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THERMOELASTIC ANALYSIS OF SPENT FUEL AND
HIGH LEVEL RADIOACTIVE WASTE REPOSITORIES
IN SALT - A SEMI-ANALYTICAL SOLUTION

Christopher M. St. John
April 1977

MINERAL RESOURCES RESEARCH CENTER
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ABSTRACT

An underground repository containing heat generating, High Level Waste or Spent Unreprocessed Fuel may be approximated as a finite number of heat sources distributed across the plane of the repository. The resulting temperature, displacement and stress changes may be calculated using analytical solutions, providing linear thermoelasticity is assumed. This report documents a computer program based on this approach and gives results that form the basis for a comparison between the effects of disposing of High Level Waste and Spent Unreprocessed Fuel.

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1. Introduction

The current emphasis of the U.S. program for radioactive waste resulting from both the weapons program and commercially operated nuclear power plants is towards underground, geologic disposal. Historically, attention has been directed towards disposal in salt and as a result some twenty years of laboratory and field investigations of the many technical aspects of disposal in this material have been completed. The terms of reference for much of this work were that the High Level Waste materials resulting from the commercial nuclear energy program would consist of reprocessed fuel assemblies. The recent change in emphasis from terminal disposal of such High Level Waste (HLW) to recoverable storage of Spent Unreprocessed Fuel (SURF) assemblies has led to reappraisal of criteria previously established for conceptual design of HLW disposal facilities.

This report concerns rock mechanics aspects of underground waste repository design. In particular it examines the thermo-mechanical response of the rock mass in which a repository is sited and compares results for repositories identical in every respect except for the waste form stored.

The work reported here was performed in response to a scoping statement requiring thermoelastic analyses of both Spent Fuel and High Level Waste repositories. Three completely different solution procedures were to be adopted by the parties represented at a meeting held in Denver between 9th and 11th February, 1977. The analysis within this report draws on work relating to displacement discontinuity models that has been carried out largely at the University of Minnesota. However, none of the three dimensional displacement discontinuity codes available at the time of this study were directly applicable to the problem defined. A completely new code was therefore generated and part of this report is devoted to a discussion of the formulation of the code and

some necessary verification studies. Following these essential preliminaries, results for representative conditions and times for both Spent Fuel and High Level Waste repositories are presented.

2. Heat Sources at Finite Depth

2.1 Introduction

Simulation of the effect of time varying heat sources at finite depth has been discussed by Nowacki (1). The complete solution for temperatures, displacements and stresses around instantaneous, constant and exponentially decaying sources has been given by Crouch and McClain (2). In this section of the report the necessary equations are presented briefly and results for a number of test cases are given.

2.2 Solution for instantaneous, constant and decaying heat sources

Suppose an instantaneous heat source of magnitude Q is sited at the point $x = y = z = 0$ in an infinite region. The resulting temperature, at any time t and location x, y, z is given by the equation

$$T(R, t) = \frac{Q}{(\pi\beta)^{3/2}} \exp\left(-\frac{R^2}{\beta}\right)$$

where $R = \sqrt{(x^2 + y^2 + z^2)}$

and $\beta = 4Kt$, in which K is the thermal diffusivity. The solution for any time varying heat source ($Q(\tau)$) may be obtained by replacing this equation by

$$T(R, t) = \frac{1}{(4\pi K)^{3/2}} \int_0^t Q(\tau) (t-\tau)^{-3/2} \exp\left[\frac{-R^2}{4K(t-\tau)}\right] d\tau$$

The displacements and stresses resulting from an instantaneous source may be derived from the thermoelastic potential

$$\phi(R, t) = \frac{mQ}{4\pi R} \operatorname{erf}\left(\frac{R}{\sqrt{\beta}}\right)$$

where $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\eta^2} d\eta$ is the error function

and $m = \alpha(1+\nu)/(1-\nu)$, in which α is the coefficient of linear thermal expansion of the body.

For a time varying heat source ($Q(\tau)$) the above equation for thermo-elastic potential is replaced by

$$\phi(R,t) = \frac{m}{4\pi R} \int_0^t Q(\tau) \operatorname{erf} \left[\frac{R}{\sqrt{4K(t-\tau)}} \right] d\tau$$

It is observed that complete equations (Appendix B) for the temperature, displacement and stress distributions involve functions $V_1(R,t)$, $V_2(R,t)$, and $V_3(R,t)$. The functions are dependent upon the type of the heat source. For an instantaneous source they are:

$$V_1(R,t) = Q \operatorname{erf}(R/\sqrt{\beta})$$

$$V_2(R,t) = Q \beta^{-1/2} \exp(-R^2/\beta)$$

$$V_3(R,t) = Q \beta^{-3/2} \exp(-R^2/\beta)$$

For time dependent sources these expressions become:

$$V_1(R,t) = \int_0^t Q(\tau) \operatorname{erf}(R/\sqrt{\beta}) d\tau$$

$$V_2(R,t) = \int_0^t Q(\tau) \beta^{-1/2} \exp(-R^2/\beta) d\tau$$

$$V_3(R,t) = \int_0^t Q(\tau) \beta^{-3/2} \exp(-R^2/\beta) d\tau$$

The necessary integrals are given in Crouch & McClain (2). For completeness, they are reproduced in Appendix C.

2.3 Solutions for a half space

The solutions given above refer to heat sources in an infinite elastic region. For a heat source at the point 0, 0, H in a half space $z \geq 0$ the

following boundary conditions must be met:

$$\sigma_{zz} = 0, \quad \sigma_{rz} = 0, \quad T = 0 \quad \text{on } z = 0$$

where σ_{zz} , σ_{rz} are normal and shear stress and T is the temperature.

The first and last of these are satisfied by adding the effects of a positive source at the point 0, 0, H and a negative source, or sink, of the same magnitude at the point 0, 0, -H. However, on $z = 0$ the shear stresses σ_{rz} for both the source and the sink are of the same sign and magnitude. They must therefore be removed by adding the solution to the following boundary value problem for the half-space.

$$\sigma_{rz}^* = -2\sigma_{rz}, \quad \sigma_{zz}^* = 0 \quad \text{on } z = 0$$

Where σ_{rz} is the shear stress on the plane $z = 0$ for either the source or the sink.

The simplest way of achieving the required solution is to replace the shear stresses σ_{rz} by equivalent point shear forces T_{xz} and T_{yz} , where:

$$T_{xz} = \int_{-L/2}^{+L/2} \int_{-L/2}^{+L/2} \sigma_{rz} \frac{x}{r} dx dy$$

$$T_{yz} = \int_{-L/2}^{+L/2} \int_{-L/2}^{+L/2} \sigma_{rz} \frac{y}{r} dx dy$$

Where $r = \sqrt{x^2 + y^2}$ and the surface is divided into square zones of dimensions $L \times L$. These integrals represent the average shear stresses over a zone. However, within the computer code an approximation to these integrals is obtained by using the shear stresses at the center of each zone. The shear stresses σ_{rz}^* therefore correspond to point shear forces T_{xz}^*, T_{yz}^* acting at the

center of each zone and are given as:

$$T_{xz}^* = -2T_{xz}$$

$$T_{yz}^* = -2T_{yz}$$

Cerruti solved the problem of concentrated shear forces on the plane surface of a semi-infinite solid (4). The necessary equations for the combined effect of T_{xz}^* and T_{yz}^* are given in the Appendix D(a).

A better approximation to the exact solution is obtained by considering a uniformly distributed shear force over each zone. Necessary equations for this case are obtained through area integration of Cerruti's solution and are given in Appendix D(b). As for the point shear case, the density of distributed shear force over a zone is taken as the shear stress at the center of the zone. Improved results could be obtained very simply by numerical integration over each zone if desired.

2.4 Model verification studies

A computer code based on the equations referred to in sections 2.2 and 2.3 was written. The program is described and briefly documented in Appendix A, and this section deals with a number of preliminary runs intended to demonstrate that both the equations and the coding were correct.

2.4.1 Verification of basic solution for an instantaneous heat source in an infinite region.

An infinite region may be considered to be a special case of a half space. By placing the heat source very far from the free surface the effects of the superposed sink and the surface tractions become insignificant. Nowacki (1) gave results from his solution for an instantaneous source in an infinite region. The agreement between his results and those from the current computer code are excellent (Figure 2.1).

2.4.2 Verification of solutions for constant and exponentially decaying sources.

The first stage in this verification is to demonstrate that the results presented in Section 2.4.1 for an instantaneous source, can be reproduced by replacing the instantaneous source by a constant source of limited duration. This can be achieved by superposing positive and negative constant sources slightly separated in time. Results given in Figure 2.2 indicate that agreement between the two cases is excellent.

Since the solution for the constant source has been shown to be correct, partial validation of the solution for the exponentially decaying source may be achieved by showing that the latter reduces to the former as the decay constant approaches zero. (This is, of course, a necessary but not sufficient condition to demonstrate the validity of the solution). Evaluation of the V functions for a range of values of R and t is all that is necessary as it has already been observed (section 2.2) that the solutions are identical except for the V functions. Table 2.1 shows that the solutions do converge as the decay constant decreases.

2.4.3 Verification of solution for a half space.

Results were obtained for the case of a single constant source in a half space. These were compared with results for the same source in an infinite region. Four factors indicative of a correct solution procedure may be quoted.

- a) The temperature boundary condition $T = 0$ on $z = 0$ is satisfied for the half space model.
- b) The stress boundary condition $\sigma_{zz} = 0$ on the free surface ($z=0$) is also satisfied. Figures 2.4(a) and 2.4(b) give stress profiles, along the line indicated in figure 2.3, obtained using the point shear force solution, respectively.
- c) Vertical displacements on the surface $z = 0$ of a half space region are approximately twice those for the infinite region. (It is exactly twice before the surface tractions are applied).

At R=1000. t=100.yrs.	Constant Source	Decay Constant λ		
		10^{-2} (Half life = 6.9yrs)	10^{-5} (6.9×10^3 yrs)	10^{-11} (6.9×10^9 yrs)
V1	$.8735 \times 10^9$	$.9831 \times 10^5$	$.9881 \times 10^8$	$.8736 \times 10^9$
V2	$.2091 \times 10^5$	$.9747 \times 10^1$	$.7314 \times 10^4$	$.2091 \times 10^5$
V3	$.7501 \times 10^{-1}$	$.2780 \times 10^{-4}$	$.2289 \times 10^{-1}$	$.7500 \times 10^{-1}$

Table 2.1 Comparison of V functions for constant and exponentially decaying heat sources.

- d) Figure 2.5 shows the shear stress profile immediately under the center of a surface zone obtained using the point shear force and integrated shear force for removing the surface shears. Except methods very close to the free surface, both methods give practically same result. Close to the surface, the point shear force method results in poor approximation. This is because Cerruti's solution is singular at the point of application of the load and the stress profile was taken immediately under such a point. On the basis of such observations it was decided that the point shear solution should be used only when stresses were calculated for points more than one zone width below the free surface.
- e) The analytical solutions for point shear load and distributed shear load were checked against results, presented by Holl (5), for a uniform horizontal load applied over a rectangular area on the surface of a half space. Agreement was exact when using the distributed shear solution.

2.5 Conclusions

It is believed that reasonable steps have been taken to verify the correctness of both the equations given in this section and the computer code that was generated from them. The results obtained indicate that the model is free from errors. Changes could be made to better represent continuously distributed heat sources but such changes would have effect only close to the heat sources. There are no approximations made within the interior of the half space, other than that the material is considered to be isotropic and linearly elastic.

