Pattern recognition, inner products and
correlation filters

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ABSTRACT

In this paper, we review correlation filters as an approach to pattern recognition with a special emphasis on the consequences of normalizing the correlation to achieve intensity invariance. Intensity invariance is effected using the Cauchy-Schwarz inequality to normalize the correlation integral. We discuss the implications of this criterion for the application of correlation filters to the pattern recognition problem. It is shown that normalized phase-only and synthetic discriminate functions do not provide the recognition/discrimination obtained with the classical matched filter.

INTRODUCTION

Pattern recognition is an extremely complicated field. At present, a general solution to the problem of recognizing an arbitrary object in an arbitrary background does not exist. Many approaches to the problem have been suggested and have been or are continuing to be investigated.

The evaluation of a correlation integral is a popular approach to optical pattern recognition.¹ The volume of literature on the subject has become quite large and it would not be appropriate to attempt a complete survey of the literature in this paper. However, the references in the papers cited by this paper offer an extensive coverage of the field. For those new to the field, the review paper by Flannery and Horner¹ provides an excellent introduction.

In 1964, Vander Lugt² proposed a Fourier optical system for pattern recognition that was capable of implementing the matched filter (autocorrelation). The Fourier-plane filter was recorded on photographic film. Primarily for hardware reasons, Horner and Gianino³ introduced the phase-only filter (POF). Further research has suggested the use of binary phase-only filters (BPOFs),⁴⁻⁶ ternary valued filters⁷⁻¹⁰ quad-phase-only filters (QPOFs)^{10,11} and complex ternary matched filters (CTMFs).¹²

Considerable research effort has been directed toward the analysis of the performance of these partial information filters.¹³⁻²⁴ A major portion of the analysis has been directed toward the evaluation of filter performance with respect to stochastic noise. Significant attention has also been given to the sharpness of the peak of the correlation response for these filters. There has been less analytical effort in the area of filter performance with respect to pattern recognition or discrimination. This is due mainly to the difficult nature of the pattern recognition problem.

To address this problem, we assume that the measurement of the angle between the vectors in a Hilbert (function) space is axiomatic to the correlation approach to pattern recognition. In fact, we believe that it is a tacit assumption in most of the papers on the subject. The angle measurement assumption requires that the correlation integral be normalized. Normalization effects intensity invariance which is inherently different from other invariances such as scale or rotation invariance.

Equivalently, intensity invariance is a requirement of pattern recognition. Normalization of the correlation integral to obtain intensity invariance implies the measurement of the angle between

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vectors in a function space. Intensity invariance is a criterion that allows for the comparison of correlation filters.

In the next section, we briefly discuss filter performance criteria. The normalization of the correlation function is discussed in Section 3. Applications of the theory to filter performance with respect to pattern recognition are discussed in Section 4. A summary of the work is given in Section 5.

2. FILTER PERFORMANCE CRITERIA

In this section, we briefly discuss various correlation filter performance criteria from the standpoint of the pattern recognition problem. That is, we attempt to assess their significance with respect to pattern recognition.

2.1 SIGNAL-TO-NOISE RATIO

If $c(x)$ is the output of the correlation filter, the stochastic signal-to-noise ratio (SNR) is appropriately defined by

$$\text{SNR} = \frac{\left| E\{c(x_p)\} \right|^2}{\text{var} \{c(x_p)\}}, \quad (1)$$

where $E\{ \}$ and $\text{var}\{ \}$ denote the ensemble average and the variance, respectively. In Eq.(1), x_p is the point at which the correlation response has its peak value in the absence of noise. For many correlation filters, the peak output occurs at the origin; however, that is not always the case.¹⁷ When Eq. (1) is evaluated at the origin ($x_p = 0$), the SNR in the frequency domain can be shown to be given by

$$\text{SNR} = \frac{\left| \int F(\nu) H(\nu) d\nu \right|^2}{\int P_n(\nu) |H(\nu)|^2 d\nu}, \quad (2)$$

where upper case letters denote Fourier transforms, $f(x)$ is the input function, $h(x)$ is the filter function and $P_n(\nu)$ is the noise power spectral density. In obtaining Eq. (2), additive, stationary ergodic noise was assumed. It is well known that the SNR given by Eq. (2) is maximized by the matched filter

$$H_{mf}(\nu) = \frac{F^*(\nu)}{P_n(\nu)}. \quad (3)$$

Clearly, any filter (linear or nonlinear) should exhibit reasonable noise performance since noise or uncertainty exists to some degree in all systems. In fact, noise is a major factor that limits the performance of any system. For example, a sensor's maximum operating range is frequently limited by noise. The noise performance of correlation filters is well treated in the literature.^{8,12,14-18,24}

