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MODIFICATION OF TOKAMAK BALLOONING STABILITY BOUNDARIES BY ALPHA POPULATIONS*

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Abstract

For $T_\alpha/T_i < 150$ (where T_α is the α -mean energy and T_i the ion temperature) the trapped α - precession resonance destabilizes the kinetic ballooning mode of the bulk plasma. Using a slowing down distribution and both circular and noncircular equilibrium models, the eigenfrequencies of the resonant ballooning integral equation are computed. One finds lower critical betas and a widening of the unstable region, particularly at higher electron temperature and for flat q , steep pressure profiles typical of ignited sawtooth plasmas.

I. Energetic trapped alpha particles interact through their precessional drift with high mode number ballooning modes. In the low frequency regime $\omega_r \ll \omega_{d\alpha}$ (where ω_r is the real frequency of the kinetic ballooning mode and $\omega_{d\alpha}$ the α -banana precession frequency which is proportional to the α -energy) the α -species is decoupled from the bulk plasma. Figure 1 shows this as a progressive shrinking of the unstable region in s - α space (to be defined in more detail below) with increasing hot particle "temperature" ratio T_H/T_i . (For the sake of simplicity the α -s are modelled as a deeply trapped Maxwellian, in this figure only). Thus, it is the medium α -energy group $T_H/T_i \leq 150$ which is of concern in what follows.

In the intermediate frequency regime $\omega_r \leq \omega_{d\alpha}$ a destabilizing coupling of the ballooning mode with the alpha population can occur. Its strength depends on the energy and pitch angle dependence of the α -slowing down distribution and those other parameters of the bulk plasma which affect the drift resonance.

The destabilization widens the unstable region in s - α space (where $s \equiv rq'/q$ is the global shear and $\alpha_c \equiv \frac{8\pi Rq^2}{B^2} p'_c$, where p'_c is the "cold" i.e., bulk plasma component pressure gradient) and increases the growth rate. An accurate calculation of these effects is needed in order to delineate the effective MHD stability boundaries including the burning plasma and to provide information for the stable choice of operating conditions (e.g., T_e and plasma profiles) in the presence of the destabilizing alphas ($\alpha_H \equiv (8\pi Rq^2/B^2)p'_\alpha$).

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FIG. 1. Stability boundaries versus s and α_c for $\alpha_H/\alpha_c = 0.3, b_i = 0.05, q = 2, \theta_{\max} = 3\pi$, and $T_H/T_i = \infty, 150, 100$. (The solid line is the case without hot particles).

II. In this work, we present results for a collisional slowing down α -distribution for various electron temperatures (entering through $v_c \sim T_e^{1/2}$) and (a) deeply trapped, (b) isotropic pitch angle α -distributions. Equilibria based both on the shifted circular flux surface models [1,2], and numerical solutions of the Grad-Shafranov equation for non-circular geometries have been used. In the former case, the ensuing s - α stability diagrams are universal (except for our assumption of $k_{\perp} \rho_i = 0.28, q(\psi) = 2$ and $\epsilon_p^{-1} \equiv -\frac{R}{p_c} \frac{dp_c}{dr} = 6$, inside the plasma), while in the latter, flux surfaces are analyzed for stability as a function of β for specific devices.

The equations solved are described in detail in [3]. Basically, we follow [4] extending the ideal MHD ballooning equation to include ω_* - effects for the cold and hot components plus, most importantly for the present purpose, the destabilizing integral term containing the hot particle precession resonance

$$\int d^3v \frac{\mu^2 (\bar{e} \cdot \nabla F_{\alpha})}{\langle \omega_{d\alpha} \rangle - \omega} \langle Q_{\parallel} \rangle \quad (1)$$

This term requires two velocity integrations plus a bounce average over the θ -dependence of the eigenfunction (θ is the extended poloidal angle variable). The exact set of two equations for $Q_{\parallel} \equiv \frac{e}{c} B_{\parallel} + \left(\frac{\bar{e} \cdot \nabla B}{B} \right) \Phi$, and Φ , can be solved numerically for the eigenmodes

and eigenvalues ω . For multiple parameter scans we also use an approximate reduction to one eigenmode equation for Φ as discussed in [5], which can be justified a posteriori by comparison with the results of the 2 equation system.

III. Results

(a) Deeply trapped pitch angle distribution. A highly peaked (about $\mu B_{\min}/E = 1$) Gaussian model for the alpha pitch angle distribution has been used in the deeply trapped limit. This, coupled with the fact that the ballooning eigenfunction is highly peaked about $\Theta = 0$, (cf Fig. 2), allows the bounce average in Eq (1) to be evaluated at the midplane and pulled outside the velocity integral.

FIG. 2. Typical ballooning eigenfunction

Since the deeply trapped α -s are located entirely in the bad curvature region, the results are overly pessimistic. Nevertheless, they permit rapid insight into the parameter trends in the s - α diagrams.

Figure 3 shows the strongly destabilizing effect of increasing electron temperature T_e . On one hand, from the dependence of f_α on T_e : $f_\alpha \propto (v^3 + v_c^3)^{-1}$, $v_c^3 \propto T_e^{3/2}$ one expects a decoupling effect with increasing T_e . On the other hand, however, increasing $T_e \simeq T_i$ increases ω_r (through ω_s , cf. Fig. 6 of Ref. [3]) thus maintaining the destabilizing resonance condition $\omega_r \sim \omega_{d\alpha}$ at higher energies.

