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FLOW IN GEOTHERMAL WELLS:
PART IV. TRANSITION CRITERIA FOR
TWO-PHASE FLOW PATTERNS

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MASTER

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Part IV. Transition Criteria for
Two-phase Flow Patterns

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1. Introduction

The present report contains detailed considerations justifying the criteria for the transitions between flow patterns which we summarized in Section 3 of ref. [1]. The material presented here supplements the definition of the flow model given there.

2. Transition from bubble to plug (or slug) flow

In order to estimate the point of transition between bubble and plug (or slug) flow, we adopt the following physical model [2]. Bubble flow consists of an array of spherical gaseous elements which are dispersed over a liquid stream. The bubbles move upwards through the vertical channel with a velocity which exceeds that of the adjacent liquid. This relative slip causes a perturbation of the basic flow pattern and gives rise to a wake behind each bubble. The mean fluid velocity in each wake is also higher than that in the adjacent liquid. In some parts of the flow field there occur situations in which a bubble (say bubble number 1 in Fig. 1) follows another bubble (bubble number 2) and accelerates in the wake of number 1. This creates a tendency for bubbles to catch up with predecessors and to coalesce. If the distance, a , between two bubbles of which one trails the other exceeds a certain characteristic (or effective) length ℓ , the two cease to interact, and the tendency to coalesce is absent. For values of $a < \ell$ this tendency exists, bubbles coalesce and eventually form elongated, bullet-shaped, Taylor bubbles. From the practical point of view, the transition is not sharp, because both quantities, i.e. a and ℓ , are stochastic in nature and follow some kind of probability distribution. The model for the calculation of the effective length ℓ proposed by Bilicki [2] will be discussed presently.

Fig. 1. Notations

In the model under consideration, it has been assumed that transition occurs when the statistical (or ensemble) averages $\bar{L}$ and $\bar{a}$ of the above two quantities become equal. It is clear that the absolute velocity of a bubble has an axial as well as a radial component. In our model the assumption is made that the radial displacement of a bubble does not lead to coalescence and that only the axial component counts.

We begin our analysis by considering a single spherical body which is dragged through a stationary fluid with a velocity U_∞ thus creating a wake behind it, Fig. 2. We assume that the velocity distribution $u(z, r)$ in the wake is adequately described by Prandtl's theory of turbulent wakes [3-5]. Essentially, Prandtl's theory amounts to a dimensional analysis which is based on several well-proven physical hypotheses. This leads to the conclusion that the velocity $u_1(z, r)$ in the wake measured relative to the moving sphere,

$$u_1 = U_\infty - u \quad (1)$$

is proportional to the group

$$u_1 \sim U_\infty \left(\frac{C_D d^2}{\beta^2 z^2} \right)^{1/3} \quad (2)$$

Here C_D is the drag of the sphere and β is an unspecified constant of proportionality between the mixing length of the flow and the diametral width of the jet. The coefficient of proportionality between u_1 and the group depends on r , and assumes some unspecified value when u_1 is measured along the flow axis, z .

In turn, it is supposed that the relative wake velocity, u_1 , is proportional to the local friction velocity

$$v_* = (\tau_w / \rho)^{1/2}, \quad (3)$$

Fig. 2. Sphere dragged with velocity U_∞ through fluid at rest. (Wake relative to liquid when sphere moves with slip)

the implication being that it is related to the correlated turbulent fluctuations

$$\tau_w \sim -\rho \overline{u'v'} \quad (4)$$

at the wall. Hence

$$u_1 \sim v_* \quad \text{implies} \quad U_\infty \left(\frac{C_D d^2}{\beta_z^2} \right)^{1/3} \sim \left(\frac{\tau_w}{\rho} \right)^{1/2} \quad (5)$$

If a bubble is aligned with the wake of a preceding bubble along the flow axis, it will be drawn in if $u_1 < U_\infty$, and the attraction will cease for $u_1 \approx U_\infty$ sufficiently closely. The length ℓ which corresponds to this situation, Fig. 3, will satisfy the dimensionally established eqn. (5), except that a new coefficient, A_1 , to be determined experimentally, will appear in it. Hence the limiting ratio ℓ/d can be obtained from

$$A_1^{2/3} \left(\frac{C_D d^2}{\ell^2} \right)^{1/3} U_\infty = v_* \quad (6)$$

or

$$\frac{\ell}{d} = A_1 \frac{C_D^{1/2} \rho^{3/4} U_\infty^{3/2}}{\tau_w^{3/4}} \quad (7)$$

We assume that this ratio adequately approximates the ratio of stochastic averages $\bar{\ell}/\bar{d}$.

