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A SMALL,
OCTOPOLE-STABILIZED TANDEM MIRROR REACTOR

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A SMALL, OCTOPOLE-STABILIZED TANDEM MIRROR REACTOR

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ABSTRACT

It is shown that the use of octopole stabilization in a tandem mirror allows a large reduction in end-cell length. A novel feature of the method proposed in this report is the placement of the minimum $|B|$ region considerably off axis, thus rendering the core plasma more axisymmetric. The region from the core to the field minimum is bridged by a mirror-confined hot-electron mantle. Low β , ideal magnetohydrodynamic stability, as evaluated with the interchange criterion, yields an upper limit to the required mantle peak β of 78% for a center-cell peak β of 75% and barrier peak β of 30%. Estimates for the worst type of classical radial diffusion - with stochastic displacements per bounce - show that such radial losses are negligible for this configuration. First estimates of power balance indicate $Q \sim 10$ for a reactor producing 500 MW of fusion power and $Q \sim 25$ for one producing 1000 MW, using conservative assumptions regarding mantle β requirements and synchrotron radiation losses.

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MASTER

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1. INTRODUCTION

A design for a tandem mirror reactor (MARS) producing 1200 MW of electricity from 2600 MW of fusion power was recently completed.¹ The MARS design uses C-coil magnets in the yin-yang configuration at each end of the cylindrical center cell to stabilize against magnetohydrodynamic (MHD) instabilities. Constraints on the magnetic field arising from considerations of radial transport of fuel ions, as well as the need to minimize the generation of axial current passing through the center cell, required the use of six C coils in each end cell. As a result, the main end-cell length, L_p , is 21.8 m with an additional 4.5 m needed to recircularize the flux for the direct converter. The long end cell has two disadvantages. First, the loss rate of center-cell (fuel) ions due to trapping in the end cell is proportional to the end-cell length. The 21.8 m length in MARS requires center-cell lengths $L_c > 100$ m for alpha-particle heating to balance the trapping losses, i.e., to obtain ignition of the center cell.² Second, the long MARS end cells result in large end cell magnet costs, which in turn requires large central cell fusion power to achieve reasonable capital cost per kilowatt.

Although subsequent studies have indicated that some reduction could be made in MARS-type end-cell lengths a very large reduction is necessary for ignition in reactors with much smaller fusion power ($P_f < 1000$ MW). We can gain an idea of the necessary reduction in end-cell length over MARS with the assumption that alpha heating and fusion power must be proportional to the ion trapping losses, which in turn scale as the end-cell length. We find that the length should obey the relation $L_p = L_p(\text{MARS}) P_f/P_f(\text{MARS})$. For $P_f = 500$ MW the length should be 4.2 m. But the requirement for adiabaticity of the hot ions in the end cell (see Section V) sets a lower limit on the length of about 7 m. Thus, at 500 MW we would need some reduction in P_f/L_c in addition to L_p compared to MARS (smaller wall loading).

Since a large reduction in the length of the yin-yang end cells does not appear possible, designing an economical reactor with lower fusion power requires alternatives to quadrupole stabilization. A possible stabilization method suggested by Baldwin³ and investigated for the Tandem Mirror Experiment (TMX) by Hooper⁴ uses higher-order multipoles obtained with multiple conductors (Ioffe bars) in planes parallel to the principal axis. Multipoles higher than quadrupole have lower on-axis fields, thus rendering

the core fields less asymmetric. This feature suggests that radial transport and current generation should be less of a problem. As a result of the smaller effect near the axis, the minima in $|B|$ occur off axis and with period $\cos(n\theta)$ where $2n$ is the number of bars. The near-axis multipole field varies as r^{n-1} , so the higher the number of bars, the more axisymmetric the near-axis field will be. There is, however, a limit on the number of bars which can be used because of the need for access for neutral beams, etc. We propose here the use of octopole fields for the stabilization, although hexapoles might also be practical to use. Furthermore, we propose adjusting the fields to place the minimum in magnetic field $|B|$ outside of the flux tubes which link to the center-cell plasma, bridging the region between the core and the minimum- $|B|$ region with a mantle of hot, nearly constant-pressure electrons, which are confined between the plug mirrors.

The concept can be more clearly explained with the aid of Fig. 1, which shows the magnetic field, $|B|$, and electrostatic potential, ϕ , along the principal axis and Fig. 2, which gives a schematic picture of the radial variation of B and plasma pressure, $p = p_{||} + p_{\perp}$, in the center and end cells. In the center cell, the plasma pressure decreases with radius (or magnetic flux $\psi = r^2 B/2$) while B is nearly constant. This core plasma maps to the end cell where the field decreases with radius. The criterion for flute stability of low- β plasmas with the paraxial approximation to the magnetic field, and neglecting finite-Larmor-radius (FLR) effects, is

$$\int_{-L/2}^{L/2} \frac{\kappa}{rB^2} \frac{\partial p}{\partial \psi} ds < 0 \quad (1)$$

where κ is the normal curvature of the vacuum field and the integral runs over the entire length, L , of the machine. In the essentially axisymmetric core plasma, the average $\kappa < 0$ in the plugs and central cell, and the above criterion would not be satisfied if $\partial p / \partial \psi < 0$. To satisfy Eq. (1), we add sufficient plasma pressure to form a positive pressure gradient in the end cell out to the edge of the core, as indicated in Fig. 2. If the gradient is large enough, it will satisfy Eq. (1) and ensure interchange stability. Note that the region of positive pressure gradient extends axially over a limited length of the end cell; the pressure gradient must be large enough to balance out the negative gradients in other portions of each field line in the core.

From the edge of the core to the region of positive curvature, i.e., minimum $|B|$, the gradient can be zero or very small but positive according to Eq. (1).

Satisfaction of the interchange criterion does not guarantee stability to ballooning at high β . However, FLR effects are expected to stabilize all but $m = 1$ ballooning. Calculations for this mode in quadrupole-stabilized tandem mirror designs with close walls have yielded β limits remarkably close to limits found from the interchange criterion.⁵ In addition, FLR should make the $m = 1$ mode rigid, so that Eq. (1) may in fact be pessimistic as a local condition on each field line.

