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**The Spin-Rotation Parameter Q
for Elastic Scattering
of 800 MeV Polarized Protons
from ^{16}O , ^{40}Ca , and ^{208}Pb**

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Abstract

The spin-rotation parameter Q was measured for ^{16}O , ^{40}Ca , and ^{208}Pb using the 800 MeV proton beam produced at the Clinton P. Anderson Meson Physics Facility. The experiment was carried out using the High Resolution Spectrometer equipped with a focal-plane polarimeter to determine the scattered polarization components in all three directions. These data (when combined with previous cross section and analyzing power data) determine the amplitude describing the elastic scattering of protons from these spin-zero nuclei to within an overall phase. Q is shown to be more sensitive than either the cross section or analyzing power to differences between the nonrelativistic and relativistic scattering theories that describe the proton-nucleus reaction in terms of fundamental proton-nucleon input. The nonrelativistic predictions for Q generally lie below the data but consistently have the correct structure. The relativistic predictions for the ^{40}Ca and ^{208}Pb Q data are quite good (better than the nonrelativistic predictions). The relativistic predictions for the ^{16}O Q data show much more structure than is evident in the data.

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THE SPIN-ROTATION PARAMETER Q FOR ELASTIC SCATTERING OF
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I. INTRODUCTION

One of the primary objectives of medium energy physics is the formulation of a microscopic theory of nucleon-nucleus scattering in terms of nucleon-nucleon amplitudes and underlying nuclear structure. The difficulties inherent in formulating a quantum-mechanical, many-body problem in terms of elementary two-body input are many. Success in this endeavor could, however, lead to a fuller utilization of the unique properties of the many different types of probes that are, and will soon be, available to examine the nucleus and its reaction mechanisms. Although each probe has its own peculiarities that must be taken into account when attempting to extract information about the nucleus, a fundamental method for dealing with the quantum-mechanical, many-body problem is essential to the framework of any successful microscopic description of the scattering process.

A parallel goal for medium energy proton-nucleus physics is the extraction of information about the neutron density distributions of nuclei. Proton density distributions are well known from the results of experiments with electromagnetic probes; however, the neutron distributions are extremely difficult to

extract from such experiments. It is hoped that the effective interaction between the incident proton and the bound nucleons is sufficiently simple at intermediate energies to allow precise and accurate determinations of the neutron density distributions using high quality data, known proton matter distributions, and a microscopic, parameter-free, description of the scattering process.

Since it is impossible to measure the effective proton-nucleon interaction directly, the path that has been followed is to make the provisional assumption that the effective nucleon-nucleon interaction is identical to the free interaction (the "impulse approximation"). The free scattering amplitudes are directly measurable and are believed to be reasonably well known at intermediate energies.

Attempts to extract ground-state neutron density distributions from proton-nucleus data using microscopic theories have run into serious difficulties, however. Accurate determinations of the proton-nucleon scattering amplitudes constrain parameter-free microscopic theories in such a way that they are unable to produce good quantitative agreement with proton-nucleus analyzing power data, especially at low momentum transfer.⁽¹⁾ This is puzzling since theoretical considerations indicate that the impulse approximation should work best in exactly this momentum transfer region. In addition, the neutron density that is necessary to produce the best fit to the data is distinctly energy dependent.

It is unclear, judging from these observations, whether the problems are due to the inadequacy of the impulse approximation, to some previously unconsidered factors in the scattering description, or to both. A recently proposed model in which the Dirac equation, rather than the usual Schrödinger equation, is used to describe the scattering process led to good descriptions of 500 MeV proton-nucleus spin observables.⁽²⁾ Later studies showed, however, that the extraction of neutron densities with this model produced results that are even more energy dependent than those produced using the Schrödinger equation as the starting point.⁽³⁾ Modifications to the model that directly influence its energy dependence are presently under investigation.⁽⁴⁾ The results of these studies on the extracted structure information are not yet available.

The recent implementation of a focal-plane polarimeter (FPP) for the High Resolution Proton Spectrometer (HRS) at the Clinton P. Anderson Meson Physics Facility (LAMPF) allowed, for the first time at intermediate energies, measurements of the scattered-beam polarization components in all three orthogonal directions for excitation of individual nuclear states. Such data, of course, further constrain the scattering theories. It is hoped that these new data will shed light on the theoretical difficulties and lead to improved theoretical description of proton-nucleus scattering. In any event, it can be shown that the measurement of one of the scattered-beam polarization components, together with

previous cross section and analyzing power measurements, leads to a complete empirical determination of the proton + spin-zero-nucleus scattering amplitude (apart from an overall phase). A parameter that describes this polarization component is commonly given the label $Q^{(36)}$ and referred to as the "spin-rotation parameter". Therefore cross section, analyzing power and spin-rotation data will provide the most complete test of any theory describing elastic scattering of protons from spin-zero nuclei.

In May of 1981 an experiment (assigned number 654) to measure the spin-rotation parameter Q for 800 MeV proton + ^{16}O and ^{40}Ca elastic scattering was proposed and approved by the HRS Program Advisory Committee. The ^{40}Ca data were obtained in June of 1982 during a continuous running period of 116 hours. In this time 220 data tapes were obtained at 10 overlapping angle settings of the HRS spanning 2° to 15.5° (lab). The time spent taking data was approximately equally divided between incident-beam in-plane polarization directions along the beam momentum and transverse to it. This was done in order to allow for independent systematic checks on the results. In January of the next year 160 hours were allocated for the running of the ^{16}O portion of the experiment. An ice target was used to provide the ^{16}O nuclei for this part. During this run approximately 250 tapes of data were taken at 13 spectrometer settings from 2° to 21° . Once again the time was divided equally between incident-beam polarizations in the longitudinal and transverse in-plane directions.

In November of 1983 the type of beam required for use by a scheduled experiment in another experimental area was of such a nature as to discourage the scheduling of a number of the "usual" types of experiments that run at HRS. The beam polarization at the HRS target had longitudinal and transverse in-plane components simultaneously. In addition, the sign of the polarization changed at a rate of 30 Hz rather than the more conventional rate of once every several minutes. This type of beam does not preclude a measurement of the parameter Q, however. It was also noted that the measurement of Q for ^{208}Pb would provide needed data at 800 MeV for a large A nucleus. Accordingly, 176 hours of beam time were approved by the Director of LAMPF to obtain Q data for ^{208}Pb with the uncharacteristic beam just described. In this time approximately 270 data tapes were obtained at 9 spectrometer angles between 3.5° and 15.5° . As before, data were taken with the incident beam polarized in two in-plane orthogonal directions; however, this time the two directions were a fixed and known rotation from the transverse and longitudinal directions used previously.

It was hoped that these measurements of Q with 800 MeV protons on a variety of nuclei would shed some light on the theoretical problems that have been outlined and constrain as tightly as possible present, and possible future, proton-nucleus scattering theories. This dissertation is a description of these measurements and a report on the results.

II. THEORETICAL BACKGROUND

This chapter is a brief presentation of the nomenclature and theoretical framework necessary for an understanding of the reasons for, and the results of, this experiment. The first section is a discussion of the solution of the Schrödinger equation for the proton-nucleus system in terms of a scattering matrix, the form of which is deduced from physical symmetry arguments. The next section describes how this matrix can be determined by a few conceptually simple experiments. In the following section a method is described in which fundamental nucleon-nucleon (NN) amplitudes and a minimum of experimentally determined information about the bulk properties of the nucleus can be used to construct a description of the proton-nucleus elastic scattering using the Schrödinger equation as the starting point. The introduction of a relativistic equation of motion (the Dirac equation) to describe the scattering is presented in the last section.

II-A. Scattering Matrix

The equation most often used to describe nucleon-nucleus elastic scattering has been the Schrödinger equation. It is believed that the Schrödinger equation accurately describes the behavior of non-relativistic systems. Medium-energy protons are clearly relativistic ($v/c = 0.84$ at 800 MeV), however.

Traditionally relativistic kinematics have been used in the Schrödinger equation in an attempt to account for the most obvious effects of relativity in the medium energy realm. The success of this approach in arriving at solutions that describe a number of experimental results provides some support for this procedure.

The Hamiltonian that describes the system of a proton incident on a nucleus of "A" nucleons is quite complicated. One method for dealing with the A+1 nucleon system is to treat the incident proton as if it were distinguishable from the protons in the nucleus. Calculations have been done which show that, at intermediate to high incident projectile energies, exchange between the incident and nuclear protons is, in fact, small.⁽⁵⁾ This is due mostly to their wide disparity in momentum. If any three-body forces (which are also believed to be small⁽⁶⁾) are neglected, the Schrödinger equation can be written in the form:

$$(T_p + H_{nuc} + \sum_{j=1}^A v_{pj})\Psi = E\Psi \quad (II-1)$$

In this equation T_p is the kinetic energy operator for the incident proton and H_{nuc} is the Hamiltonian for the "A" nucleons in the nucleus. The interaction between the incident proton and the j^{th} nucleon in the nucleus is represented by v_{pj} while Ψ is the spin-dependent wave function for the proton-nucleus system. The total energy of the system is given by E.

The formal asymptotic solution of this equation for the case of elastic scattering from a spin-zero nucleus can easily be shown to be (Sec. II-C)

$$\psi \underset{r \rightarrow \infty}{=} C \phi_{gs} \left[e^{i\vec{k} \cdot \vec{r}} + \frac{e^{ikr}}{r} M \right] \chi_I, \quad (\text{II-2})$$

where $\vec{r}$ is the position (relative to the nucleus) and $\vec{k}$ the (center-of-momentum) momentum of the incident proton. C is a normalization constant. ϕ_{gs} is the ground-state wave function of the nucleus (the lowest energy solution to the Schrödinger equation for the nucleus: $H_{\text{nuc}} \phi_{gs} = \epsilon_0 \phi_{gs}$), and χ_I is the two component spinor that describes the incident spin state of the proton. M is an operator that transforms the initial spin state of the proton into the final spin state and is referred to as the "scattering matrix" or "scattering amplitude".

The first term of this solution is a plane wave that describes the incident proton. Note that, although it is not explicitly indicated in Eq. II-2, this term will vanish when the wave function is evaluated outside the envelop of the beam. The second term describes an outgoing spherical wave centered on the nucleus. All of the information describing the proton-nucleus scattering process is contained in M . All of the angular and spin structure of the outgoing wave is completely described by the final-state wave function χ_S :

$$\chi_S \equiv M\chi_I \quad . \quad (II-3)$$

Since M operates on the two-component spinor χ_I , it can be written in general as a complex 2×2 matrix. If the Pauli spin matrices are used as a basis set in which to expand the scattering matrix, the expansion can be written as

$$M = aI + b\sigma_x + c\sigma_y + d\sigma_z \quad ,$$

where the coordinate system used is arbitrary and a , b , c and d are the expansion coefficients. A coordinate system can be defined for the scattering problem discussed above in terms of the incident- and scattered-particle momenta $\vec{k}$ and $\vec{k}'$. The unit vectors that define the usual system are $\hat{K}$, $\hat{N}$, and $\hat{P}$ as shown in Fig. II-1. The defining equations for these vectors are given below.

$$\hat{K} \equiv \frac{\hat{k}' - \hat{k}}{|\hat{k}' - \hat{k}|} \quad \hat{P} \equiv \frac{\hat{k}' + \hat{k}}{|\hat{k}' + \hat{k}|} \quad \hat{N} \equiv \frac{\hat{k} \times \hat{k}'}{|\hat{k} \times \hat{k}'|} \quad (II-4)$$

The "scattering plane" is defined as the plane containing the momentum vectors of the incident and scattered particle. Thus, $\hat{N}$ is the normal to this plane while $\hat{P}$ and $\hat{K}$ lie in it. Written in terms of this orthogonal coordinate system, the scattering matrix is

Fig. II-1

The $(\hat{K}, \hat{N}, \hat{P})$ Coordinate System.

$$M = aI + b(\vec{\sigma} \cdot \hat{K}) + c(\vec{\sigma} \cdot \hat{N}) + d(\vec{\sigma} \cdot \hat{P}) \quad . \quad (\text{II-5})$$

A number of experiments have been done that indicate that the strong interaction conserves parity to a great extent.⁽⁷⁾ In terms of the solution to the Schrödinger equation in the form of Eq. II-2, one must then expect the scattering matrix to be invariant under a parity transformation since it is M that defines the wave function outside the envelop of the beam. Since the

expansion of M in terms of the Pauli spin matrices yields four terms that are independent, it is required that each term separately be invariant. It is easily shown that the second and fourth terms in Eq. II-5 do not satisfy this criteria. The vectors $\hat{K}$ and $\hat{P}$ (sums and differences of momenta) change sign with a space inversion while $\vec{\sigma}$ (which behaves like an angular momentum) does not. The expansion coefficients are merely numerical constants (for a given $\vec{k}'$) and are thus invariant. The vector $\hat{N}$ is also invariant since it depends on the cross product of two momenta. From these observations it can be concluded that the scattering matrix can contain, at most, a constant term and a term dependent upon the $\hat{N}$ projection of the spin of the incident beam. For convenience an "i" ($i \equiv \sqrt{-1}$) is usually factored out of the coefficient of the spin-dependent term and written explicitly. Thus the most general form for the scattering matrix describing the elastic scattering of a proton from a spin-zero nucleus is given by

$$\begin{aligned}
 M &= aI + ic\vec{\sigma} \cdot \hat{N} , \\
 &= \begin{pmatrix} a & c \\ -c & a \end{pmatrix} .
 \end{aligned}
 \tag{II-6}$$

To completely specify the proton-nucleus wave function of Eq. II-2, and thus the results of a proton + spin-zero-nucleus elastic scattering experiment, all that need to be determined are

the coefficients a and c . These coefficients are referred to as the components of the scattering amplitude. Since the overall phase of the wave function has no physical consequences, one might expect that three experiments which measure independent observables would allow determination of the scattering amplitude apart from an overall phase. The next section deals with the problem of determining a convenient set of such experiments. (A "convenient" set since there is no reason to believe that any set of measurements which determine M is unique.)

II-B. Observables

Before discussing the observables of the system, it is first useful to introduce more convenient coordinate systems that can be used in place of the one described in the previous section. Since the scattering matrix depends only on the direction of the normal to the scattering plane, the form of the matrix will be identical in any system that maintains this normal. The lab coordinate system $(\hat{s}_0, \hat{n}_0, \hat{l}_0)$ shown in Fig. II-2 is a convenient one in which to measure the polarization of the incident beam. The $\hat{l}_0$ unit vector points along the beam momentum and $\hat{n}_0$ is the normal to the nominal scattering plane. This system does not maintain the true normal to the scattering plane, however. A related system that does maintain this normal is shown as $(\hat{s}, \hat{n}, \hat{l})$. It is most convenient to measure the polarization of the scattered beam in the

Fig. II-2

Experimental Coordinate Systems.

$(\hat{s}', \hat{n}', \hat{l}')$ system of Fig. II-2, although it is sometimes simpler to discuss the scattered-beam polarization in terms of components along the axes of the incident beam system $(\hat{s}, \hat{n}, \hat{l})$. Care must be taken to specify which of these systems is meant in any discussion of a scattered-beam polarization component.

To introduce the notation and formalism in which scattering problems are usually discussed consider Fig. II-3. In this figure a beam of protons is incident on a single target nucleus; some of these protons elastically scatter and are detected by the detector of area A located at a distance r from the target. The number of particles dN that scatter from the nucleus into the detector at θ

Fig. II-3

Schematic of a Cross Section Measurement.

per unit time will be proportional to the incident proton flux at the target, F_I , and the detector solid angle $d\Omega$. The "constant of proportionality", which contains all the information on the proton-nucleus interaction, is called the differential cross section, $\sigma(\theta)$. Thus

$$dN = F_I \times d\Omega \times \sigma(\theta) ,$$

which can be rewritten as

$$\sigma(\theta) = r^2 \frac{dN}{A} \frac{1}{F_I} . \quad (\text{II-7})$$

Recognizing dN/A as the flux scattered through the detector (F_S) gives

$$\sigma(\theta) = r^2 \frac{F_S}{F_I} . \quad (\text{II-8})$$

The quantum mechanical "particle flux" (or probability-density current) at a position $\vec{r}$ for a system described by a wave function Ψ and reduced mass μ is given by the relation⁽⁸⁾

$$\text{"Flux"} = \frac{\hbar}{2\mu i} \left(\Psi^*(\vec{r}) \nabla_{\vec{r}}[\Psi(\vec{r})] - \Psi(\vec{r}) \nabla_{\vec{r}}[\Psi^*(\vec{r})] \right) . \quad (\text{II-9})$$

Evaluating this expression for the incident and scattered "flux" gives the following:

$$\begin{aligned}
 |F_I| &= \left| \frac{\hbar}{2\mu i} \left[\left(C^+ \phi_{gs}^+ \chi_I^+ e^{-i\vec{k} \cdot \vec{r}} \right) \nabla_{\vec{r}} \left(C \phi_{gs} \chi_I e^{i\vec{k} \cdot \vec{r}} \right) \right. \right. \\
 &\quad \left. \left. - \left(C \phi_{gs} \chi_I e^{i\vec{k} \cdot \vec{r}} \right) \nabla_{\vec{r}} \left(C^+ \phi_{gs}^+ \chi_I^+ e^{-i\vec{k} \cdot \vec{r}} \right) \right] \right| \\
 &= \left| \frac{\hbar}{2\mu i} (i\vec{k} + i\vec{k}) C^+ C \phi_{gs}^+ \phi_{gs} \chi_I^+ \chi_I \right| \\
 &= \frac{\hbar k}{\mu} C^+ C \phi_{gs}^+ \phi_{gs} \chi_I^+ \chi_I \\
 |F_S| &= \left| \frac{\hbar}{2\mu i} \left[\left(C^+ \phi_{gs}^+ \chi_S^+ \frac{e^{-ikr}}{r} \right) \nabla_{\vec{r}} \left(C \phi_{gs} \chi_S \frac{e^{ikr}}{r} \right) \right. \right. \\
 &\quad \left. \left. - \left(C \phi_{gs} \chi_S \frac{e^{ikr}}{r} \right) \nabla_{\vec{r}} \left(C^+ \phi_{gs}^+ \chi_S^+ \frac{e^{-ikr}}{r} \right) \right] \right| \\
 &= \left| \frac{\hbar}{2\mu i} \left(ik \frac{1}{r^2} - \frac{1}{r^3} + ik \frac{1}{r^2} + \frac{1}{r^3} \right) \hat{r} C^+ C \phi_{gs}^+ \phi_{gs} \chi_S^+ \chi_S \right| \\
 &= \frac{\hbar k}{\mu} \frac{1}{r^2} C^+ C \phi_{gs}^+ \phi_{gs} \chi_S^+ \chi_S.
 \end{aligned}$$

Substituting these results into Eq. II-8 and assuming that the spin state of the incident proton is normalized to unity produces

$$\sigma(\theta) = \chi_S^+ \chi_S = |\chi_S|^2 \quad . \quad (II-10)$$

This result can also be written in terms of a density matrix ρ_j , defined for any two component spinor χ_j , as ⁽¹⁷⁾

$$\rho_j = X_j X_j^\dagger = \begin{pmatrix} |a_{1j}|^2 & a_{1j} a_{2j}^* \\ a_{2j} a_{1j}^* & |a_{2j}|^2 \end{pmatrix}, \quad (\text{II-11})$$

where $X_j = \begin{pmatrix} a_{1j} \\ a_{2j} \end{pmatrix}$. Taking the trace of this matrix one can see that

$$\text{Tr}(\rho_j) = |a_{1j}|^2 + |a_{2j}|^2 = X_j^\dagger X_j. \quad (\text{II-12})$$

An incoherent mixture of "N" states can be represented by an average density matrix ρ as follows:

$$\rho = X X^\dagger = \frac{1}{N} \sum_{j=1}^N \rho_j \quad (\text{II-13})$$

where

$$X = \frac{1}{\sqrt{N}} \sum_{j=1}^N X_j. \quad (\text{II-14})$$

These averages will be used in this presentation since the observables of interest are defined in terms of the results of measurements of particle ensembles. Thus if X is chosen to be X_S , then from Eqs. II-10 and II-12,

$$\text{Tr}(\rho_S) = \sigma(\theta). \quad (\text{II-15})$$

It can easily be verified that the expectation value of any operator $\hat{O}$ for the state X can be written in terms of the density matrix ρ as

$$\langle \hat{O} \rangle = X^\dagger \hat{O} X = \frac{\text{Tr}(\rho \hat{O})}{\text{Tr}(\rho)} . \quad (\text{II-16})$$

If the density matrix corresponding to the incident-beam state is expanded in terms of the Pauli spin matrices with expansion coefficients b_i ,

$$\rho_I = \sum_{i=0}^3 b_i \sigma_i , \quad (\text{II-17})$$

the orthogonality of the Pauli matrices and Eq. II-16 can be used to determine b_i :

$$b_i = \frac{1}{2} \langle \sigma_i \rangle_I . \quad (\text{II-18})$$

The expectation value of the Pauli spin matrix in this equation is evaluated using the incident-beam state and is thus the component of the beam polarization, $\vec{P}^b$, in the i^{th} direction (for $i=1,2,3$). Thus

$$\begin{aligned}\rho_I &= \frac{1}{2} (I + \sum_{i=1}^3 p_i^b \sigma_i) \\ &= \frac{1}{2} (I + \vec{p}^b \cdot \vec{\sigma}) \quad .\end{aligned}\tag{II-19}$$

Using this result and Eq. II-15,

$$\begin{aligned}\sigma(\theta) &= \text{Tr}(\rho_S) = \text{Tr}(X_S X_S^\dagger) \\ &= \text{Tr}[(MX_I)(MX_I)^\dagger] \\ &= \text{Tr}[M X_I X_I^\dagger M^\dagger] \\ &= \text{Tr}\left[M \left[\frac{1}{2} (I + \vec{p}^b \cdot \vec{\sigma})\right] M^\dagger\right] \quad .\end{aligned}\tag{II-20}$$

If the beam is unpolarized ($\vec{p}^b=0$), the cross section is given by

$$\sigma(\theta) \Big|_{\vec{p}^b=0} = \sigma_0(\theta) = \frac{1}{2} \text{Tr}(M M^\dagger) \quad .\tag{II-21}$$

By substituting the explicit form for the scattering matrix (Eq. II-6) into this equation one obtains the well known result

$$\sigma_0(\theta) = |a|^2 + |c|^2 \quad .\tag{II-22}$$

This result can also be obtained from Eq. II-10 by a much more direct method: by simply substituting in the explicit form of the scattering matrix and averaging over the incident spin states (to describe the results produced by a net unpolarized incident beam). The point of introducing the density matrix is that it implicitly takes the averaging process into account (Eqs. II-13 and II-14) so that polarizations enter into the results that follow in a straightforward manner.