The preceding relations apply to wakes formed behind solid spheres in homogeneous fluids. In the case of two-phase flow, the assumption is made that the same relations remain valid, except for the assignment of numerical values and a modified interpretation of symbols. First, we assume a constant value

$$C_D = 0.45, \quad (8)$$

noting that actually the product $A_1 C_D$ is determined by experiment rendering

Fig. 3. Conditions for presence and absence of coalescence of two spheres.

the choice of the above numerical value largely irrelevant. For the wall-shearing stress, we accept the proposal made by Nakariakov and Armand [6], according to which in bubble flow

$$\tau_w = (1 - \phi)^{-m} \tau_{sp}, \quad (9)$$

with the single-phase value

$$\tau_{sp} = \frac{1}{2} f \rho' w'^2 \quad (10)$$

and

$$f = 0.023 (w' D / \nu')^{-0.2}, \quad (11)$$

according to the Darcy-Weissbach correlation [4]. Here

$$m = 1.53 + 3.5 \exp(-w'_0{}^{1.5}/R), \quad (12)$$

and

$$R = 0.05 (D/d)^{0.75}, \quad (12a)$$

according to the correlations established by Nakariakov and Armand [6] as already stated.[#]

The reinterpretation of symbols in eqn. [7] starts with the recognition that U_∞ represents the slip velocity $\Delta = w'' - w'$, eqn. (2) of [7]. Similarly, ρ is the density of the liquid, $\rho \equiv \rho'$.

The coefficient A_1 has been determined by extensive experiments performed by Bilicki [2]. This was done by probing radially the velocity field in the wake behind a solid cylinder with the aid of a hot-wire anemometer. The distance ℓ was determined by the criterion that the velocity fluctuations

[#] Equation (12) is quoted here in its original form even though the argument of the exponential function is not dimensionless, as it should be. Here w'_0 should be expressed in m/s. The situation can be remedied by writing

$$R = w^{*1.5} (D/d)^{0.75} \quad (12b)$$

$$\text{with } w^* = 0.136 \text{ m/s} \quad (12c)$$

so that

$$m = 1.53 + 3.5 \exp \left[- \left(\frac{w'_0}{w^*} \right)^{1.5} \left(\frac{d}{D} \right)^{0.75} \right]. \quad (12d)$$

in the traverse became equal to those external to the wake and the traverse failed to detect the existence of a wake. The factor turned out to be constant within a confidence level of 95%, and its recommended value is

$$A_1 = 0.012 \pm 0.0024 \text{ (bubble flow) .} \quad (13)$$

The stochastic distance $\bar{a}$ between bubbles is calculated by assuming a regular array of bubbles and postulating that it must produce the given void fraction ϕ . The recommended array is shown in Fig. 4. In it the bubbles simply place themselves at the vertices of cubes. The ratio a/d for this and other (discarded) arrays was compared with extensive photographs from which the frequency of pairs of bubbles was plotted against the ratio a/d . The maximum of a standard distribution representing the experiments differed from the calculation based on the cubic model by 10%, whereas the standard deviation of the measurements was 25%. In the face of this result we recommend the correlation

$$\frac{\bar{a}}{d} = \frac{0.8}{\phi^{1/3}} - 1 \quad (14)$$

obtained by a geometric calculation based on the cubic array.

It follows that transition occurs when $\ell/d=a/d$, or when ϕ_{lim} is given by the implicit equation

$$\frac{0.8}{\phi_{lim}^{1/3}} - 1 = A_1 \frac{C_D^{1/2} \delta^{3/2}}{\tau_{sp}^{3/4}} (1 - \phi_{lim})^{3m/4} , \quad (15)$$

with

$$\phi < \phi_{lim} \quad (16)$$

signifying bubble flow.

Fig. 4. Array for the calculation of stochastic distance ratio $\bar{a}/\bar{d}$.

3. Transition from plug flow to froth flow

The diagram in Fig. 5 shows the configuration known as plug (or slug) flow in which bullet-like, vapor-filled "Taylor bubbles" move axially in a statistically distributed linear array, being separated by plugs of liquid. The four diagrams a-d represent qualitatively the velocity distributions in the four cross-sections (see also [8]). It should be noted that the thickness of the liquid layer adjacent to the wall, δ , can become quite thin thus bringing into play the effects of surface tension. Behind the Taylor bubble there forms an axially symmetric wake which has the same structure as the one behind a sphere discussed in Section 2, except for the value of the coefficient A_1 in eqn. (7) and the replacement of the bubble diameter, d , by that of the pipe, D . According to the measurements reported in [2] and [8], we recommend the value

$$A_2 = 0.01 \pm 0.003 . \quad (\text{plug flow}) \quad (17)$$

In addition, eqns. (12, 14) must be modified to read

$$\begin{aligned} m &= 1 , \\ \frac{\bar{a}}{d} &= 7 \left(\frac{1}{\phi} - 1 \right) . \end{aligned} \quad (18)$$

Here the assumption has been made that the most stable length of a Taylor bubble is

$$b = 7d \approx 7D , \quad (18a)$$

as proved by Govier and Aziz [9] as well as Bilicki [2].

