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**A METHOD FOR DETERMINING POLOIDAL
COIL CONFIGURATIONS FOR TOKAMAK DEVICES**

Kenneth Evans, Jr.

December 1990

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TABLE OF CONTENTS

	<u>Page No.</u>
ABSTRACT.....	1
1. INTRODUCTION....	2
2. THEORETICAL FORMULATION.....	4
2.1. MHD Equilibrium Basis.....	4
2.2 Calculation of Desired External Field.....	5
2.3 Determination of Coil Currents.....	6
2.4 Coil Locations.....	14
2.5 Ohmic Heating Systems.....	16
2.6 Ferromagnetic Materials.....	17
3. PRACTICAL CONSIDERATIONS AND EXPERIENCE.....	20
3.1 Closeness of External Field Match.....	20
3.2 Typical Design Sequence.....	26
3.3 Other Examples.....	29
4. COMPUTER PROGRAM.....	35
4.1 Compiling, Running, and Obtaining Output.....	35
4.2 Input File.....	36
4.3 MHD File.....	41
4.4 Output.....	43
ACKNOWLEDGMENTS.....	46
REFERENCES.....	46
APPENDIX A, INDUCTANCE CONSIDERATIONS.....	A1
A.1 Flux Function.....	A1
A.2 Calculation of Inductances.....	A3
A.3 Equivalent Inductances.....	A6
APPENDIX B, INPUT FILE.....	B1
APPENDIX C, OUTPUT FILE.....	C1
APPENDIX D, PLOT OUTPUT.....	D1

LIST OF FIGURES

<u>Figure No.</u>		<u>Page No.</u>
2-1	ψ , ψ_{int} , and ψ_{ext} contours for an $R_0 = 6.25$, circular, high-beta equilibrium.....	7
2-2	Required external fields for several plasma cross sections..	8
2-3	Stored energy vs. field error for several plasma cross sections.....	15
3-1	Coil configurations and currents for several values of field error.....	24
3-2	Comparison of desired and calculated plasma boundaries for several values of field error.....	25
3-3	Steps in arriving at a final set of coil locations. (a) Uniformly-spaced coils all around. (b) Determining locations of outer coils. (c) Determining locations of upper coils. (d) Final configuration.....	28
3-4	Poloidal coil set for APEX with an iron core.....	30
3-5	Poloidal coil set for STARFIRE with a normal ohmic-heating system.....	31
3-6	Poloidal coil set for WILDCAT, steady-state version.....	33
3-7	Poloidal coil set for WILDCAT, pulsed version with full ohmic-heating system.....	34
4-1	Grid for MHD variables.....	44
A-1	Coordinate system and reference surface for definition of the flux function.....	A1

LIST OF TABLES

<u>Table No.</u>		<u>Page No.</u>
3-1	Sensitivity to Field Error.....	22

A METHOD FOR DETERMINING POLOIDAL COIL CONFIGURATIONS FOR TOKAMAK DEVICES

Kenneth Evans, Jr.

ABSTRACT

This paper presents a method for obtaining the locations and currents of the poloidal coil systems for a tokamak, given a desirable magnetohydrodynamic equilibrium for the device. The method involves a simultaneous minimization of the match to the desired poloidal field and the stored energy in the coils, subject to the constraints necessary to achieve decoupling of the equilibrium and inductive-current-drive (ohmic-heating) systems and to achieve a given coupling of the current-drive system with the plasma. A compendium of mutual and self-inductance formulas as they apply to tokamak systems is presented, as well as examples of how the method has been used in the design of several tokamaks. Finally, a user manual for a computer code that implements this method is provided.

1. INTRODUCTION

The poloidal coil system for a tokamak is one of the major subsystems that must be designed whenever a new device is contemplated. This system is necessary to keep the plasma in magnetohydrodynamic (MHD) equilibrium, that is, to provide the forces to keep the plasma in balance. Typically this system, for tokamaks, consists of several coils which run symmetrically around the device in the azimuthal direction. The design effort consists in determining the locations of these coils and the currents in them as well as in the construction of the coils and the design of the power supplies and energy storage systems to drive the currents. The design must be coordinated with the design of the toroidal coils and must take into account other engineering constraints, such as access to the various components of the device and the location of vacuum ducts and other piping. In this paper we will only be concerned with the first two tasks, the determination of the locations and currents.

The poloidal coil system depends intimately on the MHD equilibrium, since the forces needed to balance the equilibrium depend sensitively on the characteristics of the equilibrium. We have found that it is most practical to design the equilibrium first, then determine a coil configuration that will support it. This makes the equilibrium calculations much simpler and more robust, and allows the maximum freedom in achieving desirable properties, such as high β . It allows one to make fixed-boundary MHD calculations, which are speedier and more amenable to parameter surveys, making it easier to determine optimum values for the MHD parameters.

In order for this approach to work, it must be possible to find an attractive poloidal configuration that will support the optimized equilibrium after it is determined. We have found the procedures described in this paper serve that purpose well. We have applied these procedures to a wide variety of tokamaks from small to very large ones, with and without iron cores, from circular to highly shaped, and with and without extensive inductive-current-drive (commonly called ohmic-heating) systems. We have found it has always been possible to relatively easily design a coil system, that had it been used in a free-boundary MHD calculation, would have given the same equilibrium. In

addition, there is typically a great deal of freedom in choosing the system, leaving the opportunity to optimize it.

The basic method used, a least-squares minimization of the poloidal field error and a simultaneous minimization of the stored energy, was first presented in Ref. 1 by A.M.M. Todd. The minimization of the stored energy was ostensibly to improve the conditioning of the least-squares matrix. This approach has been considerably extended into an extremely useful, general tool for designing poloidal systems, both for equilibrium and for inductive current drive. The purpose of this paper is to present a detailed description of the considerations involved in using the method to determine the currents and locations of the poloidal coils as well as several extensions to the basic method.

The material in Sec. 2 is a reasonably rigorous presentation of the formulas necessary to calculate the matrix elements of the least-squares minimization. These mostly pertain to mutual and self inductances and to the inductive coupling between various groups of coils with each other and with the plasma, which is in a sense a coil as well. While these formulas represent electromagnetic theory that has been known for years, and hence cannot be considered "new" work, it is quite convenient to have them collected in one place as they apply to tokamaks. Furthermore, there is a great deal of "bookkeeping" involved with the various coils and their groupings, so that it is an advantage to have the definitions of the corresponding tokamak-relevant equivalent currents and inductances in one place. This material should be considered as reference material and referred to as necessary, rather than as material that must be understood before proceeding. It will be more or less transparent depending on the experience of the reader. It is all essential for a true understanding of the interactions of the poloidal coils and the plasma, however. The more basic of the inductance considerations for an azimuthally symmetric device have been relegated to Appendix A.

The material in Sec. 3 represents practical experience in using the method after all the equations and features have been incorporated into a computer code. This material should be easier to follow and should give a feel for what can be and has been done.

Section 4 is a user manual for the code that has been developed at Argonne to implement this method.

2. THEORETICAL FORMULATION

The basic method used in the paper to design an equilibrium-field (EF) or ohmic-heating (OH) system for a tokamak consists of the following steps:

- A. Pick a fixed-boundary MHD plasma equilibrium.
- B. Determine that part of the equilibrium field that must be produced by the EF coils by subtracting the part produced by the plasma currents.
- C. Fix the EF coil positions and determine the EF coil currents so as to simultaneously minimize the deviation from the desired field and the stored energy, subject to the constraint (if desired) that the EF and OH systems be decoupled.
- D. Optionally, refine the design by varying the number of EF coils and their positions.