We propose to furnish the additional pressure in the end cell with hot, electron-cyclotron-heated (ECH) electrons. These electrons must be able to respond to MHD instabilities in the center cell and not "rigid" as in ring-stabilized devices. A simple criterion for MHD response of the electrons is $\omega_d < \gamma_{MHD}$, where ω_d is the drift frequency of the hot mantle electrons and γ_{MHD} is the growth rate of the instabilities in the center cell. Here the appropriate growth rate is that for $m = 1$ instabilities of the center cell.

The mantle electrons must be axially confined by the magnetic field. If the mirror ratio for these electrons is small, their distribution function will be highly anisotropic leading to the possibility of enhanced losses due to anisotropy driven instabilities such as the whistler. As a guard against these instabilities, we specify a mirror ratio of at least two on the field lines threading the mantle electrons. For confinement of sloshing ions in the end cell a higher mirror ratio, around 3 or higher is needed in the core. A magnet design for the end cell which satisfies these criteria is described in Section II. The mantle electrons must also follow closed drift surfaces satisfying the above-mentioned criterion on drift frequency. In the third section we present results on the electron drift and then consider heating of these electrons. In the next section we evaluate radial transport of center-cell fuel ions, as caused by the octopole field. In the fifth and sixth sections we cover MHD stability and plasma parameters which can be expected in this configuration.

II. MAGNET DESIGN

As described in the introduction, we wish to create a minimum- $|B|$ region external to the core plasma with octopole fields. All field lines

passing through the minimum- $|B|$ region should have a mirror ratio of at least two to allow development of a sufficiently broad electron distribution function. A coil set which satisfies these requirements is shown in Figs. 3 and 4. It consists of a high-field (24 T) hybrid coil similar to that proposed in the MARS design,¹ a small octopole transition coil, a barrel-shaped main octopole coil, and a 7.5 T outer mirror coil. In an earlier attempt at meeting the above requirements we aligned the Ioffe bars parallel to the z-axis. We found it difficult to obtain a mirror ratio of two for the mantle electrons with this configuration. Consideration of the magnetic fields generated by the coils in the vicinity of the outer mirror suggests that the main Ioffe bars should bow outward in the middle. The octopole coil shown in Figs. 3 and 4 is an approximation to the desired barrel shape.

The bars of the main octopole coil are placed at $\theta = 22.5^\circ, 67.5^\circ$, etc. to the vertical (x) axis in Fig. 3. In the $\theta = 0^\circ, 45^\circ, 90^\circ$, etc. planes, the field lines remain within the plane, i.e., have zero geodesic curvature. Contours of the magnetic field at $0^\circ, 22.5^\circ$ and 45° from the x-z plane, as obtained with the aid of the EFFI code,⁶ are shown in Figs. 5-7. Also shown are the trajectories of field lines which map to several center-cell radii. We note that the field lines in the 0° plane are most strongly radially deflected by the octopole field. For comparison we show in Fig. 8 the B-field contours and field lines for the $\theta = 22.5^\circ$ plane for the same coils without the transition octopole and in Fig. 9 with no octopole field at all. The transition coil clearly aids in gathering field lines from the center cell so they pass through the outer mirror.

There is no unique prescription for adjusting the size and current in the transition coil. We chose the size quite arbitrarily, only desiring to keep it small and with adequate clearance between the coil and both the halo plasma and the mantle electrons. The transition-coil current was then adjusted to map the center-cell plasma surface into a circle at the midplane of the main end-cell octopole. On either side of the chosen current, the surface deforms into one resembling a four-pointed star with rounded points. We found that small changes in the transition-coil current and position could cause substantial changes in the shape of the flux surfaces in the end cell. This coil would usually be constructed of superconductor, but since it need carry only about 1 MA in the present configuration, it could be constructed of copper and shaped to fit closely to the plasma surface. Since the octopole fields scale as r^3 ,

considerable reduction in current might be obtained with such a design. We have not yet explored this possibility in detail.

Figure 10 shows the contours of $|B|$ in a quadrant of the x - y plane at the midplane of the main octopole. We see one of the four minima in $|B|$ which occur in the octopole. The hot mantle electrons fill the region with near constant pressure between the indicated surface of the core plasma and the minima. Pressure should rapidly decrease at radii beyond the minima.

For purposes of comparison with the MARS coils, we collect some parameters of interest in Table I. We see that the mirror-to-mirror length and total current have each been reduced over the values for MARS by a factor approaching three, while the volume of the superconductor and the stored energy are down by a factor of better than four. It should be noted that the volume does not include the choke coils which would be similar for the two reactors, nor does it include the coils used in MARS to recircularize the flux bundle for the direct converter. We expect that the latter coils would also be smaller in the octopole configuration.

The main octopole coil in Figs. 3 and 4 was designed with sharp bends only to ascertain if it is possible to obtain the desired field structure with such a coil. Because of the absence of smooth bends at the ends of the straight bars, the coil could not actually be built in this form. A new coil shape with smooth and continuous bends and more flexibility is shown in Fig. 11. This coil shape and others are currently under study for use as octopole magnets.

III. PROPERTIES OF MANTLE ELECTRONS

The mantle electrons bridge the region between the core plasma in the end cell and the region of minimum $|B|$. Throughout this region they must drift on closed surfaces in order not to introduce a channel for losses of electrons. The energy of the electrons should be chosen to minimize losses--high energies reduce scattering and drag losses but also increase synchrotron radiation. The energy for minimum loss rate depends on the density of background cold electrons. Assuming all the synchrotron radiation is lost and not reabsorbed, we find the minimum near 800 keV (Section V); in EBT, where the cold electron density must be much higher, the minimum occurs near 1.5 MeV.⁷ We need to compute the maximum energy that will satisfy the requirement $\omega_d < \gamma_{MHD}$.