There are those that would argue that noise need not be considered when evaluating correlation filter performance with respect to pattern recognition. A counter argument is that if noise is not to be considered, we do not need to consider correlation filters of the type frequently suggested. For example, recognition/discrimination might be achieved by evaluating a select number of points in the frequency domain. The zeros of the Fourier Transform might be used since they characterize objects of finite support (extent).²⁵⁻²⁷ A relatively small number of points would need to be considered if the number of objects to be discriminated is small.

The question of noise tolerance is also encountered when addressing object location or tracking. This is discussed in the following section.

2.2 Peak sharpness

Peak sharpness is another measure that is often suggested as an attribute of correlation filters. A suitably broad and analytically tractable measure of peak sharpness is the peak-to-sidelobe ratio given by ¹⁹

$$\alpha = \frac{\int_{-I/2}^{I/2} |g(x)|^2 dx}{\int_{-\infty}^{\infty} |g(x)|^2 dx}, \quad (4)$$

where $g(x)$ is the correlation given by $G(\nu) = H(\nu)F(\nu)$ in the Fourier-transform domain and I is a resolution interval in the correlation plane. A related measure defines the peak to correlation energy (PCE)²⁸ as

$$\text{PCE} = \frac{|g(0)|^2}{\int |g(x)|^2 dx}, \quad (5)$$

this is clearly obtained by dividing Eq. (4) by I and letting I approach zero.

The peak sharpness of the POF has received considerable analytical attention. It has been shown²⁰ that the PCE of the POF is equal to or greater than that of the matched filter. It was also shown that when the peak-to-sidelobe ratio of Eq.(4) is used, the matched filter can give ratios greater than for the POF. Further, it has been shown¹⁹ that the POF is the unit modulus filter that simultaneously optimizes the SNR of Eq. (2) for white noise and the peak-to-sidelobe ratio of Eq. (4).

For reference, it should be pointed out that the band limited function that maximizes Eq. (4) is given by¹⁹

$$g(x) = \Psi_0(x, c), \quad (6)$$

where $c = \pi\nu_o I/2$, ν_o is the bandwidth, and Ψ_o is the zero-indexed prolate spheroidal wave function. By using the Fourier-transform properties of $\Psi_o(x,c)$, it can be shown that the filter maximizing Eq. (4) for band-limited input function $f(x)$ is

$$H(\nu) = \begin{cases} \frac{\Psi_o[I(\nu/\nu_o), c]}{F(\nu)} & \text{for } |\nu| \leq \nu_o/2 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Note that this is simply an inverse filter multiplied by Ψ_o and includes the phase of the matched filter. Inverse filters can be very sensitive to noise.

Despite these theoretical results, there does not appear to be an inherent reason to use peak sharpness as a measure of performance with respect to pattern recognition. Peak sharpness is essentially a resolution criterion. It is not clear that resolution is as appropriate to a correlation plane as it generally is in an image plane. It has been argued that peak sharpness is needed to prevent weak targets from being lost in the sidelobes of strong targets. However, the problem of strong targets masking weak targets is essentially an intensity invariance (normalization) problem.

Another attribute of correlation filters that is associated with pattern recognition is the ability to locate recognized objects accurately. It has been assumed that better location accuracies are possible if the correlation peaks are sharper. It turns out that location accuracy does not correspond to peak sharpness. It has been shown²⁴ that the peak location error (PLE) for correlation filters is minimized by the matched filter. In formulating the PLE problem, random noise was taken to be the source of the location error. The only restrictions on the noise process were that it be stationary and ergodic. The peak of any function can be located with arbitrary precision if there is no noise or uncertainty involved in the process.