3. Model Description and Data Selection

3.1 Introduction

In this section the situations to be studied are briefly defined and discussed with reference to the computer model to be used. Consideration is then given to selection of appropriate values for input to the code.

3.2 Required geometry and boundary conditions

In the scoping statement (3) for this study it was assumed that the repository would be modelled using either a finite element or a finite difference computer code intended for the analysis of axially symmetric structures. The geometry and boundary conditions were defined accordingly. For the code discussed in this report there are no boundaries other than the free surface. The initial conditions are, however important. These were taken to be:

Free Surface temperature	Isothermal at 60°F
Temperature gradient	0.015°F per ft.
Stresses (Lithostatic)	150 lb/ft ² per ft.

The last figure was obtained by taking the average density of all rocks overlying the repository. This was done because the computer code calculates only stress changes. These changes are subsequently added to an initial stress state, which could itself be calculated taking into account density changes with depth.

For axially symmetric analysis, the repository was to be modelled as having a radius of 3500 ft. In this study the total area of the repository was maintained by modelling it as 6204 ft. square. By assuming that heat sources are symmetrically placed, only one quarter of this area need be represented in the model. That area was split into 36 zones each 517 ft. square and containing a concentrated heat source at the center. (This arrangement

implies that the entire repository is represented by 144 heat sources).

3.3 Material Properties

The stratigraphic columns provided for use in models of repositories in bedded salt, showed sandstone, shale and sandstone and anhydrite horizons above the top of the salt bed. Since the current model is for an isotropic, homogeneous material some selection of material properties was necessary. On the basis that the very large body of salt immediately surrounding the repository (extending from 3000 ft. to 1250 ft., with the repository horizon at 2000 ft.) is likely to dominate the response of the entire mass, the material properties selected correspond to those listed for the salt. They were:

Youngs modulus	$288 \cdot 10^6 \text{ lb/ft}^2$
Poissons ratio	0.4
Coefficient of linear thermal expansion	$22 \cdot 10^{-6}/^\circ\text{F}$
Diffusivity	$876.6 \text{ ft}^2/\text{yr}$
Heat capacity	$28.83 \text{ Btu/ft}^3 \text{ } ^\circ\text{F}$

(Note that all time is measured in years of 8766 hours. Distances are measured in feet and, to be consistent, stresses are in lb/ft^2 . The computer code will work with any consistent set of units.)

3.4 Heat source characterization

Appendix A of the scoping statement (3) consisted of a tabulation of power and energy generated as a function of time for representative Spent Fuel and High Level Waste canisters. As the computer model is capable of modeling exponentially decaying sources it was logical to attempt to reproduce the given relationship between power and time as a summation of several exponentially decaying components. An adequate representation was obtained by manual

curve fitting. The equations deduced were as follows:

Spent Fuel:

$$Q1 = 9.0 * \exp(-2.3 \cdot 10^{-5} T)$$

$$Q2 = 17.98 * \exp(-9.95 \cdot 10^{-5} T)$$

$$Q3 = 167.05 * \exp(-1.75 \cdot 10^{-3} T)$$

$$Q4 = 805.49 * \exp(-2.163 \cdot 10^{-2} T)$$

$$Q5 = 60.0 * \exp(-4.014 \cdot 10^{-2} T)$$

$$Q6 = 130.0 * \exp(-2.915 \cdot 10^{-1} T)$$

High Level Waste:

$$Q1 = 0.255 * \exp(-1.5 \cdot 10^{-5} T)$$

$$Q2 = 0.65 * \exp(-7.49 \cdot 10^{-5} T)$$

$$Q3 = 2.97 * \exp(-1.12 \cdot 10^{-3} T)$$

$$Q4 = 7.13 * \exp(-2.34 \cdot 10^{-3} T)$$

$$Q5 = 849.8 * \exp(-2.42 \cdot 10^{-2} T)$$

$$Q6 = 77.59 * \exp(-1.17 \cdot 10^{-1} T)$$

$$Q7 = 64.71 * \exp(-4.83 \cdot 10^{-1} T)$$

where Q1, Q2 etc. are components of the composite source and T is measured in years from the date of emplacement of the waste. Although better approximations could be achieved, the results given in Appendix E suggest that these equations are adequate for the purposes of the present study.

The above equations describe how the power generation of an individual cannister varies with time. These must be scaled to represent the actual strength of the heat sources to be used in the computer model. The necessary scaling factors may be derived by starting from a calculation of the strength of the source: Strength Q of the heat source = Q_0 / C

where Q_0 is the heat generation per segment, at which the source is centered, and C is the heat capacity of the Salt.

Assuming a loading of 150 kw per acre we have, for segments 517 ft. square:

$$Q_0 = \frac{150 \text{ kw/acre}}{1.05506 \text{ kJ/Btu}} * \frac{517 \text{ ft}^2}{43560 \text{ ft}^2/\text{acre}} * 31557600 \text{ s/yr}$$

$$= 2.753 * 10^{10} \text{ Btu/yr}$$

Hence:

$$Q = \frac{2.753 * 10^{10} \text{ Btu/yr}}{28.83 \text{ Btu/ft}^3 \text{ } ^\circ\text{F}}$$

The scaling factors for the components of the initial thermal output are given then by:

$$\text{Scaling Factor} = \frac{\text{Total strength of segment}}{\text{Initial cannister power}}$$

which gives the values:

$$8.028 * 10^5 \text{ - Spent Fuel}$$

$$9.520 * 10^5 \text{ - High Level Waste}$$

For example, the first component of the spent fuel characterization becomes:

$$Q_1' = 8.028 * 10^5 * 9.0 * \exp(-2.3 * 10^{-5} T)$$

$$= 7.225 * 10^6 * \exp(-2.3 * 10^{-5} T)$$

Finally, it should be observed that the temperature, displacements and stresses are linearly dependent upon the strength of the heat source if the material is homogeneous. Calculations for both Spent Fuel and High Level Waste repositories were performed for a loading of 150 kw/acre and results for any other cases may be obtained by appropriate scaling.

4. Results from Analysis for Spent Fuel and High Level Waste Repositories

4.1 Introduction

This section of the report presents data obtained from a series of computer runs intended to provide the information specified in the scoping statement for this study. It is important to note when looking at the results presented that the output from the code discussed in this report is fundamentally different from that obtained using a finite element or finite difference code for analysis of axially symmetric structures. The distinction is that output is always for a particular horizon rather than for a vertical section. (Minor code changes could be made to permit vertical sections to be output instead). Attention is therefore directed largely towards plans which show variations in temperature, displacement and stress across a given horizon. This form of output should be particularly useful in assessing the implication of the assumption of axial symmetry that has been made for other analyses conducted as part of the "Spent Fuel and High Level Waste Repository Study."

It will be noted that results are only given for repositories with an initial power generation of 150 kw/acre. Since, for the computer model used in this study, the temperature, displacement and stress changes caused by the repository are all directly proportional to the initial power generation there would be no sense in performing a complete calculation for each different, possible waste density.

4.2 Temperature history

The temperature history is represented in two ways. Figures 4.1 and 4.2 show the time variation of temperature profiles for the spent fuel and high level waste repositories respectively. Profiles are given at the repository

horizon, 2000 ft. below the surface, and also at 1000 ft. below the surface. It will be noted that although the profiles extend right across the repository at the 1000 ft. level they are curtailed actually at the repository horizon where they would be directly affected by the representation of the repository as a limited number of concentrated heat sources.

It is clear from figures 4.1 and 4.2 that the temperatures are significantly higher for a Spent Fuel repository and that these high temperatures persist for a very much longer time than for a High Level Waste repository of the same initial power generation. Further distinction may be made between the thermal effects of disposing of these two types of waste, from examination of Figures 4.3 to 4.18. These are all plots of temperature distributions at various horizons. They give a reasonably complete picture of the temperature variation within the rock mass above and surrounding each type of repository 500 years after disposal. (The timing was chosen to roughly correspond to a period of considerable effect for both cases). Each plan represents an area approximately 7250 ft. square, and is therefore part of the 10340 ft. square represented by the computer model. In every case the repository is sited at the top left hand corner, as indicated in figure 4.3, and the key is as follows:

Letter	Range of Values ($^{\circ}\text{F}$)
D	60 - 70
E	80 - 90
F	100 - 110
G	120 - 130
H	140 - 150
I	160 - 170
J	180 - 190
K	200 - 210
L	220 - 230
M	240 - 250
N	260 - 270
O	280 - 290
P	300 - 310
Q	320 - 330
R	340 - 350
S	350 - 360
T	380 - 390

Particular figures of interest are the temperatures calculated immediately above the repository on the 1950 ft. horizon. These are 386°F and 161°F respectively for the Spent Fuel and High Level Waste cases 500 years after emplacement. Since the initial temperature at this depth is 89.25°F these are rises of approximately 297°F and 72°F .

4.3 Displacement history

Displacement values of most interest are those that would be observed at the surface. Figures 4.19-4.32 show the distribution of vertical displacements at the surface over both a Spent Fuel and a High Level waste repository. The area represented in these figures is the same as that in the temperature distribution plot (figure 4.3). The different letters have the following interpretation:

Letter	Range of Values (ft.)
A	0 - 0.25
B	.5 - 0.75
C	1.0 - 1.25
D	1.5 - 1.75
E	2.0 - 2.25
F	2.5 - 2.75
G	3.0 - 3.25
H	3.5 - 3.75
I	4.0 - 4.25
J	4.5 - 4.75
K	5.0 - 5.25
L	5.5 - 5.75
M	6.0 - 6.25
N	6.5 - 6.75
O	7.0 - 7.25
P	7.5 - 7.75

The results of the analyses are summarized in figure 4.33, which gives the displacement histories at a point vertically above the center of a repository. It is seen that for the Spent Fuel case the greatest computed vertical, surface displacement of 11.2 ft. occurs approximately 1000 years after emplacement. For the High Level Waste case a much lower value of 4.2 ft. was recorded after 300 years. The relative timing and magnitude of these peaks corresponds

well with what would be anticipated from the temperature profiles of figures 4.1 and 4.2

4.4 The potential for failure

The potential for failure may be discussed in terms of whether the rock at any point can be expected to perform adequately under the imposed stress conditions. The simplest approach to this assessment is to examine the minor principal stress (the least stress, adopting the sign convention that tensile stresses are negative) at every point. If the minor principal stress is negative then, whatever the intact strength of rock, there exists the possibility of fractures forming or pre-existing fractures opening.

It will be recalled that for the purposes of the computer model it was assumed that the rock mass, in which the repository is sited, is homogeneous. In particular, it was assumed that the density was uniform and the initial stresses therefore increased linearly with depth. Before carrying out calculations relating to potential for failure these computed stresses were adjusted to take account of the actual, specified, stratigraphic column which, for the bedded salt models was listing downwards from the surface.

250 ft.	Sandstone - specific gravity	2.6
500 ft.	Shale and Sandstone	2.4
500 ft.	Anhydrite	2.8
2000 ft.	Salt	2.1

Principal stresses were calculated taking into account the actual initial stresses at each horizon. Results are presented in figures 4.34 - 4.37 for Spent Fuel and High Level Waste repositories after 500 and 1000 years. The figures show the minor principal stresses on the 1000 ft. horizon - which is half way between the repository and the surface. The scale of the figures is identical to those for the figures giving temperatures and displacements

and the key is as follows:

-	Tension
A	Compression, 0-500 lbf/in ²
B	Compression, 100-1500 lbf/in ²

Since the original stress state may be assumed to be hydrostatic, at approximately 1100 lbf/in², any areas not indicated by the letter B on these figures corresponds to a region of reduced compression. Under such circumstances the rock mass permeability may be expected to increase, giving rise to locally increased groundwater flow rates. The least principal stresses observed were approximately:

Spent Fuel	-	500 yrs (figure 34)	-800 lbf/in ²
		1000 yrs (figure 35)	-300 lbf/in ²
High Level Waste	-	500 yrs (figure 36)	+500 lbf/in ²
		1000 yrs (figure 37)	+900 lbf/in ²

(The minimum values for the high level waste occur some distance out from the edge of the repository. The blank area on these figures therefore corresponds to values between 500 and 1000 lbf/in²).