The result of a cross section experiment with a polarized beam (Eq. II-20) can be written in terms of the unpolarized result as follows:

$$\begin{aligned}\sigma(\Theta) &= \text{Tr} \left\{ M \left[\frac{1}{2} (I + \sum_{i=1}^3 p_i \sigma_i) \right] M^+ \right\} \\ &= \sigma_0 \left[I + \sum_{i=1}^3 p_i A_i(\Theta) \right] ,\end{aligned}\tag{II-23}$$

where

$$A_i(\Theta) = \frac{\text{Tr}(M \sigma_i M^+)}{\text{Tr}(M M^+)}$$

is the analyzing power for the i^{th} direction. Parity conservation requires that this relationship be invariant under space reflection. The quantities $\sigma(\Theta)$ and $\sigma_0(\Theta)$ (scalars) and the direction of the beam polarization and $\hat{n}$ (pseudovectors) all are

invariant under reflection. The in-plane coordinate-axis unit vectors ($\hat{s}$ and $\hat{l}$) both change sense, however. Thus the quantities $\langle \vec{\sigma} \cdot \hat{s} \rangle$ and $\langle \vec{\sigma} \cdot \hat{l} \rangle$ (p_s and p_l) both change sign under space reflection. Since these quantities are linearly related to scalars ($\sigma(\theta)$, $\sigma_o(\theta)$) by Eq. II-23, it is evident that the coefficients governing these relationships (A_s , A_l) must be zero. The analyzing power in the $\hat{n}$ direction can be non-zero and by convention is labeled A_y and simply referred to as "the analyzing power".

Substituting the explicit form for the scattering matrix into the definition of the analyzing power yields

$$A_y(\theta) = \frac{-2 \operatorname{Im}(a^* c)}{|a|^2 + |c|^2} . \quad (\text{II-24})$$

Analyzing power data can thus be used to put a second constraint on a and c (the components of the scattering amplitude). $A_y(\theta)$ is sensitive to both the magnitudes and the phases of a and c , in contrast to the cross section which is sensitive only to their magnitudes.

It is clear from the previous discussion that two independent measurements can be made when the scattered-beam polarization is not measured. To find a third independent observable it is necessary to measure the spin of the scattered beam. It is first useful to define a symbol p_j' for the final state,

$$p'_j \equiv \langle \sigma_j \rangle_S, \quad (\text{II-25})$$

to indicate polarization components [measured in either the $(\hat{s}, \hat{n}, \hat{l})$ or the $(\hat{s}', \hat{n}', \hat{l}')$ system]. Using this definition and substituting $\vec{\sigma}$ for $\hat{O}$ and χ_S for χ into Eq. II-16 gives the relationship:

$$p'_j = \frac{\text{Tr}(\rho_S \sigma_j)}{\text{Tr}(\rho_S)}. \quad (\text{II-26})$$

If ρ_S in this equation is replaced in a similar manner as was done to obtain Eq. II-20, the result is

$$p'_j = \frac{\sigma_o(\theta)}{\sigma(\theta)} [P_j(\theta) + \sum_{i=1}^3 p_i D_{ij}(\theta)] \quad (\text{II-27})$$

$$\text{where } P_j(\theta) \equiv \frac{\text{Tr}(M M^+ \sigma_j)}{\text{Tr}(M M^+)}, \text{ and } D_{ij}(\theta) \equiv \frac{\text{Tr}(M \sigma_i M^+ \sigma_j)}{\text{Tr}(M M^+)}.$$

For an incident beam which is unpolarized, this result reduces to

$$p'_j = P_j(\theta). \quad (\text{II-28})$$

$P_j(\theta)$ is thus the polarization that is induced in the j^{th} direction when an unpolarized beam scatters from a target. Again parity conservation can be invoked to show that $P_j(\theta)$ can only be non-zero for $j=\hat{n}$. Substituting the explicit form for the scattering matrix

into the definition of $P_n(\theta)$ results in the finding that $P_n(\theta)$ is identically equal to A_y . This is a direct result of the time reversal invariance of the scattering matrix and the fact that only elastic scattering is being considered. The subscript on the surviving induced-polarization component is usually dropped, leaving the symbol $P(\theta)$ to indicate this quantity. (The explicit functional dependence on θ will be retained throughout this presentation in order to clearly distinguish this quantity from either incident- or scattered-beam polarizations.)

The parameter $D_{ij}(\theta)$ is the coefficient that determines the relationship between the magnitudes of the i^{th} component of the incident-beam polarization and the j^{th} component of the scattered-beam polarization. Recalling the properties of the coordinate axes under a parity transformation constrains the values of some of these observables. For instance, since $p_s (\langle \vec{\sigma} \cdot \hat{s} \rangle_I)$ changes sign under a parity transformation and $p'_n (\langle \vec{\sigma} \cdot \hat{n} \rangle_S)$ does not, it follows that D_{sn} must be identically zero. By the same argument $D_{ln} = D_{ns} = D_{nl} = 0$. The remaining parameters are referred to as the spin-depolarization (D_{nn}) and spin-rotation ($D_{ll}, D_{ls}, D_{ss}, D_{sl}$) parameters.

When the explicit form for the scattering matrix is used to evaluate the potentially non-zero coefficients using their definitions, the results (using the $(\hat{s}, \hat{n}, \hat{l})$ coordinate system both before and after scattering) are as shown below:

$$D_{nn} = 1$$

$$D_{\ell s} = -D_{s\ell} = \frac{-2 \operatorname{Re}(a^* c)}{|a|^2 + |c|^2} \equiv Q(\theta)$$

$$D_{\ell\ell} = D_{ss} = \frac{|a|^2 - |c|^2}{|a|^2 + |c|^2} \equiv S(\theta) \quad . \quad (\text{II-29})$$

Data for Q and S , when coupled with cross section and analyzing power data, would seem to give four independent constraints on the components of the scattering amplitude. However, using the definitions of the observables in terms of the amplitudes, one can show that, if Q is known, S is determined to within a sign by the relationship

$$Q^2 + S^2 + P^2 = 1 \quad . \quad (\text{II-30})$$

Thus three parameters (σ , A_y , and Q) have been found that constrain the scattering amplitude with a minimum of additional information provided by a fourth observable (S).

In order to relate clearly Q and S to physical quantities, consider the case of an incident beam polarized along a coordinate axis in the scattering plane. If

$$\hat{P}_b = P_\ell \hat{1} \quad ,$$

then Eq. II-27 gives the following expressions for the scattered-beam-polarization components:

$$p'_Q = D_{QQ} p_Q = S(\theta) p_Q$$

$$p'_S = D_{QS} p_Q = Q(\theta) p_Q$$

$$p'_N = P(\theta) \quad .$$

This result is illustrated in Fig. II-4. Projecting the scattered-beam polarization vector onto the scattering plane shows that Q and S are directly related to the angle (β) through which

Fig. II-4

Scattered-Beam Polarization in Terms of Spin-Rotation Parameters.

the in-plane polarization component is rotated by the scattering process:

$$\begin{aligned}\tan\beta &= \frac{p'_S}{p'_l} \\ &= \frac{Q}{S} = \frac{\pm Q}{\sqrt{1 - P^2 - Q^2}}.\end{aligned}\tag{II-31}$$

An identical result is obtained when a beam polarized in the $\hat{s}$ direction is considered.

One should recall that the effects of an incident-beam polarization component in the $\hat{n}$ direction on scattered-beam components in the $\hat{s}$ or $\hat{l}$ directions is merely to scale their absolute magnitudes (by $(1+A_y p_n)^{-1}$) and not to change their relative sizes (Eq. II-27). It is clear then that the above relationship (Eq. II-31) is valid for an arbitrary incident-beam polarization if β is understood to be the angle between the projections in the scattering plane of both the incident- and scattered-beam polarizations.

As mentioned previously, the coordinate system in which it is convenient to measure the spin of the outgoing particles is $(\hat{s}', \hat{n}', \hat{l}')$. Equation II-27 is also valid in this system so long as p'_j , p_j , and the D_{ij} 's are consistently determined from polarization measurements in this mixture of systems $\{(\hat{s}, \hat{n}, \hat{l})$ before scattering and $(\hat{s}', \hat{n}', \hat{l}')$ after scattering $\}$. When measured in this manner the

spin-rotation and spin-depolarization parameters are identical to the "Wolfenstein" parameters.⁽⁹⁾ The convention is to use capital letters as the subscripts for this particular choice of coordinate systems. Thus:

$$D_{SS} = R$$

$$D_{LS} = A$$

$$D_{SL} = R'$$

$$D_{LL} = A'$$

$$D_{NN} = D \quad .$$

The parameters measured in this manner are related to the parameters determined from polarizations measured solely in the $(\hat{s}, \hat{n}, \hat{l})$ system by simple geometric relationships governing a rotation counterclockwise about $\hat{n}$ by the laboratory scattering angle θ_{LAB} . Thus, the relationships between the two sets of parameters can be written as

$$Q = D_{LS} \cos\theta_{LAB} + D_{LL} \sin\theta_{LAB} = -D_{SL} \cos\theta_{LAB} + D_{SS} \sin\theta_{LAB}$$

$$S = D_{SS} \cos\theta_{LAB} + D_{SL} \sin\theta_{LAB} = D_{LL} \cos\theta_{LAB} - D_{LS} \sin\theta_{LAB}$$

$$D_{NN} = D_{nn} \quad . \quad (II-32)$$

It is also clear from Eq. II-29 and the geometry involved that $D_{LS} = -D_{SL}$ and $D_{LL} = D_{SS}$. Relations of this form are preserved in

any orthogonal coordinate system with one axis along the normal to the scattering plane. Thus, for an arbitrary, incident, orthogonal, right-handed coordinate system $(\hat{u}, \hat{n}, \hat{v})$ (Fig. II-5) where $\hat{u}$ and $\hat{v}$ are in the scattering plane, $D_{UL} = -D_{VS}$ and $D_{US} = D_{VL}$. The expressions for Q and S for an incident beam polarized along an arbitrary direction $\hat{i}$ at an angle α , measured counterclockwise from $\hat{l}$, are given by:

Fig. II-5

The $(\hat{u}, \hat{n}, \hat{v})$ Coordinate System.

$$Q = D_{IS} \cos(\theta_{LAB} - \alpha) + D_{IL} \sin(\theta_{LAB} - \alpha)$$

$$S = D_{IL} \cos(\theta_{LAB} - \alpha) - D_{IS} \sin(\theta_{LAB} - \alpha) \quad .$$

Thus both Q and S can be determined with the incident beam polarized in any in-plane direction by measuring outgoing components in the scattering plane in two orthogonal directions.

II-C. KMT Multiple Scattering Theory

If three-body forces are neglected the Schrödinger equation for the proton-nucleus system can be written as

$$(H_p + H_A + \sum_{j=1}^A v_{pj}) \Psi(\vec{r}_p; \vec{r}_1, \dots, \vec{r}_A) = E \Psi(\vec{r}_p; \vec{r}_1, \dots, \vec{r}_A) \quad ,$$

where H_p = the kinetic energy operator for the proton $\left(\frac{-\hbar^2}{2\mu} \nabla_{\vec{r}_p}^2 \right)$,

H_A = the Hamiltonian for the target nucleus, and

Ψ = the total wave function of the proton nucleus system.

Ψ can be expanded using a complete set of states as follows (treating the projectile and target nucleons as distinct):

$$\Psi = \sum_i \chi_i(\vec{r}_p) \phi_i(\vec{r}_1, \dots, \vec{r}_A) \quad .$$

χ_i are the incident-proton wave functions relative to the nucleus, and ϕ_i are the antisymmetric solutions to the nuclear Hamiltonian

(i.e., $H_A \Phi_i = \epsilon_i \Phi_i$), and include all bound and continuum states of the nucleus.

A formal solution to the Schrödinger equation can be written in terms of a Lipmann-Schwinger equation as follows:

$$(E - H_p - H_A) \Psi = \sum_{j=1}^A v_{pj} \Psi, \quad (\text{II-33})$$

and therefore

$$\Psi = \psi_0 + \frac{1}{E - H_p - H_A + i\epsilon} \sum_{j=1}^A v_{pj} \Psi. \quad (\text{II-34})$$

ψ_0 is the lowest energy solution to the homogeneous counterpart of Eq. II-33:

$$(e_i - H_p - H_A) \psi_i = 0. \quad (\text{II-35})$$

By inspection, the solution to this equation is seen to be

$$\psi_i = \exp(i\vec{k} \cdot \vec{r}_p) \Phi_i(\vec{r}_1, \dots, \vec{r}_A). \quad (\text{II-36})$$

Thus ψ_0 in the Lipmann-Schwinger equation is the product of a plane-wave wave function for the incident proton and the ground-state wave function for the nucleus. The $i\epsilon$ term in Eq. II-34 is the usual convergence factor chosen to give the proper outgoing spherical-wave asymptotic solutions.

At this point it is useful to define an operator T (the nuclear "transition matrix" or "t-matrix") that operates on the unperturbed solution ψ_0 such that

$$T \psi_0 \equiv \sum_{j=1}^A v_{pj} \psi_j \quad (II-37)$$

Note that this matrix is related to the scattering matrix M of Secs. II-A,B through:

$$M = \frac{-E_p E_A}{2\pi(\hbar c)^2 (E_p + E_A)} T \quad , \quad (II-38)$$

where E_p (E_A) is the proton (nucleus) total energy in the proton-nucleus center-of-momentum system. Defining

$$G \equiv (E - H_p - H_A + i\varepsilon)^{-1} \quad , \quad (II-39)$$

and operating on the Lipmann-Schwinger equation (Eq. II-34) with $\sum_{j=1}^A v_{pj}$ produces

$$T \psi_0 = \sum_{j=1}^A v_{pj} \psi_0 + \sum_{j=1}^A v_{pj} G T \psi_0 \quad ,$$

or an operator equation of the form

$$T = \sum_{j=1}^A v_{pj} + \sum_{j=1}^A v_{pj} G T \quad . \quad (II-40)$$

If antisymmetrized target states are selected for the basis of intermediate states in Eq. II-40, then all subsequent matrix elements of v_{pj} (of a particular isospin type) are equivalent; hence $\sum_{j=1}^A v_{pj}$ can be replaced with Av where v is an isospin averaged proton-nucleon interaction. (Isospin corrections to this procedure have been studied and are very small.⁽⁴⁵⁾) Thus Eq. II-40 takes the form

$$T = Av + Av G \hat{A} T , \quad (\text{II-41})$$

where

$$\hat{A} = \sum_i |\phi_i\rangle\langle\phi_i|$$

is the projection operator for antisymmetric target-nucleus states.

It is now preferable to express this proton-nucleus operator equation in terms of an effective proton-nucleon interaction rather than the "bare" interaction, v . The approach of Kerman, McManus and Thaler (KMT)⁽¹⁰⁾ is to consider the fundamental proton-bound-nucleon problem independently and do identical algebra to that just done for the proton-nucleus case. A result with a similar form is produced:

$$\tau = v + v G \hat{A} \tau , \quad (\text{II-42})$$

where τ is now the transition matrix describing the scattering of the proton from any one of the bound nucleons.

Solving for the bare proton-nucleon interaction from this equation yields

$$v = \frac{1}{1 + \tau G} \hat{A} \tau . \quad (\text{II-43})$$

Now v can be eliminated from Eq. II-41 by the substitution of this result. Thus

$$T = A\tau + (A-1)\tau G \hat{A} T . \quad (\text{II-44})$$

The interpretation of this equation is clear if a few terms in the infinite series are written out:

$$T = A\tau + A(A-1)\tau G \hat{A} \tau + A(A-1)^2 \tau G \hat{A} \tau G \hat{A} \tau + \dots$$

The operator G describes the propagation of the incident proton inside the nucleus. Thus the proton-nucleus scattering operator (T) is seen as the sum of the proton-nucleon single-scattering operators ($A\tau$) plus a series of higher order terms. The second-order term represents scattering from one nucleon, propagation through the nucleus, and then scattering from another (possibly the same) nucleon. The third-order term represents scattering sequentially from three bound nucleons. Because of the

size of the nuclear force these higher order terms have significant effects on the observables and it is not reasonable to truncate this expansion.

The operator equation can be put into standard Lipmann-Schwinger form by multiplying through by $(A-1)/A$ and defining a pseudo t-matrix T' ,

$$T' = \frac{A-1}{A} T . \quad (\text{II-45})$$

This process results in Eq. II-44 transforming to

$$T' = (A-1)\tau + (A-1)\tau G \hat{A} T' . \quad (\text{II-46})$$

This equation is still the exact (formal) solution to the original Schrödinger equation. At this point it is convenient to introduce the proton-nucleus optical potential, defined by

$$T' = U + U G P T' , \quad (\text{II-47})$$

where P is the projection operator for the target-nucleus ground state. This optical potential includes the effects of intermediate scattering through all excited states of the many-body system. Solving for the exact form of this potential in terms of τ (using Eq. II-46) yields

$$U = (A-1)\tau + (A-1)\tau G Q U , \quad (\text{II-48})$$

where $Q \equiv \hat{A} - P$.