2.3 Recognition/discrimination

The pattern recognition problem can be said to be one of discriminating between different classes of objects or signals. Frequently the problem is one of recognition (detection) of a specific object (signal) in the presence of all other possible objects (signals). For example, the detection of a specific vehicle type in the presence of other vehicle types, terrain features and various man-made objects. This is a very difficult problem and a general solution may not be achievable. The major problem is one of suitably defining "all other objects." The problem is considerably simplified when the number of object to be discriminated is small. In the case of a small number of well-defined objects, there may be several relatively simple approaches to the problem.

The interest in correlation filters is, in the main, associated with the problem of recognizing an object in an arbitrary background. The problem is, then, to determine a recognition system that will cause output values for all inputs that are to be recognized as the object of interest to fall within a specified (or determined) range. For all other objects the output values fall outside this range. This may sound simple, but the problem is one of defining all inputs that are equivalent to the object of interest. For example, how big must the tail on the letter "Q" be before it is

discriminated from the letter "O"; is a white circle (or other shapes) contained in a white square?

The intensity invariant, normalized correlation discussed in the next section can map the inputs into a suitable range of outputs. The problem remains to define for a specific problem the degree of acceptable variation among the possible inputs that are to be classed as the same object. That is, what range of viewing angles, amplitude distortions, and geometrical distortion can the system (filter) accommodate?

3. NORMALIZATION

In general, the literature on the technique of using correlation filters for optical pattern recognition assumes or implies that pattern recognition can be effected by computing the magnitude of the correlation between the interrogated object and a reference (filter) function. However, a necessary requirement for this assumption to be valid is that the correlation integral be normalized to achieve intensity (signal level) invariance.

The treatment of normalization and intensity invariance in the literature is relatively sparse. Goodman²⁹ suggested the normalized matched filter as a means of achieving character recognition. Duda and Hart³⁰ suggested normalization to effect intensity invariance. As discussed in the previous paragraph, within the context of this paper, normalization is essential to the problem of pattern recognition using correlation filters. Intensity invariance is inherently different from other invariances such as rotation, scale and distortion invariance. Invariance to these types of variations can be accomplished by searching through a suitable set of filters; this is clearly not the case for intensity invariance. The authors²² have used the normalization requirement to address the pattern recognition performance of correlation filters.

The use of correlation filters in pattern recognition is essentially an inner product between two functions: the object and reference functions. These functions may be considered vectors in a Hilbert space.³¹ The normalization of the inner product (correlation integral) defines a unique angle between the reference and the object function. It is this angle that provides a measure of similarity between the object and reference functions.

The normally computed correlation function is

$$c(y-x_o) = \left| \int h^*(x) s_h(x) f(x+y-x_o) dx \right|^2 \quad (8)$$

where

$f(x)$ = input(object) function,

$h^*(-x)$ = filter impulse response,

$$s_h(x) = \begin{cases} 1, & x \in \text{support of } h(x), \\ 0, & \text{otherwise,} \end{cases}$$

x_o = coordinate that maximizes the integral.

In what follows, we will frequently use

$$s_h(x)h(x) = h(x), \quad (9)$$

$$s_h^2(x) = s_h(x). \quad (10)$$

Applying the Cauchy-Schwarz inequality to Eq.(8) gives

$$c(y-x_o) \leq \int \left| f(x+y-x_o) \right|^2 s_h(x) dx \int \left| h(x) \right|^2 dx. \quad (11)$$

The preceding suggests a normalized correlation function given by

$$\hat{c}(y-x_o) = \frac{\left| \int h^*(x) f(x+y-x_o) dx \right|^2}{\int \left| f(x+y-x_o) \right|^2 s_h(x) dx \int \left| h(x) \right|^2 dx} \leq 1. \quad (12)$$

It is a further property of the Cauchy-Schwarz inequality that the equality in Eq. (12) is obtained at $y = x_o = 0$ if and only if

$$h(x) = \lambda f(x) s_h(x). \quad (13)$$

For the case $y = x_o$, Eq. (12) is equivalent to

$$\hat{c}(0) = \cos^2(\theta) \quad (14)$$

where θ is the Hilbert space angle between $h(x)$ and the restriction of $f(x)$ to the support of $h(x)$. It is interesting to note that the normalized inner product has been proposed³² as a measure of similarity between vectors in a vector approach to automatic text retrieval.