An interesting aspect of figures 34 and 35 is the lack of radial symmetry in the results. Such asymmetry will not be observed in models of repositories that assume an axially symmetric geometry and its importance should be assessed by detailed comparison of the results of the two types of analysis.

Also using the principal stress the failure potential across the horizon may be determined. For this determination a parabolic failure envelope (6) and a quantitative definition of failure potential as depicted in Figure 4.38 were used. Accordingly, if the Mohr's circle describing the state of stress at any point is tangent to the failure envelope, the failure potential is unity, while for stress conditions under which initiation of failure could be expected the failure potential is less than unity.

For the purposes of this analysis the strength of the rock was assumed to be represented by the following properties of intact materials:

Sandstone - Uniaxial compressive strength (C_o) 6000 lbf/in²

- Tensile strength (T_o) 500 lbf/in²

Anhydrite - Uniaxial compressive strength (C_o) 13000 lbf/in²

- Tensile strength (T_o) 800 lbf/in²

Figures 4.39 to 4.46 show the failure potential plots at the surface and at 1000 ft. horizon for both a Spent Fuel and a High Level Waste repository.

Key to the plots is as follows:

Symbol	Failure Potential Minor principal stress (most tensile) exceeds the tensile strength
-	
A	0 - 0.25
B	0.5 - 0.75
C	1.0 - 1.25
D	1.5 - 1.75
E	2.0 - 2.25
F	2.5 - 2.75
G	3.0 - 3.25
H	3.5 - 3.75
I	4.0 - 4.25
J	4.5 - 4.75
K	5.0 - 5.25
L	5.5 - 5.75
M	6.0 - 6.25
N	6.5 - 6.75
O	7.0 - 7.25
P	7.5 - 7.75
Q	8.0 - 8.25
R	8.5 - 8.75

Symbol	Failure Potential
S	9.0 - 9.25
T	9.5 - 9.75
+	10.25

It will be observed from Figures 4.39 to 4.41 that on the 1000 ft. horizon, despite the zones of tensile stress already noted, the failure potential exceeds 1.0 (C or higher in the alphabet on the plot) at all points. Failure initiation is therefore not predicted for that horizon, either 500 or 1000 yrs. after waste emplacement. Figures 4.42 to 4.46 are failure potential plots for the surface above the repository. It will be observed that over a wide area the minor principal stress is sufficient to cause tensile failure of the intact rock and there is a further region of potential failure.

While these results are interesting, they should be considered in the context of the assumptions made in the analysis and their practical significance. Firstly, it should be recalled that the analysis assumed elastic material. Tensile stresses determined by such an analysis will not be observed in a real, discontinuous rock mass. Secondly, most of the materials in the stratigraphic column assumed in these analysis creep under an applied load. For long term analysis, calculations based on elasticity are therefore, strictly speaking, inappropriate. Thirdly, local cracking at the surface above the repository may be of no practical significance, particularly if the regional groundwater table is well below the surface. More detailed analyses, incorporating non-linear, time dependent material behavior, will be required before the significance of the results reported here can be adequately assessed.

5. Conclusions

This report serves two functions. Firstly it briefly documents a computer code developed to study the thermoelastic response of a rock mass surrounding a radioactive waste repository and, secondly, it presents some results obtained using that code.

It is believed that the computer code is a useful addition to codes already being used in the rock mechanics investigation of the problems associated with radioactive waste disposal. A significant feature of the code is that it permits very simple analysis for arbitrary disposal patterns at any depth below the surface. Furthermore, it is relatively inexpensive to use.

The results presented within this report refer specifically to repositories with an initial heat generation rate of 150 kw/acre. These indicate that the effect of disposing of Spent Fuel rather than High Level Radioactive waste is to very significantly increase the magnitude and duration of the influence of the repository on the surrounding rock mass. Reducing the loading density of the Spent Fuel repository will clearly bring the two cases closer in line. How much effect such a reduction will have, can be determined directly from the results so far obtained, as in this model the induced temperature, displacement and stress changes are all linearly related to the initial strength of the heat sources. Such scaling has been applied to the surface displacement histories shown in figure 5.1. As indicated in that figure, the initial thermal loading density of a spent fuel repository must be reduced from 150 kw/acre to 60 kw/acre if the peak displacement computed for the 150 kw/acre High Level Waste repository is not to be exceeded.

No significant comment can be made at this stage on the basis of failure potential calculations reported in section 4. This is because the real changes in elastic properties of the rock that occur with depth may affect the

induced stresses sufficiently for calculation of local failure potentials to be misleading. Whether this is in fact the case, it can be determined from a comparison of the results reported here with those obtained using codes for analysis of axisymmetrical, but non-homogeneous, models. Should the results be comparable, further failure potential calculations could be made. However, before that is done some attention should be given to determine whether it is reasonable to use laboratory measured moduli rather than lower values more representative of that of the rock mass.

References

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Figure 2.1 Comparison with Nowackis' solution - instantaneous heat source.

Figure 2.2 Comparison with Nowackis' solution - pulse heat source.

Figure 2.3 Computer modeling for a single heat source analysis.

Figure 2.4(a) Vertical stress due to a single heat source.

Figure 2.4(b) Shear stress due to a single heat source.

Figure 2.5 Comparison of two methods for surface shear removal.

Figure 4.1 Temperature distribution in a high level waste repository.

Figure 4.2 Temperature distribution in a spent fuel repository.

Figure 4.2 (Cont'd) Temperature distribution in a spent fuel repository.

Figure 4.3

SPENT FUEL - 150 kw/acre

Temperature at the surface after 500 years

Figure 4.4

SPENT FUEL - 150 kw/acre

Temperature at 500 ft below the surface after 500 years

Figure 4.6

SPENT FUEL - 150 kw/acre

Temperature at 1250 ft below the surface after 500 years

[illegible]

Figure 4.7

SPENT FUEL - 150 kw/acre

Temperature at 1500 ft below the surface after 500 years

[illegible]

Figure 4.8

SPENT FUEL - 150 kw/acre

Temperature at 1750 below the surface after 500 years

[illegible]

Figure 4.9

SPENT FUEL - 150 kw/acre

Temperature at 1900 ft below the surface after 500 years

[illegible]

Figure 4.11

HIGH LEVEL WASTE - 150 kw/acre

Temperature at the surface after 500 years

Figure 4.11 is a contour map showing the temperature distribution at the surface after 500 years for a high level waste repository with a power density of 150 kw/acre. The map displays temperature contours in degrees Fahrenheit, ranging from 100 to 150. The highest temperatures are concentrated in the central area, where they reach 150 degrees Fahrenheit. The temperature decreases as one moves away from the center, with contours marked at 140, 130, 120, 110, and 100 degrees Fahrenheit. The map is oriented with North at the top. The area shown is a rectangular plot, and the contours are labeled with their respective temperature values. The map illustrates the thermal profile of the waste repository over a long period, showing how the heat from the waste has dissipated into the surrounding environment.

Figure 4.12

HIGH LEVEL WASTE - 150 kw/acre

Temperature at 500 ft below the surface after 500 years

Figure 4.13

HIGH LEVEL WASTE - 150 kw/acre

Temperature at 1000 ft below the surface after 500 years

[illegible]

Figure 4.14

HIGH LEVEL WASTE - 150 kw/acre

Temperature at 1250 below the surface after 500 years

[illegible]

Figure 4.16

HIGH LEVEL WASTE - 150 kw/acre

Temperature at 1750 ft below the surface after 500 years

Figure 4.17

HIGH LEVEL WASTE - 150 kw/acre

Temperature at 1900 ft below the surface after 500 years

[illegible]

Temperature at 1950 ft below the surface after 500 years

Figure 4.19

SPENT FUEL - 150 kw/acre

Vertical displacements at the surface after 100 years

Figure 4.20

SPENT FUEL - 150 kw/acre

Vertical displacements at the surface after 300 years

[illegible]

v

Figure 4.22

SPENT FUEL - 150 kw/acre

Vertical displacements at the surface after 1000 years

Figure 4.23

SPENT FUEL - 150 kw/acre

Vertical displacements at the surface after 5000 years

Figure 4.24

SPENT FUEL - 150 kw/acre

Vertical displacements at the surface after 10,000 years

Figure 4.25

SPENT FUEL - 150 kw/acre

Vertical displacements at the surface after 50,000 years

Figure 4.26

HIGH LEVEL WASTE - 150 kw/acre

Vertical displacement at the surface after 100 years

Figure 4.27

HIGH LEVEL WASTE - 150 kw/acre

Vertical displacement at the surface after 300 years

Figure 4.28

HIGH LEVEL WASTE - 150 kw/acre

Vertical displacement at the surface after 500 years

Figure 4.29

HIGH LEVEL WASTE - 150 kw/acre

Vertical displacement at the surface after 1000 years

Figure 4.30

HIGH LEVEL WASTE - 150 kw/acre

Vertical displacement at the surface after 5000 years

Figure 4.31

HIGH LEVEL WASTE - 150 kw/acre

Vertical displacement at the surface after 10,000 years

Figure 4.32

HIGH LEVEL WASTE - 150 kw/acre

Vertical displacement at the surface after 50,000 years

Figure 4.33 Vertical uplift above the center of repository.

Figure 4.34

SPENT FUEL

Minor principal stresses at 1000 ft after 500 years

Figure 4.35

SPENT FUEL

Minor principal stresses at 1000 ft after 1000 years

Figure 4.36

HIGH LEVEL WASTE

Minor principal stresses at 1000 ft after 500 years

Figure 4.37

HIGH LEVEL WASTE

Minor principal stresses at 1000 ft after 1000 years

Figure 4.38 Failure envelope and Mohr's circle for determining Failure Potential.