These steps are discussed in detail below.

2.1 MHD Equilibrium Basis

The MHD equilibrium field configuration, which satisfies the MHD equation, $\nabla p = \vec{J} \times \vec{B}$, and Maxwell's equations, is calculated in the standard ways, for example as described in Ref. 2. The equilibrium can be characterized by various plasma sizes, pressure profiles, peak pressures, degrees of diamagnetism or paramagnetism, boundary shapes, aspect ratios, safety factors, etc. The choice of equilibrium parameters should depend on the guidelines, constraints, and design optimizations for the device. The equilibria expected in a reactor, for example, are substantially different from those found in a small, contemporary, experimental device.

In designing a device our experience has indicated it is most convenient to be able to specify the plasma boundary, which must be a poloidal flux line for the plasma to be in equilibrium. The method is hence designed to use an equilibrium calculated by the fixed boundary method. (Equilibrium calculations where the plasma boundary is specified are referred to as fixed boundary calculations. Calculations where the currents in the EF coils are specified and the plasma boundary is thence determined are referred to as free-boundary calculations.) With this approach it is not necessary to calculate the

equilibrium configuration beyond the plasma boundary. It is useful, however, to make a free-boundary calculation afterward to insure that the EF coil configuration is adequate.

The factors which influence the choice of equilibrium parameters are not the subject of this report, however. For this method the desired, fixed-boundary equilibrium is assumed given before the coil design is started.

2.2 Calculation of Desired External Field

Given the equilibrium configuration and the corresponding plasma current distribution, J_p , it is possible to determine what part of that configuration is supplied by the plasma current alone. If the surrounding materials in the device all have permeability μ , this field can be calculated from the flux function, $\psi_{int} = -2\pi R A_{int,\phi}$, where

$$\psi_{int} = \mu \frac{\int_p J(R', Z') [AK(q) - BE(q)] dR' dZ'}{\sqrt{B}} \quad (2.1)$$

$$q^2 = \frac{4RR'}{(R + R')^2 + (Z - Z')^2} \quad (2.2)$$

$$A = R^2 + R'^2 + (Z - Z')^2 \quad (2.3)$$

$$B = (R + R')^2 + (Z - Z')^2. \quad (2.4)$$

See Appendix A.1. Cylindrical coordinates with the Z-axis vertical are assumed. The quantity $A_{int,\phi}$ is the vector potential in the Coulomb gauge of the field, $\vec{B}_{int,\phi}$, due to the plasma current alone, and $K(q)$ and $E(q)$ are the complete elliptic integrals of the first and second kinds. If magnetic materials such as iron are present, more sophisticated methods are required to determine ψ_{int} .

The part of the equilibrium field that must be supplied by the EF coils is then obtained from

$$\psi_{ext} = \psi - \psi_{int}, \quad (2.5)$$

where $\psi_{\text{ext}} = -2\pi R A_{\text{ext},\phi}$ is the flux function of the required EF field, and ψ is the flux function of the total MHD equilibrium field. The quantities $A_{\text{ext},\phi}$ and A_ϕ are the vector potentials of the external and total equilibrium poloidal fields respectively. The various magnetic field vectors are tangent to the contour lines of their respective flux functions.

It is only necessary to know ψ_{ext} inside the plasma, since any EF field that adequately matches ψ_{ext} inside the plasma will hold the plasma in equilibrium. It does not matter otherwise what the EF coil configuration is. (The field could even be produced by a conducting wall.) The match inside the plasma to the desired field must, however, be good enough. An indication of what is good enough is given in Sec. 3.1.

It is assumed that the OH field is negligible inside the plasma. If this is not the case, the flux function, ψ_{OH} , due to the OH coils must also be subtracted from ψ in Eq. (2.5) in order to get the desired ψ_{ext} .

Figure 2-1 shows examples of ψ , ψ_{int} , and ψ_{ext} for the reference equilibrium for a typical circular plasma.³ Figure 2-2 shows ψ_{ext} for a variety of cross-sectional shapes.

2.3 Determination of Coil Currents

Given the desired external field in the region of the plasma, the EF coil currents are determined so as to (1) match the desired field; (2) minimize the stored energy; and (3) optionally decouple the EF and OH systems. The method used is an extension of that suggested in Ref. 1. The EF coil currents are determined by minimizing the following quantity:

$$\epsilon = (1-r) \sum_{m=1}^M w_m (\psi_{\text{EF}}^m - \psi_{\text{ext}}^m)^2 + r\gamma \sum_{i,j=1}^{N'} \frac{1}{2} L_{ij} I_i I_j + \lambda\gamma \sum_{i=1}^N \psi_{\text{OH}}^{(i)} I_i. \quad (2.6)$$

The successive terms are associated with features (1)-(3) above. The quantity, ψ_{ext}^m , is the desired external field discussed in the last section and assumed specified at m points, $m = 1, \dots, M$. ψ_{EF}^m is the field at the same m points as produced by the actual EF coils in question. The quantity w_m is a weighting factor to allow some points to have more influence than others if

Fig. 2-1. ψ , ψ_{int} , and ψ_{ext} contours for an $R_0 = 6.25$, circular, high-beta equilibrium.

$\kappa = 1.0$ $\kappa = 1.3$ $\kappa = 1.65$ $d = 0.00$ $d = 0.25$ Z R $d = 0.50$

Fig. 2-2. Required external fields for several plasma cross sections.

desired. L_{ij} is the inductance matrix of the coil pairs i and j . It is assumed that each coil above the midplane has a mirror image counterpart an equal distance below the midplane. (The formalism is easily adjusted if this is not the case.) The inductances are the equivalent inductances of the coil pairs, and the currents, I_i , are the currents in either coil of the pair. A further discussion of equivalent inductances is given in Appendix A.3. The set of coil pairs considered in the stored energy term may be larger than the set of EF coil pairs; that is, N' may be greater than N . In particular, OH coils could be included if it is desired to minimize the energy of the joint system. The quantity γ is a normalizing factor to make the first and second terms comparable in magnitude, and the quantity r is a parameter to adjust the relative importance of these two terms. Setting $r = 0$ minimizes the field difference alone without regard to the stored energy, and setting $r = 1$ minimizes the stored energy without regard to the field match. Values of r in between represent tradeoffs between the two considerations. The quantity γ is a Lagrange multiplier associated with the constraint given in Eq. (2.7).

The EF and OH systems are decoupled if

$$\sum_{i=1}^N \psi_{OH}^{(i)} I_i = 0. \quad (2.7)$$

See Appendix A.3. $\psi_{OH}^{(i)}$ is the flux function of the OH system evaluated at either member of the EF coil pair i . (The OH field is assumed to not vary appreciably over the EF coils.) It can be seen that if all the EF coils lie on an OH flux line then Eq. (2.7) reduces to the requirement that the EF currents add to zero.

In order to get relatively simple equations from the minimization procedure, it is necessary to assume a linear relation between the currents and the flux function, ψ_{EF}^m . This relation may be written as:

$$\psi_{EF}^m = \sum_{i=1}^N a_{mi} I_i. \quad (2.8)$$

It is necessary that the desired flux function, ψ_{ext} , and the produced flux function, ψ_{EF} , have the same otherwise arbitrary zero, preferably $\psi = 0$ at $R = 0$. Since the fixed boundary MHD equilibrium calculation only calculates ψ inside the boundary, its zero is not usually known and it can contain an unknown constant ψ_0 which could give rise to unphysical fields at $R = 0$ as discussed in Appendix A.1. For the purposes of matching the fields inside the plasma, it is not necessary to worry about these fields, and it is convenient to make both the flux functions be zero at one of the given points, say $m = m'$. If ψ_{ext}^0 is the value of ψ_{ext} at this point and a_{0i} is the value of a_{mi} , this can be accomplished by the transformation for all m

$$\psi_{ext}^m \rightarrow \psi_{ext}^m - \psi_{ext}^0, \quad (2.9)$$

$$a_{mi} \rightarrow a_{mi} - a_{0i}. \quad (2.10)$$

It will be assumed henceforth in this paper that ψ_{ext} and ψ_{EF} have been adjusted to have the same zero.

