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DRIFT-INTERCHANGE MODES IN CYLINDRICAL PLASMAS

By

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JANUARY 1985

PLASMA
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PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CED-3073.

THEORY OF SEMICOLLISIONAL
DRIFT-INTERCHANGE MODES IN CYLINDRICAL PLASMAS

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ABSTRACT

Resistive interchange instabilities in cylindrical plasmas are studied, including the effects of electron diamagnetic drift, perpendicular resistivity, and plasma compression. The analyses are pertinent to the semicollisional regime where the effective ion gyro-radius is larger than the resistive layer width. Both analytical and numerical results show that the modes can be completely stabilized by the perpendicular plasma transport. Ion sound effects, meanwhile, are found to be negligible in the semicollisional regime.

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1. INTRODUCTION

The simple resistive magnetohydrodynamic (MHD) theory in a sheared slab model with curvature simulated by gravity predicts an unstable resistive interchange mode with a growth rate proportional to the cubic root of the resistivity.¹ In a cylindrical model with unfavorable curvature and pressure gradient, plasma compression gives a stabilizing effect although it does not stabilize all the modes completely.²

These previous models, however, do not provide adequate descriptions for plasmas encountered in present or future machines. In particular, electron diamagnetic drift frequency ω_{*e} can be greater than or comparable to the growth rate γ of resistive interchange mode. For example, ZT-40M parameters³ give $\omega_{*e}/\gamma \approx 3 m^{1/3}$, where m is the poloidal mode number. This electron diamagnetic drift effect has been treated in the low- β ($\beta \equiv$ plasma pressure/magnetic pressure) limit where pressure is assumed to be convected (i.e., plasma compressional effects are missing in this model) and found to reduce the growth rate of the mode greatly although the mode remains unstable even for arbitrarily large ω_{*e} .⁴⁻⁶

Another effect the usual resistive MHD theories have neglected is the fact that the ion gyro-radius at electron temperature [$\rho_s = C_s/\Omega_i$ where C_s is the ion sound speed $(T_e/M_i)^{1/2}$ and Ω_i is the ion cyclotron frequency] is often larger than the nominal resistive layer width $L_r = (\rho \eta^2 a^2/n^2 B_0^2 q'^2)^{1/6}$. Again, for ZT-40M parameters,³ we have $\rho_s/L_r \approx 6 m^{1/3}$. This is the so-called semicollisional regime.⁷

Although there have been extensive studies on drift-tearing modes in the semicollisional regime,^{7,8} relatively little attention has been paid to drift-interchange modes. Recently, Finn et al.⁹ (hereafter referred to as FMA) studied the stability of drift-interchange modes in cylindrical geometry

including the effects of ω_{*e} as well as finite plasma compression and ion polarization drift. Their equations, in fact, can be shown to be valid not only in the collisional regime but also in the semicollisional regime. They found numerically that the drift-interchange mode can be completely stabilized for a moderate value of ω_{*e} by including plasma compression, and concluded that the principal stabilizing influences are plasma compression and coupling to out-going drift waves.

We have done extensive analytical and numerical studies on drift-interchange modes using the same equations FMA used. Specifically, an analytical dispersion relation has been derived via asymptotic matching and agrees excellently with direct numerical solutions. We find, furthermore, that the principal stabilizing effect is due to the perpendicular plasma transport as well as plasma compression instead of coupling to drift waves as indicated by FMA. In fact, we find that while parallel compression of plasma (i.e., ion sound effects which stabilize drift waves) reduces the growth rates of the modes in the collisional regime, it has a negligible effect in the semicollisional regime.

In Sec. 2, we present the eigenmode equations and explain the physical meanings of various terms. An analytical dispersion relation for the semicollisional drift-interchange mode is then derived in Sec. 3. Properties of that dispersion relation are discussed in Sec. 4. Section 5 contains direct numerical solutions which are essentially the same as those of FMA and are shown to agree excellently with the analytical dispersion relation. Final conclusions and discussions are given in Sec. 6.

2. BASIC EQUATIONS

In cylindrical geometry, the equations for resistive modes including the additional effects of electron diamagnetic drifts and ion polarization drift (semicollisional effects) can be derived by the procedure outlined in FMA and will not be repeated here. The equations in the normalized variables are given by

$$\frac{\partial^2}{\partial x^2} \Xi = \frac{x}{Q^2} \frac{\partial^2}{\partial x^2} \Psi - \frac{1}{Q^2} \Gamma, \quad (1)$$

$$\frac{\partial^2}{\partial x^2} \Psi = Q(\Psi + x\Xi) + iQ_*\Psi + \frac{2iqx}{s} \Gamma, \quad (2)$$

$$\frac{\eta_{\perp}}{\eta_{\parallel}} \frac{\partial^2}{\partial x^2} \Gamma = Q\left(1 + \frac{x^2}{Q^2} + \frac{2}{\Gamma\beta}\right)\Gamma + Q\left(s - D - \frac{2D}{\Gamma\beta}\right)\Xi + \frac{D}{Q} x\Psi - 2iQ^2q \frac{\partial^2}{\partial x^2} \Xi, \quad (3)$$