Figure 4.39

SPENT FUEL

Failure potential at the surface after 500 years

Figure 4.40

SPENT FUEL

Failure potential at 1000 ft below the surface after 500 years

```

-----AAA-----BBB-----CC-----DD-----E-----FF-----GG-----HH-----II-----JJ-----KK-----
-----AAA-----BBB-----CC-----DD-----E-----FF-----GG-----HH-----II-----JJ-----KK-----
-----AAA-----BBB-----CC-----DD-----E-----FF-----GG-----HH-----II-----JJ-----KK-----
-----AAA-----BBB-----CC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----
-----AAAA-----BBB-----CC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----
-----AAAA-----BBB-----CC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----
-----AAAA-----BBB-----CC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----
-----AAAA-----BBB-----CC-----DD-----EE-----F-----GG-----HH-----II-----JJ-----KK-----
-----AAAAAA-----BBB-----CC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----L
-----AAAAAA-----BBB-----CCC-----DD-----E-----FF-----GG-----HH-----II-----JJ-----KK-----L
-----AAAAAA-----BB-----CC-----D-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----L
-----AAAAAA-----BB-----CC-----DD-----EE-----F-----GG-----HH-----II-----JJ-----KK-----LL
-----AAAAAAAA-----BBB-----CC-----DD-----E-----FF-----GG-----HH-----II-----JJ-----KK-----L
-----AAAAAAAAAAAAAAAA-----BBB-----CC-----DD-----EE-----F-----GG-----HH-----II-----JJ-----KK-----LL
AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA-----BBB-----CC-----DD-----E-----FF-----GG-----HH-----II-----JJ-----KK-----L
AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA-----BBB-----CC-----DD-----EE-----F-----GG-----HH-----II-----JJ-----KK-----LL
                BBBB-----CC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----L
                BBBBBB-----CCC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----LL
BBB BBBB BBBB BBBB BBBB BBBB BBBB-----CCC-----DDD-----EEE-----FF-----GG-----HH-----II-----JJ-----KK-----LL
BBB BBBB BBBB BBBB BBBB BBBB BBBB-----CCC-----DDD-----EEE-----FF-----GG-----HH-----II-----JJ-----KK-----LL
                CCCCCC-----DD-----EE-----FF-----GG-----HH-----II-----JJ-----KK-----L
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC-----DDD-----EEE-----FF-----GG-----HH-----II-----JJ-----KK-----L
CCCCCCCCCCCCCCCCCCCC-----DDDD-----EEE-----FFF-----GG-----HH-----II-----JJ-----KK-----L
                DDDDDDD-----EEE-----FF-----GG-----HH-----II-----JJ-----KK-----L
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDD-----EEEE-----FFF-----GG-----HH-----II-----JJ-----KK-----L
                EEEEEEE-----FFF-----GG-----HH-----III-----JJ-----KK-----L
                EEEEEEEEEEEEEEEEEEE-----FFFF-----GGG-----HH-----II-----JJ-----KK-----LL
EEEEEEEEEE-----FFFFFF-----GGG-----HHH-----II-----JJ-----KK-----LL
                FFFFFFFFF-----GGGG-----HHH-----III-----JJ-----KK-----LL
FFFFFFFFFFFFFFFF-----GGGGG-----HHHH-----III-----JJ-----KK-----LL
                GGGGGGGGG-----HHHH-----III-----JJ-----KK-----LL
GGGGGGGGGGGGGGGGGGGGGGGGGGGGGG-----HHHHH-----III-----JJ-----KK-----LL
                HHHHHHH-----III-----JJJ-----KK-----LL
HHHHHHHHHHHHHHHHHHHHHHHHHHHHHH-----IIII-----JJJ-----KKK-----LL
HHHH-----IIIIII-----JJJJ-----KKK-----LLL-----
                IIIIIIIII-----JJJJJ-----KKK-----LLL-----
IIIIIIIIII-----JJJJJJ-----KKKK-----LLL-----
                JJJJJJJJJ-----KKKK-----LLL-----
JJJJJJJJJJJJJJJJJJJJJJJJJJJJJJ-----KKKKK-----LLLL-----
                KKKKKKKKK-----LLLL-----
KKKKKKKKKKKKKKKKKKKKKKKKKKKKKK-----LLLLL-----
                LLLLLLL-----

```


Figure 4.41

SPENT FUEL

Failure potential at the surface after 1000 years

Figure 4.42

SPENT FUEL

Failure potential at 1000 ft below the surface after 1000 years

Figure 4.43

HIGH LEVEL WASTE

Failure potential at the surface after 500 years

Figure 4.44

HIGH LEVEL WASTE

Failure potential at 1000 ft below the surface after 500 years

[illegible]

Figure 4.45

HIGH LEVEL WASTE

Failure potential at the surface after 1000 years

Figure 4.46

HIGH LEVEL WASTE

Failure potential at 1000 ft below the surface after 1000 years

```

RBBBBBBBBBBB      CCC  DDD  EEE  FF  G  HH  I  J  KL  M  OP  QR  T  ++++++
BBBBBBBBBB      CCC  DDD  EEE  FF  GG  HH  I  J  L  MN  OP  QR  T  ++++++
BBBBBBB      CCC  DDD  EEE  FF  GG  HH  I  J  L  MN  OP  QR  T  ++++++
BBBBB      CCCC  DDD  EEE  FF  GG  H  I  JK  L  MN  OP  QR  T+++++
BBB      CCCC  DDD  EEE  FF  GG  H  II  JK  L  M  P  QR  T+++++
B      CCCC  DDD  EE  FF  GG  H  I  K  L  N  PQ  RS  ++++++
P      CCCC  DDD  EE  F  GG  H  I  J  K  LM  NO  PQ  S  ++++++
      CCCCC  DD  EE  FF  G  H  I  J  K  M  O  QR  ST  ++++++
      CCCCCC  DD  EE  FF  GG  HH  I  J  L  MN  OP  Q  T+++++
      CCCCCC  DDD  EEE  FF  GG  H  I  JK  L  N  P  QS  ++++++
      CCCCCCCC  DD  EE  FF  GG  HH  I  J  K  M  O  Q  ST  ++++++
      CCCCCCCCCC  DDD  EE  FF  G  H  I  J  K  L  MN  OP  QR  T+++++
CCCCCCCCCCCCCCCCC  DDD  EE  FF  GG  H  I  J  K  M  NO  PQ  S  ++++++
CCCCCCCCCCCCC  DDD  EE  FF  G  H  I  J  K  L  MN  OP  QR  T  ++++++
CCCCCCCCCCCC  DDD  EEE  F  GG  H  II  J  K  L  MN  O  Q  S  ++++++
DDDDDDDDDDDDDDDDDDDD  EEE  FF  G  HH  I  J  K  L  MN  O  P  R  T+++++
DDDDDDDDDDDDDDDD  EE  F  G  HH  II  J  K  L  MN  O  P  R  ST  ++++++
      EEEE  FFF  GG  HH  II  JJ  K  L  MN  O  P  QR  S  ++++++
EEEEEEEEEEEEEEEEEE  FF  G  H  II  JJ  K  L  MN  O  P  Q  S  T+++++
EEEEEEEEEEEEEE  FFF  GG  H  I  JJ  KK  L  MN  O  P  Q  S  T+++++
      FFFF  GG  H  II  J  KK  L  MN  O  PQ  R  S  T+++++
FFFFFFFFFFFFFFFFFF  GG  HH  I  JJ  K  LL  MN  O  P  QR  S  T+++++
      GGGG  H  II  J  KK  L  MN  O  PP  QR  S  ++++++
GGGGGGGGGGGGGGGG  HHH  II  JJ  K  LL  MN  O  P  QR  S  T  ++++++
      HHHH  II  J  KK  L  M  O  P  QR  RS  T  ++++++
HHHHHHHHH  III  J  KK  L  MN  O  P  QR  S  T  ++++++
IIIIIIIIIIII  JJJ  KK  L  MN  O  P  QR  RS  T  ++++++
      JJJJJJ  KK  LL  MN  N  O  P  Q  R  S  T  ++++++
      KKKKK  LLL  MM  NN  OO  P  Q  R  S  T  ++++++
      LLLLL  MM  NN  OO  PP  Q  R  S  T  ++++++
      MMMMM  INN  OO  P  Q  RR  S  T  ++++++
      NNNNNN  OOO  PP  QQ  KR  S  T  ++++++
      CCCCCC  PPP  QQ  RR  SS  T  ++++++
PPPPPPPPPP  QQ  PR  S  TT  ++++++
      QQQQ  PPP  SS  TT  ++++++
PPPP  SSSS  TT  ++++++
      TTTT  ++++++
+++++
+++++
+++++
+++++
+++++

```

Figure 5.1 Surface displacement history

APPENDIX A

A computer code for thermoelastic analysis of radioactive
waste repository site

A.1 Introduction

This appendix briefly documents a computer code written specifically to simulate the complete thermal and stress history for a rock mass surrounding a radioactive waste repository. In concept, the code has a great deal in common with the displacement discontinuity models being developed at the University of Minnesota. However, since there was no need to model the excavation configuration on the disposal horizon, it was appropriate to develop a completely independent code. It is believed that this effort has produced a simple and practical method of rapidly assessing the consequences of different strategies for radioactive waste disposal.

A.2 Code Description.

The code is built around the solution for a single heat source in a half space region. That in turn is derived by superposition of the effects of a source and image sink, with free surface shear stresses removed by addition of appropriate surface tractions. In essence, the construction of the code is extremely simple but some complexity is introduced to minimize computer storage and time requirements. Figures A.1 and A.2 give the outline flowcharts for the main code and the subroutine obtaining the solution for a single heat source, respectively. These should serve to clarify the procedure and assist in interpretation of the program listing.

Fundamental to the solution procedure is division of the disposal horizon into squares all of the same size. This division is mirrored on the free surface and on any calculation horizon, to the extent that calculations of temperatures, displacements and stresses are always made for the centers of those squares. Furthermore, heat sources, even though they may be uniformly

Figure A.1 Flow chart for the main program

FLOWCHART II.

Subroutine CALC

Figure A.2 Flow chart for the subroutine CALL

distributed in practice, are considered to be concentrated at the center of any specified squares on the disposal horizon. To facilitate economy of computer storage and time it is assumed that waste emplacement occurs symmetrically about the center of the repository. Under those conditions, calculations need be carried out for only one quarter of the whole model. (Figure A.3).

It should be noted that there are two approximations made within the computer model. The first of these is that heat generation is concentrated at relatively few sources. This concentration will effect the values obtained for temperatures, displacements and stresses in the immediate vicinity of the sources. Remote from the sources the effect of the concentration will disappear. Also, there is a cut-off on the region over which surface tractions are applied. This limitation could be overcome by having special zoning arrangement for the free surface, but such a procedure was not considered essential for the current analysis.

A.3 Computer Requirements

The computer program listed runs on the CDC Cyber 74 at the University of Minnesota within 75K octal words of core storage. Run times depend very heavily on which method is used to remove the surface shear stresses. For a 20 by 20 grid model with 36 exponentially decaying heat sources each with 6 components run times for a single horizon and time were:

Using point shear force solution - 215 seconds

Using distributed shear force solution - 1226 seconds

The long run times arise because temperatures displacements and stresses are calculated right across the horizon. Such a procedure is useful for examining the effect of repository shape or waste emplacement sequence but may be unnecessary in many circumstances. Later versions of this program determine

profiles rather than horizon plans and execute very much faster.

Finally, it should be noted that the program makes use of a library routine to evaluate the error function. Calls to that routine take the form:

```
CALL MERF(X, ERF)
```

where ERF is assigned the value $\text{erf}(x)$. The complex error function is not widely available as a library routine and has therefore been included within the listed coding.

A.4 Input instructions.

Since it is assumed that the rock mass surrounding the repository is homogeneous, isotropic and linearly elastic the input requirements for the model are minimal. They are listed below, with explanation as required.

Card 1 - Environment details - Format (2F8.0, F16.0, 6F8.0)

Cols.

1-8	Repository depth below surface	
9-16	Thermal diffusivity	} of the host rock
17-32	Elastic modulus	
33-40	Poissons ratio	
41-48	Coefficient of expansion	
49-56	Surface temperature	(Isothermal surface)
57-64	Geothermal gradient	
65-72	Unit weight of host rock*	
73-80	Ratio of horizontal to vertical initial stress*	

* The initial stresses at any point at depth D are:

$$\sigma_{zz} = D \cdot \text{Unit weight of host rock}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} \cdot \text{Ratio (cols.73-80)}$$

Card 2 - Modelling instructions - Format (F5.0, 6I5)

Cols.