At this point it is reasonable to consider approximations to the full proton-nucleus many-body-problem solution represented in Eqs. II-47 and II-48. Since the second- and higher-order terms in Eq. II-48 correspond to virtual target nucleus excitations, they are considerably smaller than the first-order term. The series expansion of U in terms of τ converges quite rapidly for the intermediate-energy proton-nucleus system. Hence, in the following discussion only the first-order term will be retained.

The first-order optical potential $U^{(1)}$ is then simply related to the proton- bound-nucleon transition matrix τ . The first-order relationship is not simply $U^{(1)} = A\tau$ because of the normalization change that was made to produce an equation in the Lipmann-Schwinger form. In order to "realize" this operator relationship into momentum space one must multiply on the right by the incident-beam state and on the left by the scattered-beam state.

$$\begin{aligned} U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') &\equiv \langle \vec{k}', \Phi_{\text{gs}} | U^{(1)} | \Phi_{\text{gs}}, \vec{k} \rangle \\ &= (A-1) \langle \vec{k}', \Phi_{\text{gs}} | \tau | \Phi_{\text{gs}}, \vec{k} \rangle, \end{aligned} \quad (\text{II-49})$$

where $|\vec{k}'| = |\vec{k}|$ because only elastic scattering is being considered. If identities of the form

$$1 = \int^A d\vec{r}_1 \dots d\vec{r}_A |\vec{r}_1 \dots \vec{r}_A \rangle \langle \vec{r}_A \dots \vec{r}_1| \quad (\text{II-50})$$

are introduced into Eq. II-49, where $\int^A$ indicates integration over the coordinates of the A bound nucleons, the relationship becomes

$$\begin{aligned} U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') = (A-1) \int^{A'} \int^A & \left(\langle \vec{k}', \phi_{\text{gs}} | \vec{r}_1' \dots \vec{r}_A' \rangle \right. \\ & \times \langle \vec{r}_A' \dots \vec{r}_1' | \tau | \vec{r}_1 \dots \vec{r}_A \rangle \langle \vec{r}_A \dots \vec{r}_1 | \phi_{\text{gs}}, \hat{k} \rangle \Big) \\ & \times d\vec{r}_1' \dots d\vec{r}_A' d\vec{r}_1 \dots d\vec{r}_A . \end{aligned} \quad (\text{II-51})$$

Now recall that τ is the transition matrix for any of the bound nucleons. With complete generality τ is selected to operate on nucleon 1, such that the matrix element in Eq. II-51 becomes

$$\langle \vec{r}_A' \dots \vec{r}_1' | \tau | \vec{r}_1 \dots \vec{r}_A \rangle = \langle \vec{r}_1' | \tau | \vec{r}_1 \rangle \delta(\vec{r}_2' - \vec{r}_2) \dots \delta(\vec{r}_A' - \vec{r}_A) . \quad (\text{II-52})$$

Thus the first-order optical potential is

$$U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') = (A-1) \int d\vec{r}_1 \int d\vec{r}_1' \langle \vec{k}', \vec{r}_1' | \tau | \vec{r}_1, \vec{k} \rangle \rho(\vec{r}_1', \vec{r}_1), \quad (\text{II-53})$$

where the one-body density $\rho(\vec{r}_1', \vec{r}_1)$ is defined as

$$\begin{aligned} \rho(\vec{r}_1', \vec{r}_1) = \int^{A-1} d\vec{r}_2 \dots d\vec{r}_A & \langle \phi_{\text{gs}} | \vec{r}_1', \vec{r}_2 \dots \vec{r}_A \rangle \\ & \times \langle \vec{r}_1 \dots \vec{r}_2, \vec{r}_1 | \phi_{\text{gs}} \rangle . \end{aligned} \quad (\text{II-54})$$

In obtaining Eq. II-53 from Eq. II-51 it is assumed that spin-saturated targets are being considered. It follows that only

those components of τ which do not depend on the target nucleon spin will survive in Eq. II-53 and all subsequent expressions. If expressions of the form

$$1 = \frac{1}{(2\pi)^3} \int d\vec{k}_1 |\vec{k}_1\rangle \langle \vec{k}_1| \quad , \quad (\text{II-55})$$

where $\vec{k}_1$ is the momentum of bound nucleon 1, are inserted into the equation for the optical potential (Eq. II-53), the result is

$$\begin{aligned} U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') &= \frac{A-1}{(2\pi)^6} \iiint \rho(\vec{r}_1', \vec{r}_1) \langle \vec{r}_1' | \vec{k}_1' \rangle \langle \vec{k}', \vec{k}_1' | \tau | \vec{k}_1, \vec{k} \rangle \\ &\quad \times \langle \vec{k}_1 | \vec{r}_1 \rangle d\vec{r}_1' d\vec{r}_1 d\vec{k}_1' d\vec{k}_1 \\ &= \frac{A-1}{(2\pi)^6} \iiint \exp[i(\vec{k}_1' \cdot \vec{r}_1' - \vec{k}_1 \cdot \vec{r}_1)] \rho(\vec{r}_1', \vec{r}_1) \\ &\quad \times \langle \vec{k}', \vec{k}_1' | \tau | \vec{k}_1, \vec{k} \rangle d\vec{r}_1' d\vec{r}_1 d\vec{k}_1' d\vec{k}_1 \quad . \quad (\text{II-56}) \end{aligned}$$

Momentum conservation ($\vec{k}_1' = \vec{k}_1 + \vec{k} - \vec{k}'$), and integration over $\vec{k}_1'$, produces the following:

$$\begin{aligned} U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') &= \frac{A-1}{(2\pi)^3} \iiint \left(\exp[i\vec{k}_1 \cdot (\vec{r}_1' - \vec{r}_1)] \right. \\ &\quad \times \exp[i(\vec{k} - \vec{k}') \cdot \vec{r}_1] \rho(\vec{r}_1', \vec{r}_1) \\ &\quad \times \langle \vec{k}', \vec{k}_1 + \vec{k} - \vec{k}' | \tau | \vec{k}_1, \vec{k} \rangle \left. \right) d\vec{r}_1' d\vec{r}_1 d\vec{k}_1 \quad . \quad (\text{II-57}) \end{aligned}$$

This is an exact expression for the first-order optical potential. If it is now assumed that the transition matrix is independent of the initial momentum of the bound nucleon the integral over $\vec{k}_1$ can be done explicitly, yielding

$$U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') = (A-1) \iint d\vec{r}_1 d\vec{r}_1' \delta(\vec{r}_1 - \vec{r}_1') \exp[i(\vec{k} - \vec{k}') \cdot \vec{r}_1] \\ \times \langle \vec{k}' | \tau | \vec{k} \rangle \rho(\vec{r}_1', \vec{r}_1) . \quad (\text{II-58})$$

The transition matrix can now be factored out leaving an integral that is recognized as the Fourier transform of $\rho(\vec{r}_1)$. Defining the momentum transfer $\vec{q} \equiv \vec{k}' - \vec{k}$ gives

$$U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') = (A-1) \langle \vec{k}' | \tau | \vec{k} \rangle \tilde{\rho}(\vec{q}) . \quad (\text{II-59})$$

At this point the first-order optical potential is still non-local. For intermediate-energy proton-nucleus scattering at forward angles it is adequate to localize the optical potential by assuming that the scattering from state $|\vec{k}\rangle$ to $|\vec{k}'\rangle$ depends only on the difference between the two momenta; thus

$$U_{\text{opt}}^{(1)}(\vec{k}, \vec{k}') \approx U_{\text{opt}}^{(1)}(q) = (A-1) \tau(q) \tilde{\rho}(q) . \quad (\text{II-60})$$

The simplest application of Eq. II-60 involves replacing the complicated many-body operator τ by the free-scattering proton-nucleon t-matrix. The so-called "impulse approximation"

should become valid for sufficiently high-energy incident protons. The free proton-nucleon scattering matrix f can be shown to have the following general form based on physical symmetry arguments: ⁽¹¹⁾

$$f_{pj} = a_{pj} + ic_{pj}(\vec{\sigma}_p + \vec{\sigma}_j) \cdot \hat{N} + m_{pj}(\vec{\sigma}_p \cdot \hat{N})(\vec{\sigma}_j \cdot \hat{N}) \\ + g_{pj}(\vec{\sigma}_p \cdot \hat{K})(\vec{\sigma}_j \cdot \hat{K}) + h_{pj}(\vec{\sigma}_p \cdot \hat{P})(\vec{\sigma}_j \cdot \hat{P}) \quad , \quad (\text{II-61})$$

where j indicates either a proton or a neutron. The coefficients a_{pj} , c_{pj} , f_{pj} , g_{pj} , and h_{pj} are complex amplitudes that can be determined experimentally by doing proton-nucleon scattering. These amplitudes are defined in the proton-nucleon c.m. system. The t-matrix τ must be expressed in the proton-nucleus c.m. system, however. The transformation from one to the other (making the impulse approximation) is given by ⁽¹⁰⁾

$$\tau = \eta f \quad , \quad (\text{II-62})$$

where η is defined to be

$$\eta = \frac{-2\pi(\hbar c)^2}{E_L} \frac{k_L}{k} \frac{1 + \frac{2A}{A^2 + 1} \frac{E_L}{m}}{1 + \frac{A}{A^2 + 1} \left(\frac{E_L}{m} + \frac{m}{E_L} \right)} \quad ,$$

and k_L (E_L) is the incident momentum (total energy) of the projectile in the lab and k is the proton-nucleon c.m. momentum (as

before). Substituting Eqs. II-61 and II-62 into Eq. II-60 and including explicit isospin dependence the proton-nucleus optical potential for spin-saturated targets takes the form:

$$U_{\text{opt}}^{(1)}(q) = \frac{A-1}{A} \eta \sum_{j=p,n} (a_{pj} + ic_{pj} \vec{\sigma}_p \cdot \hat{N}) \tilde{\rho}_j(q) \quad . \quad (\text{II-63})$$

The sum over j extends over all protons and neutrons in the target. The quantities $\tilde{\rho}_p(q)$ and $\tilde{\rho}_n(q)$ are the Fourier-transformed proton and neutron matter densities, respectively. This potential has the form

$$V(q) = V_c(q) + iV_{so}(q) \vec{\sigma}_p \cdot (\vec{k} \times \vec{k}') \quad , \quad (\text{II-64})$$

which after Fourier transformation yields a local optical potential in coordinate space of the form (only the on-shell portion of $\vec{k} \times \vec{k}'$ is used):

$$V(r) = \tilde{V}_c(r) - \frac{1}{r} \frac{d}{dr} \tilde{V}_{so}(r) \hat{q} \cdot \hat{s} \quad , \quad (\text{II-65})$$

where, as before, $\sim$ indicates Fourier transformation. Noting that

$$\hat{n} \equiv \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|} \quad ,$$

the optical potential in coordinate space is then given by

$$\begin{aligned}
 U_{\text{opt}}^{(1)}(r) &= \frac{A-1}{A} \frac{\eta}{(2\pi)^3} \sum_{j=p,n} \left[\int a_{pj} \tilde{\rho}_j(q) e^{i\vec{q} \cdot \vec{r}} d\vec{q} \right. \\
 &\quad \left. - \frac{1}{r} \frac{d}{dr} \int \frac{c_{pj}}{|\vec{k} \times \vec{k}'|} \tilde{\rho}_j(q) e^{i\vec{q} \cdot \vec{r}} d\vec{q} \hat{l} \cdot \hat{s} \right] \\
 &= \frac{A-1}{A} \frac{\eta}{(2\pi)^3} 4\pi \sum_{j=p,n} \left[\int q^2 a_{pj} \tilde{\rho}_j(q) j_0(qr) dq \right. \\
 &\quad \left. - \frac{1}{r} \frac{d}{dr} \int q^2 \frac{c_{pj}}{|\vec{k} \times \vec{k}'|} \tilde{\rho}_j(q) j_0(qr) dq \hat{l} \cdot \hat{s} \right] . \quad (\text{II-66})
 \end{aligned}$$

The solution to the Lipmann-Schwinger equation in which this optical potential was introduced (Eq. II-47) will now be described. The ground-state projection operator P is multiplied through on both sides of Eq. II-47 in order to select the elastic scattering channel:

$$P T' P = P U P + P U P G P T' P . \quad (\text{II-67})$$

Defining the transition matrix for elastic scattering from the ground state, T'_{00} ($T'_{00} \equiv P T' P$), and its corresponding optical potential, U_{00} ($U_{00} \equiv P U P$), yields

$$T'_{00} = U_{00} + U_{00} \frac{|\phi_{gs}\rangle \langle \phi_{gs}|}{E - T_p - e_{gs} + i\epsilon} T'_{00} . \quad (\text{II-68})$$

H_A has been replaced by its eigenvalue e_{gs} since only states in which the nucleus is in its ground state are being considered. Now an arbitrary state $|\xi\rangle$ is chosen and multiplied through on the right yielding

$$T'_{oo} |\xi\rangle = U_{oo} |\xi\rangle + U_{oo} \frac{P}{E - T_p - e_{gs} + i\epsilon} T'_{oo} |\xi\rangle , \quad (II-69)$$

and a state $|\eta\rangle$ is chosen such that

$$T'_{oo} |\xi\rangle \equiv U_{oo} |\eta\rangle . \quad (II-70)$$

Thus

$$U_{oo} |\eta\rangle = U_{oo} |\xi\rangle + U_{oo} \frac{P}{E - T_p - e_{gs} + i\epsilon} U_{oo} |\eta\rangle , \quad (II-71)$$

or equivalently,

$$|\eta\rangle = |\xi\rangle + \frac{P}{E - T_p - e_{gs} + i\epsilon} U_{oo} |\eta\rangle . \quad (II-72)$$

This equation can also be written as

$$(E - T_p - e_{gs}) |\eta\rangle = (E - T_p - e_{gs}) |\xi\rangle + U_{oo} |\eta\rangle . \quad (II-73)$$

Since the choice of $|\xi\rangle$ was arbitrary a particular state can be selected such that

$$(E - T_p - e_{gs})|\xi\rangle \equiv 0 \quad . \quad (II-74)$$

The solution to this equation is clearly

$$\xi(\vec{r}) = e^{i\vec{K} \cdot \vec{r}}$$

where K is the proton-nucleus c.m. momentum. This reduces Eq. II-73 to the easily soluble Schrödinger-like equation

$$(E - T_p - e_{gs} - U_{oo})|\eta\rangle = 0 \quad . \quad (II-75)$$

Before summarizing this process for calculating the observables for proton-nucleus scattering it is necessary to consider some additional contributions to the KMT optical potential which have not been mentioned explicitly. The most obvious of these is the static Coulomb interaction. Another is the interaction between the magnetic moment (MM) of the proton and the Coulomb field of the nucleus. This electromagnetic spin-orbit potential must also be included for accurate predictions of the behavior of the spin of the scattered proton.^(38,44) The potential that must be used in solving Eq. II-75 is then the sum of the

potentials resulting from the strong, static Coulomb, and magnetic moment interactions.

To summarize, the steps in the process of extracting the elastic-scattering observables from this multiple scattering theory in first order are as follows:

1. Construct the optical potential of Eq. II-66 using proton-nucleon scattering amplitude and proton and neutron matter distributions.
2. Solve Eq. II-75 for $\eta(r)$ with $U_{oo}(r)$ approximated by $U_{opt}^{(1)}$ (of Eq. II-66) + U_{Coul} + U_{MM} .
3. Construct $\xi(r)$.
4. Match $\eta(r)$ to the incoming plane wave, $\xi(r)$, and outgoing spherical wave to obtain the pseudo-scattering amplitude.
5. Renormalize the pseudo-scattering amplitude by $A/(A-1)$ to obtain the final scattering matrix M .
6. Construct the observables as described in Sec. II-B.

II-D. Relativistic Impulse Approximation

A fully relativistic treatment of the proton-nucleus system requires the consideration of a relativistic quantum field theory. A much simpler approach which maintains contact with conventional nonrelativistic multiple scattering theory but which allows a covariant treatment of the projectile is obtained by replacing the

incident-proton Hamiltonian in the many-body Schrödinger equation with the Dirac Hamiltonian,

$$[\vec{\alpha} \cdot \vec{p} + \beta(m + \sum_{i=1}^A v_{pi}) + H_A] \Psi = E \Psi \quad . \quad (\text{II-76})$$

In Eq. II-76 $\vec{\alpha}$ and β are the usual Dirac matrices and $\vec{p}$ is the momentum operator for the incident proton. In this equation the projectile is treated relativistically and the nucleus nonrelativistically. The projectile and target nucleons are assumed to be distinct as before. The method for solving this model of the proton-nucleus scattering system is similar to that employed in solving the Schrödinger equation in Sec. II-C. As in the nonrelativistic case the formal solution of this equation can be written in terms of a new propagator G' ,

$$G' \equiv (\beta E - \beta \vec{\alpha} \cdot \vec{p} - m - \beta H_A + i\epsilon)$$

and a T-matrix that is defined to be

$$T\psi_0 \equiv \sum_{i=1}^A v_{pi} \Psi \quad ,$$

where ψ_0 is now the product of a four component plane-wave solution of the Dirac equation for the incident proton and the nonrelativistic ground-state wave function for the nucleus. Thus

the equation of motion for the proton-nucleus system can be rewritten in Lipmann-Schwinger form as

$$T = \sum_{i=1}^A v_{pi} + \sum_{i=1}^A v_{pi} G' T . \quad (\text{II-77})$$

This is identical to the result for the Schrödinger equation formalism presented in Sec. II-C. The procedure from this point in obtaining a solution to the scattering problem is then identical to that described previously.

In this formal treatment, however, any relativistic effects in the target would be automatically excluded as a direct result of the form of Eq. II-76. In order to allow relativistic effects in the target wave function (i.e. virtual pair creation) to enter into the description of the scattering process, relativistic single-particle wave functions can be used to describe the target in place of the customary nonrelativistic wave functions that would otherwise result from Eq. II-76. This arbitrary substitution is the basis of the relativistic impulse approximation (RIA) model⁽¹²⁾ of the proton-nucleus scattering process. The ground-state relativistic wave function can be written as

$$\phi_{gs}(\vec{r}_1 \dots \vec{r}_A) = \frac{1}{\sqrt{A!}} \det \prod_{i=1}^A \zeta_{\{\alpha\}}(\vec{r}_i) ,$$

where

$$\zeta_{\{\alpha\}}(\vec{r}_i) = \begin{pmatrix} \phi_{nlj}(r) \\ -i\vec{\sigma} \cdot \vec{r} \lambda_{nlj}(r) \end{pmatrix} \mathcal{Y}_{lj}^{\mu}(\hat{r})$$

are the Dirac single-particle wave functions specified by the set of quantum numbers $\{\alpha\}=n,l,j$ for all filled states. $\mathcal{Y}_{lj}^{\mu}(\hat{r})$ is the usual spin-angle function formed from the coupling of the orbital- and spin-angular momenta:

$$\mathcal{Y}_{lj}^{\mu}(\hat{r}) = \sum_{m_s} (l, \mu+m_s, 1/2, -m_s | j\mu) Y_l^{\mu+m_s}(\hat{r}) X_{1/2, -m_s}.$$

An important consideration in a relativistic treatment of the nucleon-nucleus interaction is that the Lorentz invariant NN amplitude be used (in contrast to the "Pauli form" of Sec. II-C). A procedure for doing this is discussed at length in Ref. 13 where the following Lorentz invariant form for the NN amplitude is proposed:

$$\begin{aligned} \hat{F}(s,t) = & F_S(s,t)I_1I_2 + F_P(s,t)\gamma_1^5\gamma_2^5 + F_V(s,t)\gamma_1^{\mu}\gamma_{2\mu} \\ & + F_A(s,t)\gamma_1^5\gamma_1^{\mu}\gamma_2^5\gamma_{2\mu} + F_T(s,t)\sigma_1^{\mu\nu}\sigma_{2\mu\nu}. \end{aligned} \quad (\text{II-78})$$

In Eq. II-78 s and t are the Mandelstam kinematic variables and I , γ^5 , γ_{μ} , $\gamma^5\gamma_{\mu}$ and $\sigma_{\mu\nu}$ represent the usual set of 16 linearly independent 4×4 matrices. The subscripts 1 and 2 label the two nucleons. F_S , F_P , F_V , F_A and F_T are the Lorentz invariant scalar, pseudoscalar, vector, axial vector, and tensor components of the amplitude, respectively. These Lorentz NN amplitude components can

be related to those in the Pauli representation by requiring that the NN scattering probabilities be independent of the representation.