where, $Q = \gamma/Q_r$, $x = x/L_r$, $D = -2p'a/B_0^2 R^2 q'^2$, $\beta = 2p(a)/R^2 q'^2$, $s = 4/R^2 q'^2$,

$$\Psi = \frac{-ia\tilde{B}_r}{nB_0 q' L_r},$$

$$\Gamma = \frac{-2Ba\tilde{B}_{\parallel}}{R^2 q'^2 B_0^2} = \frac{2a\tilde{p}}{R^2 q'^2 B_0^2},$$

$$L_r = \left(\frac{p\eta_a^2}{n^2 B_0^2 q'^2}\right)^{1/2},$$

$$Q_r = \left(\frac{\eta n^2 B_0^2 q'^2}{\rho a^2}\right)^{1/3},$$

$$q = \frac{(aV_A/\eta)^{1/3} |nq'a|^{1/3} V_A}{|kq'| Q_1 a},$$

and the rest of the notations are standard. In deriving Eqs. (1) - (3), one has assumed $T_i \ll T_e$, $d(p_i/\rho\Gamma)/dt = 0$, and electrons are isothermal. We have

differentiated $\eta_{\perp}$ and $\eta_{\parallel}$ in Eq. (3) in order to isolate the effects of the perpendicular resistivity.

Now, we explain the physical meanings of each equation. Equation (1) is the equation of motion in the radial direction. The first term on the right-hand side (RHS) is the field line bending term and the second term is the pressure gradient times curvature (i.e., the interchange instability driving term); while the left-hand side (LHS) describes the inertia. Equation (2) is the parallel Ohm's law. The first term on the RHS is the parallel electric field, the second term comes from the gradient of the equilibrium electron pressure along the magnetic field tilted by perturbations, and the last term describes the gradient of the perturbed electron pressure along the equilibrium magnetic field direction. Meanwhile, the LHS is the resistivity times parallel current. (Note that resistivity is absorbed into normalized variables.) The last equation, Eq. (3), is the pressure evolution equation. ∂T and $\partial(s - D)\bar{E}$ represent the temporal and convective changes of the pressure, respectively. $(2Q/T\beta)(T - D\bar{E})$ comes from the divergence of the perpendicular displacement. $(\eta_{\perp}/\eta_{\parallel})(\partial^2 T/\partial x^2)$ represents the effect of plasma diffusion due to the perpendicular resistivity. $x^2 T/Q$ and $Dx\bar{V}/Q$ are ion sound terms. The last term in Eq. (3) is the ion polarization drift term which is neglected in the collisional regime. This term represents the semicollisional (finite ρ_s) effect. In the low- β limit Eq. (3) reduces to $T - D\bar{E} = 0$. In this limit, not only compressional effects in both the parallel and the perpendicular directions (including ion polarization drift) but also perpendicular resistivity do not appear in the equations. On the other hand, the full sixth order system of Eqs. (1), (2), and (3) is hard to treat analytically. This system of equations can be reduced to a fourth order system by Fourier transformation. We present those Fourier transformed

equations in Sec. 5 where we discuss the numerical results. If we neglect the ion sound terms in Eq. (3), we can further reduce the order of the system to second order. The second order equation we obtain is

$$\frac{d}{d\theta} \frac{1+\theta^2}{1+[i\lambda/(\omega-\omega_*)](1+\theta^2)} \frac{d}{d\theta} \tilde{\phi} + \left(\frac{\omega-\omega}{\omega-(1-\epsilon)\omega_*} \right) \frac{(1-\epsilon)D+\omega(\omega-\epsilon\omega_*)(1+\theta^2)+(i\lambda_{\perp}\Gamma\beta/2)\omega(1+\theta^2)^2}{1+\{(\epsilon\omega\omega_*^2/D+i\lambda_{\perp}\Gamma\beta/2)/[\omega-(1-\epsilon)\omega_*]\}(1+\theta^2)} \tilde{\phi} = 0; \quad (4)$$

where we have used [although k_{θ}^2 has been neglected in Eqs. (1) - (3)] $\nabla_{\perp}^2 = -k_{\theta}^2 + d^2/dr^2$ to remove the singularity at $\theta = 0$. Equation (4) is written in terms of variables adopted by Connor et al.¹⁰; i.e.,

$$\tilde{\phi} = (2\pi)^{-1/2} \int_{-\infty}^{\infty} dr \phi(r) \exp(-ik_x r),$$

$\omega_{\text{new}} = \omega_{\perp s}/v_A$, $\omega_* = \omega_{*e} L_s/v_A$, $\epsilon = s\Gamma\beta/2D$, $\lambda = v_{ei} v_A L_s b_{\theta}^2/v_{te}$, $\lambda_{\perp} = \eta_{\perp} \lambda/\eta_{\parallel}$, $\theta = k_x/k_{\theta}$, and L_s is the shear length. Here, ϕ , which is the nonadiabatic part of the electron density response,¹¹ is related to the radial displacement ξ_r by

$$\phi = \frac{1+\{(\epsilon\omega\omega_*^2/D+i\lambda_{\perp}\Gamma\beta/2)/[\omega-(1-\epsilon)\omega_*]\}(1+\theta^2)}{1+(i\lambda_{\perp}\Gamma\beta/2\omega)(1+\theta^2)} \xi_r.$$

Equation (4) is the eigenmode equation upon which our analytical theory is based. It will be shown in Sec. 4 that ion sound contribution is indeed negligible in the semicollisional regime of interest here.