1-5	Half-width of squares for division of horizon
6-10	Number of heat sources (in one quarter model)
11-15	Number of components of each heat source
16-20	Number of times for computer output
21-25	Number of horizons for computer output
26-30	Number of rows for quarter model
31-35	Number of columns for quarter model

Card 3 - Composition of heat sources - Format (3F10.0)

One card for each component of the heat sources (see section 3.4)

Cols.

1-10	Time at which this component starts
11-20	Q_c
21-30	λ - zero or positive

The general form of the equation describing each component of the heat source is:

$$Q(t) = Q_c e^{-\lambda t}$$

If it is required to specify a constant source then λ is set to zero. (In these circumstances the integrals are evaluated in a different subroutine). Note that sources may be removed at any time by addition of a second source, of the same magnitude but opposite sign. For an

exponentially decaying source Q'_C would be given by

$$Q'_C = -Q_C e^{-\lambda t}$$

where Q_C is the initial strength of the source and t the time at which it is to be removed.

Cards 4 - Position of heat sources - Format (10I5)
Row and column positions for five sources per card.

Cols.

1-5 Row number of square in which heat source is sited
6-10 Column number of square in which heat source is sited
etc

Note that heat sources are specified for one quarter of the repository only. Hence for every source specified there are three others implied.

Cards 5 - Timing for output - Format (5F10.0)
Five times per card

Cols.

1-10 Time for output
etc

Cards 6 - Horizons for output - Format (5F10.0)
Five horizons per card

Cols.

1-10 Horizon for output (measured from the free surface)
etc

```

PROGRAM JUDITH(INPUT,OUTPUT,TAPE5=INPUT,TAPE7)
C
C THERMO ELASTIC ANALYSIS OF RADIO-ACTIVE REPOSITORY
C
      DIMENSION T(25,25),U(25,25),V(25,25),W(25,25),SIGMA(25,25,6),
      * TS(26,26),US(26,26),VS(26,26),WS(26,26),SIGS(26,26,6),TX(26,26),
      * TY(26,26),IQ(50),JQ(50),TIM(50),ELEV(50),ISGR(4),ISGC(4)
      COMMON DEPTH,DIFF,E,PR,ALPHA,SW,NROW,NCOL,NTYPE
      COMMON QTA(10),QCA(10),QEXPA(10)
      DATA (ISGR(I),I=1,4)/1,-1,-1,1/, (ISGC(I),I=1,4)/1,-1,1,-1/
      IR=25
      IC=25
      JR=26
      JC=26
      REWIND 7
C READ MODEL DATA
      READ(5,1000)DEPTH,DIFF,E,PR,ALPHA,TSURF,TGRAD,DEN,RATIO
1000  FORMAT(2F8.0,F16.0,6F8.0)
      PRINT 2000, DEPTH,DIFF,E,PR,ALPHA,TSURF,TGRAD,DEN,RATIO
2000  FORMAT(58H1THERMO-ELASTIC ANALYSIS OF RADIO-ACTIVE WASTE REPOSITOR
      * Y / 15H0DEPTH ,F10.0/
      * 15H DIFFUSIVITY ,E12.4/
      * 15H ELASTIC MOD. ,E12.4/
      * 15H POISSON RATIO ,E12.4/
      * 15H COEFF OF EXPAN,E12.4/
      * 15H SURFACE TEMP ,E12.4/
      * 15H TEMP GRADIENT ,E12.4/
      * 15H ROCK DENSITY ,E12.4/
      * 15H INT. H/V RATIO,E12.4)
C READ MODEL CONTROL INFORMATION
      READ(5,1001) SW,NSORC,NTYPE,NTIMES,NLEV,NROW,NCOL
1001  FORMAT(F5.0,6I5)
      NR=NROW+1
      NC=NCOL+1
      PRINT 2001,SW,NSORC,NTYPE,NTIMES,NLEV,NROW,NCOL
2001  FORMAT(34H0SEMI WIDTH OF ELEMENTS ,F10.2/
      * 34H NUMBER OF HEAT SOURCES ,I10/
      * 34H NUMBER OF SOURCE COMPONENTS ,I10/
      * 34H NUMBER OF TIMES FOR OUTPUT ,I10/
      * 34H NUMBER OF LEVELS FOR OUTPUT ,I10/
      * 34H NUMBER OF ROWS FOR QUARTER MODEL ,I10/
      * 34H NUMBER OF COLS FOR QUARTER MODEL ,I10)
C READ SOURCE DATA
      READ(5,1002) (QTA(I),QCA(I),QEXPA(I),I=1,NTYPE)
1002  FORMAT(3F10.0)
      PRINT 2002, (QTA(I),QCA(I),QEXPA(I),I=1,NTYPE)
2002  FORMAT(22H SOURCE DATA - TIME ON ,F15.2,9H STRENGTH,
      * E12.4,6H EXP(- ,E12.4,1H))
      READ(5,1004) (IQ(I),JQ(I),I=1,NSORC)
1004  FORMAT(10I5)
      PRINT 2009, (IQ(I),JQ(I),I=1,NSORC)
2009  FORMAT(24H0COORDINATES - X,I LIST /(10(3X,2I5)))
C READ TIMES AND LEVELS
      READ(5,1003) (TIM(I),I=1,NTIMES)
1003  FORMAT(5F10.0)

```



```

      PRINT 2003, (TIM(I),I=1,NTIMES)
2003 FORMAT(24H0TIMING FOR OUTPUT (YRS)  /(3X,10F10.2))
      READ(5,1003) (ELEV(I),I=1,NLEV)
      PRINT 2004, (ELEV(I),I=1,NLEV)
2004 FORMAT(34H0ELEVATION OF HORIZONS FOR OUTPUT  /(3X,10F10.1))
C
C OBTAIN OUTPUT FOR EACH TIME REQUIRED
C
      DO 400 I=1,NTIMES
      PRINT 2007, TIM(I)
2007 FORMAT(44H1TEMPERATURES DISPLACEMENTS AND STRESSES AT ,E12.4,
* 6H YEARS //)
      TM=TIM(I)
C
C START BY CALCULATING CORRECTIVE SHEAR LOADS , FOR A SINGLE HEAT SOURCE
C , THAT WILL BE APPLIED TO THE FREE SURFACE TO MAKE IT TRACTION FREE
C
C
C INITIALIZE SHEAR FORCE ARRAYS
C
      DO 10 K=1,NR
      DO 10 L=1,NC
      TX(K,L)=0.0
      TY(K,L)=0.0
10 CONTINUE
      CALL SHEAR(TM,TX,TY,JC,JR)
C
C NOW OBTAIN A SOLUTION FOR A SINGLE HEAT SOURCE LOCATED AT THE POINT
C (0,0,H) - FOR THE APPROPRIATE TIME AND HORIZON
C
      DO 200 J=1,NLEV
      EL=ELEV(J)
      PRINT 2005, EL
2005 FORMAT(21H0ELEVATION OF HORIZON ,F10.2)
C
C INITIALIZE ARRAYS
C
      DO 20 K=1,NR
      DO 20 L=1,NC
      TS(K,L)=0.0
      US(K,L)=0.0
      VS(K,L)=0.0
      WS(K,L)=0.0
      DO 20 K1=1,6
      SIGS(K,L,K1)=0.0
20 CONTINUE
      CALL CALC(TM,EL,TS,US,VS,WS,SIGS,TX,TY,JC,JR)
      PRINT 2006
2006 FORMAT(11H0 ROW COL ,6X,1HT,11X,1HU,11X,1HV,11X,1HW,11X,
* 16H(SIGMA(I),I=1,6))
C NOW SUPERIMPOSE SOLUTIONS TO ACCOUNT FOR ALL HEAT SOURCES
C START BY INITIALIZING
C
      TP=TSURF+EL*TGRAD
      SP=EL*DEN

```

```

SPH=SP*RATIO
DO 30 K=1,NROW
DO 30 L=1,NCOL
T(K,L)=TP
U(K,L)=0.0
V(K,L)=0.0
W(K,L)=0.0
SIGMA(K,L,1)=SPH
SIGMA(K,L,2)=SPH
SIGMA(K,L,3)=SP
SIGMA(K,L,4)=0.0
SIGMA(K,L,5)=0.0
SIGMA(K,L,6)=0.0
30 CONTINUE
C
C FOR EACH SQUARE IN TURN SUM THE EFFECT OF ALL HEAT SOURCES
C
DO 100 K=1,NROW
DO 100 L=1,NCOL
C
C FOR EACH HEAT SOURCE
C
DO 90 NS=1,NSORC
MQ=IQ(NS)
NQ=JQ(NS)
DO 80 NQUAD=1,4
SK=1.0
MQ1=MQ*ISGR(NQUAD)
IF(MQ1.LT.0) MQ1=MQ1+1
KMQ=K-MQ1
IF(KMQ.LT.0) SK=-1.0
KQ=IABS(KMQ)+1
IF(KQ.GT.NR) GO TO 80
SL=1.0
NQ1=NQ*ISGC(NQUAD)
IF(NQ1.LT.0) NQ1=NQ1+1
LNQ=L-NQ1
IF(LNQ.LT.0) SL=-1.0
LQ=IABS(LNQ)+1
IF(LQ.GT.NC) GO TO 80
T(K,L)=T(K,L)+TS(KQ,LQ)
U(K,L)=U(K,L)+US(KQ,LQ)*SL
V(K,L)=V(K,L)+VS(KQ,LQ)*SK
W(K,L)=W(K,L)+WS(KQ,LQ)
SIGMA(K,L,1)=SIGMA(K,L,1)+SIGS(KQ,LQ,1)
SIGMA(K,L,2)=SIGMA(K,L,2)+SIGS(KQ,LQ,2)
SIGMA(K,L,3)=SIGMA(K,L,3)+SIGS(KQ,LQ,3)
SIGMA(K,L,4)=SIGMA(K,L,4)+SIGS(KQ,LQ,4)*SK*SL
SIGMA(K,L,5)=SIGMA(K,L,5)+SIGS(KQ,LQ,5)*SK
SIGMA(K,L,6)=SIGMA(K,L,6)+SIGS(KQ,LQ,6)*SL
80 CONTINUE
90 CONTINUE
100 CONTINUE
PRINT 2008, ((M,N,T(M,N),U(M,N),V(M,N),W(M,N),(SIGMA(M,N,L),L=1,6)
• ,N=1,NCOL),M=1,NROW)

```

Program Listing

Judith - Page 4

```

2008 FORMAT(1X,2I5,10E12.4)
WRITE (7) NROW,NCOL,TM,EL,T,U,V,W,SIGMA
200 CONTINUE
400 CONTINUE
REWIND 7
STOP
END
SUBROUTINE CALC(TM,EL,TS,US,VS,WS,SIGS,TX,TY,JC,JR)
C
C CALCULATES TEMPERATURE , DISPLACEMENTS AND STRESSES RESULTING FROM
C A SINGLE HEAT SOURCE IN A SEMI INFINITE REGION AT THE POINT (0.0,DEPTH
C FOR HORIZON EL
C
COMMON DEPTH,DIFF,E,PR,ALPHA,SW,NROW,NCOL,NTYPE
COMMON QTA(10),QCA(10),QEXPA(10)
DIMENSION TS(JR,JC),US(JR,JC),VS(JR,JC),WS(JR,JC),SIGS(JR,JC,6),
* TX(JR,JC),TY(JR,JC)
DIMENSION CSIG(6),ISGX(4),ISGY(4)
DATA (ISGX(I),I=1,4)/1,-1,-1,1/, (ISGY(I),I=1,4)/1,-1,1,-1/
DATA PI1,PI2/0.564189583547756,0.179587122125166/
C1=(ALPHA*(1.0+PR)/(1.0-PR))*0.159154943091895
G=E/(2.0+2.0*PR)
C2=C1*G
NR=NROW+1
NC=NCOL+1
C
C FOR EVERY SQUARE IN ONE QUADRANT CALCULATE TEMPERATURE , DISPLACEMENTS
C AND STRESSES
C
DO 400 I=1,NR
C Y DISTANCE OF SQUARE
Y=FLOAT(2*I-2)*SW
DO 400 J=1,NC
C X DISTANCE OF THE SQUARE
X=FLOAT(2*J-2)*SW
C CONSTANTS RELATING TO DISTANCES
X2=X*X
Y2=Y*Y
RS2=X2+Y2
RS=SQRT(RS2)
C
C LOOP SO THAT CALCULATION IS CARRIED FOR SOURCE AND IMAGE
C
PM=-1.0
DO 200 K=1,2
PM=-PM
C
C DISTANCE BETWEEN HORIZON AND SOURCE OR IMAGE
C
Z=EL-PM*DEPTH
R2=RS2+Z*Z
IF(R2.EQ.0.0) GO TO 200
R=SQRT(R2)
R3=R2*R
R5=R2*R3

