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APPLICATIONS OF GAUGE-FIXED CORRELATION FUNCTIONS OF QUARKS AND GLUONS *

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Two uses of fixed gauge correlation functions involving quarks and gluons—as probes of effective quark and gluon parameters and their gauge dependence and as constraints for matching coefficients—are examined. Effective quark and gluon masses are found to decrease with increasing gauge parameter λ . A fixed gauge technique is used to determine $\bar{s}d$ matching coefficients. Despite gauge variance of effective quark masses, results of this matching—modulo statistics—are gauge invariant. Accordingly this technique is applied to the Delta I=1/2 Rule. We are however unable to obtain physically useful results at present in this case because of large fluctuations.

1. INTRODUCTION

Computation of gauge variant correlation functions in lattice QCD has been the subject of several recent papers.^{1,2,3} The gluon propagator was studied in Ref. 1 and the quark propagator was studied in Ref. 2 on quenched Wilson lattices. Beyond demonstrating that quark and gluon propagators have sensible behavior containing apparent mass gaps even if quarks and gluons are not experimentally observable states, these studies display what may be fundamental features of nonperturbative QCD. Indeed, the effective quark mass is found to decrease linearly with decreasing m_π^2 to a critical value which is nonzero.²

Here we examine two more uses for quark and gluon correlation functions:

- I. as probes for effective quark/gluon masses and vertices and their gauge dependence;
- II. as constraints for matching between lattice and continuum operators.

For these applications, the following fixed-gauge correlation functions are being computed on the lattice:

$$\bullet \quad \langle 0 | \psi(x) \bar{\psi}(0) | 0 \rangle \quad (1)$$

$$\bullet \quad \langle 0 | A^\mu(x) A^\nu(0) | 0 \rangle \quad (2)$$

$$\bullet \quad \langle 0 | \prod_{0 \leq t' \leq t} U_i(\vec{x}, t') | 0 \rangle \quad (3)$$

$$\bullet \quad \langle 0 | \psi(y) \bar{\psi}(0) \Gamma \psi(0) \bar{\psi}(x) | 0 \rangle \quad (4)$$

$$\bullet \quad \langle 0 | s(y) (\bar{s}(0) d(0)) \bar{d}(x) | 0 \rangle \quad (5)$$

$$\bullet \quad \langle 0 | s(y) O_\pm[\Gamma_1 \Gamma_2] \bar{d}(x) | 0 \rangle \quad (6)$$

Effective quark and gluon masses are extracted by projecting (1) and (2) down to their zero momentum, physical polarizations and averaging the log-ratio of successive propagator timeslices over the "plateau."² Global symmetries suppress all components of (3) except

$$\frac{1}{3} \text{ReTr} \langle 0 | \prod_{0 \leq t' \leq t} U_i(\vec{x}, t') | 0 \rangle \equiv Z_h e^{-V_h t} \quad (7)$$

where V_h may be interpreted as $(M_q - m_{\text{curr}})$ of a static heavy quark. Function (4) gives for arbitrary Dirac matrix Γ the quark's effective electromagnetic radius, magnetic moment, and other couplings.⁴ Functions (5) and (6) are needed for application II, described below.

We present here results from two lattices: sixteen $\beta = 5.7$ $16^3 \times 24$ gauge configurations ($16^3 \times 39$ quark propagators) and eight $\beta = 6.0$ $24^3 \times 40$ gauge configurations ($24^3 \times 39$ quark

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Figure 1: Ratios of effective masses of $\lambda = 2$ to $\lambda = 1$. Despite the fact that the individual effective masses do not plateau until the $\sim 8^{\text{th}}$ timeslice, $M_q^{\lambda=2}/M_q^{\lambda=1}$ reaches the apparent plateau value of $\sim 90\%$ by the 5^{th} timeslice and stays there.

propagators).

2. MASSES AND GAUGE DEPENDENCE

Since QED is infrared free, the electron mass m_e appears as a pole of *gauge invariant* Greens functions consisting of an electron and positron separated widely enough that interaction between them is negligible. Then the pole of the electron propagator can be renormalized to be m_e . Since m_e is derived from a gauge invariant Green's function, m_e is manifestly gauge invariant.⁵

Due to photon fluctuations, the nonperturbative electron is a composite particle whose propagator has branch cuts rather than an isolated pole. To leading order in continuum perturbation theory,⁶ the Euclidean electron propagator is

$$\int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{e^{ip_0 t}}{(p_0^2 + m_e^2)^{1+b}} \xrightarrow{(m_e t) \rightarrow \infty} A t^{-b} e^{-m_e t} \quad (8)$$

where $b \equiv \alpha_{\text{em}}(3 - \xi)/2\pi$ and ξ is the covariant gauge parameter. Consequently the effective mass $M_e = m_e + \frac{b}{t} \log t$ picks up a soft ξ dependence. Nonetheless, the limiting value

$$\lim_{t \rightarrow \infty} M_e = m_e \quad (9)$$

is gauge invariant.