Since the bounce frequency is much greater than the drift frequency, the electrons should drift on surfaces of constant J . We first compute the J surfaces and later consider the actual drift motion. Although the electrons are relativistic, we chose to use the non-relativistic form for J to obtain an initial idea of the shape of the surfaces,

$$J = \oint v_{\parallel} ds \quad (2)$$

where $v_{\parallel}$ is the velocity component parallel to the magnetic field. The integral is evaluated over a complete bounce period. Using the energy equation with no electrostatic potential we rewrite Eq. (2) in the form

$$J = \oint (\epsilon - \mu B)^{1/2} ds \quad (3)$$

where ϵ is the total energy and $\mu = m v_{\perp}^2 / 2 B$ is the magnetic moment. In general, J is a function of the field line coordinates, say r and θ at the midplane of the end cell, μ and ϵ . However, we are not interested in the magnitude of J , only in the shape of the constant- J surfaces. We can, therefore, divide through by ϵ and plot J surfaces as a function of r and θ for various values of μ/ϵ . We choose to use R , the mirror ratio where the electrons reflect, in place of μ/ϵ as the independent variable.

The calculation of J has been carried out with the code GCDRIFT⁹ which can also be used to compute the non-relativistic drift motion of ions and electrons in the vacuum magnetic field. The fields are computed with filamentary approximations to the conductors. For all but the octopoles, multiple filaments were used for each conductor. For the octopoles of Figs. 3 and 4, the straight bars are continuous at the connections only along the center line, and so it is possible to use only a single filament.

We find that the constant- J surfaces are nearly circular near the axis with small variations in radius of period 4θ , where θ is the azimuthal angle. At increasing radii, the radial excursions grow in magnitude and the periodicity changes to 8θ . This behavior can be anticipated from considerations of the expansion of J near the axis for octopole fields which reads:

$$\begin{aligned}
J(r, \theta, z, \mu/e) = & J_0(r, z) \\
& + r^4 \cos(4\theta) J_4(r=0, z, \mu/e) \\
& + r^8 \cos(8\theta) J_8(r=0, z, \mu/e) + \dots
\end{aligned} \tag{4}$$

The 8θ azimuthal dependence is shown in the lower parts of Figs. 12 and 13 which display portions of the J surfaces for electrons reflecting at mirror ratios of 1.05 and 2.0, respectively. Since the field contains octal symmetry, only an octant of the surfaces is shown. In both figures we see axis-encircling J surfaces at both large and small radii, separated by islands which occur near the radius of the minimum $|B|$ islands (compare Fig. 10).

However, just outside the islands, the surfaces for $R = 1.05$ maintain the 8θ behavior while the next component, 16θ becomes important for the surfaces for $R = 2$. The latter electrons sample fields closer to the octopole bars.

At first sight, it is surprising that the drift surfaces displayed in Figs. 12 and 13 contain one island per octant while the $|B|$ contours in Fig. 10 display one per quadrant. Electrons drifting in a plane must drift parallel to the $|B|$ contours since $\vec{v}_D \propto \vec{B} \times \text{grad } B$, but the bounce motion samples different grad B so the J contours will, in general, deviate from the $|B|$ contours. As bounce motion is more limited axially, the J and $|B|$ contours should approach. We have verified by similar calculations that the J islands do indeed coalesce into a single island per quadrant for mirror ratio = 1.01 electrons.

For this stabilization scheme to be successful, the constant-pressure mantle electrons must extend to at least the minimum $|B|$ location after which their density may fall to zero. We assume that they extend to about 1.05 m at $\theta = 0$ where they are axis encircling. To determine the maximum electron energy for non-rigid behavior, we have computed the actual (non-relativistic) drift of 3.6 MeV electrons reflecting at $R = 1.05$ and $R = 2$. The (non-relativistic) drift frequencies for these two cases are 3.3×10^5 and 1.4×10^6 rad/s, respectively. With the aid of the TEASCO code,^{5,11} we found a growth rate for $m = 1$ instabilities in the center cell of $3.1 \times 10^5 \text{ s}^{-1}$ at an average β of 0.50. This is the same size as the curvature drift frequency for the bulk of the mantle electrons at $R = 1.05$. Also, electrons drifting on the islands (non-encircling electrons) can presumably have higher energy. However, relativistic effects increase the drift frequency by the factor γ , which is about 8 for 3.6 MeV electrons. From the criterion $\omega_d < \gamma \omega_{MHD}$,

we expect an upper limit of approximately 1 MeV for the energy of the mantle electrons.

In Fig. 14 we show a cross section of the drift surface at the octopole midplane for the 3.6 MeV electrons reflecting at $R = 2$. We clearly see the two peaks per octant which were already observed in the J surfaces of Fig. 12. A similar drift-surface cross section corresponding to the electrons of Fig. 13 shows the much smoother behavior expected for the electrons confined by a smaller mirror ratio.

Lowest order theory says that J is constant on a drift surface, but more detailed results show that there are oscillations about the constant mean value as the particle drifts. As an example of such oscillations we show in Fig. 14 a plot of J vs. time for electrons reflecting at $R = 1.05$. J values are plotted only at every half bounce period, so only two points on the oscillations at the bounce frequency are apparent. We also notice other components which oscillate at fractions of the drift frequency.

Although we have not yet performed calculations on electron-cyclotron heating of the mantle electrons, we have a general picture of how this might be accomplished. As previously discussed, the plug radial pressure profile should increase to a maximum at the minimum $|B|$ point and then decrease beyond. As long as scattering dominates the hot-electron losses, minimizing the ECH power required to maintain the hot electrons requires maximizing the hot-electron energies, subject to the MHD constraint on their drift frequencies. At the peak pressure point (minimum $|B|$ point) the curvature drifts change sign and so in the vicinity of this point the drift frequency does not constrain the energy, but rather synchrotron radiation places a soft limit around 1 MeV for ECH power minimization. The hot electron density is constrained only in the core, where density mapping to plug the central cell determines the barrier density. Thus we expect the radial density profile for the hot electrons to decrease monotonically with radius, while the pressure rises to the minimum $|B|$ point, due to the increasing energy with radius.