In the case where all of the space surrounding the plasma has the same permeability, μ , the matrix, a_{mi} , can be written

$$a_{mi} = - [M_{m,1} + M_{m,2}] - a_{0i}, \quad (2.11)$$

where

$$M_{ab} = \mu \sqrt{R_a R_b} \left[\left(\frac{2}{k} - 1 \right) K(k) - \frac{2}{k} E(k) \right] \quad (2.12)$$

$$k^2 = \frac{4 R_a R_b}{(R_a + R_b)^2 + (Z_a Z_b)^2}. \quad (2.13)$$

The notation used in Eq. (2.11) means that for the index m , the coordinates (R_m, Z_m) of the point where ψ_{EF}^m is evaluated are to be used in Eqs. (2.12) and (2.13); and for the index 1 or 2, the coordinates (R_{i1}, Z_{i1}) or (R_{i2}, Z_{i2}) of the individual coils of the coil pair i are to be used. Equations (2.8) and (2.11)-(2.13) are a special case of Eq. (2.1) for thin coils. The validity of this approximation depends on the coils being sufficiently small that the integrand in Eq. (2.1) does not vary significantly over the coils. (The quantity M_{ab} is the mutual inductance of two thin coils located at (R_a, Z_a) and (R_b, Z_b) . See Appendix A.2.1.) Again, in the case where magnetic material is present, more sophisticated methods than in Eqs. (2.12) and (2.13) must be used, and the linearity of Eq. (2.8) must be checked.

If Eq. (2.8) is substituted into Eq. (2.6) and ϵ is minimized with respect to all or some of the currents I_i and λ by setting

$$\frac{\partial \epsilon}{\partial I_i} = 0 \quad (2.14)$$

$$\frac{\partial \epsilon}{\partial \lambda} = 0, \quad (2.15)$$

then a system of linear equations for I_i and λ results:

$$2(1-r) \sum_{m=1}^M w_m \left[\sum_{j=1}^N a_{mj} I_j - \psi_{ext}^m \right] a_{mi} + r\gamma_1 \sum_{j=1}^{N'} L_{ij} I_j + \lambda \gamma \psi_{OH}^{(i)} = 0 \quad (2.16)$$

$$\gamma \sum_{j=1}^N \psi_{OH}^{(j)} I_j = 0. \quad (2.17)$$

These are the basic set of equations to be solved.

It is desirable to consider two options before the equations are presented in their final form. The first is that it may be desirable to specify the currents in some of the EF coils instead of determining them. Sometimes, for example, the minimization procedure puts so much current in a

single coil that if the current density is kept constant, that coil becomes inordinately large, perhaps even overlapping adjacent coils. It is convenient in this case to specify the current in this coil at a value which is commensurate with a reasonable size of the coil and leave it out of the minimization.

The second option is that it may be desired to minimize the stored energy in all of the system (EF coils, OH coils, and plasma) rather than in the EF system alone. It is convenient to do this by treating the plasma and OH system as coils for which the currents are specified and for which the matrix elements, a_{mi} , in the minimization are zero.

The equations are then of the form $Bx = C$ with

$$x = (I_{\alpha}, \lambda) \quad (2.18)$$

$$B_{\alpha\beta} = 2(1-r) \sum_{m=1}^M w_m a_{m\alpha} a_{m\beta} + r\gamma L_{\alpha\beta} = B_{\beta\alpha} \quad (2.19)$$

$$B_{\alpha\lambda} = \gamma \psi_{OH}(\alpha) = B_{\lambda\alpha} \quad (2.20)$$

$$B_{\lambda\lambda} = 0 \quad (2.21)$$

$$C_{\alpha} = -2(1-r) \sum_{m=1}^M w_m \left[\sum_{\alpha'} a_{m\alpha'} I_{\alpha'} - \psi_m^{\text{ext}} \right] a_{m\alpha} - r\gamma L_{\alpha\alpha'} I_{\alpha'} \quad (2.22)$$

$$C_{\lambda} = -\gamma \sum_{\alpha'} \psi_{OH}(\alpha') I_{\alpha'}, \quad (2.23)$$

where the indices α and β range over EF coils whose currents are to be determined, the index α' ranges over coils whose currents are specified and which may include the plasma and OH coils, and the index λ refers to the terms associated with the Lagrange multiplier. These equations are easily inverted

by standard methods if the matrices, a_{mi} and L_{ij} , are given, the normalizing factor γ is specified, and a value of r is chosen.

For the case of thin coils with no magnetic material present, the inductance matrix of the coil pairs is given by:

$$L_{ij} = \begin{cases} 2 (L_{i1} + M_{i1,i2}) & i = j \\ 2 (M_{i1,j1} + M_{i1,i2}) & i \neq j \end{cases}, \quad (2.24)$$

where

$$L_a = \mu R_a \left[\log \frac{8R_a}{a_a} - \frac{7}{4} \right], \quad (2.25)$$

and M_{ab} is given by Eqs. (2.10)-(2.11). R_{i1} is the major radius of one coil of the pair i , a_{i1} is the minor radius, and Z_{i1} is the height above the midplane. These coordinates with 1 replaced by 2 refer to the second coil of the pair. A further discussion of inductances is given in Appendix A.

It should be noted that the treatment of the coils by pairs is for convenience only. The coils can be treated as single coils with no up-down symmetry if the inductance matrix is given by

$$L_{ij} = \begin{cases} L_i & i = j \\ M_{ij} & i \neq j \end{cases}, \quad (2.26)$$

and Eq. (2.11) is replaced by

$$a_{mi} = -\frac{1}{2} M_{mi}. \quad (2.27)$$

If it is desired to keep the current density constant in the coils, the coil radius, a_i , cannot be known before the currents are calculated. This problem can be overcome by a simple iteration. If arbitrary, reasonable values of the a_i are picked for the first determination of the currents, I_i , and the values for the n -th iteration are taken as

$$a_i^{(n)} = \left[\frac{I_i^{(n-1)}}{\pi J_i} \right]^{1/2}, \quad (2.28)$$

where J_i is the desired current density in the coil pair i , then a consistent set of a_i and I_i is obtained after a few iterations because of the weak dependence of the inductance on a_i .

A normalizing factor which makes the values of r for a given field error relatively independent of sizes, fields, number of coils, and number of field points is

$$\gamma = \frac{(M/N) \sum_{m=1}^M \sum_{i=1}^N w_m a_{ni}^2}{\sum_{i=1}^N L_{ii}^2}. \quad (2.29)$$

By varying r it is possible to get a relation between stored energy and field error. A graph of such relations for several cross-sectional shapes is shown in Fig. 2-3. In general, the stored energy does not change much for large field errors, and there is a critical value of stored energy and field error beyond which increasing the stored energy gives little improvement in field error. The maximum field error necessary for an adequate match to the plasma boundary is discussed in Sec. 3.1.

2.4 Coil Locations

The procedure described assumes the number and positions of the EF coils are known beforehand. The coil currents and quantities such as the stored energy are then determined by solving the equations. If the geometrical layout of the coils is poorly chosen, the currents and stored energy may be excessive or else the field match may be poor. To choose an optimum geometrical configuration remains a trial and error procedure.