3. DERIVATION OF DISPERSION RELATION

In this section, we derive a dispersion relation for the semicollisional drift-interchange modes. First, we neglect the perpendicular resistivity, but will include it as a perturbation in the next section. Neglecting the perpendicular resistivity, Eq. (4) becomes, noting $b_0 = \epsilon \omega_*^2 / D$,

$$\frac{d}{d\theta} \frac{1 + \theta^2}{1 + [i\lambda/(\omega - \omega_*)](1 + \theta^2)} \frac{d}{d\theta} \tilde{\phi} \sim$$

$$\left[\frac{\omega - \omega_*}{\omega - (1 - \epsilon)\omega_*} \right] \frac{(1 - \epsilon)D + \omega(\omega - \epsilon\omega_*)(1 + \theta^2)}{1 + \omega b_0(1 + \theta^2)/[\omega - (1 - \epsilon)\omega_*]} \tilde{\phi} = 0. \quad (5)$$

We note that this equation can be also derived from the gyro-kinetic approach.¹¹

Although ion sound effects are neglected in Eq. (5), perpendicular compression of plasma is represented by the $\omega b_0(1 + \theta^2)/[\omega - (1 - \epsilon)\omega_*]$ term which comes from the divergence of polarization drift velocity in the pressure evolution equation. The inertial term $\omega(\omega - \epsilon\omega_*)(1 + \theta^2)$ has a factor $\omega - \epsilon\omega_*$ which makes the real frequency of the unstable mode approach $\epsilon\omega_*$ as we increase ω_* or decrease λ . This is the reason why the mode appears to be drift-wave-like.

From Eq. (5), we can identify at least the following three scaling lengths. $\theta_\lambda^2 \equiv |(\omega - \omega_*)/\lambda|$ corresponds to the inverse of resistive layer width where the stabilizing field line bending term [the first term in Eq. (5)] becomes small due to resistive dissipation at large θ . By balancing the inertia and the driving term $D(1 - \epsilon)$, we have $\theta_D^2 \equiv |D(1 - \epsilon)/\omega(\omega - \epsilon\omega_*)|$. The semicollisional (finite ρ_s) effect becomes non-negligible when $\theta^2 \sim \theta_b^2 \equiv |[\omega - (1 - \epsilon)\omega_*]/\omega b_0|$.

To make further analytical progress, we make the following subsidiary orderings. First, we note that for the collisional drift-interchange mode one has $\omega \sim D^{2/3} \lambda^{1/3}$ and, thus, $\theta_D > \theta_\lambda$ when $D < 1$. In the semicollisional regime, where ρ_s is greater than the resistive layer width, we have $\theta_D < \theta_\lambda$. As a result, we adopt the following orderings for the semicollisional drift-interchange mode

$$\theta_b \ll \theta_\lambda \ll \theta_D. \quad (6)$$

With orderings given by Eq. (6), Eq. (5) can be approximated by the following equations in the three different asymptotic regions. In region (i), where $\theta \ll \theta_\lambda$, we have

$$\frac{d}{d\theta} (1+\theta^2) \frac{d}{d\theta} \tilde{\phi}_i + \left(\frac{\omega - \omega_*}{\omega - (1-\epsilon)\omega_*} \right) \frac{(1-\epsilon)D}{1 + \omega b_\theta (1+\theta^2) / [\omega - (1-\epsilon)\omega_*]} \tilde{\phi}_i = 0. \quad (7)$$

In region (ii), where $\theta_b \ll \theta \ll \theta_D$, we have

$$\frac{d}{d\theta} \frac{\theta^2}{1 + i\lambda\theta^2(\omega - \omega_*)} \frac{d}{d\theta} \tilde{\phi}_{ii} + \frac{D(1-\epsilon)(\omega - \omega_*)}{b_\theta \omega \theta^2} \tilde{\phi}_{ii} = 0. \quad (8)$$

Finally, in region (iii), $\theta > \theta_\lambda$, we have

$$\frac{d^2}{d\theta^2} \tilde{\phi}_{iii} + \frac{i\lambda}{b_\theta \omega} \frac{D(1-\epsilon) + \omega(\omega - \epsilon\omega_*)\theta^2}{\theta^2} \tilde{\phi}_{iii} = 0. \quad (9)$$

Note that these three equations have common regions of validity and, hence, we can perform asymptotic matching.