It follows that eqn. (15) is now replaced by

$$7 \left(\frac{1}{\phi_{\text{lim}}} - 1 \right) = A_2 \frac{C_D^{1/2} \delta^{3/2}}{\tau_{\text{sp}}^{3/4}} (1 - \phi_{\text{lim}})^{3/4} , \quad (19)$$

with condition (16) retained.

A-A

B-B

C-C

D-D

Fig. 5. Axial velocity profiles behind a Taylor bubble.

Bearing in mind the structure of plug flow, it can be understood that transition to froth flow may occur by one of two mechanisms. The first is the same as that for the disappearance of bubble flow and is due to the coalescence of Taylor bubbles. The criterion for it is that of eqns. (19) with (16). The second cause of disappearance is the onset of instability in the layer of thickness δ . This we base on the theory of the stability of thin liquid films [10,11]. Absence of stability manifests itself in the disruption of the liquid film so that the gas phase fills the whole cross-section ("flooding") and slip no longer occurs.

The detailed theory is based on the assumptions that:

- (a) The flow in the film is potential
- (b) The perturbation amplitude is small
- (c) The inertial forces in the film may be neglected.

The equation of motion in the film is

$$\frac{\partial \Psi}{\partial x} = \frac{\partial \eta}{\partial t} - \frac{\partial \Phi}{\partial x} \frac{\partial \eta}{\partial x}, \quad (20)$$

in which Ψ , Φ are the stream and potential functions, respectively, and η denotes the perturbation amplitude.

Instability sets in due to the fact that the liquid film is excited externally by a flow of vapor of velocity w'' and causes it to assume a wavy form. Equation (20) possesses a real solution for η provided that

$$w'' + w'_b \leq w_{lim}, \quad (\text{algebraic sum}), \quad (21)$$

where

$$w_{lim} = \sqrt{\frac{2\pi}{10} \frac{\sigma}{\rho''\delta}} \approx \sqrt{\frac{\sigma}{\rho''\delta}}. \quad (22)$$

Here σ denotes the surface tension of water in the presence of steam and w'_b is the velocity of the descending film.

The velocity w'_b in the film follows from an equation of continuity. This is formulated with respect to the control volume shown in Fig. 6 on the assumption that the Taylor bubble retains its shape (no evaporation or condensation). Thus

$$(w'_o + w''_o)A = w'_b A_f + w''_b A_s . \quad (23)$$

Hence

$$w'_b = \frac{w''(1 - 2\delta/D)^2 - (w'_o + w''_o)}{4\delta/D(1 - \delta/D)} \quad (24)$$

with δ specified by Ozgu's [12] equation

$$\delta = 1.5 \sqrt{\frac{3\mu' w'_b}{\rho' g}} \quad (25)$$

which gives the thickness of a layer of fluid of viscosity μ' moving in potential flow in a vertical gravitational field. The factor 1.5 is empirical and accounts for departures from potential flow conditions.

The complex relation in eqn. (24) can be represented by the purely numerical correlation

$$w'_b = K_1 \ln(\phi/B) + K_2 , \quad (26)$$

with

$$K_1 = (3/D) (3\mu'/g\rho')^{1/2} , \quad (27)$$

$$K_2 = 5.5 (w'_o + w''_o); \quad R = 0 \quad \text{for} \quad w'_o + w''_o < 0.3 ; \quad (28a)$$

$$K_2 = 1.12 (w'_o + w''_o); \quad R = 1.06 \quad \text{for} \quad w'_o + w''_o > 0.3 . \quad (28b)$$

Transition to froth flow occurs for the lower value of ϕ_{lim} from eqns. (19) and (21).

$$(w_O' + w_O'')A = w_b' A_f + w'' A_s$$

Fig. 6. Control volume for the slug flow.