A useful procedure for optimizing the geometrical configuration is to first choose a set consisting of a large number of coils approximately equally spaced around the plasma and as close to the plasma as engineering constraints allow. One can then begin eliminating coils that are calculated to have little current in them. The system can be "fine tuned" by making small

Fig. 2-3. Stored energy vs. field error for several plasma cross sections.

changes in the remaining coil locations after the coils have been reduced to a manageable number. An example of this procedure is given in Sec. 3.2.

To some extent picking the geometrical configuration is an art, and one's ability to design an efficient system increases with experience.

2.5 Ohmic Heating Systems

Essentially the same formulation can be used to design ohmic heating systems (perhaps more appropriately called current induction systems). In this case, the desired external field in the plasma is zero, and there is typically a requirement that the system produce a given number of volt seconds, $\Delta\phi_{OH}$. The first requirement can be fulfilled by replacing ψ_{ext}^m in Eq. (2.6) by zero.

The second requirement is equivalent to the constraint that:

$$M_{OH,P} I_{OH} = \sum_{i=1}^M M_{iP} I_i = \Delta\phi_{OH}, \quad (2.30)$$

where $M_{OH,P}$ is the equivalent mutual inductance of the ohmic heating system and the plasma, I_{OH} is the equivalent current, and M_{iP} is the mutual inductance of each coil pair of the OH system and the plasma. (Equivalent inductances and currents are discussed in Appendix A.3.) The solenoid that regularly appears in OH systems can be treated as a single coil, or may be divided into smaller segments. Care must be taken in using the thin coil approximations in any event. This constraint can be incorporated into Eq. (2.6) with an additional term

$$v_Y \sum_{j=1}^M (M_{jP} I_j - \Delta\phi_{OH}),$$

where v is a second Lagrange multiplier. The resulting changes are an additional term, $v_Y M_{iP}$, in Eq. (2.16); an additional linear equation

$$\gamma \sum_{j=1}^M M_{jP} I_j - \Delta\phi_{OH} = 0 \quad (2.31)$$

to go with Eqs. (2.16) and (2.17); additional matrix elements

$$B_{\alpha v} = \gamma M_{\alpha, P} = B_{v\alpha} \quad (2.32)$$

$$B_{\lambda v} = B_{v\lambda} = 0 \quad (2.33)$$

$$B_{vv} = 0 \quad (2.34)$$

for the expanded solution vector $x = (I_{\alpha}, \lambda, v)$; and an additional element in C

$$C_v = \gamma \Delta\phi_{OH} - \gamma \sum_{\alpha'} M_{\alpha', P} I_{\alpha'}. \quad (2.35)$$

2.6 Ferromagnetic Materials

If ferromagnetic materials, such as an iron core are present in the device, the relatively simple formulas in Eqs. (2.1), (2.2), (2.11)-(2.13), (2.24), and (2.25) for the flux functions (or vector potentials) and inductances are no longer valid. All of these formulas are special cases of the general form given in Eq. (2.1) derived under the assumption of azimuthal symmetry and constant permeability, μ .

The formalism developed above still applies for systems with ferromagnetic materials, but it is necessary to calculate the flux function for each EF coil, the OH system, and the plasma in the presence of the magnetic materials by some other means rather than using the formulas given. There exist codes such as TRIM⁵ capable of doing this numerically.

The following changes are necessary:

- (1) ψ_{int} is the flux function calculated for the plasma current distribution in the presence of the magnetic materials instead of that given by Eq. (2.1). It must be evaluated at the points where ψ_{ext}^m , $m = 1, \dots, M$ is given.
- (2) $\psi_{\text{OH}}(i)$, $i = 1, \dots, N$ must be calculated with the magnetic materials included. (The OH system itself will be different than if there were no magnetic materials.)
- (3) The matrix a_{mi} in Eq. (8) cannot be calculated by Eqs. (2.11)-(2.13). The matrix elements a_{mi} are the flux functions calculated with the magnetic material included for unit current in the coil pairs i . They must be evaluated at the points where the desired external field is given. The linear relation in Eq. (2.8) may not be true if magnetic materials are present. It should be checked that the EF coil configuration is adequate in the end.
- (4) The single coil self-inductances, L_a , in Eqs. (2.24) and (2.26) must be calculated by some other means. If the total energy, U_a , of the field due to the coil a is known or if the flux function, ψ_a , due to the coil is known sufficiently well inside the coil, the self-inductance can be obtained from

$$\frac{1}{2} L_a I_a^2 = U_a = \frac{1}{2} \int J_a \psi_a dR' dZ', \quad (2.36)$$

provided ψ_a is normalized to be zero at ∞ (see Appendix A). Alternatively if U_i is the total energy of the field of the coil pair i and ψ_i is the flux function, then

$$\frac{1}{2} L_{ii} I_i^2 = U_i = \frac{1}{2} \int_{\text{both coils}} J_i \psi_i dR' dZ', \quad (2.37)$$

provided ψ_i is normalized as above. These formulas can also be used for the plasma and OH system.

- (5) The single coil mutual inductances, M_{ab} , in Eqs. (2.24), (2.26), and (2.27) must be calculated by some other means. If J_a is the current density in coil a and ψ_b is the flux function in the presence of the magnetic materials for coil b, then

$$M_{ab} I_a I_b = - \int J_a \psi_b dR' dZ', \quad (2.38)$$

provided ψ_b is normalized as above. For thin coils this reduces to

$$M_{ab} = - \frac{\psi_b(a)}{I_b}. \quad (2.39)$$

Alternatively, if ψ_j is the flux function of the pair j then

$$M_{ij} = - \frac{2\psi_j(i)}{I_j}. \quad (2.40)$$

where ψ_j is evaluated at either coil of the pair i. These relations can also be used to calculate mutual inductances with the plasma and OH system provided they are symmetric about the midplane.

3. PRACTICAL CONSIDERATIONS AND EXPERIENCE

3.1 Closeness of External Field Match

As was discussed in Sec. 2.3, one can emphasize either the closeness of field match or the minimization of stored energy in the EF system by changing the parameter r in Eq. (2.6). Typical results are shown in Fig. 2-3. One, of course, does not want to waste stored energy getting a closer-than-needed match to the field. It is therefore useful to determine a maximum acceptable field error which in general adequately produces a desired MHD equilibrium and plasma shape. One can then find the value of r and the resulting coil currents corresponding to this field error. The associated set of currents should then have the lowest practical stored energy for the given coil locations. Less stored energy would imply an insufficient match to the equilibrium, and more stored energy would imply an unnecessarily close match to the equilibrium.

In order to illustrate how the coil currents and the resulting MHD equilibrium vary with the closeness of field match, we will consider the field error variation for a particular case. The coil configuration used for this illustration is a possible coil set for DEMO⁶ and the toroidal coils and MHD equilibrium are those for DEMO. The sample coil configuration consists of ten coils evenly spaced about 1 m outside the toroidal coils. This coil set, while not a practical one, has the flexibility to demonstrate the typical features of the error variation. The plasma boundary for DEMO is centered at a major radius of 5.2 m with a half-width of 1.3 m and a half-height of 2.08 m. The coils are located at $(R, Z) = (1.1, 0.5), (1.1, 2.2), (1.35, 4.0), (2.5, 5.4), (4.0, 6.32), (5.8, 6.9), (7.5, 7.0), (9.2, 5.35), (10.6, 5.4),$ and $(11.8, 4.0)$, where the coordinates are in meters. The first two coils were limited to a minor radius of 0.34 m or a current of 6.3 MA.