The mantle electrons will therefore likely be at lower density and more relativistic than in the core (see Section V), and the ECH absorption efficiency for a single pass may be 10% or less in the mantle. We envision a set of mirrors arranged around the mantle so as to reflect a quasi-optical microwave beam for several passes through the mantle. Off-resonant frequencies can be used to control the hot-electron mantle β because absorption at

relativistic energies is strongly doppler- and mass-shifted. To control the mantle electron density, a separate microwave beam aimed at an appropriate cold electron resonant zone intersecting the mantle can be used to control the lower energy electron population feeding the hotter anisotropic electrons. Since the mantle electrons are in a stable minimum |B| field, there is no hot electron interchange (Berk-Dominguez) mode requirement that the cold electron background density be dominant. We expect ECH feed requirements will allow the cold density to be much less than the hot density, $n_{ec} \ll n_{eh}$, the origin of the cold electrons being ionization of a low-pressure ($< 10^{-6}$ Torr) gas background. Calculations with Fokker-Planck and Monte-Carlo codes are currently under way to check this expectation.

III. ION RADIAL TRANSPORT DUE TO DRIFTS

In order to avoid excessive losses due to radial transport of center-cell ions in MARS, it was necessary to cancel deflections of passing ions (i.e., those center-cell ions which pass into the end cell) by adding C coils to symmetrize the geodesic curvature in the end cell. To obtain a short end cell, such added coils must be avoided. In this section we estimate the confinement time of center-cell ions in the magnet shown in Figs. 3 and 4. For this estimate we use the fastest diffusion coefficient due to classical processes, namely that for stochastic diffusion,

$$D = a^2 / (4 \tau_b R) \quad (5)$$

where a is the maximum deflection of an ion on a single reflection from the end cell as measured in the center cell, τ_b is the bounce time for a complete cycle, and R is the mirror ratio for the ions, in this case 24/3.3. The mirror ratio factor occurs in Eq. (5) because $\sim 1-1/2R$ of the ions are reflected by the large mirror field and do not see the octopole fields.

If the paraxial approximation to the magnetic fields is applicable, the radial deflection of a passing ion on a single reflection in the end cell will obey the relation

$$\delta r = a \cos(4\theta) \quad (6)$$

and the amplitude a will scale as r^3 . In order to verify that this relation is valid and to ascertain the value of a , we have computed deflections with the aid of the GCRIFT code.⁵ Ions were started with various pitch angles in the axisymmetric region of the central cell, reflected elastically by a wall at the approximate location of the axial confining potential (see Fig. 1) and then followed back to the z location of their starting point. The difference in radial position is then the deflection δr . As for the electron drifts reported in Section II, only vacuum fields with no electrostatic potentials were used.

It was found that δr was very nearly independent of the initial pitch angle if the pitch angle was small enough for the ions to pass through the choke coil. Consequently, we used only a single pitch angle near the loss cone for this study. The amplitude, a , of the deflection was computed for several values of initial radius in the center cell between .1 and the plasma edge at .45 m and azimuthal angles from 0 to 90°. Fig. 16 shows that δr follows closely the sinusoidal function given in Eq. (6) with some distortion at large r . The deflection is given vs. radius in Fig. 17. Also shown for comparison is the deflection without a transition coil. We see that the deflection does scale as r^3 as expected and that the addition of the transition coil substantially reduces the deflection. The transition coil also makes the field more paraxial in nature as demonstrated by closer agreement with r^3 scaling.

We can define a radial confinement time, t_r , at any radius by

$$t_r = \frac{L}{2\pi} \frac{\int_0^R n(r) r dr}{r D \left| \frac{\partial n}{\partial r} \right|} \quad (7)$$

where n is the ion density. For the quartic density profile which we use to model reactors, Eq. (7) becomes

$$t_r = (r_p^4 / 8r^2 D) [1 - (1/3) (r/r_p)^4] \quad (8)$$

where r_p is the core plasma radius. Diffusion coefficients and confinement times are given in Table II. We see that the confinement time is high, even at the outer edge of the plasma. For comparison, it should be noted that passing ions trap in the end cell with characteristic time of 1 s. Thus, radial transport should be of no concern in octopole-stabilized end cells and there is no need for additional coils to cancel the radial deflection.

We can gain additional insight into the radial transport from considerations of the geodesic curvature. In analyzing quadrupole-stabilized tandem mirrors, it has been found convenient to form normal and geodesic components of the field line curvature

$$\kappa = R \text{ grad } \psi + \mathcal{H} \text{ grad } \theta \quad (9)$$

with R the normal curvature and $\mathcal{H}$ the geodesic curvature. ψ and θ , the magnetic flux and angular position of a field line in the center cell, where flux surfaces are circular, are the generalized field-line coordinates. The deflection of a passing ion in one reflection in the end cell can be written

$$\delta \psi = 2(m/q) \int_0^{s_t} \mathcal{H} (v_{\parallel} + v_{\perp}^2/2v_{\parallel}) ds \quad (10)$$

where s_t is the location of the turning point. Only ions with $v_{\parallel} \gg v_{\perp}$ are able to make it through the choke coil so the quantity in parentheses is nearly constant. Thus, the radial deflection is approximately proportional to an integral of a field property along the field line.

The quantities $\mathcal{H}$ and $\int \mathcal{H} ds$ are plotted for the field line at 22.5° in Figs. 18 and 19. Values are shown both with and without the transition octopole. Comparing the curves in Fig. 18, we see that the addition of the transition magnet has (1) introduced a new region of positive $\mathcal{H}$, (2) reduced the magnitude of $\mathcal{H}$ near its maximum by a factor of about four, and (3) rendered the maximum and minimum curvature almost symmetric about the midplane of the main octopole. As a result, the integral, shown in Fig. 19, is reduced by a factor of about 15. If so desired, we could reduce the integral to zero by adjustment of the transition current and/or position. Such adjustments would then probably cause the flux tube from the center cell to be non-circular at the midplane of the end cell.