As was done for the nonrelativistic case, the NN amplitude must be transformed to the nucleon-nucleus frame of reference before it is input into the multiple scattering theory. This procedure is also discussed in Ref. 13, with the result that

$$t_i(q) = -2\pi \frac{(\hbar c)^2}{m} \frac{R(q)}{R(0)} P_{LAB} \hat{F}_i(q) \quad (II-79)$$

where $R(q)$ contains several kinematic factors while the ratio $R(q)/R(0)$ is of the order of unity and increases monotonically with q (momentum transfer). P_{LAB} is the laboratory momentum of the incident proton. The subscript i incorporates the Lorentz character and isospin dependence of each component of the scattering amplitude ($i=S,P,V,A,T$ for pp and pn).

A point of current theoretical investigation is the form of the NN amplitude. Additional terms can be added to this equation which are not constrained by NN scattering measurements but which contribute to the optical potential in nucleon-nucleus scattering. Such terms involve negative energy states of either or both of the nucleons and the off-energy-shell (non-energy-conserving) dependence of $\hat{F}$. In order to determine or constrain these terms a theoretical model of the NN scattering system is needed. Since a

reliable, covariant theory of NN scattering is not available at the present the form of the NN amplitudes given in Eq. II-78 is commonly used as a starting point.

Using the same procedure and approximations which lead from Eq. II-40 to Eq. II-60 in Sec. II-C for the nonrelativistic case, Eq. II-77 is used to obtain the following form for the first-order Dirac optical potential,

$$U_{\text{opt}}^{(1)}(q) = \sum_i \tau_i(q) \bar{\rho}_i(q) \quad (\text{II-80})$$

where $i=S,P,V,A,T$. $\tau_i(q)$ is replaced by the free NN amplitude $t_i(q)$ (Eq. II-79) in the impulse approximation. Three terms (scalar, vector, and tensor) contribute to elastic scattering from a spin-zero nucleus in this relativistic treatment, each involving different densities. These are defined to be

$$\rho_S^{(i)}(r) \equiv \sum_{n\ell j} \frac{2j+1}{4\pi} (|\phi_{n\ell j}|^2 - |\lambda_{n\ell j}|^2) ,$$

$$\rho_V^{(i)}(r) \equiv \sum_{n\ell j} \frac{2j+1}{4\pi} (|\phi_{n\ell j}|^2 + |\lambda_{n\ell j}|^2) ,$$

$$\rho_T^{(i)}(r) \equiv 2 \sum_{n\ell j} \frac{2j+1}{4\pi} \phi_{n\ell j} \lambda_{n\ell j} ,$$

where (i) indicates either a proton or neutron density. Fourier transforming the optical potential (Eq. II-80) into coordinate space yields

$$U_{\text{opt}}^{(1)}(r) = \frac{-2\pi(\hbar c)^2}{m} \frac{R(q)}{R(0)} P_{\text{LAB}} \times \left[U_S(r) + \beta_1 U_V(r) - 2i(\vec{\alpha}_1 \cdot \hat{r}) U_T(r) \right] \quad (\text{II-81})$$

where

$$U_j(r) \equiv \frac{1}{(2\pi)^3} \sum_{i=p,n} \int e^{-i\vec{q} \cdot \vec{r}} F_j^{(i)}(q) \tilde{p}_j^{(i)}(q) d\vec{q} ,$$

for $j=S,V$ and

$$U_T(r) \equiv \frac{1}{(2\pi)^3} \sum_{i=p,n} r \int e^{-i\vec{q} \cdot \vec{r}} F_T^{(i)}(q) \tilde{p}_T^{(i)}(q) d\vec{q} .$$

This form for the optical potential can be used to find the solution of the Lipmann-Schwinger equation as in Sec. II-C. In practice, however, the tensor term in the optical potential is small and is usually omitted.

The result analogous to Eq. II-72 is

$$|\eta\rangle = |\xi\rangle + \frac{P}{\beta E - \vec{\alpha} \cdot \vec{p} - m - \beta e_{gs} + i\epsilon} U_o \cdot |\eta\rangle .$$

Here $|\xi\rangle$ is defined to be the solution of

$$(E - \vec{\alpha} \cdot \vec{p} - \beta m - e_{gs}) |\xi\rangle = 0$$

and the equation that must finally be solved is then

$$(E - \vec{\alpha} \cdot \vec{p} - \beta m - e_{gs} - \beta U_{oo}) |\eta\rangle = 0 \quad . \quad (\text{II-82})$$

As in the nonrelativistic case the potential that must be inserted into this equation has several pieces that have not yet been written down. The static Coulomb interaction must be included, of course. Since the Dirac equation describes the motion of a spin-half point particle in an external field, the effects due to such a particle's intrinsic magnetic moment are implicitly included in this Dirac formulation. When used to describe protons, however, the strength of this implicit interaction is too small because the internal structure of the proton gives rise to a magnetic moment which is roughly 2.8 times greater than that of a spin-half point particle with the mass of a proton. In order to handle the enhanced strength of this interaction a term must be added to the potential that is proportional to the difference between the proton's predicted magnetic moment, based on point-like structure (its "normal" magnetic moment), and its true value. This difference is referred to as the anomalous magnetic moment (AMM).

Thus the potential to be used in solving Eq. II-82 is of the form

$$U_{oo} \approx U_{opt}^{(1)} + U_{Coul} + U_{AMM}$$

where $U_{\text{opt}}^{(1)}$ is defined by Eq. II-81. The procedure for construction of the observables from this point is identical to that presented at the end of Sec. II-C.

III. EXPERIMENTAL ENVIRONMENT

This chapter contains a description of the hardware used in the experiment. The first two sections deal with the production of a polarized proton beam and its transport to the HRS target. The last three sections contain brief descriptions of the experimental apparatus in Area C as well as the electronics.

III-A. Polarized Ion Source

A schematic description of the production of the polarized H^- (P^-) beam at LAMPF is shown in Fig. III-1.⁽¹⁹⁾ This type of source is referred to as a Lamb-shift source because the

Fig. III-1

Schematic for the Polarized Ion Source.

polarization technique makes use of the Lamb-shift in the $2S_{1/2}$ state of hydrogen (as opposed to polarizing atoms in the ground state directly, as is the case for an atomic beam source). A duoplasmatron is used to supply H^+ ions. These ions are extracted by an electrode and enter a cesium cell where the charge-exchange reaction produces some neutral hydrogen atoms. The remaining H^+ ions (and any H^- ions that have been created in the cesium cell) are electrostatically separated and discarded. A sizable fraction (optimally 43%) of the neutral hydrogen atoms that are produced are in the metastable $2S$ state which has a lifetime of ~ 1 s. The neutral beam then passes through a spin filter (described below) which makes use of the hyperfine structure of hydrogen to pass only those $2S$ state atoms with a given nuclear spin orientation. All atoms in other $2S$ substates are quenched to the ground state. This mixture of ground-state and polarized $2S$ state atoms then enters a cell containing argon where the $2S$ state atoms preferentially capture an electron. A small portion of the ground state atoms will also capture electrons in the argon; this process limits the polarization of the ion beam produced. It is necessary to maintain an axial magnetic field in this cell to keep the atoms in the $2S$ state from depolarizing while capturing electrons.

The spin filter used in the polarized ion source at LAMPF will now be described. The splitting of the $2S_{1/2}$ and $2P_{1/2}$ levels in hydrogen is shown in Fig. III-2. The $2P_{3/2}$ level is off-scale by approximately 10,000 MHz. When an external magnetic field is

Hyperfine Splitting of Hydrogen.

applied to the atom, the hyperfine structure of the levels becomes evident. The $2P_{1/2}$ state can decay by electric dipole radiation to the ground state and has a lifetime of ~ 1.6 ns. The $2S_{1/2}$ state is metastable with a lifetime of ~ 0.1 s. In an applied magnetic field of ~ 575 gauss the $2S_{1/2}$ β and the $2P_{1/2}$ e states are degenerate so a small transverse dc electric field is all that is needed to mix these states (the Stark effect). Thus the population of atoms originally in the $2S_{1/2}$ β states will decay to the ground state.

In order to be left with a system of known nuclear spin one of the α states must also be eliminated. One method for doing this is to pump energy (in the form of an axial rf field) equal to the separation energy between the α^- and e states (at a given magnetic field strength) into the system. This will induce transitions from the α^- to the e states which will then decay rapidly to the ground state. Thus the result of applying the axial magnetic and electric fields and the dc electric field is to quench any atom not in the α^+ state (proton and electron polarized along the momentum).

Changing the direction of the magnetic fields in the spin filter and argon cell changes the sign of the polarization. This is typically done every one to two minutes to allow for the elimination of some of the instrumental asymmetries involved in data collection. When a much shorter cycle is desired (for more accurate systematic corrections) a switching magnet is placed between the spin filter and the argon cell. This magnet is capable of switching the sign of the polarization at 30 Hz. This latter mode is referred to as rapid reversal. When the magnetic fields in the argon and cesium cells are aligned such that the polarization is produced parallel to the momentum of the beam, the source is said to be in the normal mode; when the fields (and thus the resulting polarization) are switched, the source is said to be in the reverse mode.

Changing the frequency of the rf in the spin filter can quench both α states to the ground state leaving only an unpolarized beam. This is done periodically during an experiment to measure the amount of unpolarized beam that is ionized in the argon cell. The measurements of the polarized and quenched beam currents can then be used to determine the absolute beam polarization (as described in Sec. IV-C-1-b).

After the argon cell the P^- beam is accelerated to 750 keV with a Cockroff-Walton accelerator. It then passes through a spin precessor in which the spin can be aligned in any direction desired by the experimental areas. After this precession the labels normal and reverse refer to whether the polarization is parallel or antiparallel to a desired coordinate axis in a particular beam line. After precession the pulsed beam (with pulse length 500 μ s) is injected into the main accelerator.

III-B. Linac and Beam Lines

This section describes the transport of the P^- beam from the polarized ion source to the HRS target. This is done in three stages. The first is the accelerator which brings the kinetic energy of the protons up to 800 MeV. The second is Line X which prepares the beam for input into areas B, EPB, and C. The third stage is Line C which transports the beam to the target and

disperses it on target for operation of the HRS in the "energy-loss" mode.

III-B-1. Linac

The first section of the accelerator is an Alvarez-type drift-tube linac (Fig. III-3).⁽²⁰⁾ The protons are accelerated from the injector energy of 750 keV to 100 MeV in this section. The protons are injected in phase with the rf supplied to this section

Fig. III-3

Alvarez-Type Linac.

in such a way that they are accelerated by the positive potential difference between the tubes (for P^-) and are passing through the tubes (and thus shielded) at times when a negative potential difference occurs. Both P^- and H^+ are accelerated simultaneously by injecting them at different points on the rf structure.

The second section of the accelerator is a side-coupled cavity system (Fig. III-4)⁽²¹⁾ that was developed at LAMPF. In this system power is supplied to internal (accelerating) cavities

Fig. III-4

Side-Coupled-Cavity Linac.

indirectly by way of external cavities which receive power from several dozen klystrons operating at 805 MHz. The external cavities visible in Fig. III-4 are strongly electromagnetically coupled to pairs of internal cavities and serve to shift the phase between the internal cavities by $\pi/2$ (thus eliminating the need for a drift tube between them). Each external cavity can be physically distorted separately to shape the field in each internal cavity. This process can be used to tune each of the nearly 5000 units in the half-mile-long linac. The 805 MHz frequency of the power supplies gives the beam a periodicity on the order of 1 ns. These 1 ns pulses are referred to as micropulses. This "microstructure" is superimposed on the 500 μ s "macrostructure" formed by the pulsed nature of the source.

III-B-2. Line X

The H^+ and P^- beams are separated magnetically in the switchyard (Fig. III-5). The P^- beam is diverted into Line X and steered and focused in the direction of Line C and the nucleon physics areas (EPB, BR) which exclusively utilize this beam. The beam can be collimated in Line X by the use of "strippers" that strip the electrons from the unwanted portions of the beam. This technique can be used to improve the transmitted beam quality with little or no increase in background.

Fig. 1III-5
LAMPF Switchyard.

III-B-3. Line C

The layout of Line C is shown in Fig. III-6. The proton beam is produced by a stripper in Line X and is separated from the P^- by three small dipole magnets (LCBM01, LCBM02, LCBM03). The percentage of the beam stripped can be controlled by changing the dimensions and composition of the stripper. The beam is then dispersed horizontally by two large dipoles (LCBM04, LCBM05). The HRS requires a vertically dispersed beam so the horizontal dispersion produced by these dipoles must be rotated 90° . This is accomplished by the "twister" section, which consists of a series of quadrupoles (QM04-QM11) rotated 45° about the beam axis.

Moving the beam horizontally or vertically can be accomplished at various places in Line C by the use of steering magnets. The beam intensity and phase space can also be controlled by the use of sets of collimators at various places in the beam. Monitoring of the effects of magnet and collimator adjustments is done with several strategically placed insertable phosphors which can be observed remotely with TV cameras.

III-C. Area C Hardware

This section describes the non-focal-plane hardware in common use in Area C (Fig. III-7). Included are descriptions of the Line C polarimeter, scattering chamber equipment and spectrometer.

Fig. III-6

Line C.

Fig. III-7
Area C.

III-C-1. Line C Polarimeter

The Line C polarimeter is positioned after the twister section and roughly 10 meters upstream of the target, and is used to determine the beam polarization components in the $\hat{s}$ and $\hat{n}$ directions simultaneously. It consists of a set of thin CH_2 targets of varying thickness that can be put into the beam remotely, and eight detection arms (Fig. III-8⁽⁴²⁾ in both the horizontal and vertical planes) each consisting of a scintillator

Fig. III-8

One Plane of the Line C Polarimeter.

and photo-multiplier tube. Pairs of scintillators are positioned to detect, in coincidence, the forward-scattered and recoil protons for $^1\text{H}(p,p)$ at a forward scattering angle of 17° in the lab (45° in the c.m.) in the beam left, right, up, and down directions. At this angle the analyzing power of the reaction is known to be large over a wide incident-energy range.⁽¹⁴⁾ The beam polarization is determined from the asymmetry in the number of particles detected by opposing pairs of arms (i.e. left and right). (The details of this calculation are given in Sec. IV-C-1-a.) Since there is no energy discrimination in the polarimeter counters, the asymmetries contain contributions from quasi-elastic scattering from carbon. The inclusive analyzing power for the polarimeter target must then be used in determining the beam polarization. This inclusive analyzing power can be determined by cross calibrating to the quench-ratio beam polarization result (described in Sec. IV-C-1-b) or by scattering from a target in the HRS scattering chamber to a state of known analyzing power and adjusting the polarimeter analyzing power to produce the correct result for the incident-beam polarization. Both of these methods produce a result for the inclusive analyzing power of the polarimeter target of 0.475 ± 0.010 for a beam energy of 800 MeV.⁽¹⁵⁾

III-C-2. Scattering Chamber and Targets

The scattering chamber of the HRS is a 2 meter diameter aluminum tank containing a target wheel and two ion chambers. The target wheel is a remotely-driven disk with slots in it to accommodate a number of targets. The ion chambers are used to determine the relative beam current. If necessary they can be cross calibrated to another current monitor and then used to determine the absolute beam intensity, but such a process was not necessary for this experiment. The desired quantities in this experiment are scattered-beam polarizations and not normalized yields, so the ion chambers were used only to determine the incident-beam polarization by the quench-ratio technique. For the ^{208}Pb part of the experiment, which was run in the rapid-reversal mode without a quench cycle, the ion chambers were not used at all.

Electrical feedthroughs to the scattering chamber allowed for the refrigeration and monitoring of the status of the ice target used in the 160 part of this experiment.⁽¹⁸⁾ A schematic of the ice target is shown in Fig. III-9. Its thickness was approximately 1/8" which corresponds to $\sim 280 \text{ mg/cm}^2$ of 160 (compared to 100 mg/cm^2 for the ^{40}Ca and ^{208}Pb targets used). During this experiment the ice target was maintained at a temperature of -50°C by a refrigeration unit.

Fig. III-9

Ice Target.

III-C-3. High Resolution Spectrometer

The High Resolution Spectrometer (Fig. III-10)⁽¹⁶⁾ consists of a quadrupole (HSQM01) and two large dipoles (HSBM01, HSBM02) with a 3.5 meter bend radius. A multipole magnet is also present between the dipoles for use in very high resolution work but it was not used in this experiment. The optics of the HRS, which are point-to-point in the dispersion direction (x) and parallel-to-point in y, are discussed in some detail in Refs. 16 and 51.

III-D. Focal Plane Detection System

A system of detectors is used to determine the identities and trajectories of particles at the exit of the HRS. These particles are then rescattered by a carbon analyzer of known analyzing power and their new trajectories measured in order to determine their polarization. This overall system of detectors is shown in Fig. III-11 and is loosely referred to as being the "focal plane." Its components will be described individually in the following sections.

Fig. III-10

High Resolution Spectrometer.

HRS Focal Plane.

Fig. III-11

III-D-1. Scintillators

The fundamental purpose of the focal-plane scintillators is to provide a "trigger" for the data acquisition system (to signal that a high-energy proton has gone through the system). This is done by requiring that signals appear in several of the scintillators with the correct time structure for an ~800 MeV proton (loosely called a "coincidence"). For this experiment the required coincidence was

$$\text{EVENT} \equiv S1 \cdot S2 \cdot SP \quad .$$

Scintillators S1 and S2 are each $10 \times 56 \times 0.80 \text{ cm}^3$ and are viewed by photomultiplier tubes at both ends. The SP scintillator consists of two overlapping double-ended scintillators (SP1 and SP2). Two scintillators are used in order to improve light collection efficiency. Each of these scintillators is $38 \times 76 \times 0.64 \text{ cm}^3$ and they overlap by ~3 cm. The SP signal for the EVENT coincidence is then given by

$$SP \equiv SP1 + SP2 \quad .$$

III-D-2. Wire Chambers

Accurate position and angle information at the focal plane is provided by two sets of multiwire-proportional chambers.⁽²²⁾ These sets are each 10×60 cm² and each chamber consists of a plane of horizontal and a plane of vertical wires with a uniform wire spacing of 0.8 cm. Two separate chambers are housed in a single aluminum frame and are separated by 0.95 cm. The open sides of the frame are covered with a thin sheet of mylar and the inside of the chamber is flooded with a mixture of gases (65% argon, 35% isobutane, 0.10% freon and 0.50% isopropyl alcohol by volume) which ionize when a charged particle passes through. Within a given plane of wires, alternate wires are connected together and a high voltage applied (~ 2150 V). The other wires in that plane are grounded⁽⁴³⁾ (Fig. III-12). Thus when a charged particle passes through the chamber the high voltage wire nearest its trajectory will "fire" (discharge) shortly thereafter. Since all the charged wires in a plane are connected together, some effort is required to determine which wire fired and where the particle passed through.

The determination of the wire that fired is accomplished by evenly spacing the connections to the high voltage wires over a length of wire that contains a time delay of roughly 2.5 ns/cm (called a "delay line"), as in Fig. III-12. By sensing the discharge at both ends of the delay line and recording the relative times of the discharges the position of the particle trajectory can be determined as follows. Assume " l " wires connected to a delay

Fig. III-12

Wire Chamber Schematic.

line whose delay between connections is Δt (a constant). The time at which the left (right) end of the delay line fires is t_1 (t_2). The time it takes the ionization cloud to travel through the chamber to the nearest high voltage wire is t_{drift} . Thus if wire "m" from the left fires

$$t_1 = m \Delta t + t_{drift} \quad , \text{ and}$$

$$t_2 = (l-m) \Delta t + t_{drift} \quad .$$

Thus

$$m = \frac{1}{2} \left(\frac{t_1 - t_2}{\Delta t} + l \right) , \text{ while}$$

$$t_{\text{drift}} = \frac{1}{2} (t_1 + t_2 - l\Delta t) .$$

The wire that fired (m) can then be determined from the time difference between the arrival of the signals at the ends of the delay line while the "drift time" can be determined from the time sum. The drift time can be converted to a drift distance easily since a chamber gas is chosen that allows a fairly linear (in time vs. distance) drift of the electrons toward the nearest high voltage wire. Typical drift velocities for the mixture of chamber gases used at the HRS are 0.050 mm/ns.