Coil currents have been obtained for six values of the RMS error in the field match. The RMS error is defined as:

$$\epsilon_{\text{RMS}} = \left[\sum_{m=1}^M (\psi_{\text{EF}}^m - \psi_{\text{ext}}^m)^2 \right]^{1/2} / (M \psi_{\text{ext,max}}), \quad (3.1)$$

where $\psi_{\text{ext,max}}$ is the maximum absolute value of the given flux function and M is the number of coil pairs. (The arbitrary constant in the flux function is determined by picking $\psi_{\text{ext}} = 0$ at a grid point closest to the center of the plasma as discussed in Sec. 2.3.) The coil currents and other properties of the resulting EF systems are shown in Table 3-1. These include the absolute error:

$$\epsilon_{\text{abs}} = \left[\sum_{m=1}^M \left| \psi_{\text{EF}}^m - \psi_{\text{ext}}^m \right| \right] / M \psi_{\text{ext,max}}; \quad (3.2)$$

the maximum error, ϵ_{max} , at any point; the stored energy, U_{stor} ; the total Ampere turns, I_{total} ; the total coil volume, V_{coil} ; and the coupling, $\Delta\phi_{\text{EF,p}}$, of the EF system with the plasma. The numbers are shown to three significant figures although no claim is made that they can in practice be determined that accurately. The coil configurations are shown graphically in Fig. 3-1. As expected, the stored energy, Ampere turns, and coil volume increase as the field match becomes better. The coupling, in contrast, is not monotonically related to the error.

The desired equilibrium was calculated with the Argonne, fixed-boundary equilibrium code² to have the desired equilibrium properties and boundary shape for, in this case, DEMO. Using the calculated coil currents for the six error values, free boundary calculations were performed in order to see how the plasma properties and boundary shapes match those desired. Several of these properties as calculated with the Princeton, free-boundary, equilibrium code⁷ are shown in the lower part of Table 3-1, along with the values of the desired equilibrium. The plasma current, the peak pressure, and the pressure profile parameters have been specified in these calculations and are the values for the desired equilibrium. (The definitions of the MHD equilibrium properties listed are given in Ref. 2. The functions in the Princeton code have been modified to be consistent with those in Ref. 2.) The plasma boundaries from the free boundary calculations are compared with the desired boundary in Fig. 3-2.

The $\epsilon_{\text{RMS}} = 10^{-2}$ case is clearly inadequate for matching the plasma. For the other cases the plasma parameters for the most part become successively

Table 3-1. Sensitivity to Field Error

RMS Error, ϵ_{RMS}	1×10^{-2}	3×10^{-3}	1×10^{-3}	3×10^{-4}	1×10^{-4}	1.14×10^{-5}	Desired Values
Absolute error, ϵ_{abs}	2.15×10^{-1}	5.54×10^{-2}	1.80×10^{-3}	5.09×10^{-3}	1.71×10^{-3}	1.59×10^{-4}	--
Maximum error, ϵ_{max}	5.99×10^{-1}	2.06×10^{-1}	7.52×10^{-2}	2.47×10^{-2}	9.29×10^{-3}	1.78×10^{-3}	--
Parameter, $-\log_{10} r$	1.22	2.52	3.29	3.93	4.61	9.00	--
Stored energy, U_{stor} , GJ	0.37	1.97	3.34	4.06	4.32	17.8	--
Total current, I_{total} , MA	11.6	34.2	60.2	67.5	67.2	243.0	--
Coil volume, V_{coil} , m^3	26.5	67.0	99.4	112.0	118.0	396.0	--
Coupling, $\Delta\phi_{\text{EF},p}$, V-s	-19.2	-38.9	-37.1	-34.5	-35.3	-40.6	--
Coil currents, I_1 , MA	1.01	-2.81	-6.34	-6.34	-6.34	-6.34	--
I_2 , MA	0.73	0.99	0.20	-0.30	-0.91	-6.34	--
I_3 , MA	0.20	2.68	5.34	5.57	3.83	-3.79	--
I_4 , MA	-0.04	2.28	5.38	6.42	5.89	28.61	--
I_5 , MA	-0.28	1.12	3.38	4.50	4.90	-29.84	--
I_6 , MA	-0.47	0.02	1.01	1.69	2.23	26.67	--
I_7 , MA	-0.57	-0.67	-0.24	0.01	0.28	2.64	--
I_8 , MA	-0.70	-1.44	-1.47	-1.36	-1.18	-11.67	--
I_9 , MA	-0.81	-2.12	-2.66	-2.87	-2.94	-2.05	--
I_{10} , MA	-0.98	-2.97	-4.11	-4.68	-5.12	-3.47	--

Table 3-1. Sensitivity to Field Error (continued)

RMS Error, ϵ_{RMS}	1×10^{-2}	3×10^{-3}	1×10^{-3}	3×10^{-4}	1×10^{-4}	1.14×10^{-5}	Desired Values
Poloidal beta, β_p	0.01	1.01	1.36	1.58	1.57	1.56	1.59
Toroidal beta, β_t	0.1070	0.0746	0.0725	0.0735	0.0741	0.0739	0.0750
Safety factor (limiter), q_L	0.01	1.05	1.67	2.10	2.05	2.01	2.00
(axis), q_0	0.03	0.58	0.82	0.93	0.92	0.92	1.00
Average pressure, $\bar{p}$, MPA	0.987	0.687	0.667	0.667	0.682	0.681	0.690
Magnetic axis, R_{mag} , m	6.31	5.41	5.42	5.44	5.44	5.45	5.46
Inner boundary, R_{max} , m	6.12	3.98	3.85	3.85	3.87	3.89	3.90
Outer boundary, R_{max} , m	6.50	6.50	6.50	6.50	6.49	6.49	6.50
Height, Z_{max} , m	0.15	1.41	1.83	2.08	2.08	2.08	2.08
Diagnostic wall depth, δ	-42.7	-0.0010	0.0672	0.0876	0.0867	0.0884	0.0889
Plasma volume, V_p , m ³	4.0	185.0	248.0	281.0	277.0	276.0	273.0
Plasma circumference, C_p , m	1.14	8.40	10.0	10.9	10.8	10.8	10.8
Flux function (limiter), ψ_L , Wb	72.9	6.48	32.2	30.2	30.1	29.8	29.6

Fig. 3-1. Coil configurations and currents for several values of field error.

Fig. 3-2. Comparison of desired and calculated plasma boundaries for several values of field error.

closer to the desired values as the field error decreases. The differences that exist for the lowest value, $\epsilon_{\text{RMS}} = 10^{-5}$, of error obtainable for this equilibrium and coil configuration are partly due to the use of different grids and the way the currents are distributed among grid points in the Princeton free-boundary code. The plasma boundary gets successively closer to the desired boundary as the error decreases. At $\epsilon_{\text{RMS}} = 10^{-5}$ the maximum deviation is ~ 2 cm, certainly a very close match when one considers that in reality the plasma edge is quite fuzzy.

On the other hand, the currents for the $\epsilon_{\text{RMS}} = 10^{-5}$ case are impractically large. This case illustrates the typical occurrence of adjacent large currents of opposite sign which give canceling fields to provide the fine shaping of the plasma EF field necessary for small errors. The intermediate value, $\epsilon_{\text{RMS}} = 3 \times 10^{-4}$, gives a reasonably close fit to the plasma parameters, certainly within the accuracy with which the code describes a real plasma. The plasma boundary for this case is within 10 cm of the desired boundary and at most places much closer. Based on this and other similar sensitivity studies, an RMS error of 3×10^{-4} has been found to be a good choice for adequately matching the field.