IV. MHD STABILITY

MHD stability theory of the octopole configuration is currently under development.¹⁰ As described in the introduction, only $m = 1$ modes are of concern, and, when a close-fitting wall is present, as it is in a reactor, ballooning may be suppressed and the mode becomes flute-like. We can at least

show here that this octopole configuration can meet the classical low- β interchange stability condition of Eq. (1). We apply this relation to the core plasma to obtain the required increment in hot-electron pressure at the edge of the core. We assume that maintenance of this pressure out to the minimum- $|B|$ region will serve to stabilize the entire machine.

To apply Eq. (1) to the octopole we need to specify a pressure function. We choose p to be separable in ψ and B ,

$$p = \omega(\psi) \hat{p}(B) \quad (11)$$

For the core plasma we use a quartic profile in radius,

$$\omega = 1 - (\psi/\psi_p)^2 \quad (12)$$

and for the extra hot electrons in the end-cell core which are needed to stabilize the core by reversing the total pressure gradient, we take

$$\omega = (\psi/\psi_p)^2 \quad (13)$$

where ψ_p is the magnetic flux at the core plasma surface. For the pressure function, $\hat{p} = \hat{p}_{||} + \hat{p}_{\perp}$, in the core, we use the following model functions¹¹:

Center-cell ions ($0 < z < z_{m1}$)

$$\hat{p}_{||} = \hat{p}_{\perp} = \frac{1}{2} \beta_c B_c^2 \quad (14)$$

Passing ions ($z_{m1} < z < z_b$)

$$\hat{p}_{||} = \frac{1}{2} \beta_c B_c^2 \frac{1}{4} \left[5 \frac{B}{B_{m1}} - \left(\frac{B}{B_{m1}} \right)^5 \right] \quad (15)$$

$$\hat{p}_{\perp} = \frac{1}{2} \beta_c B_c^2 \frac{B}{B_{m1}}^5 \quad (16)$$

Passing ions ($z_b < z < z_{m2}$)

$$\hat{p}_{||} = \frac{1}{2} \beta_c \beta_b^2 \left[\frac{5}{4} \frac{\beta_b}{\beta_{m1}} - \frac{1}{4} \left(\frac{\beta_b}{\beta_{m1}} \right)^5 \right] \left(\frac{\beta}{\beta_b} \right)^{1+\delta} \left(\frac{\beta_{m2} - \beta}{\beta_{m2} - \beta_b} \right)^2 \quad (17)$$

$$\hat{p}_{\perp} = \left(\frac{2\beta}{\beta_{m2} - \beta} - \delta \right) \hat{p}_{||} \quad (18)$$

$$\delta = \frac{2\beta_b}{\beta_{m2} - \beta_b} + \frac{4\beta_b^4}{\beta_b^4 - 5\beta_{m1}^4} \quad (19)$$

Sloshing ions and hot electrons ($z_{m1} < z < z_{m2}$)

$$\hat{p}_{||} = \frac{1}{2} \beta_b \beta_b^2 \frac{(\beta_{m2} - \beta)^2}{\beta_{m2}^2 - \beta_b^2} \quad (20)$$

$$\hat{p}_{\perp} = \frac{1}{2} \beta_b \beta_b^2 \frac{\beta_{m2}^2 - \beta^2}{\beta_{m2}^2 - \beta_b^2} \quad (21)$$

$$\hat{p}_{||} = p_{\perp} = 0, \quad \beta > \beta_{m2} \quad (22)$$

β_c and β_b are input values for the plasma properties, and β_c , β_{m1} , β_b , and β_{m2} are magnetic fields at key locations (see Fig. 1).

The "extra" stabilizing hot electrons in the core are expected to be narrower in axial extent than the combination of hot electrons and sloshing ions represented by Eqs. (20)-(22). We use the same pressure functions as in Eqs. (20)-(22) but with the replacement of β_b by β_m and β_{m2} by β_{max} , where the mirror ratio β_{max}/β_b is chosen to be two.

We can now substitute Eqs. (11)-(22) in Ineq. (1) and solve for the *minimum* β_m as a function of β_c and β_b . Results are displayed in Fig. 20. The stability criterion is linear in β and we can fit it with the function

$$\beta_m = 1.2 \beta_b + 0.56 \beta_c \quad (23)$$

Typical reactor parameters are $\beta_c = .75$, $\beta_b = .30$ for which the mantle electrons must have $\beta_m = .78$. Note that this is the total β at the edge of the plasma (based on the center-line vacuum field), so the increase in β from the center line to the edge is $.78 - .30 = .48$. This plasma pressure is assumed constant out to the edge of the mantle electrons.

The above analysis is based on the interchange relation while we expect $m = 1$ ballooning to set the limit on stability. The $m = 1$ ballooning is a rigid-body type of motion and stabilization comes from a ψ (radial) average of the forces.⁵ It is possible that the ballooning analysis, which is currently under development,¹⁰ will allow radial profiles of mantle pressure which are not monotonically increasing. In other words, p_m could actually decrease from the core to just inside the region of minimum $|B|$ and then increase to a high electron pressure which would prevent rigid motion of the plasma column.

An additional stabilizing mechanism not explicitly included here arises from the presence of conducting walls near the edge of the plasma. If the center-cell average β is high enough, of order 0.90, the image currents in the wall will stabilize against $m = 1$ ballooning.¹² If the center-cell is fully stabilized in this way, then, by Eq. (23), β_m need be only some 20% higher than β_b . Even if the wall stabilization is only partially effective, it could reduce β_m below the value found from Eq. (23).

V. PLASMA PARAMETERS

Computation of the plasma parameters proceeds in much the same way as described in the MARS report¹ and in more detail in Ref. 13. Of course the end cell of the octopole tandem mirror is composed of only a single cell instead of the three - transition, anchor and plug - in MARS. The anchor is missing, or, rather, it has been replaced by the mantle electrons; the transition section is absent, at least as a distinct region with its own properties. Only the plug remains with the hot, barrier electrons (at point b in Fig. 1), the sloshing ions (injected at point a'), and the warm electrons (at point a). Added to the plug are the mantle electrons which could require considerable power input. As a first step in computing the plasma properties, we consider the computation of the hot-electron power.