The solution of Eq. (9), which behaves well at large θ , is given by

$$\tilde{\phi}_{iii} = \theta^{1/2} K_p \left\{ -i \left[\frac{i\lambda(\omega - \varepsilon \omega_*)}{b_\theta} \right]^{1/2} \theta \right\}, \quad (10)$$

where K_p is the modified Bessel function, and

$$p^2 = \frac{1}{4} - \frac{i\lambda D(1-\varepsilon)}{\omega b_\theta}. \quad (11)$$

In Eq. (10), we require

$$\operatorname{Im} \left[\frac{i\lambda(\omega - \varepsilon \omega_*)}{b_\theta} \right]^{1/2} > 0$$

to ensure proper behavior as $\theta \rightarrow \infty$. In the small- θ limit, Eq. (10) leads to

$$\frac{d\tilde{\phi}_{iii}}{d\theta} \sim \theta^{-1/2-p} - e^{-p\pi i} \frac{1/2 + p\Gamma(1-p)}{1/2 - p\Gamma(1+p)} \left[\frac{i\lambda(\omega - \varepsilon \omega_*)}{4b_\theta} \right] p \theta^{-1/2+p}. \quad (12)$$

Here, we remark that it is easier to match $d\tilde{\phi}/d\theta$ rather than $\tilde{\phi}$.

We now examine the solution of region (ii). To solve Eq. (8), we make the following transformation $t = 1/\theta$ and

$$U = - \frac{\theta^2}{1 + i\lambda\theta^2(\omega - \omega_*)} \frac{d}{d\theta} \tilde{\phi}_{ii} = \frac{t^2}{t^2 + i\lambda(\omega - \omega_*)} \frac{d}{dt} \tilde{\phi}_{ii}. \quad (13)$$

Equation (8) then becomes

$$\frac{d^2}{dt^2} U + \left[\frac{D(1-\varepsilon)(\omega - \omega_*)}{b_\theta \omega} + \frac{i\lambda D(1-\varepsilon)}{\omega b_\theta t^2} \right] U = 0, \quad (14)$$

which has a solution

$$U = t^{1/2} [z_{1p}(\delta t) + c_2 z_{2p}(\delta t)], \quad (15)$$

where Z_{1p} and Z_{2p} are Bessel functions to be specified and

$$\delta^2 = \frac{D(1-\epsilon)(\omega-\omega_*)}{b_0\omega} . \quad (16)$$

From Eq. (13), we have

$$\frac{d\tilde{\phi}_{ii}}{d\theta} = \left(\frac{i\lambda}{\omega-\omega_*} + \frac{1}{\epsilon^2} \right) \epsilon^{-1/2} [Z_{1p}\left(\frac{\delta}{\theta}\right) + c_2 Z_{2p}\left(\frac{\delta}{\theta}\right)] . \quad (17)$$

But in the region (ii), where Eq. (8) is valid, $\delta \ll \theta$ is satisfied such that the small-argument limit of Z_p can be used in Eq. (17) both in the small- and large- p limits of Eq. (8). Matching the large- θ solutions of Eq. (17) with that given by Eq. (12), we have

$$Z_{1p}\left(\frac{\delta}{\theta}\right) + c_2 Z_{2p}\left(\frac{\delta}{\theta}\right) \sim \theta^{-p} - e^{-p\pi i} \frac{1/2 + p\Gamma(1-p)}{1/2 - p\Gamma(1+p)} \left[\frac{i\lambda(\omega-\epsilon\omega_*)}{4b_0} \right]^p \theta^p . \quad (18)$$

Since Eq. (17) is valid even in the small- θ limit of Eq. (14), we then have

$$\begin{aligned} \frac{d\tilde{\phi}_{ii}}{d\theta} &\sim \theta^{-5/2} (Z_{1p} + c_2 Z_{2p}) \sim \epsilon^{-5/2-p} e^{-p\pi i} \frac{(1/2) + p\Gamma(1-p)}{(1/2) - p\Gamma(1+p)} \\ &\quad \times \left[\frac{i\lambda(\omega-\epsilon\omega_*)}{4b_0} \right]^p \theta^{-5/2+p} . \end{aligned} \quad (19)$$

We now consider the solution of Eq. (7). Since we are interested in a situation where the system is stable to ideal MHD modes, Suydam's criterion¹² requires $D < 1/4$ and we can treat the second term of Eq. (7) perturbatively. We then have

$$\frac{d}{d\theta} (1 + \theta^2) \frac{d}{d\theta} \tilde{\phi}_{i0} = 0 , \quad (20)$$

and

$$\frac{d}{d\theta} (1+\theta^2) \frac{d}{d\theta} \tilde{\phi}_{11} + \left[\frac{\omega-\omega_*}{\omega-(1-\epsilon)\omega_*} \right] \frac{(1-\epsilon)D}{1+[\omega b_\theta (1+\theta^2)]/[\omega-(1-\epsilon)\omega_*]} \tilde{\phi}_{10} = 0. \quad (21)$$

