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ABSTRACT

A fusing plasma with coherently polarized spin nuclei can be subject to instabilities due to the anisotropy of the reaction product distributions in velocity space, which is a result of their polarization. The characteristics of these instabilities depend strongly on the plasma spatial inhomogeneities and a significant rate of spin depolarization can be produced by them if adequate fluctuation amplitudes are reached. The results of the relevant analysis are, in addition, of interest for plasma heating processes with frequencies in the range of the cyclotron frequencies of the considered nuclei.

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1. Introduction

An important point in the analysis of the ignition conditions of a thermonuclear plasma is the fact that the contribution of the nuclear cross section to the fusion reaction rate can have a strong dependence on the polarization state of the spins of the fusing nuclei. For example, in the case of a D-T plasma, the presence of an excited state $J^P \approx \frac{3}{2}^+$ (i.e., with angular momentum equal to 3/2 and even parity) of the (unstable) He^5 isotope makes the fusion reactions occur almost entirely through the channel in which the deuteron and the triton spins add up to form a state with total spin 3/2.

Quantitative estimates of the possible rates of spin depolarization in a magnetically confined plasma have been published only recently. In fact, the energy difference between the spin states is minute compared to the plasma thermal energy. Single particle processes, such as binary collisions and irregularities of the equilibrium magnetic field, were shown in Ref. [1] to give rather long relaxation times. In addition, magnetic field fluctuations in the frequency range around the nuclear spin precession frequencies, with levels not significantly above thermal, were found to lead to the same conclusion.

On the other hand, confined plasmas are known to be able to sustain higher fluctuation amplitudes [2] as the distribution functions of their components are intrinsically nonthermal. In fact, a net nuclear spin polarization of a plasma is in itself a nonthermal feature which is magnified by the effects of the occurring fusion reactions. In a D-T plasma the maximum fusion

rate is obtained when both the deuteron and the triton spins are aligned either parallel or antiparallel to the magnetic field in a configuration which corresponds to a state of total spin $3/2$. Then the conservation of parity and of angular momentum requires the neutron and the α -particle produced by the decay of the unstable He^5 isotope to be in a d-state of orbital angular momentum. This leads to an anisotropic distribution of the neutrons and of the α -particles in velocity space. After summing over the possible orientations of the neutron spin, this distribution is found to be proportional to $\sin^2 \psi$ where ψ is the angle between the particle velocity and the magnetic field direction. The anisotropy of the α -particle distribution is preserved during the first phase of their slowing down.

Waves in the frequency range of the ion cyclotron frequency and of its harmonics can induce spin flip transitions either of the deuteron spins, which have a precession frequency $\Omega_D^P \approx 0.86 \Omega_D$ with $\Omega_D = \Omega_\alpha$ the cyclotron frequency of the deuterons and of the doubly-charged α -particles, or of the triton spins whose precession frequency is $\Omega_T^P \approx 5.96 \Omega_D$. This relaxation mechanism of the nuclear polarization, due to collective oscillations driven essentially by the polarization itself, has been first pointed out in Ref. [2]. We recall that in order to induce spin flip transitions, when the nuclear spins are aligned along the equilibrium magnetic field, the fluctuating magnetic field must have a left circularly polarized component perpendicular to the equilibrium field and a frequency ω , as seen in the particle reference frame, that equals the spin precession frequency. In the laboratory frame the resonance condition becomes

$$\omega - n\Omega_{D,T} - k_{\parallel} v_{\parallel} = \Omega_{D,T}^P \quad (1)$$

where $n\Omega_{D,T} + k_{\parallel} v_{\parallel}$ gives the transverse and longitudinal Doppler shifts due to the ion motions. The weight of the resonances with $n \neq 0$ is small in the case of waves with perpendicular wavelengths longer than the Larmor radius of the deuterons and tritons.

2. Relevant Plasma Modes

The modes considered in Ref. [2] propagate almost perpendicularly to the equilibrium magnetic field and their dispersion relation can be derived using the cold plasma conductivity tensor to account for the contributions of the electrons and the main ion species.

