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Applications of Bent Cylindrical Mirrors to X-Ray Beamlines*

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ABSTRACT

Bent cylindrical mirrors are considered as substitutes for paraboloidal and ellipsoidal mirrors in x-ray beamlines. Analytic and raytracing studies are used to compare their optical performance with the corresponding ideal elements. Particular emphasis is placed on obtaining the practical limitations in the application of bent cylinders to typical beamline configurations.

I. Introduction

For high reflectivity at x-ray energies, mirrors must operate at extreme grazing angles. This results in very long mirrors. Table I gives the mirror lengths needed to collect a horizontal divergence W_h for three common figures. These formulas assume no vertical extent to the beam. Thus, for a real beam a factor W_v/θ must be added to these formulas where W_v is the vertical width of the beam at the mirror and θ is the glancing angle. For typical applications, mirror lengths can exceed 1 m. Since paraboloids and ellipsoids can be difficult and expensive to produce in such lengths, this paper examines the possibilities of approximating these ideal elements with bent flat and cylindrical mirrors.

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Most existing x-ray beamlines have used bent optical elements.⁽¹⁻⁴⁾ Ellipsoids are approximated by bent cylindrical mirrors and focusing in one dimension is accomplished by bending flat mirrors. The flats are usually bent in segments to minimize the difficulty of bending and supporting a long plate. It has also been shown that a plate can be bent to parabolic or elliptical cylinders with high accuracy by an appropriate contouring prior to bending.⁽⁵⁻⁷⁾ Since either of these techniques is capable of high accuracy, this paper will concentrate on bent cylindrical approximations to the more complicated paraboloids and ellipsoids.

II. Single Cylinders

The geometry of a cylindrical mirror is shown in Fig. 1a. To approximate an ellipsoid or paraboloid the cylinder is bent to a large radius R_m along its length. For an ellipsoid

$$R_s = \frac{2F_1 F_2}{F_1 + F_2} \sin \theta \quad R_m = \frac{R_s}{\sin^2 \theta}$$

A collimating paraboloid is the limit as F_2 goes to infinity for which R_s becomes $2F_1 \sin \theta$.

In order to evaluate the performance of the approximate optical components, it is important to understand the limitations of their ideal counterparts. This is discussed in detail in reference 8 and only the relevant results are given here. For ellipsoids, the important feature is the quality of focus. The principal aberration is coma which causes an off axis point to be imaged as a circular arc of length $4M \phi d$ where

$$\sin \phi = \frac{M+1}{2m} \frac{W_h}{2\theta}, \quad M = F_2/F_1$$

and d is the distance off axis. For paraboloids, coma can be made negligible, and the important quality is the degree of collimation which is ideally zero for a point source. For real sources, the vertical source height gives a contribution $\Delta\theta_v = S_v/F_1$, where S_v is the vertical source size. For rays with horizontal divergence $\pm \frac{W_h}{2}$ the horizontal source size, S_h , gives a contribution $\Delta\theta_p = W_h S_h / 2F_1\theta$ for paraboloids. To find the actual distribution for the output angle, these two terms must be averaged over S_v , S_h and W_h for the source being considered.

The simplest approximation to these figures is an unbent cylinder. However, since it is unbent a cylinder provides no collimation in the vertical and is useless as a paraboloid. A short section of a cylinder is a better approximation to an $M=1$ ellipsoid. Again there is no vertical focusing, but a point source with horizontal divergence $\pm \frac{W_h}{2}$ will be focused to a "Vee" with dimensions

$$\begin{aligned}\Delta x &= \pm \frac{W_h^3 F_1}{8\theta^2} \\ \Delta y &= \frac{W_h^2 F_1}{4\theta}\end{aligned}\tag{1}$$

In most cases Δx is small and the limitation on the cylinder approximation to an ellipsoid is the amount of vertical spread which can be tolerated.

If the cylinder is bent, much better results are obtained. For a cylinder bent to a toroidal figure, the results in equation (1) become,⁽⁸⁾

$$\begin{aligned}\Delta x &= \pm \frac{5}{8} \frac{W_h^2}{F_1} \\ \Delta y &= - \frac{2\theta R_h}{F_1}\end{aligned}\tag{2}$$

where l_h is given in Table I. In this case the vertical divergence is well focused and the spread at the focus comes from the horizontal divergence W_h accepted by the mirror. Typically, the spread in the focus is unacceptable when $W_h \sim \theta$, although this depends strongly on the application. Equations (1) and (2) are for the $M=1$ case. The more complicated expressions for $M \neq 1$ are given in reference 8.

