distribution of emission is represented by one value for each ring. Each cell in the imaging device receives an amount of radiation proportional to the sum of the intensity in each ring, multiplied by the length of the line of sight in that ring. Considering all of the cells in the image, one has a set of simultaneous equations for the emission intensity in the rings. If E_j represents the emission intensity in the j -th ring, S_i the signal in the i -th image cell, the L_{ij} the length of the i -th line of sight in the j -th ring, then one can write

$$S_i = \sum_{j=1}^M L_{ij} E_j, \text{ for } i=1 \text{ to } N \quad (1)$$

Note that i and j range over all positions in both the horizontal and vertical directions (except that the problem can be separated into 2 portions, those above and below the center of the imaging device). The emission intensities in the rings can be found by

solving the set of equations.

In the horizontal direction the number of angles, α , for lines of sight may be conveniently taken as the same as the number of rings (cylinders). In the vertical direction one needs more angles than the number of levels, because, with one angle per level [β_1 to β_5 in Fig. 2b], the uppermost lines of sight would not pass thru the inner rings on the upper levels, and no lines of sight would pass thru those rings. I have taken the number of vertical angles to be twice the number of levels [β_1 thru β_{10}]. In this work I have used 20 cylinders and 12 levels. Therefore, $M = 240$ and $N = 480$.

A matrix method has been tried (least-squares solution of the overconstrained system), but it runs into difficulties with this large system.

A relaxation method has proved successful. The values of E_j are adjusted to reduce the error in the equations, one equation at a time. The adjustment to each E_j is weighted in proportion to L_{ij} . After a small number (5-10) of passes thru the system of equations, the set of values of E_j converges very well.

The algorithm is derived as follows. At any stage one has a set of values E_j . The error in the equation under consideration, the i -th equation, is

$$d = \sum_{j=1}^M L_{ij} E_j - S_i \quad (2)$$

Now, one wishes to have a set of corrections, ΔE_j , such that

$$\sum_{j=1}^M L_{ij} \Delta E_j = -fd \quad (3)$$

where f is a convergence factor and such that, if K is some constant,

$$\Delta E_j = K L_{ij} \quad \text{for all } j \quad (4)$$

Combining (3) and (4) and eliminating K leads to

$$\Delta E_j = \frac{-fd}{\sum_j L_{ij}^2} L_{ij} \quad (5)$$

After the E_j are adjusted, some values are less than zero. These values are set to zero, and the total negative amount is subtracted from the positive E_j in proportion to the corresponding L_{ij} .

Note that the techniques just described should also work for a cylindrical region viewed from the end thru one point, as long as there is no axial variation.

Tests

Test data were generated by assuming a distribution in the u, z -plane and calculating the projection that a tangentially viewing device would see.

Random noise was added to the projection data to test the sensitivity of the method.

The assumed and reconstructed distributions, and the calculated and noisy projections, were compared in 3 ways: tabular form, surface plots of intensity as a function of position, and simulated "pictures" of intensity (density of dots proportional to intensity).

Statistics were compiled on the differences between the assumed and reconstructed distributions.

For orientation, Fig. 3 shows a sketch of a hollow, glowing torus, viewed tangentially. Since the problem is separable into top and bottom halves, I have worked with top half only.

Three cases are shown: a uniform ring of emission (Figs. 4 and 5), two regions of emission near the midplane (Fig. 6), and a bright center with a ring with enhanced emission on inner and outer sides near the midplane (Fig. 7).

Results (Table 1)

With no noise added, errors in the reconstruction are very small.

With random noise added, the percentage error in the reconstruction, compared point-by-point with the assumed distribution, is roughly 2-3 times the percentage of the noise.

However, features of reasonable size in the distribution are still quite distinct with noise in the projection of 5-10%.

RECONSTRUCTION FROM RADIAL PROJECTIONS

I have looked at 2 methods:

- 1) Fitting the projections of a function $E(r, \theta)$ to the projection data by a relaxation method.
- 2) Assuming a Fourier series representation of $E(r, \theta)$, for which the individual terms can be transformed from projection space separately.

Fitting:

Method

In general one can use any function of minor radius and poloidal angle, $E(r, \theta)$, which has some number of adjustable parameters, P_j . Along any line of sight the integrated emission is given by

$$F_i = \int_{\text{line of sight } i} E(r, \theta) \, d\ell \quad (6)$$

The F_i are, of course, dependent on the P_j .