Since QCD is confining, it is not possible to isolate a manifestly gauge invariant m_q

Figure 2: The effective quark mass decreases with increasing λ . Although not shown, this feature is reproduced on our 24by40 ($\beta = 6.0$) lattices.

in gauge invariant Greens functions—constituent quark masses come with binding energy. What about extracting m_q directly from the quark propagator? The problem is the QCD analogue of Eq. (8) is beyond perturbation theory, so it is not guaranteed that the $t \rightarrow \infty$ limit of M_q is gauge invariant. Furthermore, since it is not measurable as the pole of gauge invariant Greens functions, m_q may not even exist.

As a practical matter, even if Eq. (9) does hold for QCD, the path to m_q will be gauge dependent. Thus, unless the limit is reached quickly enough gauge invariance will not be observed within the lattice. Hence we study two questions: (i) How does M_q depend on gauge? (ii) Does M_q contain a gauge invariant component m_q at large times?

Our λ gauges are the lattice analogues of $\lambda \partial_0 A_0 + \nabla \cdot \vec{A} = 0$ and are obtained by selecting gauge transformations Ω_x to drive

$$\epsilon(\lambda, \mathcal{U}) \equiv \frac{1}{\# \text{sites}} \sum_{\mu, x} \text{Tr}(\lambda_{\mu} \partial_{\mu} A_{\mu})^2 \quad (10)$$

down by a factor of 10^4 from the original configurations before gauge fixing (“ Σ gauge”²). In (10), $\mathcal{U}$ represents the configuration of gauge links, and $\lambda_0 \equiv \lambda$, $\lambda_i \equiv 1$. We have checked that the Landau gauge quark propagator is stable under an additional factor of 10 decrease in ϵ .

λ gauge fixing can be viewed as freezing Ω_x fluctuations according to the $\xi \rightarrow 0$ limit of the

Figure 3: The effective gluon mass decreases with increasing λ . Although not shown, this feature is reproduced on our 24by40 ($\beta = 6.0$) lattices.

action⁷

$$S_{gf} = -\frac{1}{2\xi} \sum_{x,\mu} \text{ReTr}[\lambda_\mu \Omega_x U_\mu(x) \Omega_{x+\hat{\mu}}^\dagger]. \quad (11)$$

Then the λ gauge quark propagator

$$P(t) = \sum_{\vec{x}} \langle 0 | \Omega_x \psi(x) \bar{\psi}(0) \Omega_0^\dagger | 0 \rangle \quad (12)$$

is an “ $\Omega_x \psi(x)$ ” propagator with respect to Σ gauge. The gauge transformations Ω_x —like quarks in the hopping constant (K) expansion—hop between neighboring sites with amplitude $\lambda_\mu/2\xi$ leaving behind a Wilson line trail. The integration over $\mathcal{U}$ suppresses all trails except those where the quark and Ω trails meet to form Wilson loops. In the strong coupling limit ($\beta \rightarrow 0$, $\xi \rightarrow \infty$), only the timelike, arealess Wilson loops survive, and we get a mass $M_q \sim \log(N\xi/K\lambda)$, where N is the number of colors (coming from Ω_x integrations). Therefore, at strong coupling M_q decreases with increasing λ .

As Figures 1 and 2 show, our $\beta = 5.7$ and $\beta = 6.0$ numerical results follow this strong coupling trend. Though the dependence of M_q on λ is mild, it is statistically significant. Figure 3 shows that the effective gluon mass has similar behavior. Likewise V_h (not shown) also decreases with increasing λ .⁴ Our Landau gauge gluon mass values (~ 500 MeV) are consistent with earlier lattice results¹ and a continuum estimate.⁸

In an effort to uncover an underlying gauge invariant mass m_q , we have done correlated fits of

Figure 4: M_ρ/M_q is independent of K within our resolution. The $M_\rho/M_q = 2$ line is to guide the eye.

the quark propagator to (i) $Ze^{-M_q t}$, (ii) $At^B e^{-m_q t}$ and (iii) $Ce^{-m_q t} + De^{-m_q' t}$. In general, the propagator is not a good fit to (i) for the limited range of times available, but (ii) and (iii) with $D \sim -C$ fit well.⁴ Moreover, ratios of any two propagators (same K , different λ) fit well to Et^F . This suggests that (ii) is a good fit for all available λ when $m_q^{\lambda=\lambda_0}$ (for any choice of λ_0) is kept fixed and only A and B are varied with λ . However, when A , B and m_q are simultaneously varied the best fit m_q decreases with increasing λ .