Knowing the drift distance and the wire that fired does not uniquely determine the position of the incident particle, however, because no information is available to determine whether the position was to the left or the right of the wire. This is called the left-right ambiguity and resolving this ambiguity involves using information from other chambers. The second chamber in each aluminum frame has its high voltage wires offset from those in the first chamber by one-half of a wire spacing, as shown in Fig. III-13. A particle firing the first chamber should also fire the second, and the position of the track in the second chamber can be determined as described previously. Now four trajectories for

Fig. III-13

Illustration of the Chamber "Left-Right" Ambiguity.

the particle are possible (Fig. III-13) since the left-right ambiguity also exists in the position identification in the second chamber. To resolve these ambiguities a crude determination of the detected particle's position is made in a second set of chambers using only the wire number position information (and not the drift time). A crude trajectory is then constructed from this position and a crude position in the original set. This crude trajectory is used to determine which of the original four possible trajectories

is the correct one -- thus resolving the left-right ambiguity in the first and second chambers simultaneously.

III-D-3. Focal Plane Polarimeter

The HRS focal-plane polarimeter⁽²³⁾ consists of a carbon analyzer to rescatter the protons and two 60×60 cm² wire chambers sets (separated by 25 cm) to allow for the analysis of the azimuthal distribution of these protons, and thus the determination of the polarization of the scattered protons. The details of this calculation are given in Section IV-C.

The carbon analyzer is a set of carbon slabs each of which is $44 \times 67 \times 3$ cm³. The number of carbon slabs can be adjusted for the needs of a particular experiment (up to a total thickness of 27 cm). The thickness of carbon chosen for a given measurement depends on the energy of the scattered protons as well as on the analyzing power for protons that scatter by the nuclear interaction. It is desired that the carbon analyzer be as thick as possible in order to maximize the number of protons that scatter. A limit on the thickness is given by the magnitude of the multiple-coulomb-scattering angle, which increases with the carbon thickness. One does not want the angular range dominated by multiple-coulomb-scattered events to infringe on the angular range where the (inclusive) analyzing power and cross section for nuclear scattered events is large. This latter region is where the most information about the focal-plane polarizations is gained. To

determine the polarization of the scattered protons the polarization for each polarimeter polar scattering angle (θ_{carbon}) bin is calculated and then a weighted average of the results is taken. The weighting factor for each region is given by $\sigma \times (A_y)^2$. (This is discussed in more detail in Sec. IV-C-2.) This unnormalized "figure of merit" for the nuclear scattered events at 800 MeV is displayed in Fig. III-14. From this figure one can see that the most information about the focal-plane polarization at 800

Fig. III-14

Nuclear Figure of Merit for 800 MeV Protons Incident on C.

MeV comes from the θ_{carbon} region between 4° and 10° . Thus the thickness of carbon was chosen to give a multiple-coulomb-scattering dominated region extending out to 3.5° . At 800 MeV this thickness is roughly 27 cm. An acceptance test is then placed on the data which prevents the acquisition of events that scatter inside of this region (Sec. III-E-3). In the course of data acquisition, the number of carbon blocks is changed to maintain roughly the same multiple-coulomb-scattering angle as the spectrometer angle (and thus the energy of the scattered protons) changes. For this experiment, however, the energy of the outgoing particles never varied more than 10 MeV so a thickness of 27 cm was used during the entire experiment.

III-E. Data Stream

This section describes the process by which focal-plane detector information is written to magnetic tape to allow for later analysis. It includes brief descriptions of the electronics and logic as well as descriptions of the functions of the CAMAC, MBD and PDP-11/45 during data acquisition.

III-E-1. Electronics

The anode signals from the scintillator photomultipliers and the wire chamber preamplifier outputs feed leading edge discriminators which are then input into time-to-digital converters (TDC's). The anode pulses are also fed into amplitude-to-digital converters (ADC's). The TDC start times are derived from scintillator S2. The TDC's and ADC's are "buffered" to allow the information from up to 32 events to be stored before it is necessary for them to be read out. This greatly enhances the data-taking rate since readout of these devices takes roughly 5 μ s for each of the ~60 data words that are formed for each event, for a total "dead time" of 300 μ s per event. Since the beam comes in 500 μ s macropulses, at most two events can be read out in a single macropulse. With the buffered electronics, 32 events can be acquired during a macropulse and then be read out between macropulses.

The data acquisition logic formed by various coincidence units falls into two broad categories. The first is the "slow, gating" logic. This is the logic that enables or inhibits data taking. It consists of a host of such things as whether the magnet power supplies are turned on and whether an event occurred when the beam was on. It is also used to flag events as either NORMAL or REVERSE corresponding to whether a gate (received from the polarized ion source) indicates that the source is in that particular mode when the data was taken. The second broad category

of data-acquisition logic is the "fast, trigger" logic. This logic determines whether an occurrence at the focal plane has the correct timing to merit recording it if the gating logic indicates that data taking is enabled. This is the logic that forms the EVENT (discussed above) and TRIGGER (discussed below) signals. A simplified logic diagram for the fast trigger logic is shown in Fig. III-15.

III-E-2. CAMAC

The CAMAC (Computer Automated Measurement and Control)⁽²⁴⁾ system is used to read the scalers, TDC's, and ADC's. When a TDC or an ADC stores an event in its buffers it sends a signal to a device known as the Event Trigger Module to indicate that this has occurred. The Event Trigger Module then generates a flag as a signal to the CAMAC to read and clear a buffer in the TDC's and ADC's. This readout can occur simultaneously with the conversion of another event by the TDC and ADC. The CAMAC also periodically receives a message (typically every 15 s) from the computer to read the scalers.

The TDC's and ADC's cannot simultaneously acquire and digitize signals, so a gate is needed to inhibit data acquisition when conversion for each event ($\sim 12 \mu\text{s}/\text{event}$) is occurring. This is somewhat confusingly called the COMPUTER BUSY gate and an EVENT that occurs when this gate is off is referred to as a TRIGGER:

HRS Fast Trigger Logic.

Fig. III-15

TRIGGER $\neq$ EVENT $\cdot$ COMPUTER BUSY .

III-E-3. MBD

The CAMAC system cannot be interfaced directly to a computer. A go-between, referred to as a branch driver, is necessary. At the HRS a micro-programmable branch driver (MBD)⁽²⁵⁾ is used to effect this interface. The calculational abilities of the MBD are limited; nevertheless, because of the MBD's placement in the data stream, it greatly enhances data-taking abilities. By performing simple calculations on the TDC and ADC data : MBD can quickly reject many undesired events before they are passed to the computer to be taped. Since taping speed limits the speed of data acquisition for many experiments, including many focal-plane-polarimeter experiments such as this one, the filtering of triggers before taping greatly enhances data-taking efficiency.

For this experiment the MBD was used to throw out events that did not rescatter in the focal-plane-polarimeter carbon analyzer outside of the multiple-coulomb-scattering dominated region (Sec. III-D-3). This cut by 2/3 the number of tapes of data needed to obtain a given statistical accuracy for the spin-rotation parameters.

For the purposes of calculating quantities such as efficiencies, it is useful to use events that are untested by the MBD. Thus the instructions to the MBD included a provision to pass

one TRIGGER in 20 without performing any test. These untested events were flagged as such in order to allow for their identification during the analysis.

The TRIGGERS that are passed on by the MBD to the computer are referred to as TAPED TRIGGERS since no further tests are performed to determine whether these events are interesting before they are written to tape.

$$\text{TAPED TRIGGER} \equiv \text{TRIGGER} \cdot (\text{MBD TEST PASSED} + \text{MBD TEST SKIPPED})$$

III-E-4. PDP-11/45

The tasks of the PDP-11/45 computer during data acquisition are to tape data received from the MBD and to provide an interface between the experimenter and the data acquisition process. In addition, a program running in the computer during data acquisition (referred to as the "analyzer") will perform "on-line" analysis of the data. This on-line analysis must take a back seat, however, to the acquiring and taping of data. The fraction of events analyzed on-line thus depends directly on the data-taking rate. In this experiment the data-taking rate was high enough (because of the large elastic cross sections at forward angles) that little data (~5%) was analyzed on-line.

IV. DATA REDUCTION

This chapter describes the processing of the taped data and the extraction of the spin-rotation observables. Discussions of general-use computer programs employed in data analysis and the details of the calculation of the spin-rotation parameters are included.

IV-A. Analyzer

Roughly sixty raw data words that describe each event are written to tape during data acquisition (TDC and ADC outputs). Over two hundred calculated data words are then formed from these data words by a computer program known as the analyzer. These calculated data words progress in complexity from chamber positions to angles to target quantities. The target quantities of interest ("missing mass," or excitation energy, and scattering angle) are calculated from the focal-plane quantities by the use of a semi-empirical transport matrix. This matrix is determined in first order by optics of the HRS.

IV-B. Auxiliary Software Packages

When the data words for an event have been calculated several auxiliary programs are called by the analyzer to allow for processing of the data words and display of the results. The first of these routines called by the analyzer is the test package. ^(26,27)

The test package checks specified data words to see if they fall within certain limits that are determined by the experimenter. If an event passes a test then a counter for that test is incremented and a flag is set to indicate that this has occurred for the event being processed. The presence or absence of this flag can be used by the dotplotting and histogramming systems to be described shortly. In addition to testing the data words, the test package can form logical combinations of the results of previous tests. Thus events which fulfil a complicated set of logical preconditions can be flagged as such and treated appropriately in the higher levels of the analysis.

If the limits to be used for each test are known beforehand, they can be written directly into an input file for the test package. It is also possible to specify interactively the data word and limits for tests that may change a number of times during the course of an experiment. This is done using a separate software package that can form histograms of the events being processed. These histograms can be displayed graphically or examined numerically as desired. The histogram package ^(28,29) has provisions for the user to send test limits to the test package

interactively by specifying those limits on the histogram display. The data word to be used in the test is naturally the data word of the histogram being displayed.

Similarly, interactive tests can be formed for two data words simultaneously. To specify the data words and limits interactively requires the graphic display of both variables simultaneously. Although there are several forms which such plots can take, the kind used exclusively in the analysis for this experiment is known as a dotplot.^(28,30) A dotplot is a planar display of two variables in which the occurrence of an event whose two data words simultaneously occur within the limits of the display is signified by a dot at that point on the display. It is the grouping of these points that convey the information about distributions of events in the two variables. Interactively placing a box around certain groups of events on this plot allows any subsequently processed events that fall within the box to be treated as a group (for acceptance or rejection) by means of a flag in the test file. The types of interactively-specified two-variable tests done for this experiment will now be described briefly.

Displayed in Fig. IV-1 is a sample dotplot used for proton particle identification by another HRS experiment.⁽³⁹⁾ The x-axis is the mean pulse height of two focal-plane scintillators used in that experiment. This quantity is proportional to the energy loss (related to the square of the charge) of the particle detected in

Fig. IV-1

Particle Identification Dotplot.

the scintillator. The y-axis is the time-of-flight (linearly related to the mass) of the particle between two focal-plane scintillators separated by 2 m. In Fig. IV-1 it is apparent that the display of these two quantities allows clear separation between protons, deuterons and tritons. Particles with similar masses and different charges (i.e. ^3H and ^3He) become separated along the x-axis of this display because of the charge dependence of the energy loss. For this experiment the time-of-flight used in

checking particle identification was that measured between S1 and either SP1 or SP2 (separated by ~1.25 m) and the pulse height used was the mean of the heights from both ends of scintillator S1. No particles other than protons were detected at the focal plane. This is due to the much smaller cross section for the pickup reactions which produce outgoing particles whose momenta are close enough to the momentum of elastically-scattered protons that they make it through the HRS.

Fig. IV-2

Spectrometer Acceptance Test Dotplot.

Figure IV-2 is a dotplot of vertical position on the focal plane (x-axis) and essentially the vertical scattering angle at the target (y-axis). A uniform region of vertical scattering angle is selected by the box shown. Those particles that traversed the HRS on the edges of the vertical acceptance are eliminated from the analysis by this test. This is done because the magnetic fields are the least uniform in this region and difficulties arise in determining target quantities for these events. No similar test is necessary to limit the horizontal acceptance of the HRS. Horizontal scattering at the target is essentially the true scattering angle of the particle and can be limited very easily in the higher levels of analysis if necessary. Since the x position of a particle at the HRS focal plane is directly related to the excitation energy of the residual nucleus, the individual nuclear states of the ^{208}Pb target are evident in this dotplot (Fig. IV-2).

Shown in Fig. IV-3 is a dotplot with a box that is used to flag events that appear to have scattered once and only once in the focal-plane-polarimeter analyzer. The quantities in this plot are calculated from the incident trajectory of the particle at the focal plane (as determined by chambers 1-4) and the scattered focal-plane trajectory (as determined by chambers 5-8). The quantity "dclose" (on the y-axis) is the distance of closest approach of the two trajectories. The quantity "zclose" (on the x-axis) is the position along the z-axis in the focal-plane coordinate system of this distance of closest approach. If the two

Fig. IV-3

FPP Scattering Angle Test Dotplot.

trajectories do not appear to be coplanar with an intersection within the carbon analyzer, the event is eliminated from any further analysis. This is done to exclude events that scatter in the focal-plane scintillators and chambers as well as events that multiple scatter in the carbon analyzer.

IV-C. Determination of the Spin-Rotation Observables

The observables of interest in this experiment are obtained from incident- and scattered-beam polarizations. The scattered-beam polarization is not directly measured by the FPP since the HRS magnets precess the scattered particles' spin. This complicates the analysis of the data, but it does allow for the determination of the scattered particles' polarization in the $\hat{q}'$ direction at the target. The first two sections of this chapter are detailed discussions of the means by which the incident- and focal-plane polarizations are extracted from the data. The last two sections deal with the determination of the spin-rotation observables once these polarizations are known.

IV-C-1. Incident Beam Polarization

The determination of the beam polarization can be done in several ways. This section describes two methods that are in common use at LAMPF. The first method makes use of the relationship developed in Sec. II-B between the cross section and beam polarization. Measuring the difference between the cross sections at two azimuthal angles for a given polar angle allows the beam polarization to be inferred. The second method involves measurements of the beam current with and without the polarized part of the beam present. Thus the percentage of the protons in the original beam whose spins are aligned can easily be determined.

IV-C-1-a. Beam-Line Polarization

The differential cross section for scattering from a target of analyzing power A_y is given by Eq. II-2:

$$\sigma(\theta, \phi) = \sigma_o(\theta)[1 + (\vec{P}^b \cdot \hat{n})A_y] \quad .$$

where $\hat{n}$ is the normal to the scattering plane and θ (ϕ) is the polar (azimuthal) scattering angle. It is convenient to describe the incident polarization in terms of components along a fixed set of axes ($\hat{s}_o, \hat{n}_o, \hat{l}_o$) (Fig. II-2) rather than along a set of axes that vary for each particle. In terms of the fixed coordinate system these equations can be rewritten as

$$\begin{aligned} \sigma(\theta, \phi) &= \sigma_o(\theta)[1 + \vec{P}^b \cdot (\hat{n}_o \cos\phi - \hat{s}_o \sin\phi)A_y] \\ &= \sigma_o(\theta)[1 + (p_n^b \cos\phi - p_s^b \sin\phi)A_y] \quad . \end{aligned} \quad (IV-1)$$

(Note that $\vec{P}^b \cdot \hat{n}_o \equiv p_n^b$ and $\vec{P}^b \cdot \hat{s}_o \equiv p_s^b$.) If the cross section is measured at $\phi=0$ (beam left) and $\phi=\pi$ (beam right) the results are:

$$\begin{aligned} \sigma(\theta, \phi=0) &\equiv \sigma_L = \sigma_o(\theta)[1 + p_n^b A_y] \\ \sigma(\theta, \phi=\pi) &\equiv \sigma_R = \sigma_o(\theta)[1 - p_n^b A_y] \quad . \end{aligned}$$

Eliminating the unpolarized cross section and solving for p_n^b yields

$$p_n^b = \frac{1}{A_y} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} . \quad (\text{IV-2})$$

The polarization component in the $\hat{s}_o$ direction can likewise be determined by measuring the cross section at $\phi=\pi/2$ (up) and at $\phi=3\pi/2$ (down):

$$\sigma(\theta, \phi=\pi/2) \equiv \sigma_U = \sigma_o(\theta)[1 - p_s^b A_y]$$

$$\sigma(\theta, \phi=3\pi/2) \equiv \sigma_D = \sigma_o(\theta)[1 + p_s^b A_y]$$

$$p_s^b = \frac{1}{A_y} \frac{\sigma_D - \sigma_U}{\sigma_D + \sigma_U} . \quad (\text{IV-3})$$

Note that the equation for the $\hat{s}$ component of the beam polarization can be easily derived directly from that for the $\hat{n}$ component by the substitutions $\sigma_L \rightarrow \sigma_D$ and $\sigma_R \rightarrow \sigma_U$ as a result of rotation symmetry.

Actual measurements require a detector of finite solid angle; thus a nominal measurement at $\phi=0$, for instance, is actually obtained by averaging between $\phi=-\delta$ and $\phi=+\delta$, where 2δ is the azimuthal angular width of the detector. This can be taken into account as follows:

$$\begin{aligned} \sigma_L \equiv \frac{1}{2\delta} \int_{\phi=-\delta}^{\phi=+\delta} \sigma(\theta, \phi) d\phi \\ = \frac{\sigma_o(\theta)}{2\delta} \left[2\delta + p_n^b A_y \int_{-\delta}^{+\delta} \cos\phi d\phi + p_s^b A_y \int_{-\delta}^{+\delta} \sin\phi d\phi \right] \end{aligned}$$

$$= \frac{\sigma_o(\theta)}{2\delta} [2\delta + p_n^b A_y (2\sin\delta)]$$

$$\sigma_R = \frac{1}{2\delta} \int_{\phi=\pi-\delta}^{\phi=\pi+\delta} \sigma(\theta, \phi) d\phi = \frac{\sigma_o(\theta)}{2\delta} [2\delta + p_n^b A_y (-2\sin\delta)]$$

Again solving for p_n yields

$$p_n^b = \frac{1}{A_y} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{\delta}{\sin\delta} \quad (IV-4)$$

The result for p_s is then

$$p_s^b = \frac{1}{A_y} \frac{\sigma_D - \sigma_U}{\sigma_D + \sigma_U} \frac{\delta}{\sin\delta} \quad (IV-5)$$

by rotation symmetry. It should be noted that in the limit $\delta \rightarrow 0$ these results reduce to Eqs. IV-2 and IV-3.