The relevant mode frequency is given by

$$K(\omega) = \frac{\omega^2}{k^2 v_A^2} \frac{\omega^2 - \bar{\Omega}^2}{\omega^2 - \Omega_h^2} = 1 \quad (2)$$

and corresponds to magnetosonic (compressional Alfvén) waves modified by the so-called two-ion hybrid resonance [3] that occurs in a plasma with more than one species of ions (deuterons and tritons in this case). In Eq. (2) the frequencies $\bar{\Omega}$ and Ω_h are defined by $\bar{\Omega} = \alpha_D \Omega_T + \alpha_T \Omega_D$, with $\alpha_D = n_D/n_e$ and $\alpha_T = n_T/n_e$ the concentrations of deuterium and tritium, and $\Omega_h = (\Omega_D \Omega_T \bar{\Omega} / \bar{\Omega})^{1/2}$, with $\bar{\Omega} = \alpha_D \Omega_D + \alpha_T \Omega_T$. The Alfvén velocity is given by $\bar{v}_A = (\Omega_e \bar{\Omega})^{1/2} d_e$, where $d_e = c/\omega_{pe}$ is the electron inertial skin depth, and $\bar{\Omega}$ and Ω_h are the cutoff and hybrid resonance

frequency, respectively. For $\alpha_D = \alpha_T = 1/2$, $\bar{\Omega}^2 = 25/36 \Omega_D^2$ and $\Omega_n^2 = 2/3 \Omega_D^2$. A short derivation of Eq. (2) is sketched in Appendix A, where the polarization of the perturbed electromagnetic fields is also given.

Since the α -particles are a small minority in the plasma, only their mode-particle resonance contribution need be retained. This resonance condition is given by

$$\omega - p\Omega_\alpha - k_{||} v_{||} - \omega_{D\alpha} = 0 \quad (3)$$

Here p labels the harmonics of the α -particle cyclotron frequency Ω_α , $\omega_{D\alpha}$ is their magnetic curvature drift frequency, and $k_{||} v_{||}$ is the longitudinal Doppler shift introduced by the α -particle motion along the magnetic field lines in the case of waves with a finite parallel wave number. In order to satisfy condition (1), with $n=0$, and (3), we need $p=1$ for the resonance $\omega \sim \Omega_D^p$ with the deuteron spins, and $p=6$ for the resonance $\omega \sim \Omega_T^p$ with the triton spins. Depending on the relative magnitude of the two terms, $k_{||} v_{||}$ and $\omega_{D\alpha}$, the resonant surfaces in velocity space change from planes $v_{||} = v_{||\text{res}} = \text{const}$ to ellipsoids $v_{||}^2 + \frac{1}{2} v_\perp^2 = \text{const}$ as indicated in Fig. 1. As a general rule, for higher frequencies, such as $\omega \sim \Omega_T^p$, which require larger values of the perpendicular wave number as shown by Eq. (2), the magnetic curvature drift frequency $\omega_{D\alpha}$ becomes the most important term in determining the width $\delta\omega = \omega - p\Omega_\alpha$ of the resonance.

The resonant contribution of α -particles modifies Eq. (2) into $K(\omega) = 1 + iR_\alpha$ and the relevant growth rate is given by

$$\gamma = R_\alpha (\partial K / \partial \omega)^{-1} \quad (4)$$

Here

$$R_\alpha = \frac{2\pi\omega\bar{\Omega}}{k^2 v_A^2} \frac{n_\alpha}{n_e} \frac{p\bar{\Omega}_\alpha}{k_\perp v_\alpha}^2 \int d^3v \delta[(\omega - k_\parallel v_\parallel - \omega_{D\alpha})/\bar{\Omega}_\alpha - p] \times \\ \times \frac{2\varepsilon_\alpha}{n_\alpha} \left[\int \lambda J_p \left(\frac{k_\perp v_\perp}{\bar{\Omega}_\alpha} \right) - \frac{k_\perp v_\perp}{p\bar{\Omega}_\alpha} J_p' \left(\frac{k_\perp v_\perp}{\bar{\Omega}_\alpha} \right) \right]^2 \times \frac{\partial f_\alpha}{\partial \varepsilon} + \frac{1}{B} \frac{\partial f_\alpha}{\partial \mu} \quad (5)$$

$J_p'(x) = dJ_p(x)/dx$, $\varepsilon_\alpha = 3.5$ MeV is the α -particle energy at birth, $\varepsilon = m_\alpha v^2/2$ and $\mu = m_\alpha v_\perp^2/(2B)$ and λ is the polarization factor defined by Eq. (A-4). The α -particle distribution can be written as

$$f_\alpha = n_\alpha \frac{\mu B}{\varepsilon} f_\alpha^0(\varepsilon) \quad (6)$$

where f_α^0 is isotropic and has the characteristic dependence $f_\alpha^0 \propto (\varepsilon^{3/2} + \varepsilon_c^{3/2})^{-1}$ for $0.6 \text{ MeV} \approx \varepsilon_c \leq \varepsilon \leq \varepsilon_\alpha$ and vanishes for $\varepsilon \geq \varepsilon_\alpha$. Since $\omega \partial K / \partial \omega > 0$, $\partial f_\alpha / \partial \mu$ gives a positive contribution to γ .