Earlier Argonne work (through STARFIRE⁸) used a criterion of $\epsilon_{\text{abs}} = 4 \times 10^{-3}$, which gives similar results. Use of the RMS error, however, more closely represents the function being minimized. Experience on a number of designs has indicated either of the two criteria consistently gives an adequate match to the desired equilibrium.

Given a desired field match of $\epsilon_{\text{RMS}} = 3 \times 10^{-4}$, the value of r to use in Eq. (2.16) can be obtained by iteration. Since for different designs the value of r that gives this error can vary by several orders of magnitude, it is convenient to iterate on $\log r$.

3.2 Typical Design Sequence

The method that has been described in this paper optimizes the poloidal coil system for given locations of the poloidal coils and more or less automates that process. It is still necessary to choose optimum coil locations, however. It has not been found practical to automate the coil location process because of the many design-related decisions that must be made. This section provides an illustrative example of how this process was

carried out for a typical tokamak, in this case a reactor design with a major radius of 7 m.¹⁰ There were four major steps, which are illustrated in Fig. 3-3.

First, a set of 20 coils was chosen. These were equally spaced around the toroidal coil and a distance of 1 m from it. The particular choice of 1 m was thought to be a reasonable compromise between having closer, more effective equilibrium coils and both reducing the bending moment on the toroidal coils and maximizing the toroidal field produced by the toroidal coils. (The toroidal field is reduced from the peak field for the superconductor by whatever field is produced by the poloidal coils.) It can be readily seen in Fig. 3-3a where it would be most useful to put the equilibrium coils. Typically, this is in three regions: the inboard midplane, the top inside corner, and near the outboard midplane. The top coils pull the plasma out, and the other two sets of coils push it in. The design at this point has a stored energy of 4.9 GJ, total Ampere-turns of 61 MA, and a coil volume of 156 m³. We have specified the error we want in matching the desired equilibrium field according to the discussion in Sec. 3.1, so all of the configurations tested are equally good in terms of matching the field. For this example we wanted to reduce the number of coils for cost and access reasons, so we expect the stored energy to be greater than 4.9 GJ.

In the second step, illustrated in Fig. 3-3b, the optimum position for the outboard coils was determined by keeping the top and inboard coils evenly spaced out and fixed. The outboard coils are important as they contain most of the stored energy and are the most massive. They usually have to be positioned to allow access on the outboard side, where the toroidal coils are furthest apart. The configuration shown in Fig. 3-3b was picked as most optimum in terms of lowering the stored energy and in consideration of the engineering constraints for this particular design.

A similar procedure was followed in the third step, illustrated in Fig. 3-3c. In this case the outboard coils were evenly spaced out and fixed, and the top coils were varied so both sets of coils would be treated "equally". The configuration chosen was the result of successively eliminating and moving coils in the top region and comparing the stored energy, Ampere-turns, and coil volume.

(a)

(b)

(c)

(d)

Fig. 3-3. Steps in arriving at a final set of coil locations. (a) Uniformly-spaced coils all around. (b) Determining locations of outer coils. (c) Determining locations of upper coils. (d) Final configuration.

The final design is shown in Fig. 3-3d. The inboard coils are important as they have the highest field and the worst access. They often must be limited in size, as the stored energy typically favors more current in this region than can be put there. (For this example, however, the coils were not limited.) The stored energy is usually somewhat insensitive as to where these inboard coils are located vertically, in spite of the impression that is given in Fig. 3-3a. For this reason they were not located keeping the other sets of coils evenly spaced out. There was a certain amount of fine tuning and determining the sensitivity of the configuration to small changes in the locations before the final configuration was fixed. In the final configuration the stored energy has increased to 5.6 GJ; the Ampere-turns have decreased to 50 MA; and the coil volume has decreased to 136 m. The design fulfilled the engineering design constraints and cost considerations for the device.

3.3 Other Examples

The poloidal coil configurations designed by this method are shown for three other tokamaks with differing parameters in Figs. 3-4 through 3-7. Figure 3-4 is for APEX,¹¹ a small iron-core tokamak. The APEX poloidal configuration was determined using the considerations in Sec. 2.6. The calculations with the iron core were made with TRIM.⁵ The necessity of making a separate TRIM calculation for the plasma alone, and each of the coils alone in order to get the matrix elements, a_{mi} , makes the procedure for the iron core a much more laborious one. The figure shows the final TRIM calculation for all of the fields combined. This design was built and worked as designed.

The coils are shown as rectangular for this case as such coils are commensurate with the TRIM grid. For the other cases, the coils are really the same as point coils except for the use of the correct self-inductance. The circles shown are for visualization only and could equally well be squares or any other compact shape. Their area is determined by the specified current density in the coils. If more extended coils are desired, they should be represented by a set of adjacent, discrete point coils, preferably with cross-sectional areas which add up to the real coil area.

Figure 3-5 shows the poloidal configuration for STARFIRE.⁸ STARFIRE, which is steady state with a major radius of 7 m, has a D-shaped cross section and a small ohmic-heating system providing 25 V-sec for startup. Most of

Fig. 3-4. Poloidal coil set for APEX with an iron core.

Fig. 3-5. Poloidal coil set for STARFIRE with a normal ohmic-heating system.

the plasma current is rf driven. The configuration for the steady-state version of WILDCAT,¹² a DD reactor design, is shown in Fig. 3-6. This version of WILDCAT also has a 25 V-s ohmic-heating system, but is much larger, with major radius 8.6 m, and has higher fields and currents. The difference in the placement of the coils occurs primarily because of the difference in the plasma shape. Finally, the configuration for the pulsed version¹³ of WILDCAT, which has a full ohmic-heating solenoid decoupled from the equilibrium coil system, is shown in Fig. 3-7. This configuration illustrates the simulation of an extended coil, the solenoid, by a set of discrete coils. Such a simulation is, in fact, accurate. When the straight-solenoid section for this design was made (much) longer, it accurately reproduced the long, thin, solenoid fields given in textbooks.⁴

Fig. 3-6. Poloidal coil set for WILDCAT, steady-state version.

Fig. 3-7. Poloidal coil set for WILDCAT, pulsed version with a full ohmic-heating system.

4. COMPUTER PROGRAM

The EFC code is the code developed to implement the method described in this report. It takes a required external flux function and specified locations of the coils and calculates the inductances and other parameters of the coils, performs the minimization, and determines the coil currents.

The code was originally designed to calculate EF systems. In the EF calculation the coils of an already-designed OH system and/or already-calculated plasma parameters can be specified in order to calculate the couplings and energies. The same method can be used to calculate an OH system as described in Sec. 2.5. In this case it is necessary to specify the desired coupling (volt-seconds) with the plasma, and already-calculated EF coils can be specified to calculate the couplings.

In both cases there are two modes of operation. One can calculate a series of solutions corresponding to different values of the parameter r in Eq. (2.6). These solutions will match the required field to different accuracies and will have correspondingly different stored energies, coil currents, etc. Alternatively, one can iterate to a solution with a specified accuracy of match to the required field as described in Sec. 3.1. The latter is the typical approach with the former being used when troubles arise.

There are a number of options to the code, and these are specified below along with a description of the input variables and format.