A slight variation in the definition of $s_h(x)$ is possible that can be useful in the recognition of objects with zero regions in their interior. In this case, $s_h(x)$ may equal one anywhere in the interior of the object; however, Eq. (9) is still valid. This variation is, perhaps, most useful in the case of binary objects such as in character recognition. It allows for the distinction between a circle and an annulus, both having the same outer diameter.

4. APPLICATIONS OF THE THEORY

It is assumed in this paper, as it is tacitly assumed in the literature, that a correlation system can achieve pattern recognition by virtue of the correlation peak output. That is, an object's presence is determined by whether or not the peak correlation response falls within a

predetermined range. This assumption clearly requires a normalization of the correlation response to achieve intensity invariance.

The requirement of intensity invariance allows for the comparison of the pattern recognition/discrimination capability of various correlation filters. In the following subsections, we use the normalized correlation given by Eq. (12) to evaluate correlation filters and techniques.

4.1 The matched filter

It is the form of Eq. (12) and the λ in Eq. (13) that effects intensity invariance. Thus it is the classical matched filter associated with white noise that maximizes the normalized correlation given by Eq. (12). If the filter used to identify an object function $f(x)$ is not the matched filter, a normalized correlation value less than one is obtained. In this case, there are many functions different from $f(x)$ that give the same correlation value. The difference (e.g., mean square difference) between these functions and the object function must approach zero as the normalized correlation function approaches one (see section 4.4).

For any filter other than the matched filter, the normalized correlation is less than one when the object function for which the filter was made is the input function. Further, there is always a function that gives a normalized correlation value of one. This function is just the filter impulse response. For an arbitrary filter, there are many functions that produce a correlation value equal to or greater than that produced by the object function.

Unless there are additional constraints on the set of admissible functions (e.g., limited character recognition), the preceding suggests that the matched filter provides the best recognition/discrimination performance. This does not exclude the use of preprocessing to enhance the system recognition/discrimination performance. Preprocessing is used to define the objects or features of the objects to be recognized. The correlation filter should then be designed to address the preprocessed image set. In no case is it conceivable that a correlation filter should be designed to produce a small normalized correlation response.

These ideas are readily illustrated by a few simple examples. For the object of interest, we use a model of a tank. The impulse response of the matched filter (reference function) and its support function are illustrated in Fig. 1a and Fig. 1b, respectively. Test image 1 and its associated

Fig. 1. Matched filter for model tank: (a) Impulse response, (b) Support function.

intensity variations are shown in Fig. 2a. Test image 2 in Fig. 2b includes simple false targets as well as intensity variations. Profile locations are given by the triangle marker to the left of the figures.

Fig. 2. Test images and intensity profiles: (a) Test image 1, (b) Test image 2.

Correlation of the matched filter function with test image 1 produces the results shown in Fig. 3a. The dependence on input scene intensity is apparent in the profile on the right of the figure. After normalization, all four objects produce the maximum response of Fig. 3b.

Fig. 3 Matched filter correlation with test image 1: (a) Unnormalized, (b) Normalized.

Unnormalized correlation of the matched filter function with test image 2, shown in Fig. 4a., illustrates a false object having a much stronger response than the autocorrelation. Normalization, Fig. 4b, results in the two tank images giving maximal responses. Although giving a strong response, the two false objects' responses are equalized and do not equal or exceed those of the tanks.

Fig. 4 Matched filter correlation with test image 2: (a) Unnormalized, (b) Normalized

Frequently authors illustrate their work with binary images. It is interesting to note that for binary images, the matched filter correlation of Eq. (8) is effectively normalized. In this case we have

$$\left| f(x) \right|^2 = f(x) \quad (15)$$

and

$$h(x) = s_h(x) \quad (16)$$

Applying Eq. (15) and (16) to Eq. (12) gives

$$\hat{c}(y-x_o) = \frac{\int s_h(x) f(x+y-x_o) dx}{\int s_h(x) dx}, \quad (17)$$

In this equation, the numerator is just the square root of Eq. (8) and the denominator is a constant; that is

$$\hat{c}(y-x_o) = \frac{\{c(y-x_o)\}^{1/2}}{K} \quad (18)$$

where K is a constant

4.2 The phase-only filter

Phase-only type filters are, perhaps, best treated in the frequency domain. Applying Parseval's relation to Eq. (12) gives

$$\hat{c}(y-x_o) = \frac{\left| \int H^*(\nu) F(\nu) e^{i2\pi\nu(y-x_o)} d\nu \right|^2}{\int [F(\nu) \odot F(\nu)]^* S_h(\nu) e^{-i2\pi\nu(y-x_o)} d\nu \int |H(\nu)|^2 d\nu}, \quad (19)$$

where $\odot$ denotes correlation.