Despite apparent gauge dependence, M_q has some intriguing phenomenological features reminiscent of the nonrelativistic quark model. As noted previously,² $M_q^{\lambda=1} \sim M_{\text{proton}}/3$. Additionally, the ratio of the ρ mass M_ρ to the quark mass M_q , depicted in Figure 4, is ~ 2 for all K . (Kilcup and Sharpe⁹ have found similar results.) The 1σ to 2σ scaling violation between $\beta = 6.0$ and $\beta = 5.7$ here is comparable to scaling violations seen in gauge invariant correlation functions.¹⁰ The reason for the difference from 2 of M_ρ/M_q for large masses is unclear.

3. $\Delta I = 1/2$ RULE MATCHING

Proportional to $(m_u - m_c)a^{-2}$, the $\bar{s}d$ coefficient in the lattice representation of the dimension-6 $\Delta S = \pm 1$ electroweak Hamiltonian is presumably not computable in weak coupling perturbation theory(WCPT) or any other conventional approach.^{11,12} We take advantage of

gauge fixing to compute quark and gluon matrix elements from which such matching coefficients can be extracted.

Let q be a quark of arbitrary flavor ($q \in \{u, d, s, c\}$) and define

$$O_{\pm}^{qq}[\Gamma_1\Gamma_2] \equiv \bar{s}(0)\Gamma_1 d(0) \cdot \bar{q}(0)\Gamma_2 q(0) \pm (d \leftrightarrow q). \quad (13)$$

The operators of interest are then $O_{\pm}^{uu}[\Gamma_1\Gamma_2] - O_{\pm}^{cc}[\Gamma_1\Gamma_2]$. Their K to π matrix elements can be decomposed into “figure 8” and “eye” components.

In lowest order of chiral perturbation theory (CPT_h), the amplitude ratio for the $\Delta I = 1/2$ part of $K_S^0 \rightarrow \pi^+\pi^-$ divided by the $\Delta I = 3/2$ part is given by¹²

$$\frac{3}{2} \times \left(\frac{1}{3} + R_2^+ + \frac{c_-}{c_+} (R_1 + R_2^-) \right) \quad (14)$$

where $c_{\pm}$ are QCD renormalization group coefficients,

$$R_1 \equiv \frac{\langle O_{-}^{uu}[LL] \rangle_{\text{fig8}}}{\langle O_{+}^{uu}[LL] \rangle_{\text{fig8}}}, \quad (15)$$

$$R_2^{\pm} \equiv \frac{\langle O_{\pm}^{uu}[LL] - O_{\pm}^{cc}[LL] \rangle_{\text{eye}}}{\langle O_{+}^{uu}[LL] \rangle_{\text{fig8}}} \quad (16)$$

(with $\langle \rangle$ denoting a $K^+ \mapsto \pi^+$ matrix element), and the operators are renormalized to give zero $K \rightarrow 0$ transition amplitude.¹³

To compute the $R_{1,2}^{\pm}$ on the lattice, it is necessary to know the coefficients $z_{\pm}^J$ and $z_{\pm}^{\Gamma_1\Gamma_2}$ in

$$O_{\pm}^{\text{cont}}[LL] = z_{\pm}^J J^{\text{latt}} + \sum_{\Gamma_1\Gamma_2} z_{\pm}^{\Gamma_1\Gamma_2} O_{\pm}^{\text{latt}}[\Gamma_1\Gamma_2] \quad (17)$$

where $J \equiv \bar{s}(0)d(0)$ and the sum over $\Gamma_1\Gamma_2$ is a sum over the combinations LL, SS, PP, TT, LR . Odd parity operators do not contribute and higher dimensional operators (order a terms) are suppressed in the continuum limit and ignored here. The mixing with 4-quark operators in Eqn. (17) comes only from short distance effects in figure-8 type graphs; mixing with penguin operators through eye graphs is absent¹⁴ due to GIM suppression. We neglect mixing with the color magnetic-moment operator $\bar{s}\sigma_{\mu\nu}F^{\mu\nu}d$ since it has

a tiny perturbative coefficient¹⁵ (it comes in only in two loops) and is GIM suppressed.