We see that the anisotropy of the α -particles can indeed drive electromagnetic waves unstable, but the relevant process is a weak one. This can be inferred by the fact that γ is further reduced by a numerical factor, multiplying n_α/n_e , resulting from the partial cancellation of the contribution of different portions of the resonant surface in velocity space. On the

other hand, considering that the relaxation time of the nuclear polarization that is needed to maintain the increased reaction rate over the whole duration of the discharge is relatively long, the growth rate given by Eq. (4) is sufficiently fast to produce adequate magnetic fluctuation amplitudes and induce a depolarization of the fuel nuclei at a significant rate [2]. A superficial inspection of Eq. (4) indicates that γ is higher for $p=6$ than for $p=1$, since $\omega \partial K / \partial \omega \sim 1$ for the larger frequency and is increased considerably for $\omega^2 \sim \Omega_h^2 \sim \bar{\Omega}^2$.

We also notice that, since it is reasonable to assume that fusion burning plasmas will have finite values of $\beta = 8\pi n(T_e + T_i)/B^2$, the effect of electron transit time damping should be considered [4]. Then, instead of Eq. (2), we have

$$K(\omega) = 1 + iR_\alpha - i\sqrt{\pi} \beta_e \zeta_e \exp(-\zeta_e^2) \quad (7)$$

where $\zeta_e = \omega / |k_{||}| v_{the}$ and $\beta_e = 8\pi n T_e / B^2$. In particular, if we consider modes that are localized around a magnetic surface r_0 where $a/2 \leq r_0 \leq a/\sqrt{2}$ and a is the plasma column minor radius, we may estimate $\beta_e \sim 1.3 \times 10^{-2} \times (T_e / (8 \text{ keV})) \times [n / (10^{14} \text{ cm}^{-3})] / [B / (50 \text{ kG})]^2$. Since $R_\alpha \sim n_\alpha / n_e \times \Delta$ and Δ is found to be less than unity, for the cases we have considered, the instability condition $\zeta_e \exp(-\zeta_e^2) < R_\alpha / (\beta_e \sqrt{\pi})$ requires ζ_e to be larger than unity.

3. Inhomogeneous Plasmas

The fact that magnetically confined plasmas are inhomogeneous has an overriding influence on both the existence and the characteristics of the modes of interest. Thus we undertake to demonstrate that normal modes solutions can be found, as convective modes would not be significantly amplified by the weak growth rates that we have estimated from the homogeneous theory. After normal modes are identified in both a cylindrical and a toroidal confinement configuration, the next step is to find out whether they can have sufficiently low values of $k_{||}$ (the effective wave number parallel to the magnetic field) so that the electron transit time damping does not overtake the mode growth rate.

In order to analyze the effects of inhomogeneities we have started from a model equation that refers to a toroidal plasma with a large aspect ratio, magnetic surfaces having circular cross section, and a magnetic field dependence of the form $B = B_0 [1 - (r/R_0) \cos \theta]$ that corresponds to low β . θ is the poloidal angle and R_0 the torus major radius. In particular we look for modes with the parallel component of the perturbed magnetic field of the form

$$\tilde{B}_{||} = \hat{B}(r, \theta) \exp(-i\omega t + i n^0 \phi) \quad (8)$$

where ϕ is the toroidal angle and n^0 is an integer. The longitudinal wave number in this case is represented by

$$k_{||} B \rightarrow \frac{\mathbf{B} \cdot \nabla \tilde{B}}{\tilde{B}} = \frac{B_\theta}{r} \frac{1}{\hat{B}} \frac{\partial \hat{B}}{\partial \theta} + i \frac{n^0}{R_0} B_\phi \quad (9)$$