The solid angle subtended by each detector of the Line C polarimeter is sufficiently small ($\delta \approx 1^\circ$) so as to make the $\delta/\sin\delta$ correction factor insignificant. Equations IV-2 and IV-3 can then be used to determine the beam polarization components. However, if one set of polarimeter coincidence detectors (detecting, in coincidence, forward-scattered and recoil particles) is in some way inefficient relative to its complementary set (due to scintillator, phototube, or scaler inefficiencies, radial or angular misalignment, missteered beam, etc.) a correction ("geometry factor") of one yield relative to its counterpart is required if these yields are to be used to determine the beam polarization. In

order to eliminate this factor use can be made of the fact that the sign of the beam polarization changes periodically. When an incident $\hat{n}$ -type beam is polarized NORMAL, the yield that is measured by the arm that is beam left (right) will be labeled y_L^+ (y_R^+); when the beam polarization is REVERSE it will be labeled as y_L^- (y_R^-). Thus, for a detector system with a relative inefficiency (as described above) the yields (normalized by beam current) would be

$$y_L^+ = y_0 (1 + p_n^b A_y)$$

$$y_L^- = y_0 (1 - p_n^b A_y)$$

$$y_R^+ = y_0 G (1 - p_n^b A_y)$$

$$y_R^- = y_0 G (1 + p_n^b A_y)$$

where the geometry factor G has been arbitrarily assigned to the right arm of the polarimeter. The quantities y_0 and G can be eliminated from the analysis by dealing only with the ratio

$$\frac{y_L^-}{y_L^+} = \frac{y_R^+}{y_R^-} = \frac{1 - p_n^b A_y}{1 + p_n^b A_y} .$$

Taking the product of these ratios of yields gives a result that is independent of the normal and reverse beam currents and thus allows the use of unnormalized yields in the calculation.

$$\sqrt{\frac{y_L^- y_R^+}{y_L^+ y_R^-}} \equiv \chi = \frac{1 - p_n^b A_y}{1 + p_n^b A_y} .$$

Solving for p_n^b yields

$$p_n^b = \frac{1}{A_y} \frac{1 - \chi}{1 + \chi} . \quad (\text{IV-6})$$

A similar result is obtained for p_s^b . The beam polarization can be calculated in this manner to eliminate the effects of a relative inefficiency between any two sets of polarimeter arms. An overall inefficiency of the system is unimportant since only the ratio of yields enters into the calculation.

It should be noted that the geometry factor has been assumed to be independent of the beam-polarization sign. This should be the case under normal operating conditions. On occasion a pathological beam tune will cause a lateral shift in the beam when a polarization sign change occurs. This possibility can be monitored by comparing the ratios y_R^+/y_L^- and y_R^-/y_L^+ , which should be equal if the beam currents in normal and reverse are equal. If the beam currents are not equal normalized yields must be used for this comparison.

IV-C-1-b. Quench-Ratio Technique

A second method by which the polarization of the incident beam can be measured is to make use of the quenching process described in Sec. III-A. If the beam current is measured before and during the quenching process, then the absolute beam polarization can be determined. If the beam current before quenching (which results from both the metastable $2S_{1/2}$ state and the ground state leakage through the argon cell) is I_P and the beam current during the quench cycle (which contains only the ground state leakage) is I_Q the absolute beam polarization is then given in lowest order by

$$|P^b| = \frac{I_P - I_Q}{I_P} = 1 - \frac{I_Q}{I_P} . \quad (IV-7)$$

The (extremely small) corrections to this relationship are discussed in Ref. 50.

The quench-ratio technique gives only the magnitude of the beam polarization and not its direction. It should also be pointed out that this technique determines the magnitude of the beam polarization immediately after the argon cell in the source, independent of where the beam current is being monitored. Thus it is useful in providing a check on the beam polarization that is obtained from the beam-line polarimeter, but care must be taken if

it is to be used as the only measure of the incident-beam polarization.

IV-C-2. Focal-Plane Polarization

The asymmetry method described in Sec. IV-C-1-a is used in a modified form to determine the focal-plane polarizations. The event rate is necessarily so low at the focal plane that it is desirable to use all the information that is available to determine the polarization components and not merely scatterings that happen to occur within a small predetermined angular region, as is done with the beam-line polarimeter. Moreover, the events of interest are not necessarily localized on the focal plane, which makes the use of a polarimeter similar to the beam-line polarimeter impractical.

In Sec. IV-C-3 it will be shown that to extract the scattered-beam polarization components at the target in the $\hat{n}$ and $\hat{q}$ directions it is vital to measure the focal-plane polarizations when the beam is polarized in both normal and reverse directions. In order to be able to do this an acceptance test must be placed on those particles which are to be used to calculate the focal-plane-polarization components. This can be demonstrated by referring to Fig. IV-4. The box in this figure represents the edges of one of the rear chambers (C5-8). The trajectory of a particle as determined by the front wire chambers (C1-4) is calculated and the intersection of this trajectory with the rear

Fig. IV-4

Actual and Projected Positions of an Event in a Rear Chamber.

chamber is the point designated as the "unscattered trajectory." The detected position of the event in the rear chamber is also indicated in Fig. IV-4. One can see that, for events with this trajectory, an asymmetry in the azimuthal distribution will develop because the edge of the chamber limits the acceptance for some values of the azimuthal angle ϕ (for a given θ). Thus one can see that, in order to eliminate this false asymmetry, the minimum sufficient acceptance test is one requiring, for an event that

scatters through (θ, ϕ) in the FPP, that an event at $(\theta, \phi + \pi)$ would also be within the detector acceptance. This acceptance check is called the " $\phi + \pi$ test".⁽³¹⁾ Also useful is a more restrictive test requiring, for an event at (θ, ϕ) , that any choice of ϕ for the given θ would lie within the detector acceptance. This latter test is called the "cone test".⁽³¹⁾ The acceptance or rejection by these two tests of events with different positions and carbon scattering angles is shown in Fig. IV-5.⁽⁴⁰⁾ The cross-hatched region in this figure indicates the small-scattering-angle region (in the carbon polarimeter) in which events are rejected by the MBD (Sec. III-D-3).

The relationship between the angular distribution of protons scattered by the carbon analyzer and the initial polarization is given by Eq. II-23. If the events of interest are limited to those that pass a certain acceptance test $T(\theta, \phi)$, then the observed distribution $I(\theta, \phi)$ is of the form

$$I(\theta, \phi) = I_0(\theta) [1 + (p_n^{fp} \cos \phi - p_s^{fp} \sin \phi) A_y] T(\theta, \phi) \quad ,$$

where I_0 is the distribution that would result for unpolarized protons incident upon the carbon analyzer. A_y in this equation is the (inclusive) analyzing power of the carbon. Since the extraction of the polarization components is algebraically easiest when the cone test is used, it will be discussed first.

CONE-TEST

ACCEPTED

REJECTED

$\Phi + \pi$ - TEST

ACCEPTED

REJECTED

Fig. IV-5

Event Acceptance and Rejection by the Cone and Phi + Pi Tests.

IV-C-2-ε. Cone Test

The cone test has no azimuthal dependence; its dependence on the polar angle is determined by the physical acceptance of the detector system. Thus

$$I(\theta, \phi) = I_0(\theta)T(\theta)[1 + (p_n^{fp}\cos\phi - p_s^{fp}\sin\phi)A_y(\theta)] \quad (IV-8)$$

Since the focal-plane-polarization components p_n^{fp} and p_s^{fp} are the desired quantities, the form of Eq. IV-8 suggests Fourier analysis as a means to extract them. Proceeding accordingly yields

$$p_n^{fp}(\theta) = \frac{2}{A_y(\theta)} \frac{\int_0^{2\pi} I(\theta, \phi) \cos\phi \, d\phi}{\int_0^{2\pi} I(\theta, \phi) \, d\phi}, \text{ and} \quad (IV-9)$$

$$p_s^{fp}(\theta) = \frac{-2}{A_y(\theta)} \frac{\int_0^{2\pi} I(\theta, \phi) \sin\phi \, d\phi}{\int_0^{2\pi} I(\theta, \phi) \, d\phi}. \quad (IV-10)$$

Experimentally, these integrals are approximated by sums over a finite number of events for given intervals of θ . Typically the full range of θ in the polarimeter acceptance (20°) is subdivided into 1° bins. The ratio

$$\frac{\int_0^{2\pi} I(\theta, \phi) \begin{Bmatrix} \cos\phi \\ \sin\phi \end{Bmatrix} d\phi}{\int_0^{2\pi} I(\theta, \phi) d\phi}$$

is simply the average value of $\begin{Bmatrix} \cos\phi \\ \sin\phi \end{Bmatrix}$ for a given θ bin. Thus, for the i^{th} θ bin,

$$p_n^{\text{fp}}(\theta_i) \approx \frac{2}{A_y(\theta_i)} \langle \cos\phi \rangle_i ,$$

and

$$p_s^{\text{fp}}(\theta_i) \approx \frac{-2}{A_y(\theta_i)} \langle \sin\phi \rangle_i , \quad (\text{IV-11})$$

where $\langle \rangle_i$ indicates an average over all events in the i^{th} θ bin that pass the acceptance test. This form allows for the calculation of p_n^{fp} and p_s^{fp} for each of the θ bins. The polarizations are independent of the focal-plane scattering angle, however, so the focal-plane polarization can be determined from a weighted average over the whole range of θ . The weighting factor for each bin, w_i , is proportional to the square of the reciprocal of the estimated error for that bin. The estimated error "s" in the average value of $\cos\phi$ and $\sin\phi$ measured under these conditions is given by⁽³¹⁾

$$s = \frac{1}{\sqrt{N}} .$$

The angle-dependent error in $p_n^{fp}(\theta_i)$ and $p_s^{fp}(\theta_i)$ is then proportional to

$$\frac{1}{A_y(\theta_i)\sqrt{N_i}} ,$$

where N_i is the number of events occurring in the i^{th} θ bin. The weighting factor for each bin is thus $w_i = N_i A_y^2(\theta_i)$. Employing the usual formula to calculate the weighted average, ⁽³²⁾

$$\langle x \rangle = \frac{\sum w_i x_i}{\sum w_i} ,$$

one obtains the following expressions for the polarization components averaged over all of the θ bins:

$$p_n^{fp} = \frac{2 \sum_{ev} A_y \cos \phi}{\sum_{ev} A_y^2} \quad p_s^{fp} = \frac{-2 \sum_{ev} A_y \sin \phi}{\sum_{ev} A_y^2} , \quad (\text{IV-12})$$

where the sums are now over all events that pass the acceptance test.

IV-C-2-b. $\Phi + \pi$ Test

The $\phi + \pi$ acceptance test is of the form

$$T(\theta, \phi) = T(\theta, \phi + \pi) \quad .$$

Extracting the polarization components as a function of θ is now more difficult because of the presence of the azimuthal dependence of the acceptance test. It is not possible to evaluate any of the Fourier integrals analytically. One must approximate these integrals by sums over events that pass the acceptance test for each polarimeter scattering angle region. Doing this yields

$$\begin{aligned} (\Sigma \cos \phi)_i &= p_n^{fp}(\theta_i) A_y(\theta_i) N_i (\Sigma \cos^2 \phi)_i - p_s^{fp}(\theta_i) A_y(\theta_i) N_i (\Sigma \cos \phi \sin \phi)_i \\ (\Sigma \sin \phi)_i &= p_n^{fp}(\theta_i) A_y(\theta_i) N_i (\Sigma \cos \phi \sin \phi)_i - p_s^{fp}(\theta_i) A_y(\theta_i) N_i (\Sigma \sin^2 \phi)_i \end{aligned}$$

or, in matrix notation,

$$\frac{1}{A_y(\theta_i) N_i} \begin{pmatrix} (\Sigma \cos \phi)_i \\ (\Sigma \sin \phi)_i \end{pmatrix} = \begin{pmatrix} (\Sigma \cos^2 \phi)_i & (-\Sigma \cos \phi \sin \phi)_i \\ (\Sigma \cos \phi \sin \phi)_i & (-\Sigma \sin^2 \phi)_i \end{pmatrix} \begin{pmatrix} p_n^{fp}(\theta_i) \\ p_s^{fp}(\theta_i) \end{pmatrix}$$

which can be written symbolically as

$$\vec{B}_i = F_i \vec{p}^{fp}(\theta_i) \quad .$$

If statistical fluctuations in F_i are ignored and the errors in the

averages are treated as before, then the angle-dependent weighting factor for $\vec{p}^{fp}(\theta_i)$ is found to be proportional to $A_y^2(\theta_i)N_iF$.⁽³¹⁾

Thus

$$\begin{aligned}\vec{p}^{fp} &= \left(\sum_i w_i\right)^{-1} \sum_i w_i \vec{p}^{fp}(\theta_i) \\ &= \left[\sum_i A_y^2(\theta_i)N_iF_i\right]^{-1} \sum_i A_y^2(\theta_i)N_iF_i (F_i^{-1}\vec{B}_i) \quad .\end{aligned}$$

Simplifying and substituting in explicit forms gives

$$\begin{pmatrix} \sum_{ev} A_y \cos\phi \\ \sum_{ev} A_y \sin\phi \end{pmatrix} = \begin{pmatrix} \sum_{ev} A_y^2 \cos^2\phi & -\sum_{ev} A_y^2 \cos\phi \sin\phi \\ \sum_{ev} A_y^2 \cos\phi \sin\phi & -\sum_{ev} A_y^2 \sin^2\phi \end{pmatrix} \begin{pmatrix} p_n^{fp} \\ p_s^{fp} \end{pmatrix} \quad (IV-13)$$

where, again, the sums are over all events that pass the acceptance test. After these sums are determined experimentally, a matrix inversion will determine the focal-plane-polarization components.

It should be noted that, for FPP events originating from the center of the focal plane, the same events will pass the cone and $\phi+\pi$ tests. For FPP events originating near the edge of the focal plane more events will pass the $\phi+\pi$ test than will pass the cone test and thus the statistical uncertainty should be smaller for the former. The fact that more integral approximations are involved in the $\phi+\pi$ test than in the cone test should make the latter the preferred result in cases where the statistical errors

are of the same size, although the results obtained using the two acceptance tests should be comparable. For this experiment the cone test polarizations were used to determine the spin-rotation parameters. The difference between these polarizations and those obtained using the $\phi+\pi$ test were less than 0.02 for all data.

IV-C-3. Extraction of the First-Order Observables

The extraction of the spin-rotation observables from the focal-plane polarizations is vastly simplified by considering only in-plane scattering. This approximation becomes increasingly valid as the spectrometer angle becomes larger. In addition, the analysis is simplified if precession of the outgoing polarization vector is assumed to take place only in the magnetic dipole fields of the HRS. The procedure for extracting the observables subject to these conditions is the subject of this section. The corrections to these first-order calculations that result from eliminating the above mentioned approximations are presented in the following section.

The scattered-beam polarization components ($p_s^{tgt}, p_n^{tgt}, p_q^{tgt}$) at the target measured in the $(\hat{s}', \hat{n}', \hat{l}')$ system are related to the incident-beam components (p_s^b, p_n^b, p_q^b) by the spin-rotation observables as follows:

$$\begin{pmatrix} p_s \\ p_n \\ p_l \end{pmatrix}_{tgt} = \begin{pmatrix} D_{SS} & 0 & D_{LS} \\ 0 & D_{NN} & 0 \\ D_{SL} & 0 & D_{LL} \end{pmatrix} \begin{pmatrix} p_s \\ p_n \\ p_l \end{pmatrix}_b + \begin{pmatrix} 0 \\ P(\theta) \\ 0 \end{pmatrix} . \quad (IV-14)$$

This equation is simply the matrix equivalent of Eq. II-27. The dipole fields of the HRS will precess the polarization vector about the $\hat{s}$ axis by an amount χ , given by⁽³³⁾

$$\chi = \gamma \left(\frac{g}{2} - 1 \right) \omega , \quad (IV-15)$$

where g is the magnetic moment g -factor of the proton (5.586), γ is the Lorentz factor, and ω is the bend angle. The fixed 150° bend angle of the central momentum ray of the HRS reduces this equation to the form

$$\chi \approx 269\gamma .$$

The transformation which takes this precession (from the HRS target to the FPP) into account is given by

$$\begin{pmatrix} p_s \\ p_n \\ p_l \end{pmatrix}_{fp} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\chi & \sin\chi \\ 0 & -\sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} p_s \\ p_n \\ p_l \end{pmatrix}_{tgt} . \quad (IV-16)$$

Combining Eqs. IV-14 and IV-16 for a beam polarized in the normal (+) direction yields

$$\begin{pmatrix} p_s^+ \\ p_n^+ \\ p_l^+ \end{pmatrix}_{fp} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\chi & \sin\chi \\ 0 & -\sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} D_{SS}p_s^b + D_{SL}p_l^b \\ D_{NN}p_n^b + P(\theta) \\ D_{LS}p_s^b + D_{LL}p_l^b \end{pmatrix} .$$

If the beam polarization is changed to the reverse (-) direction (leaving the magnitude constant) the resulting relationship is

$$\begin{pmatrix} p_s^- \\ p_n^- \\ p_l^- \end{pmatrix}_{fp} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\chi & \sin\chi \\ 0 & -\sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} -D_{SS}p_s^b - D_{SL}p_l^b \\ -D_{NN}p_n^b + P(\theta) \\ -D_{LS}p_s^b - D_{LL}p_l^b \end{pmatrix} .$$

If the case of either $\hat{s}$ - or $\hat{l}$ -type incident beam is singled out for consideration, these equations can be recast as

$$\begin{pmatrix} p_S^+ \\ p_n^+ \\ p_Q^+ \end{pmatrix}_{fp} = \begin{pmatrix} \pm p_i^b & 0 & 0 \\ 0 & \cos\chi & \pm p_i^b \sin\chi \\ 0 & -\sin\chi & \pm p_i^b \cos\chi \end{pmatrix} \begin{pmatrix} D_{IS} \\ P(\theta) \\ D_{IL} \end{pmatrix}, \quad (IV-17)$$

where $i(I)=s(S), l(L)$. As mentioned previously, a single determination of focal-plane polarizations with either normal or reverse beam would not allow for decoupling of $P(\theta)$ and D_{IL} since p_Q^{fp} is not a measurable quantity. By adding and subtracting the rows of Eq. IV-17 appropriately and eliminating p_Q^{fp} the following relationship can be obtained:

$$\begin{pmatrix} p_S^+ - p_S^- \\ p_n^- + p_n^- \\ p_n^+ - p_n^- \end{pmatrix}_{fp} = 2 \begin{pmatrix} p_i^b & 0 & 0 \\ 0 & \cos\chi & 0 \\ 0 & 0 & p_i^b \sin\chi \end{pmatrix} \begin{pmatrix} D_{IS} \\ P(\theta) \\ D_{IL} \end{pmatrix} \quad (IV-18)$$

By forming these sums and differences of NORMAL and REVERSE focal-plane polarizations, the parameters $P(\theta)$ and D_{IL} are now decoupled. Thus the first-order results are

$$D_{IS} = \frac{1}{p_i^b} \frac{(p_S^+ - p_S^-)_{fp}}{2}$$

$$D_{IL} = \frac{1}{p_i^b \sin \chi} \frac{(p_n^+ - p_n^-) f_p}{2}$$

$$P(\theta) = \frac{1}{\cos \chi} \frac{(p_n^+ + p_n^-) f_p}{2} \quad (\text{IV-19})$$

with $i(I)=s(S), l(L)$. These results are clearly valid for a beam whose incident polarization is along any given axis. Thus the determination of the quantities D_{US} , D_{UL} , D_{VS} , and D_{VL} (with $\hat{u}$ and $\hat{v}$ defined as in Fig. II-5) is evident. Equation IV-19 shows that any instrumental asymmetry present in the measurement of p_s^{fp} or p_n^{fp} will cancel out (in first order) when the spin-rotation parameters are determined since it will add to both the normal and reverse polarization results. Such an asymmetry will enter into in the determination of $P(\theta)$, however.