The relaxation method is to adjust the P_j by comparing F_i with the observed signal, S_i , for each line of sight, i , in the projections. Let the difference be denoted by $d = F_i - S_i$. For a non-linear dependence of the P_j , I write for the adjustments ΔP_j ,

$$\sum_j \Delta P_j \left(\partial F_i / \partial P_j \right) = -fd \quad (7)$$

where f is a convergence factor, and set

$$\Delta P_j = K \left(\partial F_i / \partial P_j \right) \quad (8)$$

Combining (8) with (7), I get

$$\Delta P_j = \frac{-fd}{\sum_j (\partial F_i / \partial P_j)^2} \frac{\partial F_i}{\partial P_j} \quad (9)$$

In order to improve the performance of the relaxation, the order of choosing lines of sight for adjustment has been randomized.

Tests

A function which appears suitable for fitting data from visible and UV spectroscopy is

$$F(r, \theta) = A(\theta) \exp \left[-\frac{1}{2} (r - B(\theta))^2 / W^2(\theta) \right] \quad (10)$$

where A, B, and W are each given by a Fourier series in θ .

This function has been tested on projections calculated from an assumed distribution. The results are shown in Fig. 8.

The case assumed was that of a bright ring with an extra bright spot at the top and bottom. Two projections were calculated. The assumed ring was centered at $r = 30$, with a width of 3, a height of 1, and an extra height of 1 in the bright spots.

In each case the starting $A(\theta) = 1$. Cases were fitted for $B(\theta) = 27, 30$, and 33 and for $W(\theta) = 2$ and 3 . (The starting parameters were all 0. for the θ -dependent terms.) Terms were used up to 50, and the convergence factor, f , was set equal to 0.25.

As might be expected, the results are strongly dependent on some of the choices of starting values of the parameters. The reason is, of course, that with limited information available (in this case, 2 projections) many interpretations

are possible. The fitting process cannot really choose between the interpretations and converges on one of them. But it does converge. It is interesting that in the two cases with starting value $B(0) = 30$ (Fig. 8(c) and (d)) the fitted function is a good representation of the assumed distribution.

The utility of this method would be in testing various hypotheses for $E(r, \theta)$.

Fourier series:

This method has been described by Pohl [5]. The distribution in r, θ -space and the data in the observed projections are each represented by a Fourier series of the form

$$f(r, \theta) = f_0(r) + f_{1c}(r) \cos \theta + f_{1s}(r) \sin \theta + \dots \quad (11)$$

In order to pick from the infinite set of possible solutions, one can assume the "simplest possible solution", that is, for N views admit terms only up to $\cos(N\theta)$. The functions of r which are coefficients of each term in θ can be transformed analytically.

Comparison of methods:

The Fourier series method gives a solution in closed form. The $E(r, \theta)$ must be represented as simple functions of θ , with arbitrary functions of r .

The fitting method allows one to specify a function of r , allowing more latitude in the function of θ . In the example above, terms were used up to 5θ , for a total of 30 parameters. This could be advantageous, for example, in spectroscopy, in which it is known that the given spectral line should appear only in a narrow range of radius.

SUMMARY

Reconstruction methods have been presented for both tangential and radial projections. Under the assumption of (at least partial) toroidal symmetry, tangential projections make possible unambiguous determination of a 2-dimensional distribution in a torus. In the case of radial views the limited number of projections usually available in plasma physics experiments means that one must make quite strong assumption about the 2-dimensional distribution in order to select from among the possible interpretations.

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TABLE 1

Statistics on Reconstruction Errors

(errors and noise are r.m.s.)

Case	Reconstruction error no noise	noise added to projection	Reconstruction error with noise	amplification: error / noise
Bright ring	2%	11%	27%	2.5
Two bright regions	4	9	13	1.4
Center + ring + two bright regions	2	3	10	3.3

FIGURE CAPTIONS

- Fig. 1 Cross section of ring-shaped regions.
- Fig. 2 Typical lines of sight thru the plasma, a) top view, b) side view.
- Fig. 3 Close-up tangential view of a glowing hollow torus.
- Fig. 4 Intensity plots for the "bright ring" case.
a) projection of b) the given distribution, c) reconstruction, d) projection with 11% noise added, and e) reconstruction from (d).
- Fig. 5 The same values as those in Fig. 4 shown as surface height, rather than intensity.
- Fig. 6 Surface plots for the case of "two bright regions". Noise added to the projection is 9%.
- Fig. 7 Surface plots for the case of "center plus ring plus two bright regions". Noise added is 3%.
- Fig. 8 Fitting a spatial distribution to two projections.
a) given, b) projections thru two points, one at 0 one at $\pi/2$ radians, of given (points) and fitted (curves) c) thru f) fitted distributions with the starting values given in the table:

Section	A	B	W
b,c	1	30	3
d	1	33	3
e	1	27	3
f	1	30	2

Fig. 1. 793601

Fig. 793599

Fig. 3. 793600

Fig. 4. 793521

Fig. 5. 793525

Fig. 6. 793523

Fig. 7. 793524

Fig. 8. 793598