and, for modes that propagate along θ , $k_{\parallel} B \rightarrow -(B_{\theta}/r) [1 - q(r)n^0/m]$ where we have written $\hat{B}(r, \theta) = \hat{B}(r) \exp(-im\theta)$, m is an integer, and $q(r) = B_{\phi} r / B_{\theta} R_0$. Thus $k_{\parallel}$ in this case can be made small if we have a mode localized around a surface $r = r_0$ and we choose $m = q(r_0)n^0$. On the other hand, if we have a standing mode in the poloidal plane with $\hat{B}(r, \theta) = \hat{B}(r) \cos m\theta$ or, in general, if a reflected component with poloidal number equal to $-m$ is present, $k_{\parallel}$ can vanish for one of its components, but not for the other. Now, if we look for modes with $\omega \approx \Omega_D^p$ we find that they undergo a reflection at the cutoff surface. On this basis we conclude that the electron transit time contribution becomes important and the growth rate of the modes that can depolarize the deuterium component can be made negative at sufficiently high values of β_e . There remains an open question regarding the existence of well defined normal modes with a very weak damping rate, which could conceivably be driven to a sufficient amplitude to cause deuteron depolarization by some form of coupling (e.g. nonlinear) to other excited modes.

For these reasons, when considering finite β_e regimes, we turn our attention to the resonance with the triton precession frequency. In this case the resonance cutoff surfaces are absent and we find modes that are localized in r but travel along θ . We then increase the value of ζ_e to the point of making the transit time damping term smaller than the α -particle resonant contribution.

The problem to be solved is inherently two dimensional as we have to take into account the plasma density dependence

on the minor radius of the plasma column and the dependence of the magnetic field on the distance from the symmetry axis. We note that in the theory of ion cyclotron heating, waves are launched from an external antenna and propagate mostly in the radial direction. Here instead we are interested in modes with a spatial structure confined by the plasma inhomogeneity to a portion of the plasma column. The model equation we have considered is

$$\frac{\partial^2}{\partial r^2} \hat{B} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \hat{B} + \frac{1}{d_i^2} \left[\frac{n(r)}{n_0} \right] \times \left[1 + \frac{r}{R_0} \cos \theta \right]^2 \hat{B} = 0 \quad (10)$$

where $d_i = \bar{v}_{A0}/\omega = [B/(4\pi n_0 m_i)]^{1/2}/\omega$, $n(r) = n_0(1 - r^2/a^2)(1 + r/R_0 \cos \theta)^2$ originates from the spatial dependence of the equilibrium toroidal magnetic field. The relevant modes have large poloidal wave number $m \sim r_0 \omega / \bar{v}_{A0}$ and are confined radially within an annulus, around $r = r_0$, that has a width of the order of $(r_0 d_i)^{1/2}$. A short derivation of Eq. (10) and its solution in the cylindrical limit, $1 + r/R_0 \cos \theta \rightarrow 1$, are given in Appendix B. In this Appendix it is also shown that the complete dispersion equation, not only its model form (10), can be cast into a quadratic form, hermitian for real ω^2 , with coefficients that, for low frequency modes around the deuteron precession frequency, become singular at the cutoff surface. For higher frequency modes, around the triton precession frequency the inclusion of the θ -dependent term $1 + r/R_0 \cos \theta$ in Eq. (10) does not change the mode structure derived in the cylindrical limit substantially. This can be shown by using the eikonal

approximation $\hat{B}(r, \theta) \approx \hat{B}(r) \exp[iS(\theta, r/R_0)]$ that is appropriate as the relevant modes are fast varying functions of the poloidal angle θ . In fact, by employing a combined expansion in powers of $1/m$ and/or r/R_0 , this representation leads, to lowest order, to the same result as in cylindrical geometry provided θ is replaced in the cylindrical analysis by $\theta - (r_0/R) \sin \theta$.