We impose the matching condition

$$\langle s | O_{\pm}^{\text{cont}}[LL] | d \rangle = 0 \quad (18)$$

as the Wilson lattice equivalent, in the parity even sector, of the requirement that the $K \rightarrow 0$ amplitude vanish. We enforce the condition by individually subtracting J^{latt} from each lattice operator ($\Theta^{\text{latt}} \rightarrow \Theta^{\text{latt}} - \alpha_{\Theta} J^{\text{latt}}$) where the weight α_{Θ} is defined by

$$\langle s | \Theta^{\text{latt}} | d \rangle \equiv \alpha_{\Theta} \langle s | J^{\text{latt}} | d \rangle. \quad (19)$$

Since $|s\rangle$ and $|d\rangle$ are not S-Matrix states, there is no guarantee that α_{Θ} defined by Eq. (19) is related to $z_{\pm}^J$ in Eq. (17). Note however that in an unconfined theory (18) would be precisely the correct condition since it prohibits the mixing of s and d quarks on shell. The best we can do here is to check that each α_{Θ} has all the properties expected of $z_{\pm}^J$: (i) uniqueness—the scalar and vector components of the quark propagators in Eq. (19) give the same α_{Θ} ; and (ii) gauge invariance— α_{Θ} must be gauge invariant. We do not see any statistically significant violations of (i) or (ii). For example, at $\beta = 6.0$ and $K = .154$

$$\frac{\alpha_{O_{+}[SS]}^{\lambda=2}}{\alpha_{O_{+}[SS]}^{\lambda=1}} = 1.00(.002); \quad \frac{\alpha_{O_{-}[LL]}^{\lambda=2}}{\alpha_{O_{-}[LL]}^{\lambda=1}} = 1.00(.092). \quad (20)$$

However, the real test of this method would be the scaling of the final continuum matrix elements after all subtractions have been made. Large fluctuations make that test meaningless at present.

Since $\bar{s}d$ doesn't contribute to figure 8 graphs, $\langle O_{\pm}^{uu}[LL] \rangle_{\text{fig8}}$ isn't affected by subtraction. Following the technique¹⁶ used to correct $\langle \bar{K}^0 | (\bar{s}d\bar{s}d)_{\text{cont}}^{\text{latt}} | K^0 \rangle$, the coefficients $z_{\pm}^{\Gamma_1\Gamma_2}/z_{\pm}^{LL}$ are determined by solving

$$\sigma_{\pm} p \cdot p = \sum_{\Gamma_1\Gamma_2} z_{\pm}^{\Gamma_1\Gamma_2} \langle (O_{\pm}^{uu}[\Gamma_1\Gamma_2])^{\text{latt}} \rangle_{\text{fig8}}. \quad (21)$$

If one neglects mixing with the color magnetic-moment operator, $z_{\pm}^{\Gamma_1\Gamma_2}/z_{\pm}^{LL}$ for the eye graphs

Figure 5: (eye graphs)/ $z_{\pm}^{LL}$ at various stages. Step 1 is the raw lattice matrix elements $\langle O_{\pm}^{uu} \rangle_{\text{eye}}$; step 2 is with $\bar{s}d$ subtraction only; step 3 is if WCPT values of $z_{\pm}^{\Gamma_1\Gamma_2}/z_{\pm}^{LL}$ are used to complete Eq. (17); step 4 is if CPT behavior is enforced nonperturbatively using the figure-8 graphs. The broad horizontal line is actually a pair of lines marking $(\langle O_{+}^{uu} \rangle_{\text{fig8}} + \sigma)$ and $(\langle O_{+}^{uu} \rangle_{\text{fig8}} - \sigma)$. Since the eye-graph errors are much larger than $\langle O_{+}^{uu} \rangle_{\text{fig8}} \pm \sigma$, the $R_{1,2}^{\pm}$ values are washed out.

are the same as for the figure 8's. Alternatively, one can try just using the perturbative (weak coupling) coefficients.^{14, 17} Figure 5 depicts the answers from both techniques. Clearly much better statistics are needed before one can adequately test this approach.

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