Equation (20) has a solution

$$\frac{d}{d\theta} \tilde{\phi}_{10} = \frac{c_0}{1+\theta^2}, \quad (22)$$

where $c_0 = 1$ for the tearing (even- ψ) mode and $c_0 = 0$ for the twisting (odd- ψ) mode. By integrating Eq. (21), we get the large- θ behavior of $\tilde{\phi}_i$

$$\begin{aligned} \frac{d}{d\theta} (\tilde{\phi}_{10} + \tilde{\phi}_{11}) \sim & \left\{ \frac{-2c_0}{\pi} + \frac{\pi D(1-\epsilon)(\omega-\omega_*)}{2b_\theta^{1/2} \omega^{1/2} [\omega-(1-\epsilon)\omega_*]^{1/2}} \right\} \theta^{-2} \\ & - \frac{D(1-\epsilon)(\omega-\omega_*)}{b_\theta \omega} \theta^{-3}. \end{aligned} \quad (23)$$

By matching Eq. (23) with Eq. (19) and applying the $p \rightarrow 1/2$ limit since $|\lambda D(1-\epsilon)/\omega b_\theta| \ll 1$ in the semicollisional regime, we get

$$\frac{\Delta'}{\pi} + \frac{\pi D(1-\epsilon)(\omega-\omega_*)}{2b_\theta^{1/2} \omega^{1/2} [\omega-(1-\epsilon)\omega_*]^{1/2}} = \frac{i^{3/2}(\omega-\omega_*)(\omega-\epsilon\omega_*)^{1/2}}{(\lambda b_\theta)^{1/2}}, \quad (24)$$

where we have replaced $-2c_0$ by Δ' according to the usual definition of Δ' in the Fourier transformed space.¹³ For the modes localized around the resistive layer, one has $\Delta' = 0$ for the twisting parity and $\Delta' = -2$ for the tearing parity mode. Equation (24) can also be expressed in the following slightly different fashion

$$\omega(\omega - \varepsilon\omega_*)[\omega - (1-\varepsilon)\omega_*] = -i\lambda\left\{\frac{\pi}{2} D(1-\varepsilon) + \frac{\Delta' b_0^{1/2} \omega^{1/2} [\omega - (1-\varepsilon)\omega_*]^{1/2}}{\pi(\omega - \omega_*)}\right\}^2. \quad (25)$$

Equation (24) or, equivalently, Eq. (25) is the desired dispersion relation for the semicollisional drift-interchange and drift-tearing modes when the perpendicular resistivity and ion sound effects are neglected. In the next section, we will discuss the properties of this dispersion relation and the effects of the perpendicular resistivity and ion sound terms.

4. PROPERTIES OF DISPERSION RELATION

For localized twisting-parity mode, $\Delta' = 0$ and Eq. (25) becomes

$$\omega(\omega - \varepsilon\omega_*)[\omega - (1 - \varepsilon)\omega_*] = -i\lambda\left[\frac{\pi}{2} D(1 - \varepsilon)\right]^2, \quad (26)$$

which in the limit $\varepsilon \rightarrow 0$ has the same form as that in the collisional regime⁴⁻⁶ except for the numerical enhancing factor $(\pi/2)^2$. Finite ε (compression in the perpendicular direction due to curvature drift) not only reduces the effective driving term $D(1 - \varepsilon)$ but also introduces a real frequency shift $(\omega - \varepsilon\omega_*)$. We can see that even a small ε changes the behavior of the eigenfrequency dramatically as $\lambda \rightarrow 0$. When $\varepsilon = 0$, we have

$$\omega = \pi(i\lambda)^{1/2} D / (2 \omega_*^{1/2}), \quad (27)$$

as $\lambda \rightarrow 0$ and $\omega \ll \omega_*$. The real frequency and growth rate are comparable and scale as $\lambda^{1/2}$. But, when ε is finite and $\lambda \rightarrow 0$,

$$\omega \approx \frac{\varepsilon \omega_* + i(\pi/2)^2 D^2 \lambda}{\varepsilon \omega_*^2}, \quad (28)$$

which shows that the real frequency of the mode approaches a finite value while the growth rate becomes very small. This is one of the reasons why even very small perpendicular resistivity can stabilize the mode completely.

For the tearing-parity mode, $\Delta' = -2$, but Δ' has a negligible effect on the eigenfrequency as long as $b_\theta \ll 1$ and ω is not very close to ω_{*e} or, more precisely, when $(\pi/2)D(1-\varepsilon) \gg (\Delta'/\pi)\varepsilon^{1/2} b_\theta^{1/2}$. This fact is further demonstrated in the numerical results to be discussed in Sec. 5.