4. Transition from froth to annular mist flow

In order to determine the criterion for the transition from froth to annular mist flow, it is usual to consider fully developed annular flow and to determine the dynamic conditions under which the thin film of thickness δ can be sustained in its upward flow by the shearing stress τ_s produced by the faster moving vapor stream of velocity w'' , Fig. 7. The condition for annular flow to persist has been studied by Wallis [13] and Pushkin and Sorokin [14]. Dimensional analysis shows that the phenomenon is governed by the Kutateladze number

$$Ku = w'' \rho''^{1/2} [g\sigma(\rho' - \rho'')]^{-1/4} . \quad (29)$$

Annular mist flow is possible when

$$Ku > 3.2 , \quad (30)$$

and below that value flooding occurs and froth flow persists.

5. Comparison between present model and that proposed by Taitel and Dukler [15].

In a well-received report, Taitel and Dukler [15] independently proposed a set of criteria for the occurrence of transition between different flow patterns which are quite similar to those presented here, even though some differences exist. We thought that a comparison between the two proposals would prove useful for future applications.

Taitel and Dukler concentrate on essentially the same basic flow patterns, viz. bubble flow, slug flow, churn (or froth) flow and annular flow. In each case the criterion for transition was based on an assumed physical model backed by observations which helped to understand the physical mechanism which leads to transition.

Fig. 7. Illustration of annular and froth flow.

Transition from bubble to slug flow is conceived as a result of coalescence of bubbles at relatively low gas velocities. The steam bubbles are treated as if they were rigid spheres distributed uniformly in a pattern for which the maximum void fraction is 0.52. It is assumed in this model that coalescence starts when the mean distance between their surfaces becomes equal to half the bubble diameter. Thus, the conclusion is drawn that transition from bubble to slug flow occurs when the void fraction $\phi=0.25$. The relative velocity of the steam bubbles is assumed to obey Harmathy's equation (16), viz.

$$\Delta = w'' - w' = 1.53 \left[g(\rho'' - \rho') \sigma / \rho' \right]^{1/4} . \quad (31)$$

Taking into account eqns. (67) and (2) of [7] and putting $\phi=0.25$ and making use of eqn. (31), we obtain the following boundary for the transition:

$$w'_0 = 3w''_0 - 1.15 \left[g(\rho' - \rho'') \sigma / \rho' \right]^{1/4} , \quad (32)$$

conceived as a line in the w'_0, w''_0 diagram. The authors assume that for a superficial velocity $w_0 = w'_0 + w''_0$ which exceeds $w_{0, \text{crit}}$, bubble flow becomes turbulent characterized by "finely dispersed bubbles" up to $\phi=0.52$, where the flow becomes annular.

As a result of coalescence, the flow develops an array of large Taylor bubbles separated by liquid plugs. The plugs still contain dispersed bubbles in them. The velocity of displacement of a Taylor bubble has been assumed to satisfy the equation proposed by Nicklin et al. [17] viz.

$$w'' = 1.2w' + 0.35(gD)^{1/2} . \quad (33)$$

Furthermore, it is assumed that the liquid velocity in the plugs is the same as that of the Taylor bubbles.

The plug-flow pattern is said to lose its stability when two neighboring Taylor bubbles cease to interact, that is when the distance between them (according to Taitel and Dukler) is larger than $8D$. The latter criterion was adopted as a result of studies concerning the jet which forms behind a Taylor bubble performed by Moissis and Griffith [8] and Govier and Aziz [9].

Transition to unstable froth flow is the result of loss of stability of slug flow. Under certain conditions, when slug flow is created artificially by circumferential injection, bubble flow does not turn into slug flow immediately, but the latter develops over an entry length which then serves to establish stable flow. This creates an intermediate region of froth-like flow whose length ℓ_E is given by

$$\ell_E/D = 40.6 \left[\frac{w_o}{(gD)^{1/2}} + 0.22 \right] . \quad (34)$$

In this inlet length, the injected bubbles form short Taylor bubbles which then coalesce into stable Taylor bubbles.

The last criterion of transition from froth to annular flow has been taken from Pushkin and Sorokin [14] and is, therefore, identical with the one described by eqn. (29) of the preceding section.

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Figure Captions

- Fig. 1. Notations
- Fig. 2. Sphere dragged with velocity U_{∞} through fluid at rest.
(Wake relative to liquid when sphere moves with slip)
- Fig. 3. Conditions for presence and absence of coalescence of two spheres.
- Fig. 4. Array for the calculation of stochastic distance ratio $\bar{a}/\bar{d}$.
- Fig. 5. Axial velocity profiles behind a Taylor bubble.
- Fig. 6. Control volume for the slug flow.
- Fig. 7. Illustration of annular and froth flow.

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