As approximations to paraboloids, toroids are less successful, but they still may be usable in some cases. The main problem is imperfect collimation. Rays originating from the source with horizontal divergence $\pm W_h/2$ will have additional contribution to the angular spread given by,

$$\Delta\theta = \frac{W_h^2}{16\theta}$$

A practical restriction is $\Delta\theta < \Delta\theta_v = S_v/F_1$ which gives a restriction on the horizontal divergence of $W_h^2 < 16\theta S_v/F_1$. This is a few milliradians in most cases.

III. Segmented Mirrors

As optical design is pushed to collect large W_h , segmented mirrors become more attractive. They can provide large effective mirror lengths from less expensive and more available short segments. Fig. 2 shows two arrangements for approximating a long mirror from shorter bent cylindrical segments. In both cases the outer segments are lower in order to intercept the beam at the same distance from the source as the inner segment. The arrows indicate the direction of the applied bending moments. In Fig. 2(b) the segments are bent as a unit. This has the advantage of simplicity, but the aberrations are the same as a single bent cylinder with the same collecting power.

A better design is shown in Fig. 2(c). In this case, each is bent separately. The direction of the bending moments all point to a common center a distance R_g above the center mirror. Also, the outer segments must be made slightly non-parallel to the center segment in order to bring all the foci into coincidence. Obviously such a mirror is more difficult to align. However, with exception of the bend, all of the alignments can be done optically prior to installation in the beamline. The aberrations are much reduced, and are determined for each segment by the horizontal divergence collected by it alone. For equal length segments, the center segment collects the most milliradians and, thus, dominates the aberrations. For bent cylinders, these are strong functions of W_h , and are, therefore, greatly reduced by segmentation.

Table II compares the performance of an ideal ellipsoidal mirror with two approximations: a single toroidal mirror and a three segment toroidal mirror with the segments bent separately. The design parameters appropriate to the NSLS x-ray ring source are used, $2\sigma_x = .75$ mm and $2\sigma_y = .25$ mm. This result shows the significant length reduction and improved performance which can be achieved using a segmented cylinder.

IV. Conclusions

This paper has shown the limitations in applying bent cylindrical mirrors to x-ray beamlines. These limitations are fairly severe for approximations to paraboloids, and bent cylinders are likely to have only limited use in this application. For focusing applications, bent cylinders are good approximations when $M \sim 1$. Typically a horizontal divergence equal to the grazing angle can be collected without serious aberrations. A larger collection angle can be achieved by using segmented mirrors to approximate very long mirrors. In this

case, the limitation is not the optical aberrations, but rather the increased mechanical complexity involved in mounting and bending a number of segments. However, much of this can be done optically prior to mounting in the beamline.

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Table I

Expressions for the mirror length l_h , necessary to collect a horizontal divergence, W_h . The vertical divergence is assumed to be zero.

Surface	l_h
Paraboloid	$\frac{W_h^2 F_1}{16\theta^2 + W_h^2}$
Ellipsoid	$\frac{M + 1}{2M} \frac{W_h^2 F_1}{8\theta^2 + W_h^2}$
Cylinder	$\left\{ W_h^2 F_1 + 4R_s \theta \left[1 - \sqrt{1 - \frac{F_1 W_h^2}{2R_s \theta} - \frac{F_1^2 W_h^2}{4R_s^2}} \right] \right\} / (4\theta^2 + W_h^2)$

Table II

Comparison of bent cylindrical and segmented bent cylindrical approximations to an ellipsoidal focusing mirror. The mirror parameters are $\theta = 5$ mr, $F_1 = 10$ m and $M = 1$. The source parameters are $W_h = 5$ mr, $2\sigma_x = 0.75$ mm, $2\sigma_y = 0.25$ mm and $E = 10$ keV.

	L	Focus:	
Ellipsoid	1447mm	.67mm	.45mm
Bent Cylinder	1450	1.03	.67
Segmented Bent Cylinder (3 segments)	828	.84	.35

FIGURE 1

- (a) Geometry of toroidal mirror at grazing incidence. For paraboloids and ellipsoids R_s varies along the mirror. The radius R_m is very large and not shown in the drawings.
- (b) Output of the mirror produced by a horizontal plane of radiation.

FIGURE 2

Two types of segmented mirrors: (a) Shows the intersection of the horizontal plane of radiation with the mirrors. The arrows mark the bending axes. In (b) all three segments are bent about the same axis while in (c) they are bent separately.