```



```

C
C ACCUMULATE CONTRIBUTIONS OF ALL COMPONENTS OF THE SOURCES
C
    V1=0.0
    V2=0.0
    V3=0.0
    DO 30 JS=1,NTYPE
    TMM=TM-QTA(JS)
    IF(TMM.LE.0.0) GO TO 30
    QC=QCA(JS)
    QEXP=QEXPA(JS)
    IF(QEXP.GT.0.0) GO TO 20
C
C CONSTANT SOURCE
C
    CALL QCON(QC,TMM,R,DIFF,VV1,VV2,VV3)
    GO TO 25
C
C DECAYING SOURCE
C
    20 CALL QDEC(QC,QEXP,TMM,R,DIFF,VV1,VV2,VV3)
    25 V1=V1+VV1
        V2=V2+VV2
        V3=V3+VV3
    30 CONTINUE
C CALCULATE TEMPERATURE
    TS(I,J)=TS(I,J)+PM*PI2*V3
C CALCULATE THE DISPLACEMENTS
    VAL=(PI1*V2-V1/(R+R))*C1/R2
    US(I,J)=US(I,J)+PM*X*VAL
    VS(I,J)=VS(I,J)+PM*Y*VAL
    WS(I,J)=WS(I,J)+PM*Z*VAL
C CALCULATE THE STRESSES
    S1=2.0*R*PI1
    S2=3.0/R2
    S3=2.0*S1
    S4=C2/R3
    S5=1.0-S2*X2
    SIGS(I,J,1)=SIGS(I,J,1)+PM*S4*(-S5*V1+S1*S5*V2-S3*(R2-X2)*V3)
    S5=1.0-S2*Y2
    SIGS(I,J,2)=SIGS(I,J,2)+PM*S4*(-S5*V1+S1*S5*V2-S3*(R2-Y2)*V3)
    S5=2.0-S2*RS2
    SIGS(I,J,3)=SIGS(I,J,3)+PM*S4*(S5*V1-S1*S5*V2+S3*RS2*V3)
    VAL=PM*3.0*C2/R5*(V1-S1*V2-S3*R2*V3/3.0)
    SIGS(I,J,4)=SIGS(I,J,4)+X*Y*VAL
    SIGS(I,J,5)=SIGS(I,J,5)+Y*Z*VAL
    SIGS(I,J,6)=SIGS(I,J,6)+Z*X*VAL
    200 CONTINUE
C
C NOW REMOVE THE SURFACE SHEARS
C
C NOTE THAT THERE IS A CUT OFF ON THE REMOVAL OF SHEAR ON THE SURFACE
    DO 300 M=1,NR
    DO 300 N=1,NC

```

110420
074011

C ACCOUNT FOR SYMMETRY ABOUT THE X AND Y AXES

```

      KL=4
      IF (M.EQ.1.OR.N.EQ.1) KL=2
      IF (M.N.EQ.2) KL=1
      DO 250 K=1,KL
      JN=ISGX(K)*N
      IF (JN.LT.0) JN=JN+2
      X1=FLOAT(J-JN)*2.0*SW
      IM=ISGY(K)*M
      IF (IM.LT.0) IM=IM+2
      Y1=FLOAT(I-IM)*2.0*SW
      TXL=TX(M,N)*FLOAT(ISGX(K))
      TYL=TY(M,N)*FLOAT(ISGY(K))
      CALL CERUTI(TXL,TYL,G,PR,X1,Y1,EL,CU,CV,CW,CSIG)

```

C ADD TO EXISTING VALUES

```

      US(I,J)=US(I,J)+CU
      VS(I,J)=VS(I,J)+CV
      WS(I,J)=WS(I,J)+CW
      DO 240 L=1,6
      SIGS(I,J,L)=SIGS(I,J,L)+CSIG(L)
240 CONTINUE
250 CONTINUE
300 CONTINUE
400 CONTINUE
      RETURN
      END

```

C
C SUBROUTINE SHEAR(TM,TX,TY,JC,JR)

C
C CALCULATES SHEAR FORCES TO BE APPLIED AT THE FREE SURFACE TO REMOVE
C SHEAR STRESS DUE TO SOURCE AND IMAGE

```

      DIMENSION TX(JR,JC),TY(JR,JC)
      COMMON DEPTH,DIFF,E,PR,ALPHA,SW,NROW,NCOL,NTYPE
      COMMON QTA(10),QCA(10),QEXPA(10)
      DATA C1/1.128379167095512/
      NR=NROW+1
      NC=NCOL+1
      G=E/(2.0+2.0*PR)
      RM=ALPHA*(1.0+PR)/(1.0-PR)
      CON=-3.0*G*RM*DEPTH*0.159154943091895
      C2=2.0*C1/3.0
      AREA=8.0*SW*SW

```

C
C CALCULATE THE SHEAR STRESSES TAU RZ DUE TO SOURCE ALONE

```

      DO 100 I=1,NR
      Y=FLOAT(2*I-2)*SW
      DO 100 J=1,NC
      X=FLOAT(2*J-2)*SW
      RS=SQRT(X*X+Y*Y)
      R=SQRT(X*X+Y*Y+DEPTH*DEPTH)

```

C
C ACCUMULATE CONTRIBUTIONS OF ALL COMPONENTS OF THE SOURCES

C

Program Listing

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```

V1=0.0
V2=0.0
V3=0.0
DO 30 JS=1,NTYPE
TMM=TM-QTA(JS)
IF(TMM.LE.0.0) GO TO 30
QC=QCA(JS)
QEXP=QEXPA(JS)
IF(QEXP.GT.0.0) GO TO 20

```

```

C
C CONSTANT SOURCE
C
  CALL QCON(QC,TMM,R,DIFF,VV1,VV2,VV3)
  GO TO 25

```

```

C
C DECAYING SOURCE
C
  20 CALL QDEC(QC,QEXP,TMM,R,DIFF,VV1,VV2,VV3)
  25 V1=V1+VV1
    V2=V2+VV2
    V3=V3+VV3
  30 CONTINUE
    TAURZ=CON*(V1-C1*R*V2-C2*R**3*V3)/R**5

```

```

C
C FORCES OF OPPOSITE SIGN TO THE STRESS - TOTAL DUE TO SOURCE AND IMAGE
C

```

```

  TX(I,J)=TX(I,J)-AREA*TAURZ*X
  TY(I,J)=TY(I,J)-AREA*TAURZ*Y
100 CONTINUE
  RETURN
  END
  SUBROUTINE CERUTI(TX,TY,G,PR,XL,YL,ZL,U,V,W,SIG)

```

```

C
C SUBROUTINE TO EVALUATE DISPLACEMENTS AND STRESSES RESULTING
C FROM SURFACE SHEARS
C

```

```

  DIMENSION SIG(6)
  X=XL
  Y=YL
  Z=ZL
  CON=1.0-2.0*PR
  PX=TX*0.159154943091895
  PY=TY*0.159154943091895
  X2=X*X
  Y2=Y*Y
  Z2=Z*Z
  R2=X2+Y2+Z2
  IF(R2.EQ.0.0) GO TO 100
  R=SQRT(R2)
  R3=R*R2
  R5=R2*R3
  RZ=R*Z
  RZ2=RZ*RZ
  GR2=2.0*G*R

```

113421

Program Listing

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```

U=(PX*(1.0+X2/R2+CON*(R/RZ-X2/RZ2))+PY*(X*Y/R2-CON*X*Y/RZ2))/GR2
V=(PX*(X*Y/R2-CON*X*Y/RZ2)+PY*(1.0+Y2/R2+CON*(R/RZ-Y2/RZ2)))/GR2
W=(PX*(X*Z/R2+CON*X/RZ)+PY*(Y*Z/R2+CON*Y/RZ))/GR2
SIG(1)=(PX*X*(3.0*X2/R2-CON/RZ2*(R2-Y2-2.0*R*Y2/RZ))+
* PY*Y*(3.0*X2/R2-CON/RZ2*(3.0*R2-Y2-2.0*R*Y2/RZ)))/R3
SIG(2)=(PX*X*(3.0*Y2/R2-CON/RZ2*(3.0*R2-X2-2.0*R*X2/RZ))+
* PY*Y*(3.0*Y2/R2-CON/RZ2*(R2-X2-2.0*R*X2/RZ)))/R3
SIG(3)=3.0*Z2*(PX*X+PY*Y)/R5
SIG(4)=(PX*Y*(3.0*X2/R2-CON/RZ2*(-R2+X2+2.0*R*X2/RZ))+
* PY*X*(3.0*Y2/R2-CON/RZ2*(-R2+Y2+2.0*R*Y2/RZ)))/R3
SIG(5)=3.0*(X*Y*Z*PX+Y2*Z*PY)/R5
SIG(6)=3.0*(X2*Z*PX+X*Y*Z*PY)/R5
RETURN

```

```

100 PRINT 2000

```

```

2000 FORMAT(38H ILLEGAL TO CALCULATE FOR FREE SURFACE )

```

```

STOP

```

```

END

```

```

SUBROUTINE QCON(QC,T,R,DIFF,V1,V2,V3)

```

```

C EVALUATE INTEGRALS FOR CONSTANT HEAT SOURCE

```

```

DATA SQPI,RSQPI/1.77245385090551,0.564189583547756/

```

```

RDIFF=1.0/(2.0*DIFF)

```

```

C1=R*R/(4.0*DIFF*T)

```

```

C2=SQRT(C1)

```

```

C3=SQRT(T/DIFF)

```

```

C EXPONENTIAL FUNCTION

```

```

EX=0.0

```

```

C KEEP EXPONENTIAL FUNCTION WITHIN RANGE

```

```

IF(C1.LT.500.) EX=EXP(-C1)

```

```

C ERROR FUCTION

```

```

CALL MERF(C2,ERF)

```

```

C COMPLEMENTED ERROR FUNCTION

```

```

ERFC=1.0-ERF

```

```

V1=QC*(T*ERF-R*R*RDIFF*ERFC+C3*R*RSQPI*EX)

```

```

V2=QC*(C3*EX-R*SQPI*RDIFF*ERFC)

```

```

V3=QC*SQPI*RDIFF*ERFC/(2.0*R)

```

```

RETURN

```

```

END

```

```

SUBROUTINE QDEC(QC,QEXP,T,R,DIFF,V1,V2,V3)

```

```

COMPLEX VAL,WOFZ

```

```

DATA SPI/1.77245385090551/

```

```

CON=QEXP*T

```

```

R2=R*R/(4.0*DIFF*T)