It is interesting to note that, in the essentially opposite limit, i.e., when $|\omega - \omega_{*e}| \ll (2/\pi^2)[(1-2\varepsilon)^{1/2}/(1-\varepsilon)](\Delta'/D)b_\theta^{1/2} \omega_{*e}$, Eq. (24) reduces to

$$\omega^{1/2}(\omega - \omega_{*e}) = i^{3/2} \frac{\Delta'}{\pi} (\lambda b_\theta)^{1/2}, \quad (29)$$

as $\varepsilon, D \rightarrow 0$ while keeping b_θ constant. Equation (29) is the dispersion relation for the semicollisional drift-tearing mode in a sheared slab geometry⁷ in the absence of a temperature gradient. If we suppose $\Delta' > 0$, Eq. (29) shows that the unstable branch corresponds to $\omega \approx \omega_{*e}$, and, in this case, pressure driven term, D , has negligible effects. Ion sound terms, on the other hand, may be important.⁸

Now we investigate the effects of perpendicular resistivity. It is more convenient to include it as a perturbation to the previous result, Eq. (24), rather than solving Eq. (4) over again with four different asymptotic regions. As one can see from Eq. (4), the perpendicular resistivity effect comes in only when θ is very large. For such a large value of θ , Eq. (4) becomes

$$\frac{d^2}{d\theta^2} \tilde{\phi} + i \frac{\lambda}{b_\theta \omega} [\omega(\omega - \varepsilon \omega_*) + i \lambda_\perp \frac{\Gamma \beta}{2} \omega \theta^2] \tilde{\phi} = 0 \quad (30)$$

The unperturbed equation (zeroth order in $\lambda_\perp$) is given by

$$\frac{d^2}{d\theta^2} \tilde{\phi}^0 + i \frac{\lambda}{b_\theta} (\omega_0 - \varepsilon \omega_*) \tilde{\phi}^0 = 0 \quad (31)$$

By keeping in Eq. (30) terms of first order in $\lambda_\perp$, we have

$$\frac{d^2}{d\theta^2} \tilde{\phi}^1 + i \frac{\lambda}{b_\theta} (\omega_0 - \varepsilon \omega_*) \tilde{\phi}^1 + i \lambda_\perp \frac{\Gamma \beta}{2} \theta^2 \tilde{\phi}^0 = 0 \quad (32)$$

Imposing the solubility condition, we have

$$\omega_1 = -i \lambda_\perp \frac{\Gamma \beta \int_{-\infty}^{\infty} d\theta \theta^2 \tilde{\phi}^0 \tilde{\phi}^0}{\int_{-\infty}^{\infty} d\theta \tilde{\phi}^0 \frac{d\tilde{\phi}^0}{d\theta}} \quad (33)$$

Since most of the contribution to the integral comes from the large- θ region where $\tilde{\phi}^0 \approx \exp \{-[i\lambda(\omega_0 - \varepsilon \omega_*)/b_\theta]^{1/2} \theta\}$, we have

$$\omega_1 \approx \frac{\lambda_\perp}{4\lambda} \frac{\Gamma \beta b_\theta}{(\omega_0 - \varepsilon \omega_*)} \quad (34)$$

ω_1 introduces a stabilizing effect since $\text{Re}(\omega_0 - \varepsilon \omega_*) > 0$ and $\text{Im} \omega_0 > 0$. Physically, the perpendicular transport of plasma smears out the pressure perturbations near the resistive layer and, hence, provides stabilization. Equation (25) with Eq. (34) gives the eigenfrequency for the semicollisional drift-interchange and drift-tearing modes. For the drift-interchange modes, we can obtain the following critical value of ω_* above which the mode is stable by balancing the approximate growth rate given by Eq. (28) against the damping rate given by ω_1 of Eq. (34); i.e.,

$$\omega_{*c} = \left(\frac{\pi^2}{2}\right)^{1/3} \left(\frac{\lambda}{\lambda_1}\right)^{1/6} \frac{\lambda^{1/3} D^{5/6}}{\epsilon^{1/2} (\Gamma\beta)^{1/6}}, \quad (35)$$

or, in terms of b_θ ,

$$b_{\theta c} = \left(\frac{\pi^2}{2}\right)^{2/3} \left(\frac{\lambda}{\lambda_1}\right)^{1/3} \frac{\lambda^{2/3} D^{2/3}}{(\Gamma\beta)^{1/3}}. \quad (36)$$

Note that both finite- β (compression) and the perpendicular resistivity are necessary for complete stabilization. However, for the drift-tearing mode,

$$\omega_0 \approx \omega_* + i^{3/2} \frac{\Delta}{\pi} \frac{(\lambda b_\theta)^{1/2}}{\omega_*^{1/2}},$$

and, hence, the damping rate available from ω_1 is always smaller than the growth rate given by ω_0 by the ratio $\lambda_1 \epsilon \Gamma \beta / 4 \lambda D$, and the effect of perpendicular resistivity is negligible.