Power to the electrons is required to make up for losses due to Coulomb collisions and radiation. Shearer¹⁴ has developed expressions for power losses (MW) due to (1) scattering off hot electrons and the neutralizing ions, (2) drag on col' electrons and (3) scattering off cold electrons and their neutralizing ions, which can be put in the form:

$$P_1 = \frac{10^{-3} \text{ eV } C_h^{(2)} A_h^{(2)} n_{eh}^2 \gamma [2/(\gamma + 1)]^{3/2} (E_h/2)}{2.4 \times 10^8 E_{eh}^{3/2} \log R_b} \quad (24)$$

$$P_2 = \frac{10^{-3} \text{ eV } C_{hc}^{(2)} A_{hc}^{(2)} n_{eh} n_{ec} \gamma [2/(\gamma + 1)]^{1/2} (E_h - 3T_{ec}/2)}{8.35 \times 10^7 E_{eh}^{3/2}} \quad (25)$$

$$P_3 = \frac{10^{-3} \text{ eV } C_{hc}^{(2)} A_{hc}^{(2)} n_{eh} n_{ec} \gamma [2/(\gamma + 1)]^{3/2} (E_{eh}/2)}{2.05 \times 10^8 E_{eh}^{3/2} \log R_b} \quad (26)$$

where E_{eh} is the hot electron energy (keV), n_{eh} is their density (cm^{-3}), n_{ec} is the density and T_{ec} the temperature of the cold electrons, V is the volume of the region in question, R_b is the mirror ratio for the hot electrons, and $C^{(2)}$ and $A^{(2)}$ are weighting factors for the averages of the densities products over the radial and axial profiles. For the purpose of this analysis, the hot-electron average energy is assumed constant in three regions: core, halo (between the core and the flux tube just clearing the inner edge of the choke coil), and the (outer) mantle (from the halo to just beyond the minimum |B| region). In reality, we would like to supply ECH so that the hot-electron energy rises from the center line to a peak in the minimum |B| region, as described in Section II. Such a profile should lower electron losses due to scattering below the values computed in this section, since the average density would be lower.

We take the density to be a separable function of radius and axial position, $n = n(r)f(z)$. A typical radial factor is then

$$C_h^{(2)} = \frac{2}{r_b^2} \int_0^{r_b} \left[\frac{n_{eh}(r)}{n_{ehb}} \right]^2 r dr \quad (27)$$

and an axial factor

$$A_h^{(2)} = \frac{1}{L_b} \int_0^{L_b} r^2(z) (B_b/B) dz \quad (28)$$

where the factor B_b/B accounts for the variation of flux-tube area along the axis, B_b being the plasma-reduced magnetic field at point b.

The loss of power through synchrotron radiation can be written in the form,¹⁵

$$P_s = 3.88 \times 10^{-4} \text{ eV } C^{(s)} A^{(s)} n_{eh} B^2 E_{eh} [1 + 2.5 (\gamma - 1) + 1.5 (\gamma - 1)^2 / (\gamma - 0.3)] \quad (29)$$

$C^{(s)}$ and $A^{(s)}$ are radial and axial averages of $n_{eh} B^2$ over radius and axial position as in Eqs. (27) and (28). Equation (29) has been derived¹⁵ for electrons with a Maxwellian distribution of velocities. Because the high-energy electrons will lose energy by radiation more rapidly than the low-energy electrons, we would expect the tail population to be depressed below that for a Maxwellian distribution and Eq. (29) to overestimate the losses.

As described above, we divide the end-cell into three regions: core, halo, and mantle. The core cold-electron density is assumed quartic in radius, as in Eq. (12), while the hot electron density is the sum of the two profiles in Eq. (12) and (13). The core volume (both ends) is given by

$$V_b = 2\pi r_b^2 L_b B_{cv} \sqrt{1 - \langle \beta_c \rangle} / B_{bv} \sqrt{1 - \langle \beta_b \rangle} \quad (30)$$

where r_c is the center-cell radius, $\langle \beta \rangle = 2\beta/3$ is the average β , and subscript v denotes the vacuum field. The radial factors are found to be

$$C_h^{(2)} = 1 + \frac{2\Delta n_{eh}}{3n_{ehb}} - \frac{1}{5} \left(\frac{\Delta n_{eh}}{n_{ehb}} \right)^2 \quad (31)$$

$$C_{hc}^{(2)} = \frac{2(1 + \Delta n_{eh}/n_{ehb})}{15} \quad (32)$$

$$C(s) = 1 + \frac{\Delta n_{eh}}{3n_{ehb}} - \frac{2\Delta\beta}{15(1 - \beta_b)} \quad (33)$$

where $\Delta n_{eh} = \beta_m B_{bv}^2 / 1.6E_{eh} - n_{ehb}$ and $\Delta\beta = \beta_m - \beta_b$. To compute the axial factors in the core we take a hot electron density distribution corresponding to the pressure distribution in Eqs. (20) and (21),

$$n_h = \frac{\beta_b B_b^2}{1.6E_{ehb}} \left(\frac{3B_{max} - B}{3B_{max} - B_b} \right) \left(\frac{B_{max} - B}{B_{max} - B_b} \right) \quad (\text{Core}) \quad (34)$$

For the cold density we use a Gaussian function inboard of the barrier point b (Fig. 1),

$$n_{ec} = n_{ecb} \exp [-(z - z_{m1})^2 / \lambda^2] \quad (35)$$

with $\lambda^2 = (z_b - z_{m1})^2 \ln[n_{ec}(z_{m1})/n_{ecb}]$ and a constant outboard of the barrier. The axial factors become

$$A_h^{(2)} = 0.192 \quad (36)$$

$$A_{hc}^{(2)} = 0.779 \quad (37)$$

$$A^{(s)} = 0.310 \quad (38)$$

The halo contains cold electrons of about 10^{12} cm^{-3} density and 50 eV energy.¹ This plasma streams out from the center cell through the end-cell to collectors in the end tanks. We assume constant cold-electron density with radius in this region. The hot electron density is taken as constant with radius in the halo region, but with a more-sharply peaked axial profile:

$$n_h = \frac{\beta B_b^4}{1.6E_{eh} B^2} \left(\frac{B_{max} - B}{B_{max} - B_b} \right)^{5/2} \quad (\text{Halo}) \quad (39)$$