```

```

R1=SQRT(R2)

```

```

EX1=0.0

```

```

EX2=0.0

```

```

IF(CON.LT.500.) EX1=EXP(-CON)

```

```

IF(R2.LT.500.) EX2=EXP(-R2)

```

```

CALL MERF(R1,ERF)

```

```

VAL=CMPLX(SQRT(CON),R1)

```

```

VAL=WOFZ(VAL)

```

```

C1=REAL(VAL)

```

```

C2=AIMAG(VAL)

```

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Program Listing

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```

V1=QC/QEXP*(-EX1*ERF+EX2*C1)
V2=QC*SPI/SQRT(4.0*DIFF*QEXP)*EX2*C2
V3=QC*SPI/(4.0*DIFF*R)*EX2*C1
RETURN
END
SUBROUTINE CERUTI1(TX,TY,G,PR,XL,YL,ZL,EW,U,V,W,SIG)
C
C SUBROUTINE TO EVALUATE DISPLACEMENTS AND STRESSES RESULTING
C FROM SURFACE SHEAR STRESSES CONSIDERED AS ACTING UNIFORMLY OVER
C EACH SEGMENT.
C CALL TO THE SUBROUTINE CERUTI1 -
C SUBROUTINE CERUTI1(TX,TY,G,PR,XL,YT,ZL,EW,U,V,W,SIG)
C
  DIMENSION PX(2),PY(2),SIG(6)
  DATA PX(1),PX(2),PY(1),PY(2),ONE/-1.,1.,-1.,1.,1./
  SW=EW
  PR3=2.*PR
  PR1=1.-PR3
  PR2=1.+PR1
  PI2=2.*ATAN(1.0)
  AREA=1./((16.*PI2*SW*SW)
  G1=-1./(2.*G)
  SX=TX*AREA
  SY=TY*AREA
  UX=SX*G1
  DY=SY*G1
  U=0.0
  V=0.0
  W=0.0
  DO 5 I=1,6
5  SIG(I)=0.0
  X0=XL
  Y0=YL
  Z=ZL
  Z2=Z*Z
C
C INTEGRATE OVER THE ELEMENT
C
  DO 100 I=1,2
  X=X0+SW*PX(I)
  X2=X*X
  AX=SIGN(ONE,X)
  DO 100 J=1,2
  Y=Y0+SW*PY(J)
  AY=SIGN(ONE,Y)
C FIRST EVALUATE THE EXPRESSIONS THAT ARE REPEATEDLY USED IN COMPUTING
C DISPLACEMENTS AND STRESSES.
C
  PXY=PX(I)*PY(J)
  Y2=Y*Y
  R2=X2+Y2+Z2
  R=SQRT(R2)
  R1=1./R

```

```

RMX=R*X
RMY=R*Y
RMZ=R*Z
XRI=X*RI
YRI=Y*RI
ZRI=Z*RI
XR=X+R
YR=Y+R
ZR=Z+R
XY=X*Y
YZ=Y*Z
ZX=Z*X
XYZ=XY*Z
X2Y2=X2+Y2
Y2Z2=Y2+Z2
X2Z2=X2+Z2
XRLOG=ALOG(XR)
YRLOG=ALOG(YR)
ZRLOG=ALOG(ZR)
THETA=(X2Z2+RMZ)/(SQRT(X2Z2)*ZR)
FI1=(PI2-ASIN(THETA))/ABS(X)*AY
THETA=(Y2Z2+RMZ)/(SQRT(Y2Z2)*ZR)
FI3=(PI2-ASIN(THETA))/ABS(Y)*AX
IF(Z.EQ.0) GO TO 10
THETA=(X2Z2+RMX)/(SQRT(X2Z2)*XR)
FI2=(PI2-ASIN(THETA))*AY
THETA=(Y2Z2+RMY)/(SQRT(Y2Z2)*YR)
FI4=(PI2-ASIN(THETA))*AX
FI5=ATAN(XY/RMZ)
GO TO 11
10 FI2=-PI2
   FI4=FI2
   FI5=SIGN(PI2,XY)
11 XR=1./(XR*R)
   YR=1./(YR*R)
   ZR=1./(ZR*R)
   X2Z2=1./X2Z2
   Y2Z2=1./Y2Z2
   THETA=(PR3*R+Z)*ZR
C
C THEN COMPUTE THE DISPLACEMENTS AND STRESSES.
C
   U= U+(DX*(PR2*X*YRLOG+2.*Y*XRLOG+Z*(FI2+FI4)-PR1*ZX*FI1)
1     -DY*(PR3*R+PR1*Z*ZRLOG))*PXY
   V= V-(DX*(PR3*R+PR1*Z*ZRLOG)-DY*(PR2*Y*XRLOG+2.*X*YRLOG+Z*
1     (FI2+FI4)-PR1*Y2*FI3))*PXY
   W=W+(DX*(PR1*(Y*ZRLOG+X2*FI1)-PR3*Z*YRLOG)
1     +DY*(PR1*(X*ZRLOG+Y2*FI3)-PR3*Z*XRLOG))*PXY
   SIG(1)=SIG(1)+(SX*(X2Z2*ZR*(PR1*R*Z2-Z*X2-PR3*R*X2)*Y-2.*YRLOG)
1     +SY*(THETA*X-PR3*XRLOG))*PXY
   SIG(2)=SIG(2)+(SX*(THETA*Y-PR3*YRLOG)+SY*(Y2Z2*ZR*(PR1*R*Z2
1     -Z*Y2-PR3*R*Y2)*Y-2.*XRLOG))*PXY

```

```

SIG(3)=SIG(3)- ZRI*(SX*YZ*X2Z2+SY*ZX*Y2Z2)*PXY
SIG(4) =SIG(4)+(SX*(THETA*X-XRLOG)+SY*(THETA*Y-YRLOG))*PXY
SIG(5)=SIG(5)+(SX*ZRI+SY*(FI5-XYZ*Y2Z2*RI))*PXY
SIG(6)=SIG(6)+(SX*(FI5-XYZ*X2Z2*RI)+SY*ZRI)*PXY

```

```

100 CONTINUE

```

```

RETURN

```

```

END

```

```

COMPLEX FUNCTION WOFZ(Z)

```

```

C
C WOFZ EVALUATES  $\exp(-Z^2) \cdot \text{ERFC}(-I \cdot Z)$  FOR  $Z=X+I \cdot Y$  IN THE FIRST
C QUADRANT OF THE COMPLEX PLANE.  $\text{ERFC}(\text{ARG})$  IS THE ERROR FUNCTION
C COMPLEMENT,  $2.0/\text{SQRT}(\pi) \cdot \text{INTEGRALFROM}(\text{ARG}) \text{ TO } (\infty) \exp(-T^2) \text{ DT.}$ 
C SEE COMMUNICATIONS OF THE ACM, VOL. 12, NO. 11 (NOV., 1969), P. 635
C AND VOL. 15, NO. 6 (JUNE, 1972), P. 465, AND SIAM JOURNAL OF
C NUMERICAL ANALYSIS, VOL. 7, NO. 1 (MAR., 1970), PP. 187-198.

```

```

C TRANSLATED FROM ALGOL TO FORTRAN ON MAR. 22, 1977 BY

```

```

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```

```

C
C COMPLEX Z

```

```

C LOGICAL B

```

```

C DATA CON/1.12837 91670 95512 37388/

```

```

C X=REAL(Z)

```

```

C Y=AIMAG(Z)

```

```

C ALAMBD=0.0

```

```

C IF(Y.LT.4.29.AND.X.LT.5.33) GO TO 10

```

```

C H=0.0

```

```

C ICAPN=0

```

```

C NU=8

```

```

C GO TO 20

```

```

10 S=(1.0-Y/4.29)*SQRT(1.0-X*X/28.41)

```

```

C H=1.6*S

```

```

C H2=2.0*H

```

```

C ICAPN=6.0+23.0*S

```

```

C ALAMBD=H2**ICAPN

```

```

C NU=9.0+21.0*S

```

```

C
C IN THE FOLLOWING 2 STATEMENTS, ALAMBD=0 COVERS THE UNDERFLOW CASE
C WHEN H.GT.0 IS VERY SMALL.

```

```

20 B=.FALSE.

```

```

C IF(H.EQ.0.0.OR.ALAMBD.EQ.0.0) B=.TRUE.