Now we can show that the ion sound term is actually negligible for the semicollisional drift-interchange mode. To demonstrate it, we take the ratio of two representative terms from Eq. (3) and find

$$\frac{k_{\parallel}^2 C_s^2}{\omega^2} \sim \frac{(x^2/Q^2)T}{(2/\Gamma\beta)T} \sim \frac{\Gamma\beta x^2}{2Q^2} \sim \frac{\Gamma\beta}{\omega^2 \theta^2} \sim \frac{\lambda(\omega_0 - \epsilon\omega_*)\Gamma\beta}{\omega_0^2} \ll 1$$

based on the semicollisional ordering and θ is taken to the typical mode width. Therefore, the ion sound term becomes negligible as we increase b_θ or decrease λ . On the other hand, $k_{\parallel}^2 C_s^2 / \omega^2 \sim \Gamma\beta / 2D$ in the collisional regime and ion sound terms cannot always be ignored.

5. NUMERICAL RESULTS

In this section, we describe the results obtained by integrating numerically the full set of equations; Eqs. (1), (2), and (3). The specific purpose is to resolve the differences between our conclusion and that of FMA's on the main stabilizing mechanism for the resistive interchange modes. As mentioned in Sec. 2, Eqs. (1), (2), and (3) can be Fourier transformed into the following fourth order system

$$\frac{d}{d\mu} \frac{\mu^2}{1+\mu^2, (Q+iQ_*)} \frac{d}{d\mu} \tilde{\phi} + (D - Q^2 \mu^2) \tilde{\phi} - (D + iQQ_* \mu^2) \tilde{\zeta} = 0, \quad (37)$$

and

$$\begin{aligned} \frac{d^2}{d\mu^2} \tilde{\zeta} + \left[\left(\frac{-Q^2}{Q+iQ_*} + \frac{\eta_{\perp}}{\eta_{\parallel}} Q + \frac{2iQ^3 g}{\Gamma D} \right) \mu^2 + \frac{D}{Q+iQ_*} + \frac{SQ^2}{D} \right] \tilde{\phi} \\ - \left(\frac{iQQ_*}{Q+iQ_*} + \frac{\eta_{\perp}}{\eta_{\parallel}} Q + \frac{2Q^2 Q_* g}{\Gamma D} \right) \mu^2 \tilde{\zeta} \\ + \left[\frac{D}{Q+iQ_*} + \left(1 + \frac{2}{\Gamma \beta} \right) Q(Q + iQ_*) - \frac{iQQ_* S}{D} \right] \tilde{\zeta} = 0 ; \end{aligned} \quad (38)$$

where

$$\tilde{\phi}(\mu) = \int_{-\infty}^{\infty} dx e^{i\mu x} \phi(x) ,$$

$$\phi(x) = D\Xi(x) + \frac{iQ_*}{Q} T(x) ,$$

and

$$\zeta(x) = D\Xi(x) - T(x) .$$

Equation (38) is apparently singular at $\mu = 0$. This singularity can be removed by including a k_0^2 contribution to ∇_1^2 . Since length has been already normalized with respect to the resistivity in Eqs. (1), (2), and (3) according to the so-called slow interchange ordering,² it is rather awkward to express k_0^2 in FMA's notation. From now on, we use the notation defined in Eq. (4). Equations (37) and (38) can then be written as

$$\frac{d}{d\theta} \frac{1 + \theta^2}{1 + [i\lambda/(\omega - \omega_*)](1 + \theta^2)} \frac{d}{d\theta} \tilde{\phi} + [D + \omega^2(1 + \theta^2)]\tilde{\phi} - [D + \omega\omega_*(1 + \theta^2)]\tilde{\zeta} = 0, \quad (39)$$

$$\begin{aligned} \frac{d^2}{d\theta^2} \tilde{\zeta} - \left[\left(\frac{-i\lambda\omega^2}{\omega - \omega_*} + i\lambda_1\omega + \frac{s\omega_*\omega^3}{\Gamma_D^2} \right) (1 + \theta^2) + \frac{i\lambda D}{\omega - \omega_*} + \frac{s\omega^2}{D} \right] \tilde{\phi} \\ + \left(\frac{-i\lambda\omega\omega_*}{\omega - \omega_*} + i\lambda_1\omega + \frac{s\omega_*^2\omega^2}{\Gamma_D^2} \right) (1 + \theta^2) \tilde{\zeta} \\ + \left[\frac{-i\lambda D}{\omega - \omega_*} + \left(1 + \frac{2}{\Gamma_D^2} \right) \omega(\omega - \omega_*) + \frac{\omega\omega_*s}{D} \right] \tilde{\zeta} = 0. \end{aligned} \quad (40)$$

Equations (39) and (40) are the basic equations for our numerical calculations. We again note that our model equation for the analytical calculation, Eq. (4), can be derived from Eqs. (39) and (40) by neglecting the ion sound terms.