IV-C-4. Corrections to the Observables

When HRS trajectories other than the central trajectory are considered, the transformation matrix between the target and focal-plane polarizations (Eq. IV-16) is greatly complicated. Care must be taken to calculate the precession of the polarization vector caused by the spectrometer quadrupole as well as the fringe fields of the HRS dipoles in order to make a high accuracy determination of the spin-rotation observables. This is done in some detail in Ref. 51. Section 5 of this reference gives a

procedure by which the process of evaluating the complicated transformation is avoided by considering the effects of deviations from the central trajectory in the vertical and horizontal directions separately and then combining the two effects sequentially. This procedure is empirically valid for all but the smallest spectrometer angles. Care must be taken, however, at very small spectrometer angles where the out-of-plane corrections can be large. This prescription greatly simplifies the analysis. The results of considering just the out-of-plane scattering are derived in Ref. 23 and given below (the out-of-plane scattering angle is labeled η as in Fig. II-2):

$$D_{SS} = \frac{(p_S^+ - p_S^-) f_p}{2p_S^b \langle \cos^2 \eta \rangle} (1 - p_{SA_y}^b \langle \sin \eta \rangle) - \frac{D_{NN} \langle \sin^2 \eta \rangle}{\langle \cos^2 \eta \rangle} + \frac{P(\theta) \langle \sin \eta \rangle}{p_S^b \langle \cos^2 \eta \rangle}$$

$$D_{SL} = \frac{(p_n^+ - p_n^-) f_p}{2p_S^b \langle \cosh \eta \rangle \sin \chi} (1 - p_{SA_y}^b \langle \sin \eta \rangle)$$

$$P(\theta) \Big|_S = \frac{(p_n^+ + p_n^-) f_p}{2 \langle \cosh \eta \rangle \cos \chi} (1 - p_{SA_y}^b \langle \sin \eta \rangle) - p_S^b \langle \sin \eta \rangle (D_{SS} - D_{NN})$$

$$D_{LS} = \frac{(p_S^+ - p_S^-) f_p}{2p_S^b \langle \cosh \eta \rangle}$$

$$D_{LL} = \frac{(P_n^+ - P_n^-) f_p}{2p_0^b \sin X} - D_{LS} \langle \sin \eta \rangle \cot X$$

$$P(\theta) \Big|_{\hat{q}} = \frac{(P_n^+ + P_n^-) f_p}{2 \cos X \langle \cosh \eta \rangle} .$$

The usual procedure for dealing with the averages over combinations of trigonometric functions in these equations is simply to accumulate the quantities $\langle \cosh \eta \rangle$ and $\langle \sin \eta \rangle$ and to make the following approximations for the averages of the squares:

$$\langle \cos^2 \eta \rangle = \langle \cosh \eta \rangle^2$$

$$\langle \sin^2 \eta \rangle = 1 - \langle \cosh \eta \rangle^2 .$$

In practice, $\langle \sin \eta \rangle$ is very small except when an acceptance test has been placed on the data which artificially skews it. In the absence of such a test, those correction terms which depend on $\langle \sin \eta \rangle$ are insignificant and can be neglected.

The corrections due to deviations from the central trajectory in the spectrometer horizontal midplane are straightforward. They give rise to differential changes as a function of horizontal scattering angle within the acceptance of the spectrometer. The illustration of this result is given in Sec. 6 of Ref. 51 with preliminary data from this experiment.

A summary of the equations used in the final analysis of the data taken during this experiment is presented below:

$$D_{SS}^1 = \frac{(p_s^+ - p_s^-) f_p}{2 p_s^b \langle \cosh \eta \rangle^2} - \frac{1 - \langle \cosh \eta \rangle^2}{\langle \cosh \eta \rangle^2}$$

$$D_{SL}^1 = \frac{(p_n^+ - p_n^-) f_p}{2 p_s^b \langle \cosh \eta \rangle \sin \chi}$$

$$P(\theta) |_{\hat{s}, \hat{l}} = \frac{(p_n^+ + p_n^-) f_p}{2 \langle \cosh \eta \rangle \cos \chi}$$

$$D_{LS}^1 = \frac{(p_s^+ - p_s^-) f_p}{2 p_l^b \langle \cosh \eta \rangle}$$

$$D_{LL}^1 = \frac{(p_n^+ - p_n^-) f_p}{2 p_l^b \sin \chi}$$

$$D_{LS} = D_{LS}^1 - \phi(1 - 2.3\gamma + .2\gamma \cos \chi) D_{LL}^1$$

$$D_{SS} = D_{SS}^1 - \phi(1 - 2.3\gamma + .2\gamma \cos \chi) D_{SL}^1$$

$$D_{LL} = D_{LL}^1 - \phi(2.3\gamma - 1) D_{LS}^1$$

$$D_{SL} = D_{SL}^1 - \phi(2.3\gamma - 1) D_{SS}^1$$

(IV-20)

V. RESULTS

This chapter discusses the data taken during this experiment. The determination of the beam polarization is discussed in the first section. The next section is a general discussion of the common ways in which false asymmetries can arise in HRS focal-plane-polarimeter experiments. The third section describes the results of various internal and external consistency checks that were made. In the next section the systematic uncertainties in the data are discussed. The final section presents the data from this experiment along with previously measured cross-section and analyzing-power data for the nuclei being studied. Results from both relativistic and nonrelativistic impulse approximation calculations are compared to the data set.

V-A. Incident Beam Polarization

The ^{40}Ca data for this experiment were obtained immediately after the quench cycle was introduced at the LAMPF accelerator. For this run the Line C polarimeter and the quench-ratio technique gave quite different results for the beam polarization. There were, however, independent reasons to believe that the fields in the source were not set correctly during the quench cycle,⁽³⁴⁾ which led to inaccurate quench-ratio polarization results. Since this condition existed during the entire course of this run and was

stable throughout, the quench-ratio results might be expected to differ from the polarimeter results by a consistent amount. In fact, during the part of the run taken with $\hat{s}$ -type incident beam, the quench-ratio results were uniformly 10% high relative to the polarimeter results. It was assumed that the same ratio held true when the beam polarization was rotated to be $\hat{l}$ -type at the HRS. Independent measurement of the beam polarization with a polarimeter in the EPB beam line (where the $\hat{l}$ -type beam was precessed to $\hat{s}$ -type) supported this assumption. Thus for $\hat{l}$ -type incident beam, the polarization used to normalize the data was the quench-ratio result multiplied by the ratio of the average Line C polarimeter to average quench-ratio $\hat{s}$ -type result. The average beam polarization for $\hat{s}$ -type incident beam was 0.752 as determined by the Line C polarimeter and was calculated to be 0.834 using the quench-ratio technique (with the number of focal-plane events being used to determine the relative beam current). These results lead to a multiplication factor of 0.902 for scaling the quench-ratio results. Using this multiplication factor gives rise to an average $\hat{l}$ -type beam polarization of 0.745. This result is in basic agreement with the average polarization measured in the EPB beam line of 0.759. [Differences of this magnitude between the beam polarization in EPB and HRS are not unusual. This difference is attributed to variations in the beam polarization across the profile of the beam in Line X. As the stripper for Line C is moved across the beam, variations in the beam polarization in Line C of

0.10 and larger have been observed.] Further support for the accuracy of this correction is demonstrated in Figs. V-3 to V-6 where spin-rotation parameters (and their sums and differences) from different beam polarization types are plotted. Since these observables are inversely proportional to the beam polarization, the fact that the relative normalization of the two data sets appear consistent in Figs. V-3,5 (or that the sums and differences do not deviate from zero in a consistent manner in both Figs. V-4,6) is evidence that this relative 10% correction to the quench-ratio polarization for $\hat{q}$ -type incident beam is reasonable. This is only a test of the relative normalization of the data from two incident-beam polarization directions, however; the absolute normalization of the data is discussed later.

By the time the ^{16}O data were taken the source problem mentioned above had been corrected. For the portion of this run with $\hat{s}$ -type incident beam the disagreement between polarimeter and quench-ratio results was less than 1.5% (average polarizations of 0.841 and 0.851 for polarimeter and quench-ratio results respectively). A discrepancy of this magnitude is not uncommon and it is difficult to determine its exact cause in any particular instance; polarimeter miscalibration, depolarization between the source and Line C, and a small $\hat{q}$ polarization component can all give rise to this effect. Again the quench-ratio polarizations were renormalized to produce the polarization used in the determination of the $\hat{q}$ -type incident-beam spin-rotation parameters.

In this case the multiplication factor resulting from the above averages was 0.987. This process produces an average $\hat{l}$ -type beam polarization of 0.841 (identical to the $\hat{s}$ -type average polarization). No comparison is possible with the EPB polarimeter in this case because the polarization was being precessed to $\hat{l}$ in the EPB beam line throughout this run. Again the normalization of the spin-rotation parameters (Figs. V-3 to V-6) indicate the consistency of this scaling process in this instance.

The ^{208}Pb data were obtained with the incident beam polarized in the $\hat{u}$ and $\hat{v}$ directions where $\hat{u}$ and $\hat{v}$ are defined in Fig. II-5 with $\alpha = 39^\circ$. Since the component of the polarization along the $\hat{s}$ -axis is significant in both cases the magnitude of the polarization can be extracted directly from the Line C polarimeter results. Thus

$$p_u^b = \frac{p_s^b}{\cos(39^\circ)}$$

$$p_v^b = \frac{p_s^b}{\sin(39^\circ)} .$$

The average magnitudes of the polarization in the $\hat{u}$ and $\hat{v}$ directions are 0.663 and 0.727 respectively. It can be noted that the polarization for this part of the experiment is moderately worse than for the other two parts. The reason for this lies in

the fact that the data for this part of the experiment were taken with the beam in the rapid reversal mode. Because of the frequency of the beam-polarization sign change (30 Hz) the magnetic fields in the switching magnet were imperfectly shaped, giving rise to some additional depolarization.⁽³⁵⁾

It should be noted that, although average beam polarizations were used in the above discussions, average polarizations were not used in the calculations of the observables. Beam polarizations used in the calculation of the spin-rotation parameters for $\hat{l}$ -type incident beam were the polarizations as determined at the time the data were taken and scaled as discussed above.

V-B. Sources of FPP False Asymmetries

Before discussing focal-plane data from this experiment and the probable causes for the specific false polarizations that are evident in the induced polarization data, it is worthwhile to discuss the various sources of false polarizations that enter into focal-plane polarization measurements at the HRS. A list of the common causes that lead to the largest false polarizations is presented below. Each is then briefly explained.

1. Internal chamber miscalibration
2. Misalignment (possibly local) of rear chambers with respect to front chambers
3. Asymmetric particle identification tests
4. Local chamber inefficiencies
5. MBD cut infringing on "good" data

Internal chamber miscalibration can cause significant false asymmetries if misassignment of wires is taking place. This occurs when the calibration of time difference (between the two ends of the delay line) to wire number (as described in Sec. III-D-2) goes awry. This can be caused by significant differential nonlinearities in the chamber delay-line or in the TDC or by nonuniform chamber wire spacing. It is most often observed for wires near the edges of a chamber. This possibility can be investigated by examining the assignment of crude position (as determined from the time differences) to wire number (the "truncated" position). (The wires are assumed to be separated by exactly 0.8 mm so position is linearly related to wire number.) Differential TDC or delay line nonlinearities can be corrected for by recalibrating the crude to truncated position assignment. Nonuniform wire spacing cannot be corrected for (unless the true wire spacing is independently determined) and remains as an intrinsic false asymmetry of the system.

Miscalibration of the drift time to drift distance conversion is not likely to cause significant false asymmetries since the same inaccurate position assignment (relative to a wire) is made for events that occur at a given distance from the wire on both sides. For example, consider a drift calibration such that events that occur at a given distance from a wire are assigned a position that is too close to the wire. This misassignment occurs for events on both sides of the wire, giving rise to some angle calculations that are larger than the true polarimeter scattering angle and an equal number that are too small by roughly the same amount (assuming a uniform distribution of events about the wire). This miscalibration will then cancel out of a determination of the focal-plane-polarization components (assuming a reasonably flat carbon analyzing power as a function of angle).

Horizontal or vertical misalignment of the rear chambers with respect to the front chambers will clearly cause a false asymmetry since the polarimeter scattering angles will be calculated to be uniformly too large in the direction of the offset and uniformly too small in the opposite direction. The alignment of the front and rear chambers has traditionally been made by using "straight through" (no carbon analyzer) quasi-elastic data to determine the positions of the rear chambers relative to the front. If an offset is present it can easily be corrected for by adjusting software offsets. Quasi-elastic data are usually used for this calibration because they uniformly illuminate the focal plane of

the HRS. The alignment that is achieved is then a global one -- an average over all wires. Thus local misalignments (caused by nonuniform wire spacings) are averaged over by this process. FPP data on a discrete nuclear state involves a small sampling of the front chamber wires (especially for heavy targets and small spectrometer angles) and a somewhat larger sampling of wires on the rear chambers. Thus local misalignments in the region of the chamber detecting events from a discrete nuclear state will give rise to false asymmetries in the data. Such a misalignment can be monitored by checking the events untested by the MBD for the state of interest. These events are primarily multiple coulomb scatterings and should display no asymmetry in their azimuthal distribution. Assuming a good global alignment, any calculated asymmetry for this distribution is due to a local misalignment (or to local chamber inefficiencies, as discussed below). This misalignment can be corrected locally by adjusting the wire position software offsets for the rear chambers to produce a zero asymmetry. Local misalignments are clearly much less important for continuum states which illuminate large portions of the focal plane. It should also be noted that recent studies⁽⁴⁸⁾ indicate that the rear chambers are rotated slightly (about $\hat{q}$) with respect to the front chambers. This rotation can be determined by observing the pattern of local misalignments determined separately for small regions of the focal plane.

The acceptance or rejection of events by the particle identification test (described in Sec. IV-B) can cause false asymmetries to occur. This is the case because the particle identification test for focal-plane polarimeter experiments is dependent on the time-of-flight from S1 to either SP1 or SP2. Placing a tighter test on the events firing one of these scintillators than on the other will generate a false asymmetry in the $\hat{n}$ -direction. This arises because SP1 and SP2 are side by side so a more loose acceptance test for particles scattering to one side produces a false asymmetry with a bias in that direction. Such an asymmetry can also arise, of course, if one SP scintillator is inefficient relative to the other.

It is clear that a lower efficiency for detection of events in one region of the rear chambers will give rise to an artificially skewed azimuthal distribution in this chamber and thus a false asymmetry. The signature of this in the data is similar to that produced by local chamber misalignments. Local inefficiencies can be partially corrected for by adjusting the software position offsets of the inefficient chamber, as described previously, to produce zero asymmetry for the multiple-coulomb-scattered events.

It is recalled that the MBD filters data before it is written to tape. It is used to reject events that multiple-coulomb scatter in the carbon analyzer in order to increase the fraction of useful (nuclear scattered) events on a data tape. Since the MBD's calculational abilities are limited, it does not determine a

precise angle inside of which events should be rejected. It instead makes a crude angle calculation and accepts or rejects events based on this calculation. The diffuseness of this cut on carbon scattering angle is on the order of 1° ; that is, some of the events in this 1° region of θ_{carbon} are accepted by the MBD and some are rejected. Events that occur in this region and are passed by the MBD must be eliminated from the analysis, usually by a test in the test package input file. One must make certain that the region of θ_{carbon} eliminated by this test is larger than the "fuzzy" region of the MBD cut. If not, the selective passing of events by the MBD may introduce a false asymmetry into the data.

V-C. Consistency Checks

This experiment was run with the incident beam polarized in two orthogonal directions in order to be able to perform a number of independent checks on the data to assure that it is both internally and externally consistent. The basis for these checks is discussed in Sec. IV-C and the results can be summarized as follows:

1. $A_Y(\theta) = P(\theta)$
2. $D_{LL} = D_{SS}$ (or $D_{VL} = D_{US}$)
3. $D_{LS} = -D_{SL}$ (or $D_{VS} = -D_{UL}$)
4. $Q^2 + S^2 + P^2 = 1$.

It should be recalled from the discussion of the first-order calculation that the measured induced polarization will be greatly affected by any false asymmetry in the focal-plane polarimeter. This effect will cause deviations from the equality of relation 1. Plotted in Fig. V-1 are the induced polarizations, determined in this experiment, and previously measured analyzing power data.^(41,46) One notes good agreement between the two sets of data in structure and phase but some discrepancies in magnitude that are presumably due to false asymmetries. These discrepancies are better displayed in Fig. V-2 where the quantity $P-A_y$ is shown. The analyzing power data were linearly interpolated to give results at the angles which correspond to those where data were obtained in this experiment.

These false polarizations have been investigated in some depth although their effects on the determination of the spin-rotation parameter Q will be shown to be small. It should first be noted that a false polarization in $P(\theta)$ of magnitude 0.10 (typical in Fig. V-2) corresponds to a false asymmetry of less than 0.02. This is due to the fact that asymmetries in $P(\theta)$ are divided by a factor of $A_y^{\text{carbon}} \cos \chi$ (Eqs. IV-11 and IV-20). At 800 MeV the event-averaged (rms), inclusive, carbon analyzing power is only 0.23 and the precession angle χ of the scattered beam in the HRS is 498° . Thus any false asymmetry enters the $P(\theta)$ data as a false polarization with an enhancement factor of approximately 5.9.

Fig. V-1

P and A_y for ^{16}O , ^{40}Ca , and ^{208}Pb .

Fig. V-2

$P - A_y$ for ^{16}O , ^{40}Ca , and ^{208}Pb .

The ^{16}O $P-A_y$ data show a large deviation from zero in exactly the region where the A_y data are changing in magnitude extremely rapidly (10.5° - 12°). In this region very small angle discrepancies between the A_y data and the $P(\theta)$ data will result in large deviations from zero. Errors in interpolating the analyzing power data in these regions of rapidly varying structure will also contribute to deviations from zero. If one ignores the data in this region for the moment it is seen that the ^{16}O data show a small (average = 0.024) deviation from zero out to an angle of 16° . At 16° the deviation changes sign and increases to an average magnitude of 0.057. This peculiar behavior is evident when examining this relationship using the value for $P(\theta)$ that is extracted separately for both incident-beam polarization types. This could be caused by a local chamber inefficiency that is passed over as the scattering angle increases and scattered-beam energy decreases. (Data for this experiment were taken with a constant HRS magnetic field. Thus, as the spectrometer angle changed so did the position of the elastically scattered protons on the focal-plane chambers.) A more probable explanation is evident upon closer examination of the ^{16}O A_y data. These data appear to become discontinuous at an angle of 16° . There seems to be a vertical displacement of the large angle ($>16^\circ$) data relative to the small angle data of roughly 0.05-0.10. This is in agreement with the discontinuity evident in Fig. V-2. Thus the large angle deviations from the small angle results for $P-A_y$ are ascribed to the A_y data.

The small average deviations from zero evident at smaller angles are likely caused by very small chamber misalignments. Extrapolation from the results of studies done on the ^{208}Pb data (to be discussed next) would indicate a misalignment in this region of <0.1 mm. It is not possible to detect a misalignment this small at a statistically significant level in the calibration data (data untested by the MBD) that were taken.

The ^{208}Pb $P-A_y$ plot shows a systematic deviation from zero with an average deviation of 0.091. This has been found to be caused by equal parts global and local chamber misalignment. The magnitude of the total misalignment is roughly 0.4 mm. Correction of this misalignment at a random selection of points brought the $P(\theta)$ data into agreement with A_y to within statistics. The effects of such misalignments on Q are small and will be quantitatively discussed later.

The discrepancies between $P(\theta)$ and A_y for ^{40}Ca are due to the MBD cut on the multiple-coulomb-scattered events. Careful analysis of the data indicates that the fuzzy region of the MBD cut infringes slightly on the data that were used to extract $P(\theta)$. This is a result of taking data with an MBD cut that was not centered properly on θ_{carbon} . To verify this interpretation the region of θ_{carbon} excluded was made large enough to completely encompass the MBD cut. This brought the $P(\theta)$ data so tested into agreement with A_y to within statistics. The results of this test for the extracted Q data were consistent with zero given the

difference in statistical error that results from excluding part of the data.

It must be emphasized that the false polarizations evident in Figs. V-1,2 do not enter in to the calculation of Q in first order. False asymmetries of the magnitude of those evident in Fig. V-2 were purposefully introduced into the data in the testing phase of the analysis to determine their effects on the spin-rotation observables. It was found that a chamber misalignment of 0.3 to 0.5 mm (an amount which causes a bias in $P(\theta)$ of ~ 0.10) introduces a bias into Q of ~ 0.005 . This bias is quite small compared to the statistical and systematic errors of the experiment. A false polarization in $P(\theta)$ of 0.10 is the magnitude of those evident in Fig. V-2 although there are larger false polarizations in some places. To be conservative then, the bias introduced into the data due to these false asymmetries will be taken as 0.01.