Two spatial effects are important in the evaluation of the mode growth rate in an inhomogeneous configuration. The first is due to the inhomogeneity of the α -particle density, that leads to $\partial f_\alpha / \partial \epsilon$ to be replaced in Eq. (5) by $(1 - \omega_{* \alpha} / \omega) \partial f_\alpha / \partial \epsilon$, where $\omega_{* \alpha}$ is the α -particle diamagnetic drift frequency. The sign of $\omega_{* \alpha}$ - i.e., the direction of propagation of the mode in the poloidal plane - can be chosen so as to reduce the stabilizing influence of $\partial f_\alpha / \partial \epsilon$ significantly. The second effect is related to the spatial structure of the modes around the triton precession frequency, which allows them to resonate with the α -particles at different locations, i.e., at different distances from the axis of symmetry corresponding to different harmonics of Ω_α . Thus in order to assess whether these modes are actually unstable, the local growth rate γ_{local} has to be summed over the different resonant regions. The number and location of these regions depend on the mode frequency and on the plasma parameters. Characteristically, one finds a few, say three, disconnected regions. In order to model this situation and simplify the relevant analysis we consider γ_{local} as a function of $x = R - R_0$ only. Then we numerically evaluate the velocity space integral (which appears in the expression for γ_{local}) at the different resonant locations. In this way we find instability with $\gamma / \omega \sim 10^{-1} n_\alpha / n_e$ for

reactor parameters such as those of an INTOR-like device

$$(n_e = 2 \times 10^{14} \text{ cm}^{-3}, B_0 = 50 \text{ kG}, R_0 = 520 \text{ cm}, a = 130 \text{ cm}).$$

This result is valid provided the assumption that $|d \ln n_\alpha / dr| > (1/0.6) |d \ln n / dr|$ is a realistic one.

An important requirement for instability is that the waves be trapped; otherwise they propagate out of the plasma at such a rate that they would not be amplified. If density fluctuations are present, they may scatter the waves onto trapped wave orbits, and thus damp them. Similarly, asymmetries in the axisymmetric field could also lead to wave detrapping and damping.

Finally we notice that all particles with "circulating" orbits sample an entire magnetic surface and those that are trapped only a part of it. If the region where the mode frequency becomes equal to the triton precession frequency falls in the outer part of the annulus over which the mode is localized, most of the trapped particles, with orbits around the magnetic surfaces contained in this annulus, and all the circulating particles will cross this region ($R + r_0 \cos \theta = \text{const.}$). The relative phase between precession and the wave is proportional to $1/2 (d\Omega^P/dt) t^2 = (t^2/2) (r_0/R) (\sin \theta) v_{||} \Omega^P / (qR)$. The incoherent change in polarization per passage through resonance is $\Omega^P [(2\pi q R^2) / (\Omega^P v_{||} r_0 \sin \theta)]^{1/2} B_\perp / B$, where $B_\perp = \tilde{B}_\perp k_{||} / k_\perp$. The rate of passage through resonance is $2v_{||} / qR$. If the wave is centered so that $\omega = 6\Omega$ at $R + r_0$, then $\sin \theta = [r_0 / (2\Delta)]^{1/2}$ where $\Delta = (R/6) (6 - 5.94)$. Averaging $\tilde{B}_\perp^2$ over the torus and considering that the width of the depolarizing region is of the order of $(r_0/d_i)^{1/2}$, we find

$$v_{\text{dep}} = \frac{2\Omega^P R}{r_0} \left(\frac{\tilde{B}_\perp^2}{B^2} \right) \left(\frac{k_{||}^2}{k_\perp^2} \right) \left(\frac{d_i}{\Delta\sqrt{2}} \right)^{1/2}.$$

We estimate $\tilde{B}_{||}/B_0 \approx \gamma/\omega \approx 0.1 n_\alpha/n_0$ since this is when wave breaking occurs. We estimate n_α/n by setting $n_\alpha \langle \epsilon \rangle / \tau_s \approx 3nT/\tau_E$ where $\bar{\epsilon}_\alpha$ is the effective alpha particle energy, τ_s the slowing down rate, and τ_E the energy replacement time. Making use of these results and $k_{||}/k_\perp \approx v_A/v_{the}$

$$v_{dep} \tau_E \approx 2 \times 10^{-2} \frac{R}{r_0} \left(\frac{Tm_\alpha}{\bar{\epsilon}_\alpha m_e} \right)^2 \frac{\Omega_p^2 n}{v_{ei}^2 (n\tau_E)} \left(\frac{v_A}{v_{the}} \right)^2 \left(\frac{d_i}{\Delta} \right)^{1/2}$$

where v_{ei} is the electron-ion collision rate. For reactor parameters and employing the Lawson criterion, we obtain $v_{dep} \tau_D \approx 5 - 50$ for $\tau_D \sim 5\tau_E$, τ_D being the particle diffusion time. Since, in practice, we consider many recycling times, depolarization is significant.