```

```

C S2=0.0

```

```

C S1=S2

```

```

C R2=S1

```

```

C R1=R2

```

```

C NUP1=NU+1

```

```

C DO 40 NN=1,NUP1

```

```

C N=NUP1-NN

```

```

C NP1=N+1

```

073011

Program Listing

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```
T1=Y+H*FLOAT(NP1)*R1
T2=X-FLOAT(NP1)*R2
C=0.5/(T1*T1+T2*T2)
R1=C*T1
R2=C*T2
IF (H.GT.0.0.AND.N.LE.ICAPN) GO TO 30
GO TO 40
30 T1=ALAMBD+S1
S1=R1*T1-R2*S2
S2=R2*T1+R1*S2
ALAMBD=ALAMBD/H2
40 CONTINUE
RE=S1
IF (B) RE=R1
RE=CON*RE
IF (Y.EQ.0.0) RE=EXP(-X*X)
AIM=S2
IF (B) AIM=R2
WOFZ=CMPLX(RE,CON*AIM)
RETURN
END
```

APPENDIX B

Temperatures, Displacements and Stresses Due to a Concentrated Heat Source in an Infinite Body.

$$T = \frac{V_3}{\pi^{3/2}}$$

$$u_x = \frac{m x}{2\pi R^2} \left\{ \frac{V_2}{\sqrt{\pi}} - \frac{V_1}{2R} \right\}$$

$$u_y = \frac{m y}{2\pi R^2} \left\{ \frac{V_2}{\sqrt{\pi}} - \frac{V_1}{2R} \right\}$$

$$u_z = \frac{m z}{2\pi R^2} \left\{ \frac{V_2}{\sqrt{\pi}} - \frac{V_1}{2R} \right\}$$

$$\sigma_{xx} = \frac{mG}{2\pi R^3} \left\{ -\left[1 - \frac{3x^2}{R^2}\right] V_1 + \frac{2R}{\sqrt{\pi}} \left[1 - \frac{3x^2}{R^2}\right] V_2 + \frac{4R}{\sqrt{\pi}} [R^2 - x^2] V_3 \right\}$$

$$\sigma_{yy} = \frac{mG}{2\pi R^3} \left\{ -\left[1 - \frac{3y^2}{R^2}\right] V_1 + \frac{2R}{\sqrt{\pi}} \left[1 - \frac{3y^2}{R^2}\right] V_2 + \frac{4R}{\sqrt{\pi}} [R^2 - y^2] V_3 \right\}$$

$$\sigma_{zz} = \frac{mG}{2\pi R^3} \left\{ \left[2 - \frac{3r^2}{R^2}\right] V_1 - \frac{2R}{\sqrt{\pi}} \left[2 - \frac{3r^2}{R^2}\right] V_2 + \frac{4R}{\sqrt{\pi}} r^2 V_3 \right\}$$

$$\sigma_{xy} = \frac{3mG}{2\pi R^5} x y \left\{ V_1 - \frac{2R}{\sqrt{\pi}} V_2 - \frac{4R^3}{3\sqrt{\pi}} V_3 \right\}$$

$$\sigma_{xz} = \frac{x}{r} \sigma_{rz} \quad \sigma_{yz} = \frac{y}{r} \sigma_{rz}$$

where

$$\sigma_{rz} = \frac{3mG}{2\pi R^5} r z \left\{ V_1 - \frac{2R}{\sqrt{\pi}} V_2 - \frac{4R^3}{3\sqrt{\pi}} V_3 \right\}$$

In which $R = \sqrt{(x^2 + y^2 + z^2)}$: $r = \sqrt{(x^2 + y^2)}$

and V_i replaces $V_i(R, t)$ etc.

APPENDIX C

Functions for Constant and Exponentially Decaying Heat Sources.

a) Constant heat sources

$$V_1(R, t) = Q_c \left\{ t \cdot \operatorname{erf} \frac{R}{\sqrt{4\chi t}} - \frac{R^2}{2\chi} \cdot \operatorname{erfc} \frac{R}{\sqrt{4\chi t}} + \sqrt{\frac{t}{\pi\chi}} \cdot R \cdot \exp\left(-\frac{R^2}{4\chi t}\right) \right\}$$

$$V_2(R, t) = Q_c \left\{ \sqrt{\frac{t}{\chi}} \cdot \exp\left(-\frac{R^2}{4\chi t}\right) - \frac{\sqrt{\pi}}{4\chi} \cdot R \cdot \operatorname{erfc}\left(\frac{R}{\sqrt{4\chi t}}\right) \right\}$$

$$V_3(R, t) = Q_c \cdot \frac{\sqrt{\pi}}{4\chi} \cdot \frac{1}{R} \cdot \operatorname{erfc}\left(\frac{R}{\sqrt{4\chi t}}\right)$$

where $\operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$

b) Exponentially decaying heat sources

$$Q(t) = Q_e \exp(-At)$$

$$V_1(R, t) = Q_e \left\{ -\frac{1}{A} \cdot \exp(-At) + \frac{1}{A} \left[\operatorname{erf}\left(\frac{R}{\sqrt{4\chi t}}\right) + \exp\left(-\frac{R^2}{4\chi t}\right) \cdot \operatorname{Re}\left\{w\left(\sqrt{At} + i \frac{R}{\sqrt{4\chi t}}\right)\right\} \right] \right\}$$

$$V_2(R, t) = Q_e \cdot \frac{\sqrt{\pi}}{\sqrt{4\chi t}} \cdot \exp\left(-\frac{R^2}{4\chi t}\right) \cdot \operatorname{Im}\left\{w\left(\sqrt{At} + i \frac{R}{\sqrt{4\chi t}}\right)\right\}$$

$$V_3(R, t) = Q_e \cdot \frac{\sqrt{\pi}}{4\chi R} \cdot \exp\left(-\frac{R^2}{4\chi t}\right) \cdot \operatorname{Re}\left\{w\left(\sqrt{At} + i \frac{R}{\sqrt{4\chi t}}\right)\right\}$$

where $w(z) = \frac{2iz}{\pi} \int_0^\infty \frac{e^{-t^2}}{z^2 - t^2} dt, \operatorname{Im}\{z\} > 0$

APPENDIX D

Displacements and Stresses due to Surface Shear Traction.

a) Point shear forces

$$u_x = \frac{-T_x}{4\pi GR} \left\{ 1 + \frac{x^2}{R^2} + (1-2\nu) \left[\frac{R}{(R+z)} - \frac{x^2}{(R+z)^2} \right] \right\} + \frac{T_y}{4\pi GR} \left\{ \frac{xy}{R^2} - \frac{(1-2\nu)xy}{(R+z)^2} \right\}$$

$$u_y = \frac{-T_x}{4\pi GR} \left\{ \frac{xy}{R^2} - \frac{(1-2\nu)xy}{(R+z)^2} \right\} + \frac{T_y}{4\pi GR} \left\{ 1 + \frac{y^2}{R^2} + (1-2\nu) \left[\frac{R}{(R+z)} - \frac{y^2}{(R+z)^2} \right] \right\}$$

$$u_z = \frac{-T_x}{4\pi GR} \left\{ \frac{xz}{R^2} + \frac{(1-2\nu)x}{(R+z)} \right\} + \frac{T_y}{4\pi GR} \left\{ \frac{yz}{R^2} + \frac{(1-2\nu)y}{(R+z)} \right\}$$

$$\sigma_{xx} = \frac{T_x x}{2\pi R^3} \left\{ \frac{3x^2}{R^2} - \frac{(1-2\nu)}{(R+z)^2} \left[R^2 - y^2 - \frac{2Ry^2}{(R+z)} \right] \right\} \\ + \frac{T_y y}{2\pi R^3} \left\{ \frac{3x^2}{R^2} - \frac{(1-2\nu)}{(R+z)^2} \left[3R^2 - y^2 - \frac{2Ry^2}{(R+z)} \right] \right\}$$

$$\sigma_{yy} = \frac{T_x x}{2\pi R^3} \left\{ \frac{3y^2}{R^2} - \frac{(1-2\nu)}{(R+z)^2} \left[3R^2 - x^2 - \frac{2Rx^2}{(R+z)} \right] \right\} \\ + \frac{T_y y}{2\pi R^3} \left\{ \frac{3y^2}{R^2} - \frac{(1-2\nu)}{(R+z)^2} \left[R^2 - x^2 - \frac{2Rx^2}{(R+z)} \right] \right\}$$

$$\sigma_{zz} = \frac{3z^2}{2\pi R^5} \left\{ T_x x + T_y y \right\}$$

$$\sigma_{xy} = \frac{T_x y}{2\pi R^3} \left\{ \frac{3x^2}{R^2} - \frac{(1-2\nu)}{(R+z)^2} \left[-R^2 + x^2 + \frac{2Rx^2}{(R+z)} \right] \right\} \\ + \frac{T_y x}{2\pi R^3} \left\{ \frac{3y^2}{R^2} - \frac{(1-2\nu)}{(R+z)^2} \left[-R^2 + y^2 + \frac{2Ry^2}{(R+z)} \right] \right\}$$

$$\sigma_{yz} = \frac{3}{2\pi R^5} \left\{ T_x xyz + T_y y^2 z \right\} \quad ; \quad \sigma_{zx} = \frac{3}{2\pi R^5} \left\{ T_x x^2 z + T_y xyz \right\}$$

where T_x and T_y are surface shear tractions

b) Distributed Shear Forces

$$u_x = \frac{P_x}{4\pi G} \left\{ 2(1-\nu)x \ln(y+R) + 2y \ln(x+R) + z^2(I_2 + I_4) - (1-2\nu)zxI_1 \right\} \\ - \frac{P_y}{4\pi G} \left\{ 2\nu R + (1-2\nu)z \ln(z+R) \right\}$$

$$u_y = \frac{P_x}{4\pi G} \left\{ 2\nu R + (1-2\nu)z \ln(z+R) + \frac{P_y}{4\pi G} \left\{ 2(1-\nu)y \ln(x+R) \right. \right. \\ \left. \left. + 2x \ln(y+R) + z^2(I_2 + I_4) - (1-2\nu)zyI_3 \right\} \right\}$$

$$u_z = \frac{P_x}{4\pi G} \left\{ (1-2\nu)(y \ln(z+R) + x^2 I_1) - 2\nu z \ln(y+R) \right\} \\ + \frac{P_y}{4\pi G} \left\{ (1-2\nu)(x \ln(z+R) + y^2 I_3) - 2\nu z \ln(x+R) \right\}$$

$$\sigma_{xx} = \frac{-P_x}{2\pi} \left\{ \frac{((1-2\nu)Rz^2 - zx^2 - 2\nu Rx^2)y}{(x^2 + z^2)(z+R)R} - 2 \ln(y+R) \right. \\ \left. - \frac{P_y}{2\pi} \left\{ \frac{(2\nu R + z)x}{(z+R)R} - 2\nu \ln(x+R) \right\} \right\}$$

$$\sigma_{yy} = \frac{-P_x}{2\pi} \left\{ \frac{(2\nu R + z)y}{(z+R)R} - 2\nu \ln(y+R) \right\} - \frac{P_y}{2\pi} \left\{ \frac{((1-2\nu)Rz^2 - zy^2 - 2\nu Ry^2)x}{(y^2 + z^2)(z+R)R} \right. \\ \left. - 2 \ln(x+R) \right\}$$

$$\sigma_{zz} = \frac{P_x}{2\pi} \left\{ \frac{yz^2}{(x^2 + z^2)R} \right\} + \frac{P_y}{2\pi} \left\{ \frac{z^2x}{(y^2 + z^2)R} \right\}$$

$$\sigma_{xy} = \frac{-P_x}{2\pi} \left\{ \frac{(z + 2\nu R)x}{(z+R)R} - \ln(x+R) \right\} - \frac{P_y}{2\pi} \left\{ \frac{(z + 2\nu R)y}{(z+R)R} - \ln(y+R) \right\}$$

$$\sigma_{yz} = \frac{-P_x z}{2\pi R} - \frac{P_y}{2\pi} \left\{ \tan^{-1} \frac{xy}{zR} - \frac{xyz}{(y^2 + z^2)R} \right\}$$

$$\sigma_{zx} = \frac{-P_x}{2\pi} \left\{ \tan^{-1} \frac{xy}{zR} - \frac{xyz}{(x^2+z^2)R} \right\} - \frac{P_y z}{2\pi R}$$

where P_x and P_y are the shear force densities over a surface zone.

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$I_1 = \frac{1}{|x|} \left\{ \frac{\pi}{2} - \sin^{-1} \left(\frac{x^2 + z^2 + zR}{(R+z)\sqrt{x^2 + z^2}} \right) \right\}$$

$$I_2 = \frac{1}{|z|} \left\{ \frac{\pi}{2} - \sin^{-1} \left(\frac{x^2 + z^2 + xR}{(R+x)\sqrt{x^2 + z^2}} \right) \right\}$$

$$I_3 = \frac{1}{|y|} \left\{ \frac{\pi}{2} - \sin^{-1} \left(\frac{y^2 + z^2 + zR}{(R+z)\sqrt{y^2 + z^2}} \right) \right\}$$

$$I_4 = \frac{1}{|z|} \left\{ \frac{\pi}{2} - \sin^{-1} \left(\frac{y^2 + z^2 + yR}{(R+y)\sqrt{y^2 + z^2}} \right) \right\}$$

APPENDIX E

Computed and Tabulated Power Generations

Time after reprocessing (Years)	Spent Fuel		High Level Waste	
	Calculated (Watts)	Tabulated (Watts)	Calculated (Watts)	Tabulated (Watts)
10	1189.5	1189.0	1003.1	1003.1
11	1136.8	1136.6	949.4	951.3
12	1092.8	1091.2	906.6	908.6
13	1055.4	1054.9	871.1	873.0
14	1023.2	1023.6	840.3	841.4
15	994.8	996.4	812.9	812.9
16	969.5	972.2	787.9	787.9
17	946.5	949.1	764.7	764.1
18	925.3	928.9	742.9	742.7
19	905.6	908.8	722.2	721.4
20	887.1	890.7	702.6	702.2
25	806.5	808.3	615.3	615.6
30	738.1	739.2	541.8	542.0
40	624.4	624.4	423.9	423.9
100	286.1	288.5	105.6	105.8
200	159.7	162.9	16.4	16.9
300	128.5	128.9	7.4	7.6
500	96.9	95.7	4.9	5.0
1000	54.6	55.4	2.5	2.7
5000	19.0	18.8	0.7	0.7
10000	13.8	13.8	0.5	0.5
25000	6.6	6.7	0.3	0.2
50000	3.0	3.0	0.1	0.1
100000	0.9	1.1	0.1	0.1