4.1 Compiling, Running, and Obtaining Output

The code was written for use on the National Magnetic Fusion Energy Computer Center (NMFECC) and uses a few routines that are specific to that environment. These would have to be changed to port the code to a new machine. In addition, the plots use DISSPLA from Integrated Software Systems Corporation, which is available at NMFECC and many other locations. The source code is called EFC and is compiled and loaded on the NMFECC machines by entering the command line:

```
RCFT I=EFC LIB=DISSPLA
```

(Upper case characters will be used in this section to describe computer variables, programs, and commands. It may be necessary, as it is on the NMFECC machines, to enter these in lower case, and the variables may appear as lower case in the listings, input, and output.)

The RCFT command above will produce an executable file, XEFC, which can be run by entering the command line:

```
XEFC I=inputfile MHD=mhdfile
```

where "inputfile" is the name of the input file, described in Sec. 4.2, and "mhdfile" is the name of the MHD file, described in Sec. 4.3.

The XEFC command will produce up to three files. The output file has the same name as the input file with an "O" attached to the end. If there is a coil output file, it will have the same name but with a "C" attached, and if there is a plot (graphics) file, it will have a "G" attached.

On the NMFECC system hard-copy output can be obtained by entering the command lines:

```
NETOUT inputfileO
NETOUT inputfileC
NETPLOT inputfileG DISSPLA
```

with whatever additional options may be necessary for the desired site, printer, etc.

4.2 Input File

The first line of the input file is a label, up to 80 characters [FORMAT(10A8)], describing the run. It will appear on the plots and output.

The next lines of input are in NAMELIST form and include the following switches and parameters:

```
ICLIM  = 1      ACOILM(I) = RCOIL(I) unless explicitly specified.
        = 2      ACOILM(I) = ACMAX unless explicitly specified.
```


IMODE = 1 Loop over $-\log_{10} r$ values from RR(1) to RR(2) in
 RR(3) increments.

 = 2 Iterate on $-\log_{10} r$ values to obtain ERR = ERRO. RR(1) is
 the best guess of $-\log_{10} r$. RR(2) is the lowest value to
 consider, and RR(3) is the highest value.

IDCPL = 0 No EF-OH decoupling.

 = 1 Approximate decoupling with $\sum_{EF} I_i = 0$.

 = 2 Decoupling via Eq. (2.7). OH coils must be specified.

ISCALE = 0,1 Do not, do scale MHD input to $R_0 = R00$ and $F_0 = F00$.

ILOOP = 0,1 Do not, do iterate on ACOIL(1) to get specified CURDEN as
 in Eq. (2.28).

IRON = 0,1 Air, iron core.

IAPNT = 0,1 Do not, do print a matrix from Eq. (2.8).

IBPNT = 0,1 Do not, do print B matrix from Eqs. (2.19)-(2.21).

IRPNT = 0,1 Do not, do print $-\log_{10} r$ iteration information for
 IMODE = 2.

IBOUND = 0,1 Use all points, only boundary points to match ψ_{ext} .

IPLOT = 0,1 Do not, do make plots.

ITRPNT = 0,1 Do not, do print information for iteration on ACOIL(1).

INDPNT = 0,1 Do not, do print inductance matrix. The inductance matrix relates each separate ITYPE = 1 or 2 coil, OH system, and plasma system, if the latter is specified. The OH system equivalent current is assumed to be -1.0 MA and the plasma equivalent current is assumed to be the plasma current.

IETOT = 0,1 Minimize EF (or OH) stored energy, total energy.

IOHSYS = 0,1 Do EF, OH system.

ICOILS = 0,1 Do not, do make a coil data set that can be used to input fixed coils.

IMODEL = 0,1,2 No, all around, outboard only model coils using DIST and NMCOILS.

ITFC = 0,1 No, standard constant-tension TF coils based on DELV, DELBSI, DELBSO, DELTFC, DELD.

RR(1) (1 = 1,3) Values of the parameter, r, according to IMODE. Default is 4.0, 0.0, 8.0. (May be specified to nearest 0.001.)

R00 MHD equilibrium input variables are scaled to major radius $R_0 = R00$ if ISCALE = 1 (m). Default is 6.0 m.

F00 MHD equilibrium input variables are scaled to $F_0 \equiv RB_{t0} = F00$ if ISCALE = 1 (T-m), where B_{t0} is the vacuum magnetic field at R_0 . Default is 30 T-m.

DELPHI Desired OH-plasma coupling. Default is 0.

CURDEN Current density in coils (MA/m^2). Default is $15.0 \text{ MA}/\text{m}^2$.

ERRO	Desired RMS error of field match. (See Sec. 3.1.) Default is .0003.
ACMAX	Maximum coil size (m). Default is 0.5 m.
DELV	Width of plasma scrape-off region (m).
DELBSI	Inside blanket/shield thickness (m).
DELBSO	Outside blanket/shield thickness (m).
DELTFC	TF coil thickness (m).
DELD	Spacing between blanket/shield and outer leg of TF coil (m).
DIST	Distance of poloidal coil centerlines from TF coil (m).
NMCOILS	Number of model coil pairs.

ROO and FOO do not need to be specified if ISCALE = 0. DELPHI does not need to be specified if IOHSYS = 0. DELV, DELBSI, DELBSO, DELTFC, and DELD do not need to be specified if ITFC = 0. DIST and NMCOILS do not need to be specified if IMODEL = 0.

The toroidal field (TF) coil model is based on Moses and Young.¹⁴ The needed quantities, DELV, DELBSI, DELBSO, DELTFV, and DELD, only need to be specified so that the inner and outer legs of the TF coils end up in the right place. If R_1 and R_2 are the coordinates of the midpoints of the inner and outer legs, respectively, then

$$R_1 = R_0 - a - \text{DELBSI} - \text{DELTFC}/2 \quad (4.1)$$

$$R_2 = R_0 + a + \text{DELBSO} + \text{DELD} + \text{DELTFC}/2, \quad (4.2)$$

where R_0 is the major radius of the plasma, and a is the minor radius. If this model is not adequate, the code could easily be modified to read in the points along the TF coil boundaries.

The last lines of the input file are the coil parameters, one line [FORMAT(I1,7F10.0)] for each coil consisting of the following parameters:

ITYPE RCOIL ZCOIL ACOILM ICOIL XL ACOIL

with the definitions:

ITYPE	Type of Coil:
	0 EF (or OH) with currents to be determined
	1 EF (or OH) with specified currents
	2 OH coils (not applicable if IOHSYS = 1)
	3 Plasma
RCOIL(I)	R coordinate of center of i-th coil (m).
ZCOIL(I)	Z coordinate of center of i-th coil (m).
ACOILM(I)	Maximum value of minor radius of i-th coil (m).
ACOIL(I)	Minor radius of i-th coil (m).
ICOIL(I)	Coil current in each member of i-th coil pair (MA).
XL(I)	Self-inductance of i-th coil (μ H).

These lines must be in order of increasing ITYPE. The quantity ITYPE determines how the coils are to be treated. If IOHSYS = 0, then ITYPE = 0 and 1 coils are EF coils and ITYPE = 2 coils are OH coils. If IOHSYS = 1, then ITYPE = 0 and 1 coils are OH coils, and there can be no ITYPE = 2 coils. ITYPE = 2 coils are necessary only if decoupling is required, and an ITYPE = 3 coil is required only if IOHSYS = 1 or decoupling with IETOT = 1 is required. Including ITYPE = 2 or 3 coils results in coupling information even if the coils are not otherwise required.