For object functions of finite extent and the POF defined by

$$H^*(\nu) = \frac{F^*(\nu)}{|F(\nu)|}, \quad \nu \in \Omega \quad (20)$$

where Ω is the support of the POF, the peak value of Eq. 7 is given by

$$\hat{c}(o) = \frac{\left[\int_{\Omega} |F(\nu)| d\nu \right]^2}{B \int_{\Omega} [F(\nu) \odot F(\nu)]^* S_h(\nu) d\nu}. \quad (21)$$

In this expression B is given by

$$B = \int_{\Omega} d\nu . \quad (22)$$

and provides a measure of the spectral content (bandwidth) of the filter.

For bounded square integrable functions³³ $f(x)$,

$$\left| F(\nu) \right| = O(1/\nu) \quad \text{as} \quad \nu \rightarrow \pm \infty . \quad (23)$$

If we apply Eq. (23) to Eq. (21) we get the limit

$$\hat{c}(0) \rightarrow 0 \quad \text{as} \quad B \rightarrow \infty \quad (24)$$

This is just the condition that allows extremely large values for the unnormalized correlation of Eq. (8). For discontinuous functions, the numerator of Eq. (21) diverges as B increases.

The discussion in Sec. 4.1, and the limits expressed in Eq. (24) suggests that the spectral support (bandwidth) for POFs should not be arbitrarily large if discrimination is considered. Equivalently, the unnormalized correlation response should not be made arbitrarily large. This is clearly the case when random noise is considered.

A further, related, complication for phase-only filters is the determination of the filter support. For a general filter, the support of the filter impulse response is not necessarily the support of the object from which the filter was made. In the case of the matched filter, the object and the filter impulse response are identical and the support function $s_h(x)$ is readily determined. However, the POF's impulse response is generally quite different from the object from which it is made.

A major problem is that the POF impulse response extends well beyond the limits of the original object.²³ This is illustrated in Fig. 5. The impulse response of the POF for the model tank is shown in Fig. 5a and the impulse of a BPOF is shown in Fig. 5b.

Fig. 5 Filter impulse response: (a) POF, (b) BPOF, α indicates the width of the original object.

There is a problem with limiting the support of the impulse response of a POF. The problem is that truncating the support of the POF impulse response, generally, results in a function whose Fourier transform is not unit modulus; that is, not phase only. Not only does the POF impulse response have little resemblance to the original object, it also occupies a large fraction of the

entire field. In effect, the local energy function is degraded toward a global energy function, allowing relatively distant objects in the scene to influence the correlation results.

4.3 Synthetic discriminant functions

Synthetic discriminant functions³⁴ (SDFs) are filter functions designed to give a specified response for a set of images. A typical set of images, commonly called the training set, is a set of images of an object made from different rotations (orientations) of the object. One application of SDFs is to achieve rotation (orientation) invariance in optical correlators used for pattern recognition.

It is instructive to consider the SDF for the training set of $f_i(x)$ that is constrained by

$$\left| \int h^*(x) f_i(x) dx \right|^2 = 1 \quad (25)$$

and

$$\int f_i^*(x) f_j(x) dx = \delta_{ij} \int |f_i(x)|^2 dx \quad (26)$$

for $1 \leq i, j \leq N$. For square integrable functions, $h(x)$ can be written as

$$h(x) = \sum_i \alpha_i \frac{f_i(x)}{\|f_i(x)\|} + \phi(x) \quad (27)$$

where $\int f_i(x) \phi(x) dx = 0$, $|\alpha_i| = 1$ and $\|f(x)\|^2 = \int |f(x)|^2 dx$.