Figures V-3 to V-6 illustrate the consistency checks 2 and 3. In general the data agree with the relationships required by parity conservation reasonably well (Fig. V-3,5). Deviations from these relationships are better displayed in Figs. V-4,6. There does not seem to be any consistent bias evident in these figures to indicate a relative normalization problem for the parameters measured separately with orthogonal incident-beam polarizations.

Fig. V-3

Off-Diagonal Spin-Rotation Parameters.

Fig. V-4

Sums of the Off-Diagonal Spin-Rotation Parameters.

Fig. V-5

Diagonal Spin-Rotation Parameters.

Fig. V-6

Differences of the Diagonal Spin-Rotation Parameters.

The results of employing the fourth consistency relationship directly are corrupted by the false asymmetry present in the induced polarization. To use the relation as a check on the absolute normalization of the spin-rotation parameters it is preferable to use A_y rather than $P(\theta)$. In addition the ^{40}Ca and ^{208}Pb analyzing power data of Ref. 46 are of a much higher statistical quality with much finer angular binning than the $P(\theta)$ data from this experiment. The results of this check are displayed in Fig. V-7. This consistency check is sensitive to the systematic errors in this experiment, as well as those present in the analyzing power measurement, and the average deviation of the plotted quantity from unity is a direct measure of net systematic error of the two (A_y and Q) measurements. It should be noted that the largest deviations from unity occur in exactly the places where the analyzing power is changing extremely rapidly. Again very small angle discrepancies or interpolation errors will cause large deviations from unity. In addition, the large angle ^{16}O A_y data appear to be discontinuous, as noted earlier. This certainly contributes to the large angle deviations from unity evident in Fig. V-2 for ^{16}O .

The average deviations from unity for this expression are -0.042, 0.025 and 0.009 for ^{16}O , ^{40}Ca and ^{208}Pb respectively. In order to estimate conservatively the systematic errors that these deviations imply for the Q data it will arbitrarily be assumed that the systematic error in the A_y data is zero. The entire apparent

Fig. V-7

Illustration of Consistency Check 4.

systematic error is then divided between Q and S equally since they are measured simultaneously. If one assumes, for the purpose of calculation, that on the average Q, S and A_y all have roughly the same magnitude then the empirically estimated systematic error in the Q data is calculated to be -0.037, 0.022, and 0.007 for ^{16}O , ^{40}Ca and ^{208}Pb respectively.

V-D. Systematics

One source of systematic error in extracting the spin-rotation parameters arises from the uncertainty in the determination of the beam polarization. The effective analyzing power of the polarimeter is believed to be known to ± 0.01 .⁽¹⁵⁾ This analyzing power uncertainty leads to an uncertainty in the spin-rotation data of $\Delta A_y/A_y$ or 0.022.

A systematic error can also be introduced into the calculation of the spin-rotation parameters if the magnitude of the beam polarization changes significantly when switching from normal to reverse. This arises because the normal and reverse focal-plane polarizations are normalized by the geometric mean of the normal and reverse beam polarizations (Sec. IV-C-1-a and IV-C-3). If the beam polarizations are significantly different in normal and reverse the corresponding focal-plane polarizations, in effect, become unequally weighted. The error introduced into the spin-rotation parameters is then proportional to the difference

between the normal and reverse beam polarizations. It is difficult to determine the magnitude of this difference using a beam-line polarimeter since a change between the normal and reverse beam-polarization magnitudes mocks the effects of a geometry factor (Sec. IV-C-1-a). One can use the quench-ratio technique for determining the beam polarization to investigate this possibility. The QUENCH gate used in data acquisition must first be replaced by two gates which indicate whether quenching is taking place while the magnetic fields in the source are aligned to produce normal or reverse beam. These two gates were unfortunately not used at the HRS for any part of this experiment due to limitations in the number of available scalars. Thus it is difficult to estimate the difference between normal and reverse beam polarizations directly from the data. Previous measurements⁽³⁴⁾ of the difference between normal and reverse beam polarization magnitudes puts the typical difference at ± 0.02 . The error in Q generated by this difference can be crudely approximated as the magnitude of the difference.

Another possible source of systematic error is uncertainty in the analyzing power of the focal-plane-polarimeter analyzer. This analyzing power has been measured for incident protons at 800 MeV with statistical errors of ± 0.01 (Fig. V-8).⁽⁴⁹⁾ For this experiment a smooth curve was drawn through the data and the analyzing power used in the analysis was read off this curve. It is important to note that errors made in the assignment of the FPP analyzing power as a function of angle enter into the data only as

Inclusive Carbon Analyzing Power.

Fig. V-8

an overall normalization change (in first order) since the weighted average of the angle-dependent (θ_{carbon}) polarizations is taken (Sec. IV-C-2). The FPP analyzing power is not expected to change a great deal over the small range of scattered-beam energy covered by this experiment (~ 10 MeV). The energy-dependent parameterization of Ransome⁽¹⁴⁾ of the inclusive carbon analyzing power data set can be used to estimate the change in the carbon analyzing power as a function of scattered-beam energy. Shown in Fig. V-8 are the Ransome predictions for scattered-beam energies of 790 and 800 MeV. The magnitude of the deviations between these two curves (~ 0.006) is taken as an approximation of the error introduced into the carbon analyzing power from not using energy dependent results in the analysis. This error is obviously much smaller for ^{208}Pb (where the change in scattered-particle energy was less than 0.5 MeV for the entire experiment) than for ^{16}O (with a corresponding energy change of ~ 10 MeV). Nevertheless the same magnitude will be assumed for all nuclei. The error in the analyzing power extracted from the data is assumed to be ± 0.01 to be consistent with the statistical errors in the data. Adding these errors in quadrature gives a total estimated systematic error of ± 0.011 associated with the carbon analyzing power. The uncertainty in Q that this error gives rise to is again $\Delta A_y/A_y$ or 0.048.

As discussed in the previous section, the contribution of false asymmetries in the FPP to systematic errors in the

determination of the spin-rotation parameters will be conservatively taken as ± 0.01 .

Summing these sources of error gives the result that the total systematic error for the experiment is roughly ± 0.054 . This result is in complete agreement with the systematic errors empirically estimated for the three data sets in Sec. V-C. The largest contribution to this error estimate is from the uncertainty in the magnitude of the carbon analyzing power, which accounts for 90% of the total estimate.

V-E. Data and Theoretical Predictions

Before presenting the data from this experiment, a brief note on the form of the presentation is in order. Since determinations of two non-independent spin-rotation parameters are made (Q and S), one can conceive of combining them in some advantageous way in order to eliminate systematic uncertainties. In particular, taking the ratio Q/S will cause any systematic uncertainties in the data to cancel. This is a particularly interesting possibility when Eq. II-31 is recalled, since this ratio is equal to $\tan\beta$ (where β is the angle between the projections in the scattering plane of the incoming and outgoing polarization vectors). Thus one might desire to discuss the more physically intuitive angle β rather than one of its components (Q or S). Unfortunately the ambiguities and confusion that are

introduced in attempting these alternate forms of presentation outweigh the benefits. In particular, if one wishes to calculate β one must choose either a principle value region or rely on the smoothness of the data to resolve the angular ambiguities of 2π . Choosing a principle value region results in no ambiguities, but when the data approach the limits of the principle value region the data display is discontinuous and confusing. Relying on the smoothness of the data to resolve the 2π ambiguities is not an improvement, however, since real (physically important) ambiguities can arise in the data presentation. There are no such ambiguities inherent in the Q or S data themselves and thus any ambiguities that become apparent are solely a result of the method of display. The display of $\tan\beta$ is out of the question since it is unbounded and infinitely discontinuous. Thus either Q or S is usually chosen to represent the third independent constraint on the scattering amplitude. The more common choice, and the one chosen here, is Q.

The Q data from this experiment along with previously measured 800 MeV cross section and analyzing power data for ^{16}O , ^{40}Ca and ^{208}Pb are displayed in Figs. V-9 to V-14. as well as results from various theoretical calculations to be discussed. The data are presented in tabular form in the Appendix. The curves labeled "KMT" are non-relativistic KMT impulse approximation predictions obtained by L. Ray using Arndt's SP82 NN amplitudes. The proton densities used in these calculations were extracted from

electron scattering results for the charge densities. The neutron densities were taken as

$$\rho_n(r) = \rho_p(r) + [\rho_n(r) - \rho_p(r)]_{\text{HFB}} .$$

The densities in the second term of this relationship are those that result from the nonrelativistic Hartree-Fock-Bogoliubov calculations of Gogny and Decharge!⁽⁴⁷⁾

The curves labeled "RIA" represent RIA predictions of L. Ray using the same NN amplitudes as for the nonrelativistic calculations. The proton vector density is that extracted from electron scattering. The remaining densities were taken as follows:

$$\begin{aligned} \rho_n^V(r) &= \rho_p^V(r) + [\rho_n(r) - \rho_p(r)]_{\text{HFB}} \\ \rho_p^S(r) &= \rho_p^V(r) + [\rho_p^S(r) - \rho_p^V(r)]_{\text{HS}} \\ \rho_n^S(r) &= \rho_n^V(r) + [\rho_n^S(r) - \rho_n^V(r)]_{\text{HS}} . \end{aligned}$$

The Dirac-Hartree theoretical scalar-vector density differences of Horowitz and Serot⁽³⁷⁾ (HS) were used to calculate proton and neutron scalar densities.

The first-order KMT results were obtained using the formalism described in Sec. II-C. The second-order results contain the second-order term in the expansion for the optical potential (Eq.II-48). This term is proportional to the two-body correlations

in the nucleus. The curves labeled "w/MM" contain the contribution due to the full proton magnetic moment as discussed in Sec. II-C. One would expect the effects of this potential to increase with proton number (since it is an interaction with the Coulomb field of the nucleus) and this result is clearly visible. It is also clear that the effects of two-body correlations on the nonrelativistic result decrease with A and are important only for the ^{16}O predictions.

The first-order RIA predictions are as described in Sec. II-D. The curves labeled "w/AMM" include the contribution of the proton anomalous magnetic moment (also described in Sec. II-D). It is expected that the effect of the proton anomalous magnetic moment on the RIA results should be less than the corresponding effect of the full proton magnetic moment in the nonrelativistic case because of the intrinsic inclusion of the normal moment in the Dirac equation. It can be seen that this is indeed the case.

Shown in Figs V-9, V-10, and V-14 are the best available KMT (second order w/MM) and RIA (first order w/AMM) predictions for the ^{16}O , ^{40}Ca , and ^{208}Pb observables. An important point to notice is that Q is much more sensitive to differences in the two approaches at this energy than either the cross section or the analyzing power. In each case the deviation between the two predictions is more significant for Q than for either of the other two observables. The KMT predictions, while displaying the correct structure and phase, generally lie below the Q data for all three

nuclei. For ^{40}Ca and ^{208}Pb it is clear that the RIA results more nearly describe the Q data. The ^{16}O RIA Q prediction has a great deal of structure near the first minimum in the cross section ($\sim 12.5^\circ$ c.m.) that is not present in the data. This structure is insensitive to changes in the proton and neutron matter densities as are the Q predictions generally.⁽³⁾ The RIA Q predictions have also been found to be insensitive to gross (factor of 2) changes in the NN amplitudes. These predictions are expected to be extremely sensitive to the scalar-vector density differences, however.⁽³⁾ One should also recall that ^{16}O is precisely the nucleus in which large second-order effects (correlations) are evident in the nonrelativistic treatment. It is possible that correlations may significantly alter the RIA predictions for this nucleus. The effect of correlations on the KMT prediction (Fig. V-11) for ^{16}O is exactly the sort of correction that needs to be applied to the RIA prediction to produce qualitative agreement with the data. It should also be noted, however, that the quality of the RIA fit to data has been observed to display a noticeable mass and energy dependence.⁽⁴⁵⁾ The RIA predictions are generally poor for heavy targets at low energies (200-300 MeV) and for light targets at higher energies (800-1000 MeV). Whether this behavior at high energies is caused by correlations or if perhaps this overall mass and energy dependence is due to some other inadequacy of the model remains to be demonstrated. For whatever reason, the data from this experiment are in agreement with this observation.

No attempt has been made here to "fit" the data set with either model by adjusting the matter densities or the scalar-vector density differences. The purpose of the theoretical results presented here is to display the parameter-free predictions of the two theories for comparison with the data. The data are not expected to be sensitive to the matter densities and attempts to extract meaningful scalar-density information must await further improvements to the RIA model.⁽³⁾

To conclude, high quality Q data were taken for 800 MeV protons elastically scattering from ^{16}O , ^{40}Ca , and ^{208}Pb . At this energy Q is more sensitive than other observables to the differences between the presently available relativistic and nonrelativistic descriptions of proton-nucleus scattering. These data are then an important addition to the medium-energy data base and will serve to test present and future theories of proton-nucleus scattering.

Fig. V-9

800 MeV Cross Section for ^{16}O , ^{40}Ca , and ^{208}Pb .

Fig. V-10

800 MeV Analyzing Power for ^{16}O , ^{40}Ca , and ^{208}Pb .

Fig. V-11

800 MeV Q for ^{16}O .

Fig. V-12

800 MeV Q Data for ^{40}Ca .

Fig. V-13

800 MeV Q Data for ^{208}Pb .

Fig. V-14

800 Mev Q with the "Best" KMT and RIA Predictions.

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Appendix

800 MeV Elastic Spin-Rotation Data for ^{16}O
(The errors given are only statistical.)

θ_{cm}	Q	ΔQ	S	ΔS
1.11	0.106	0.024	0.980	0.029
1.43	0.109	0.028	0.985	0.033
1.75	0.155	0.031	0.921	0.035
2.07	0.151	0.032	0.963	0.036
2.38	0.097	0.032	0.895	0.037
2.74	0.114	0.020	0.955	0.023
3.06	0.134	0.019	0.951	0.022
3.41	0.110	0.023	0.909	0.027
3.73	0.093	0.024	0.952	0.027
4.05	0.102	0.025	0.881	0.028
4.41	0.173	0.017	0.952	0.019
4.73	0.181	0.017	0.953	0.019
5.08	0.154	0.022	0.926	0.025
5.40	0.174	0.024	0.870	0.027
5.72	0.166	0.026	0.916	0.029
6.07	0.199	0.016	0.949	0.019
6.39	0.235	0.017	0.904	0.020
6.75	0.246	0.022	0.923	0.026
7.07	0.197	0.025	0.917	0.029
7.39	0.242	0.028	0.926	0.031
7.74	0.258	0.015	0.905	0.019
8.06	0.305	0.017	0.882	0.020
8.41	0.308	0.022	0.886	0.027
8.73	0.332	0.026	0.924	0.030
9.05	0.357	0.030	0.885	0.035
9.41	0.408	0.017	0.881	0.019
9.73	0.409	0.020	0.863	0.022
10.08	0.396	0.029	0.834	0.031
10.40	0.479	0.038	0.828	0.040
10.72	0.538	0.052	0.703	0.054
11.07	0.692	0.027	0.692	0.028
11.75	0.776	0.034	0.323	0.030
12.06	0.601	0.035	0.377	0.037
12.38	0.439	0.029	0.491	0.030
12.74	0.434	0.020	0.501	0.021
13.05	0.439	0.018	0.613	0.018
13.41	0.444	0.028	0.607	0.029
13.73	0.435	0.026	0.640	0.027

800 MeV Elastic Spin-Rotation Data for ^{16}O (cont.)

θ_{cm}	Q	ΔQ	S	ΔS
14.04	0.430	0.025	0.667	0.026
14.40	0.479	0.019	0.654	0.019
14.72	0.501	0.019	0.694	0.019
15.07	0.501	0.028	0.700	0.029
15.39	0.509	0.029	0.652	0.029
15.71	0.474	0.030	0.667	0.030
16.06	0.496	0.018	0.710	0.018
16.38	0.517	0.019	0.688	0.019
16.73	0.564	0.025	0.752	0.025
17.05	0.577	0.027	0.704	0.028
17.37	0.545	0.030	0.729	0.031
17.72	0.565	0.017	0.671	0.019
18.04	0.609	0.019	0.660	0.021
18.40	0.634	0.025	0.636	0.027
18.71	0.642	0.029	0.660	0.031
19.03	0.636	0.033	0.603	0.036
19.35	0.671	0.039	0.621	0.042
19.66	0.720	0.046	0.630	0.050
19.97	0.822	0.032	0.589	0.035
20.29	0.845	0.039	0.527	0.041
20.61	0.804	0.046	0.343	0.049
20.92	0.866	0.053	0.339	0.056
21.24	0.775	0.058	0.158	0.061
21.56	0.786	0.061	0.185	0.064
21.87	0.770	0.062	0.045	0.065
22.18	0.755	0.042	0.210	0.045
22.50	0.678	0.041	0.170	0.043
22.82	0.803	0.039	0.266	0.041
23.13	0.696	0.038	0.282	0.040
23.45	0.735	0.038	0.342	0.040
23.77	0.703	0.039	0.306	0.041
24.08	0.684	0.042	0.422	0.043

800 MeV Elastic Spin-Rotation Data for ^{40}Ca
(The errors given are only statistical.)

θ_{cm}	Q	ΔQ	S	ΔS
2.46	0.117	0.030	0.983	0.032
3.06	0.112	0.022	1.004	0.020
3.51	0.058	0.031	0.988	0.029
4.00	0.124	0.014	0.984	0.014
4.59	0.117	0.014	0.992	0.014
5.08	0.148	0.027	1.010	0.027
5.56	0.181	0.013	0.971	0.014
6.01	0.212	0.021	0.965	0.022
6.32	0.310	0.022	0.982	0.019
6.66	0.318	0.032	0.924	0.028
7.13	0.473	0.025	0.935	0.024
7.69	0.641	0.046	0.661	0.042
8.18	0.530	0.037	0.773	0.037
8.68	0.360	0.021	0.825	0.021
9.30	0.334	0.017	0.834	0.020
9.78	0.345	0.033	0.823	0.040
10.27	0.361	0.016	0.834	0.019
10.87	0.381	0.016	0.815	0.020
11.35	0.402	0.032	0.781	0.039
11.83	0.491	0.017	0.808	0.020
12.43	0.533	0.021	0.748	0.025
12.92	0.614	0.049	0.811	0.057
13.40	0.732	0.030	0.715	0.032
14.00	0.910	0.044	0.470	0.046
14.48	0.853	0.078	0.299	0.078
14.97	0.633	0.029	0.449	0.031
15.57	0.558	0.023	0.585	0.025
16.20	0.560	0.028	0.576	0.031
16.80	0.576	0.029	0.608	0.032
17.10	0.578	0.032	0.607	0.035

800 MeV Elastic Spin-Rotation Data for ^{208}Pb
(The errors given are only statistical.)

θ_{cm}	Q	ΔQ	S	ΔS
2.38	0.159	0.012	0.974	0.011
2.95	0.102	0.011	1.012	0.011
3.39	0.075	0.023	1.041	0.024
3.85	0.180	0.017	1.012	0.017
4.43	0.225	0.018	0.988	0.018
4.90	0.248	0.029	0.961	0.030
5.37	0.244	0.012	0.933	0.012
5.95	0.201	0.013	0.941	0.013
6.41	0.283	0.030	0.954	0.030
6.88	0.315	0.013	0.946	0.013
7.46	0.426	0.020	0.862	0.020
7.93	0.390	0.033	0.769	0.032
8.39	0.378	0.014	0.853	0.014
8.97	0.339	0.013	0.864	0.013
9.44	0.362	0.031	0.868	0.030
9.91	0.448	0.013	0.838	0.012
10.49	0.548	0.018	0.809	0.017
10.95	0.539	0.043	0.708	0.038
11.42	0.579	0.022	0.608	0.020
12.00	0.502	0.018	0.674	0.016
12.47	0.483	0.033	0.721	0.030
12.93	0.527	0.013	0.692	0.011
13.51	0.598	0.016	0.707	0.014
13.98	0.609	0.036	0.660	0.031
14.45	0.757	0.022	0.523	0.020
15.02	0.751	0.023	0.496	0.021
15.64	0.572	0.024	0.569	0.024
16.21	0.594	0.024	0.623	0.023
16.65	0.586	0.038	0.708	0.037

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