Note the broadening of the unstable region for low shear and high values of α_c , i.e. a flat central q -profile and peaked pressure profile which will be characteristic of low $q(a)$ ignited plasmas (particularly with large α -driven sawtoothings).

Fig. 3. Dependence of stability boundaries on T_e for $\alpha_H/\alpha_c = 0.3$, $q = 2$, $b_i = 0.04$, $\epsilon_p^{-1} = 6$, $\lambda/\alpha = 0$.

(b) Isotropic Pitch Angle Distribution

After performing the energy integration over the slowing down distribution the rigorous integral term of (1) becomes

$$I_1(\theta) = \int_{M_0^{-1}}^{M^{-1}} \frac{d\lambda F_\lambda \lambda^2 H(\lambda, \omega)}{(1 - \lambda M)^{1/2}} \langle Q_{\parallel} \rangle; \quad 2(m-1)\pi \leq \theta \leq 2(m+1)\pi, m = 0, 1, 2, \dots \quad (2)$$

where λ is the pitch angle variable and F_λ is an arbitrary α -pitch angle distribution ($F_\lambda = 1$ for the isotropic distribution). $H(\lambda, \omega)$ is the drift resonance integral (corresponding to the plasma dispersion function), $M = B/B_{\min}$, $M_0 = B_{\max}/B_{\min}$ and $\langle \dots \rangle$ denotes the (λ -dependent) bounce average over the wave function $Q_{\parallel}$. (An analogous second equation exists for Φ , containing an analogous integral term $I_2(\theta)$.) Since $Q_{\parallel}$ is strongly peaked about $\theta = 0$ (see Fig. 2) the integral I_1 can be approximated by

$$I_1(\theta) \simeq Q_{\parallel}(\theta = 2m\pi) \int_{M_0^{-1}}^{M^{-1}} \frac{d\lambda F_\lambda \lambda^2 H(\lambda, \omega)}{(1 - \lambda M)^{1/2}} \quad (3a)$$

where the function $Q_{\parallel}$ is sampled at the periodic values $\theta = 2m\pi$, $m = 0, \pm 1, \pm 2, \dots$

As mentioned, this is an excellent approximation for the deeply trapped pitch angle distribution (where F_λ is peaked about $\lambda = 1$). It is also expected to be a good approximation for the isotropic case ($F_\lambda = 1$) due to the strongly peaked nature of $Q_{\parallel}$. An alternate approximation which has been considered (cf. Ref. [5]) is:

$$I_1(\theta) \simeq \frac{2}{M} \left(1 - \frac{M}{M_0}\right)^{1/2} \langle \overline{Q_{\parallel} H(\lambda, \omega)} \rangle \quad (3b)$$

where the overbar denotes a pitch angle average.

The uniform distribution of α -pitch angles retains the partially stabilizing effect of bananas near the trapped-circulating particle boundary since these bananas extend poloidally into the good curvature region. Fig. 4 corresponds to Fig. 3 (obtained for deeply trapped particles), but note that the α -particle free ($\alpha_H = 0$) stability boundaries were not chosen identical (due to the differing values of λ/α and q). For large s , the isotropic pitch angle distribution results are more stable, but for $s \simeq 0.2$ both types of distributions prevent entry into the second stability region, at least if $T_e \geq 40$ keV. (This may provide an automatic burn control mechanism for ignited tokamaks). This flat q , peaked p -profile region is typical of ignited plasmas as mentioned previously.

Fig. 4. Stability boundaries for $\alpha_H/\alpha_c = 0.3$, $\lambda/\alpha = 0.25$, $b_i = 0.04$, $q = 1.7$. The solid line is the case without alphas.

In addition to the above circular flux surface tokamak calculations, the kinetic ballooning stability code has recently been adapted to noncircular surfaces as would characterize CIT, ETR, and TFTR in the high β supershot regime. Typical results for TFTR are given in Fig. 5. The unstable regions enlarge for increasing peak electron temperature in agreement with the qualitative dependences of Figs. 3 and 4; also, at β 's presently achieved in

Fig. 5. TFTR stability boundaries for realistic Baldur-generated profiles with and without alphas ($b_i = .01$, $p_{\alpha}/p_T = .12$).

the supershort regime, increasing fractions of the minor radius are destabilized as T_{e0} is increased.

In Fig. 6, stability boundaries for CIT are plotted both with and without alphas. Although the noncircular shaping leads to an increase of the β limit without alphas, no similar effect is observed when alphas are present. In the latter case the β limit is significantly reduced and is at a similar level as the circular TFTR case in Fig. 5.

Fig. 6. CIT stability boundaries with and without alphas ($b_i = .04$, $p_{\alpha}/p_T = .10$) for a parabolic T_e profile.

IV. In summary, our results show that (i) the bulk plasma ballooning mode boundaries are broadened by the trapped α -precession resonance, particularly in the ignition relevant low shear high pressure gradient region of the s - α diagram. Alphas in the energy range $T_\alpha/T_i < 150$ are the most destabilizing energy group. (ii) Using the collisional slowing down α -distribution, operation at $T_e > 10 - 20$ keV exacerbates this effect, lowering β_{crit} by a factor of ≥ 2 under certain circumstances (iii) shaping appears to have much less influence on stability boundaries in the presence of alphas than in the ideal case without alphas.

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