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APPENDIX A: Homogeneous Dispersion Equation

Neglecting terms of order $(k_{\parallel}/k_{\perp})^2$ and $m_e/m_i \sim \omega/\Omega_e \sim \Omega_i/\Omega_e$ the conductivity tensor can be written, in the cold plasma limit, as

$$\sigma = \begin{pmatrix} \sigma_{\perp} & 0 \\ 0 & \sigma_{\parallel} \end{pmatrix} \quad (\text{A-1})$$

where $\perp$ and $\parallel$ refer to the direction of the equilibrium magnetic field, $\sigma_{\parallel} = i\omega_{pe}^2/(4\pi\omega)$, and $\sigma_{\perp}$ is the 2×2 matrix

$$\sigma_{\perp} = -\frac{i\sigma_o}{\omega} \begin{pmatrix} 1 & +i\lambda \\ -i\lambda & 1 \end{pmatrix} \quad (\text{A-2})$$

with

$$\sigma_o = -\frac{en_e c \omega^2}{B} \sum_{i=D,T} \frac{\alpha_i \Omega_i}{\Omega_i^2 - \omega^2} \quad (\text{A-3})$$

and

$$\lambda = \sum_{i=D,T} \frac{\alpha_i \omega}{\Omega_i^2 - \omega^2} \bigg/ \left[\sum_{i=D,T} \frac{\alpha_i \Omega_i}{\Omega_i^2 - \omega^2} \right], \quad (\text{A-4})$$

which is the polarization factor.

The dispersion relation (2) for waves propagating almost perpendicularly to the equilibrium magnetic field, is obtained from Maxwell's equations and from the conductivity tensor (A-1). Neglecting the displacement current and considering

waves for which the perturbed electric field is perpendicular to the equilibrium magnetic field, one obtains

$$4\pi\sigma_0(1-\lambda^2) = k_{\perp}^2 c^2, \quad (A-5)$$

which coincides with Eq. (1).

By the same procedure, we obtain the following equations relating the components of the perturbed electromagnetic fields

$$\begin{aligned} \underline{k} \cdot \underline{\tilde{E}} &= -i\lambda \underline{b} \cdot (\underline{k} \times \underline{\tilde{E}}), & \underline{b} \cdot (\underline{k} \times \underline{\tilde{B}}) &= i\lambda \underline{k}_{\perp} \cdot \underline{\tilde{B}} \\ \underline{b} \cdot \underline{\tilde{E}} &= 0, & \underline{k}_{\perp} \cdot \underline{\tilde{B}} &= -k_{\parallel} \underline{b} \cdot \underline{\tilde{B}} \end{aligned} \quad (A-6)$$

The last equation, which just expresses the condition that $\underline{B}$ is divergence-free, is introduced as a reminder of the fact that only waves with nonzero, even if small, parallel wave number can induce spin flip transitions. The perpendicular polarization factor $\lambda(\omega)$ is equal to one for $\omega = \Omega_D$ and $\omega = \Omega_T$ (right circularly polarized waves) is equal to minus one for $\omega = \bar{\Omega}$ (left circularly polarized waves) and becomes large (almost linearly polarized waves) for $\omega \sim \Omega_h$ and for $\omega \gg \Omega_{D,T}$. For $\alpha_D = \alpha_T = 1/2$, $\lambda(\Omega_D^P) \approx 0.2$ and $\lambda(\Omega_T^P) \approx 7.1$.

APPENDIX B. Inhomogeneous Dispersion Equation

We consider, for simplicity, a cylindrical geometry, assume a constant magnetic field directed along the cylinder's axis but retain the spatial dependence of the plasma density. Even though obviously oversimplified, this model is sufficient to give an approximate picture of the spatial structure of the high frequency modes. In addition, the procedure sketched below can be generalized to more realistic configurations and to low frequency waves so as to include the spatial variation of the magnetic field, its poloidal component and the occurrence, inside the plasma column, of the cutoff resonance surfaces (see Section 3). In particular, the quadratic form derived in this Appendix can be extended to this more general case, while the explicit solution of the mode amplitude cannot be directly applied to the low frequency case.