Then for the element $f_i(x)$ of the training set as the input, the peak normalized correlation response determined by Eq. (12) for any element of the training set is

$$\begin{aligned} \hat{c}(o) &= \frac{1}{\left[\sum_{k=1}^N \frac{1}{\|f_k(x)\|^2} + \|\phi(x)\|^2 \right] \|f_i(x)\|^2} \\ &\leq \frac{1}{N} \max_{k,j} \frac{\|f_j(x)\|^2}{\|f_k(x)\|^2} \end{aligned} \quad (28)$$

If the training set consists of in-plane rotations, the right hand side of the above inequality is just $1/N$. Note that some undesired function (the matched function) will give a maximum value of one.

The result is that SDFs made for training sets with a large number of elements produce a correspondingly small normalized correlation. This suggests, as in the case for POFs, that recognition/discrimination is being degraded as N increases.

4.4 Integral squared difference

A related measure of similarity between functions in a Hilbert space that can be used for pattern recognition is the integral of the squared difference of the functions. For the same reasons, this measure should be intensity invariant. The normalized (intensity invariant) integral squared difference (ISD) is given by

$$I_{SD} = \int \left| \frac{f(x) s_h(x+y)}{||f(x) s_h(x+y)||} - \frac{h(x+y)}{||h(x+y)||} \right|^2 dx \leq 4, \quad (29)$$

where the function in the integrand are defined as before and the double bar brackets are defined following Eq. (27). The inequality in Eq. (29) is obtained by application of the triangle inequality. The minimum value of Eq. (29) is obtained if and only if

$$f(x) s_h(x+y) = \lambda h(x+y) \quad (30)$$

for any for positive λ and some y . It might be noted that the maximum value is obtained for negative λ .

Equation (29) may be expanded to give

$$I_{SD} = 2 - 2 \operatorname{Re} \int \frac{f^*(x) h(x+y) dx}{||f(x) s_h(x+y)|| ||h(x)||} . \quad (31)$$

When $f(x)$ and $h(x)$ are real and positive, Eq. (32) reduces to

$$\begin{aligned} I_{SD} &= 2 - 2 \int \frac{f(x) h(x+y) dx}{||f(x) s_h(x+y)|| ||h(x)||} , \\ &= 2 - 2 \left[\hat{c}(y) \right]^{1/2} . \end{aligned} \quad (32)$$

Thus, in this case, computing the intensity invariant ISD is equivalent to computing the normalized correlation given by Eq. (12).

5. SUMMARY

We would like to reiterate the fact that, in this paper, we have used the requirement of intensity invariance to evaluate the correlation filter approach to optical pattern recognition. This leads to the assumption (which is implied in the literature) that correlation systems achieve pattern recognition by measuring the angle between vectors in a Hilbert (function) space. Subjects in the correlation filter literature that are not addressed in this paper are not included because they were not considered to have a direct bearing on the pattern recognition problem.

It was argued in Section 2 that good (if not optimum) noise performance is a desirable attribute of correlation filters for pattern recognition as well as any other filter application. It was also argued that peak sharpness is not as significant as it might first appear. The problem of weak targets being masked by the sidelobes of strong targets seems to be one of intensity invariance.

In Section 3 we presented the mathematical basis for correlation filter normalization using the Cauchy-Schwarz inequality. The normalization was applied to POFs and SDFs in Section 4. It was shown that normalized POFs and SDFs do not generally provide the recognition/discrimination capabilities of the matched filter and suffer from the fact that there are always other functions that give a larger correlation response than that of the object function. This can be worsened by poor design of the POF or SDF such as incorporating excessive bandwidth or too many elements in the training set. It was also shown for positive images that computing normalized integrated square difference to measure similarity between functions (objects) is equivalent to computing the normalized correlation between the same functions.

The analysis in this paper was presented in one-dimensional form; but, it is easily generalized to two dimensions.

In general, it appears that if there are no hardware constraints, the matched filter is the best correlation filter for pattern recognition in almost every respect. The unnormalized phase-only filter has an advantage over the matched filter in that it is more readily implemented using current opto-electronic technology. However, it is not clear that when the filter is normalized this advantage remains. This, perhaps, suggests the need for further hardware and architecture research and development. The concepts and results in this paper apply directly to joint transform correlators in as much as they are designed to implement a correlation operation.

Finally, it should be noted that normalized correlation is not a linear filter operation or functional as is unnormalized correlation.

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Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

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