Eigenfrequencies are found using the shooting method for both the fourth order system, Eqs. (39) and (40), and the second order system Eq. (4). The parameters are chosen to be relevant to current Reversed Field Pinch experiments³ as in the work of FMA; and, therefore, direct comparison with their results is possible. In particular, effects of the perpendicular resistivity and ion sound wave are examined. To isolate the effects of the perpendicular resistivity, we examine two cases with $\lambda_1/\lambda = 0$ and $\lambda_1/\lambda = 1$,

respectively. For each case, we integrate both the full set of equations with ion sound terms, Eqs. (39) and (40), and the model equation without ion sound terms, Eq. (4). The obtained complex eigenmode frequencies are plotted in Figs. (1) to (4) as functions of ω_{*e} ; while keeping the other parameters D , β , Δ' , s , λ , λ_1 fixed.

In Figs. (1) and (2) where the perpendicular resistivity is set to zero, we do not observe the complete stabilization of the modes no matter how large ω_* is increased. This conclusion is the same whether we include ion sound terms or not. The ion sound effects, in fact, are negligible in the semicollisional regime as we have indicated in Sec. 4. We also have very good agreement between analytical [Eq. (25)] and numerical results in the semicollisional regime.

On the other hand, in Figs. (3) and (4) where the perpendicular resistivity is kept finite, we observe the complete stabilization of the modes for the moderate value of ω_{*e} whether ion sound terms are kept or not. In this case, again, the effects of ion sound waves are negligible and the agreements between analytical [Eqs. (25), (34) and (35)] and numerical results are very good in the semicollisional regime.

We also note that the behavior of the eigenfrequencies are almost the same for the even and odd parity modes because, for the pressure driven modes we are considering, the effect of the ideal MHD region (characterized by Δ') is small as we have noted in Sec. 4.

Summarizing this section, we have numerically found that the semicollisional drift-interchange modes can be completely stabilized for a moderate value of ω_{*e} as indicated in FMA. We, however, have shown that the main stabilizing effect is due to perpendicular transport of the plasma. The ion sound terms, claimed as the main stabilizing mechanism in FMA, meanwhile,

have negligible effects on the mode frequencies and stabilities.

6. CONCLUSIONS AND DISCUSSIONS

We have studied the semicollisional drift-interchange modes in a cylindrical geometry. It is shown, both analytically and numerically, that the modes can be completely stabilized for moderate value of ω_{*e} . It is, furthermore, clearly demonstrated that perpendicular transport of the plasma is the main stabilization mechanism while ion sound effects are negligible. This is because due to the small parallel resistivity compared to the collisional case, the mode is more localized near the singular surface and perpendicular transport of plasma can readily smear out the instability-driving pressure perturbations in that narrow region. This has been explicitly shown in Sec. 4. The damping rate induced by the plasma transport [$\sim$ classical plasma diffusion coefficient/(mode width)²] is given by Eq. (34). By balancing the damping rate against the growth rate given by Eq. (28), we are also able to estimate the critical value of ω_{*e} , ω_{*c} [Eq. (35)], at which the mode becomes stabilized, and the results agree with the numerical results excellently.

ACKNOWLEDGMENTS

This work is supported by the United States Department of Energy Contract No. DE-AC02-76-CHO-3073. The authors would like to thank M. S. Chance, S. C. Jardin, W. M. Tang, J. W. Connor, and G. W. Hammett for useful discussions.

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FIGURE CAPTIONS

FIG. 1 Plot of twisting-parity eigenmode frequencies versus ω_{*e} . The parameters are $\lambda = 10^{-6}$, $D = 0.14$, $\Gamma\beta = 0.0512$, $s = 0.7$, $\lambda_L = 0$, and $\Delta' = 0$. x corresponds to analytical results given by Eqs. (25) and (34). o and $\blacktriangle$ and are direct numerical results with [Eqs. (39) and (40)] and without [Eq. (4)] ion-sound effects.

FIG. 2. Plot of tearing-parity eigenmode frequencies versus ω_{*e} . The parameters are $\lambda = 10^{-6}$, $D = 0.14$, $\Gamma\beta = 0.0512$, $s = 0.7$, $\lambda_L = 0$, and $\Delta' = -2$. x corresponds to analytical results given by Eqs. (25) and (34). o and $\blacktriangle$ and are direct numerical results with [Eqs. (39) and (40)] and without [Eq. (4)] ion-sound effects.

FIG. 3. Plot of twisting-parity eigenmode frequencies versus ω_{*e} . The parameters are $\lambda = 10^{-6}$, $D = 0.14$, $\Gamma\beta = 0.0512$, $s = 0.7$, and $\Delta' = 0$. x corresponds to analytical results given by Eqs. (25) and (34). o and $\blacktriangle$ and are direct numerical results with [Eqs. (39) and (40)] and without [Eq. (4)] ion-sound effects.

FIG. 4. Plot of tearing-parity eigenmode frequencies versus ω_{*e} . The parameters are $\lambda = 10^{-6}$, $D = 0.14$, $\Gamma\beta = 0.0512$, $s = 0.7$, and $\Delta' = -2$. x corresponds to analytical results given by Eqs. (25) and (34). o and $\blacktriangle$ are direct numerical results with [Eqs. (39) and (40)] and without [Eq. (4)] ion-sound effects.

Fig. 1

Fig. 2

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Fig. 3

Fig. 4

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