We write Ampère's law (see Appendix A) in the form

$$\underline{g}^{-1} \nabla \times \underline{\tilde{B}} = (4\pi/c) \underline{\tilde{E}} \quad (\text{B-1})$$

and apply the operator $\underline{b} \cdot \nabla \times$ with $\underline{b} = \underline{B}/B$. Neglecting terms proportional to $k_{\parallel}^2$ we can disregard the contribution of the perpendicular components of the perturbed magnetic field. Then we obtain

$$\underline{b} \cdot \nabla \times \left(\underline{g}^{-1} \nabla \times \underline{\tilde{B}}_{\parallel} \right) = - \frac{4\pi}{c^2} \frac{\partial \underline{\tilde{B}}_{\parallel}}{\partial t} \quad (\text{B-2})$$

Using the cylindrical approximation described above and the definition of the tensor $\underline{g}$ given in Appendix A we obtain

$$\tilde{B}_{||} = \frac{ic^2}{4\pi\omega} \tilde{\nabla}_1 \cdot \underline{g}_1^{-1} \cdot \tilde{\nabla}_1 \tilde{B}_{||} \quad (B-3)$$

that, in polar coordinates, reads

$$\begin{aligned} \tilde{B}_{||} = & -\frac{c^2}{4\pi} \frac{1}{2} \left[\left(\frac{1}{r} \frac{\partial}{\partial r} r - \frac{i}{r} \frac{\partial}{\partial \theta} \right) \frac{1}{\sigma_0(1+\lambda)} \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \theta} \right) + \right. \\ & \left. + \left(\frac{1}{r} \frac{\partial}{\partial r} r + \frac{i}{r} \frac{\partial}{\partial \theta} \right) \frac{1}{\sigma_0(1-\lambda)} \left(\frac{\partial}{\partial r} - \frac{i}{r} \frac{\partial}{\partial \theta} \right) \right] \tilde{B}_{||} . \end{aligned} \quad (B-4)$$

This equation leads, for real ω^2 , to the hermitian quadratic form

$$\begin{aligned} \int_0^a r dr \int_{-\pi}^{\pi} d\theta \left\{ -|\tilde{B}_{||}|^2 + \frac{1}{\sigma_0(1+\lambda)} \left| \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \theta} \right) \tilde{B}_{||} \right|^2 + \right. \\ \left. + \frac{1}{\sigma_0(1-\lambda)} \left| \left(\frac{\partial}{\partial r} - \frac{i}{r} \frac{\partial}{\partial \theta} \right) \tilde{B}_{||} \right|^2 \right\} = 0 . \end{aligned} \quad (B-5)$$

Neglecting derivatives of the equilibrium quantities compared to those of the perturbed field and taking $\omega^2 \gg \Omega_h^2, \bar{\Omega}^2$, Eq. (B-4) becomes simply

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \tilde{B}_{||} + \frac{\omega^2}{v_A^2} \tilde{B}_{||} = 0 . \quad (B-6)$$

Since we have assumed $\underline{B}$ to be constant, we can take

$\partial \tilde{B}_{||} / \partial \theta = -im \tilde{B}_{||}$ where m is the azimuthal mode number and

obtain

$$\frac{d^2}{dr^2} \tilde{B}_{||} = \left[\frac{m^2}{r^2} - \frac{\omega^2}{\bar{v}_{A0}^2} \frac{n(r)}{n_0} \right] \tilde{B}_{||} \quad (\text{B-7})$$

where n_0 is the plasma density at the center of the column and $\bar{v}_{A0}$ the corresponding value of the Alfvén velocity $\bar{v}_A$. Modes with $d^2/dr^2 \ll m^2/r^2$ are obtained when the turning points of the "potential" on the right-hand side of Eq. (B-7) are close together as shown in Fig. 2.

Assuming, for example, the density profile $n(r) = n_0(1-r^2/a^2)$, Eq. (B-7) gives in this limit a radially localized mode, centered around $r = r_0 = a/\sqrt{2}$ with a radial width $\Delta r \sim m^{-1/2}/a$. The mode frequency ω is given by $\omega = \omega_0 + \delta\omega$ where $\omega_0^2 = 4m^2 \bar{v}_{A0}^2 / a^2$ and $\delta\omega/\omega_0 \sim O(m^{-1})$. For $\omega \sim p\Omega_a$ the poloidal mode number m is related to the ratio a/d_i between the minor radius a of the column and the ion inertial skin depth d_i , by $m \sim pa/d_i$.

FIGURE CAPTIONS

Figure 1: Resonant curve (solid portion of the ellipse) in the α -particle velocity space. The shaded area corresponds to the part of the velocity space where the resonant interaction is destabilizing.

Figure 2: Turning points determining the radial width of the modes.

Fig. 1.

Fig. 2.

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