We expect stronger axial peaking in this region because of the lower collision frequency and applied ECH. We take the electron energy to be

constant, typically 500 keV and compute the required power. Radial factors are unity for this case while the axial factors become

$$A^{(2)}_h = 0.117 \quad (40)$$

$$A^{(2)}_{hc} = 0.379 \quad (41)$$

$$A^{(5)} = 0.180 \quad (42)$$

The halo volume (both ends) is

$$V_h = 2\pi[r_{ch}^2 - (r_{ch} - \delta r_{ch})^2] (B_m/B_{bv})L_b \quad (43)$$

with r_{ch} the inner radius of the choke coil (ml in Fig. 3) and δr_{ch} the clearance between the choke coil and the core plasma. The hot-electron density follows from the requirement of constant pressure,

$$n_{ehh} = (\delta n_{eh} + n_{ehb})E_{ehb}/E_{ehh} \quad (44)$$

The mantle region is treated in the same way as the halo but the mantle hot electron energy is assumed higher, ca. 800 keV and cold density lower, ca. 10^{11} cm^{-3} .

In order to compute the plasma properties, we need to specify the injection energy of the sloshing ions. The maximum energy is limited by the requirement that the ions remain adiabatically confined. To estimate the adiabatic energy limit, we can use the formula¹⁶

$$E_{ad} \text{ (keV)} = \frac{10^3 \kappa^2 B_{min}^2 L_{ii}^2 Z_i^2}{M_i (1 - 0.036 \ln \Lambda)^2} \quad (45)$$

where B_{min} is the minimum magnetic field, Z_i and M_i are the charge and mass (AMU) of the ion, $L_{ii} = (2B/\partial^2 B/\partial s^2)^{1/2}$ is the scale length for a parabolic well at the minimum, and Λ , which is a function of E , and κ are defined in Ref. 16. Evaluating the parameters for the vacuum field we find $L_{iiV} = 1.77 \text{ m}$ and $E_{adv} =$

1.1 MeV. Both B and $L_{||}$ are reduced when the plasma is present. We can use the long-thin approximation, $B^2 = B_v^2 - 2p_{||}$ (B) together with Eq. (21) to find both B and $L_{||}$. For a peak β_b of .30 we find $L_{||} = 1.47$ m and $E_{ad} = 600$ keV. Since this is the ion energy measured at the midplane, the injection energy must be about 100 keV less to allow for the difference in the potential between the injection point (a' in Fig. 1) and the end-cell midplane. We have taken 475 keV for the calculations.

Properties have been computed for two cases representative of small reactors: one with a 50 m long center cell producing about 500 MW of fusion power and one twice as long producing about 1000 MW. Central cell and end-cell peak β 's were fixed at 75% and 30% respectively. From the results of Section IV, we take the conservative upper limit of $\beta_m = 78\%$ for stability. As indicated in Table IV, certain other properties such as the center-cell ion temperature, and cold-electron density and hot-electron energy in the mantle were fixed. Other properties and powers were obtained from the equations described above and in Ref. 13. The power flow for these reactors is given in Table III and a selection of the plasma properties in Table IV.

When considering these results we should note that only a limited attempt was made to optimize the parameters. The center-cell length, β_c , vacuum field and temperature were chosen to give the desired total fusion power. β_b was chosen to yield a moderate value of β_m and the mantle hot-electron energy was chosen at the approximate minimum in the mantle ECH requirements, i.e., where the scattering losses were about the same as the synchrotron radiation. Larger Q values could doubtless be obtained by a more optimum choice of parameters. In addition, credit could be taken for some reabsorption of synchrotron radiation, which we have assumed is entirely lost.

For the 500 MW reactor, the Q value of 10 indicates that there would be a substantial recirculating power fraction, while at 1000 MW the Q is essentially the same as computed for the much larger MARS reactor. Since the end-cell size is fixed, it is rather surprising that the Q isn't simply doubled on going to the larger power. In comparing the two cases, we see that the power to the core electrons has decreased from 28.9 to 17.6 MW in increasing the center-cell length. This results from two constraints on the solution: (1) $\beta_b = 0.30$ and (2) trapping rate of alpha particles in the end-cell = rate of formation of thermal alphas. To satisfy the latter condition, the TMRBAR code¹³ adjusts the trapped density in the barrier. Higher trapping rate are obtained at lower

trapped-ion density because of higher gradients across the passing-trapped separatrix in velocity space. Because of the lower density at the barrier, the hot-electron density is lower, and the first constraint allows higher hot-electron energy, thus reducing the scattering rate and the power required. It may also be noted in Table IV that the thermal alpha concentration is higher for a longer center cell which also increases the trapping rate.

VI. DISCUSSION

We have seen that the octopole stabilizing scheme allows considerable reduction of the end-cell length, and allows the design of high-Q tandem mirror reactors at much lower fusion power than previously thought possible. The results indicate that one could obtain $Q > 25$ at a power of about 1000 MW. The short end cell (7.7 m) coupled with short modules for the center cell (< 10 m) suggest that this reactor might be built out of reasonable size modules in a factory, and then assembled on site, thus reducing construction costs over conventional on-site construction. These issues will be examined in future studies.

Although we have presented estimates and discussions of most physics problems, there are several areas which will require a much more detailed study. A principal area concerns the conservative use of simple interchange conditions to determine the required β of the mantle electrons for MHD stability. This can only be resolved when codes are developed to analyze the stability of the entire flux tube. Also, the preferred electron energy profile rises from a minimum on axis to a maximum of about 1 MeV at the minimum |B| location, and requirements on the feed of cold electrons to the hot electron population, the required ECH frequencies and the wave propagation to achieve these desired profiles need to be addressed.

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Table I. Comparison of MARS¹ and Octopole end cell coils. Choke and recircularization coils for the direct converter are not included.

	<u>MARS</u>	<u>Octopole</u>
Mirror to mirror length-m	21.8	7.7
Total current-MAT	70.6	25.2
Conductor volume-m ³	87.3	19.8
Stored energy-GJ	6	1.4

Table II. Radial diffusion coefficients and confinement times at several center-cell radii.

<u>r-m</u>	<u>D-cm²/s</u>	<u>t_r-s</u>
0.10	1.9×10^{-4}	4.8×10^6
0.20	0.012	7.2×10^4
0.30	0.26	1900
0.40	2.15	117
0.45	5.61	27

Table III. Power flow for two octopole tandem mirror reactors with nominal fusion powers of 500 and 1000 MW.

Total fusion power-MW	520	990
Wall neutron power-MW/m ²	2.2	2.0
at wall radius-m	0.61	0.61
End-cell power absorbed:		
Neutral beam (475 keV)-MW	0.39	0.42
ECH at a-MW	0.66	0.52
Core ECH at b-MW	28.9	17.6
Mantle ECH in halo-MW	7.5	7.5
Mantle ECH in mantle-MW	13.3	13.3
Total power absorbed-MW	51	36
Q	10	25

Table IV. Plasma properties for two octopole tandem mirror reactors with nominal fusion power of 500 and 1000 MW. Properties marked with an asterisk are input quantities.

Center Cell:

Length*-m	50	100
Radius*-m	0.45	0.45
Vacuum magnetic field*-T	3.3	3.3
Peak β *	0.75	0.75
Ion temperature*-keV	40	40
Electron temperature-keV	27	27
Ion Density- 10^{14} cm^{-3}	2.3	2.3
Thermal alpha concentration	0.035	0.048
$n \tau$ - $10^{14} \text{ s-cm}^{-3}$	5.3	5.6
D-T fueling current-A	340	610

End Cell:

Length*-m	7.7	7.7
Radius*-m	0.59	0.59
Vacuum field*-T	1.5	1.5
Peak β *	0.30	0.30
Hot electron energies-keV		
Core	213	280
Halo*	500	500
Mantle*	800	800
Hot electron densities- 10^{12} cm^{-3}		
Core	7.0	5.7
Halo	11	11
Mantle	6.8	6.2
Cold electron densities- 10^{12} cm^{-3}		
Core	0.22	0.18
Halo*	1.0	1.0
Mantle*	0.10	0.10

FIGURES

Figure 1. Magnitude of the magnetic field, B , and electrostatic potential, ϕ , along the z axis.

Figure 2. Magnitude of the magnetic field, B , and plasma pressure, p , vs. radial coordinate $\psi = r^2 B/2$ at the midplane of the center cell (top) and end cell (bottom).

Figure 3. View of the end-cell magnets showing the hybrid mirror coil (m1), the octopole transition coil (o1), the main octopole coil (o2), and the outer mirror coil (m2).

Figure 4. View of the tandem octopole magnets showing a single modular center-magnet with the end-cell magnets.

Figure 5. Contour plot of magnetic field in the $\theta = 0^\circ$ plane in the end cell showing trajectories of field lines starting in the center cell at various radii. 1: $r = 0.15$ m, 2: $r = 0.30$ m, 3: $r = 0.45$ m (plasma edge), 4: $r = 0.61$ (wall), 5: $r = 0.67$ m.

Figure 6. Contour plot of magnetic field in the $\theta = 22.5^\circ$ plane in the end cell showing trajectories of field lines starting in the center cell at various radii. 1: $r = 0.15$ m, 2: $r = 0.30$ m, 3: $r = 0.45$ m (plasma edge), 4: $r = 0.61$ (wall), 5: $r = 0.67$ m.

Figure 7. Contour plot of magnetic field in the $\theta = 45^\circ$ plane in the end cell showing trajectories of field lines starting in the center cell at various radii. 1: $r = 0.15$ m, 2: $r = 0.30$ m, 3: $r = 0.45$ m (plasma edge), 4: $r = 0.61$ (wall), 5: $r = 0.67$ m.

Figure 8. Contour plot of magnetic field in the $\theta = 22.5^\circ$ plane in the end cell without a transition coil showing trajectories of field lines starting in the center cell at various radii. 1: $r = 0.15$ m, 2: $r = 0.30$ m, 3: $r = 0.45$ m (plasma edge), 4: $r = 0.61$ (wall), 5: $r = 0.67$ m.

Figure 9. Contour plot of magnetic field in the plane perpendicular to the z axis in the end cell without octopole coils. Also shown are trajectories of field lines starting in the center cell at various radii. 1: $r = 0.15$ m, 2: $r = 0.30$ m, 3: $r = 0.45$ m (plasma edge), 4: $r = 0.61$ (wall), 5: $r = 0.67$ m.

Figure 10. Contour plot of magnetic field in the x-y plane at the midplane of the main octopole.

Figure 11. Alternate octopole coil with smooth bends at transitions between multipole bars.

Figure 12. Portion of the J surface for mantle electrons reflecting at $R = 1.05$.

Figure 13. Portion of the J surface for mantle electrons reflecting at $R = 2$.

Figure 14. Drift surface for 3.6 MeV mantle electrons started at $r = 1.05$ m and $\theta = 0^\circ$ at the midplane of the octopole and reflecting at $R = 2$.

Figure 15. J values for approximately a quarter period of 3.6 MeV mantle electrons started at $r = 1.05$ m and $\theta = 0^\circ$ in the midplane of the main octopole and reflecting at $R = 1.05$.

Figure 16. Radial deflection of passing ions vs. azimuthal angle for initial radii of .3, .4 and .45 m.

Figure 17. Amplitude of the radial deflection of passing ions vs. initial radius with x and without 0 transition octopole coil.

Figure 18. Geodesic curvature, κ , vs. z for a field line starting at $r = .45$ m and $\theta = 22.5^\circ$ in the center cell. : with transition octopole; ---: without transition octopole.

Figure 19. ds vs. z for a field line starting at $r = .45$ m and $\theta = 22.5^\circ$ in the center cell. : with transition octopole;---: without transition octopole.

Fig. 20. Mantle β_m required to stabilize octopole tandem mirror with peak center-cell β_c and peak barrier β_b .

Figure 1a

Figure 1b

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

Figure 20