The coordinates of the coil centerline RCOIL and ZCOIL, are required and must be specified. The maximum coil radius, ACOILM, is optional. The default depends on the value of ICLIM. The current, ICOIL, in either member of the coil pair is specified in MA, but is treated throughout the code and printed in A. A specified value of ICOIL is ignored if ITYPE = 0. The coil self-inductance, XL, is specified in μH but treated and printed in H. It is required if IRON = 1 unless ITYPE = 0 and is optional otherwise. If not specified, it is calculated from Eq. (2.25). The coil radius is optional unless ITYPE = 2, IRON = 0, and XL = 0. The default is 0.15 m, the starting value for the ACOIL iterations, if ICOIL = 0.0 and is calculated as in Eq. (2.28) if ICOIL $\neq$ 0.0.

In typical use only RCOIL and ZCOIL (and possibly ACOILM) are specified for ITYPE = 0 coils, and ACOIL, ZCOIL, ICOIL, and ACOILM are specified for ITYPE = 1 or 2 coils. For the plasma, ITYPE = 3, all of the quantities are specified with ACOIL = ACOILM = plasma minor radius. The current, ICOIL, is the total plasma current for ITYPE = 3. (The plasma is not treated as a coil pair.)

A sample input file is given in Appendix B. This produces the case depicted in Fig. 3-3d. (In Sec. 3.2 the twenty original coils were positioned using IMODEL = 1, then written out using ICOILS = 1. The resulting coil data set was then modified and used as input during the design process described in Sec. 3.2, with the input file listed in Appendix B being the final result.)

4.3 MHD File

The MHD input file is read with the following lines:

```

      REAL JPHI
1     FORMAT(5F16.0)
2     FORMAT(5I16)
      READ(50,2) JMAX,KMAX,NMAX
      READ(50,1) ALPHA,BETA,ALPHAJ,CAPPA,DEITY,ASP,RO,FWALL,
1 BETAT,QSAFE,PCUR
      READ(50,1) R1,21,DR,DZ
      READ(50,1) (RJ(J),J=1,JMAX)
      READ(50,1) (ZK(K),K=1,KMAX)

```

```

READ(50,1) (PSIEXT(N),N=1,NMAX)
READ(50,1) (JPHI(N),N=1,NMAX)
READ(50,1) (IPSI(N),N=1,NMAX)

```

The MHD input variables are defined as follows:

JMAX	Number of R coordinates in grid.
KMAX	Number of Z coordinates in grid.
NMAX	Number of points in grid.
ALPHA	Pressure profile exponent.
BETA	Diamagnetic function profile exponent.
ALPHAJ	MHD equilibrium parameter.
CAPPA	Plasma elongation.
DEITY	Plasma D-shapedness parameter.
ASP	Aspect ratio.
RO	Plasma major radius, R_0 , (m).
FWALL	Vacuum diamagnetic function, F_0 , (T-m).
BETAT	Plasma toroidal beta.
QSAFE	Safety factor at the limiter.
PCUR	Plasma current (A).
R1	First R coordinate in grid (m).
Z1	First Z coordinate in grid (m).
DR	R grid spacing (m).
DZ	Z grid spacing (m).
RJ(J)	Array of R coordinates in grid (m).
ZK(K)	Array of Z coordinates in grid (m).
PSIEXT(N)	Required flux function matrix (Wb).
JPHI(N)	Current density matrix (A/m ²).
IPSI(N)	Grid parameter matrix.

The variables ALPHA, BETA, ALDHAJ, CAPPA, DEITY, ASP, BETAT, and QSAFE are not essential. They are not used in the calculations but are printed to identify the equilibrium.

The arrays PSIEXT, JPHI, IPSI, RJ, and ZK are defined on a grid. The R and Z coordinates of the grid are given by:

$$\begin{aligned} RJ(J) &= R1 + (J - 1)\Delta R & J &= 1, \dots, JMAX \\ ZK(K) &= Z1 + (K - 1)\Delta Z & K &= 1, \dots, KMAX. \end{aligned}$$

The grid points are also alternatively labeled by the single parameter, N, with

$$N = J + (K - 1) * JMAX \qquad N = 1, \dots, NMAX.$$

The parameter IPSI is defined as follows:

IPSI = 0 Point on or inside plasma boundary.
 IPSI $\neq$ 0 Otherwise (actual value is not important).

Only points in the plasma are considered, so that the array elements of the arrays except for IPSI do not need to be specified for points outside the plasma boundary.

The MHD input file is typically produced as the output of an MHD code.² For the field match any grid of points should be adequate. The coupling, however, treats the plasma as discrete current loops with cross section $\Delta R \cdot \Delta Z$. For the coupling to be accurate, the grid must be as shown in Fig. 4-1 with the extreme points of the plasma boundary half way between grid lines.

4.4 Output

The output is reasonably self-explanatory and includes all the coil parameters (coordinates, minor radius, aspect ratio, and current), and absolute and RMS errors of the match, the total Ampere turns, the stored energy, and the total coil volume. Depending on whether OH (EF) or plasma systems are input, there is also a breakdown of the coupling energies and the Volt-seconds to the plasma from each of the systems. The coupling energies are defined as $1/2 L_{IJ} I_I I_J$, where I and J denote any of the systems EF, OH, P. The mutual energy terms ($I \neq J$) appear twice in the total energy. Cautionary messages are printed when unusual situations arise. Besides the printed output, data is written into the coil output file (ending in C) for the completely specified coil data set if ICOILS = 1, and the plots are output in the graphics output file (ending in G) if IPLOT = 1.

Fig. 4-1. Grid for MHD variables.

A sample output is presented in Appendix C. It was produced from the input in Appendix B, entered in a file named SAMPLE. The MHD file which contains the MHD equilibrium information for the cases in Sec. 3.2 was named SAMPLMHD. The command line was:

```
XEFC I=SAMPLE MHD=SAMPLMHD
```

The output file was named SAMPLEO and the graphics file was named SAMPLEG. The plot from the graphics file is shown in Appendix D as produced on a TALARIS 1590 laser printer.

The first item in the printed output is the input file name followed by information about the time, date, and computer. The next item is an echo of the input file followed by a list of all of the NAMELIST input variables and the values used.

The following item is a list of the MHD parameters (IPLAS is the same as PCUR), followed by a list of magnet parameters, then a list of coordinates of the TF-coil inner and outer boundaries. Finally the coil configuration with the defaults filled in is given.

The calculation for various values of r , denoted as RRR, start next. The output for each value of RRR has the same form. For this case with IMODE = 2 there are two initial calculations to determine a case with higher-than-desired error and a case with lower-than-desired error, then there are four iterations until the final value (5.97×10^{-5}) of RRR that gives the desired RMS error (3.00×10^{-4}) is achieved.

For each value of RRR the output contains the value of RRR, $-\log_{10}(\text{RRR})$, γ [Eq. (2.29)], and CURDEN. This is followed by the coil parameters for this iteration. The value of the Lagrange multiplier is also given. The next line has the absolute error [Eq. (3.2)]; the maximum error at any field point, the rms error, and the value of the flux where the maximum error occurred. This is followed by information on the coils and couplings and then any error messages or warnings. (In this case that there is a low-aspect-ratio coil.) The last line(s) show information on the iterations. The final values are repeated on a page by themselves.

ACKNOWLEDGMENTS

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Appendix A

INDUCTANCE CONSIDERATIONS

A.1 Flux Function

For the purposes of this paper the flux function can be defined as the magnetic flux passing through the circular surface bounded by the circle $R = \text{constant}$ and $Z = \text{constant}$. (R and Z are in cylindrical coordinates). Only azimuthally symmetric systems are considered. The unit normal to the surface is assumed to be in the positive Z -direction. This surface is shown in Fig. A-1. The relation between the flux and the vector potential is then

$$\psi(R, Z) = \int_S \vec{B} \cdot d\vec{S} = \oint_C \vec{A} \cdot d\vec{\ell} = 2\pi R A_\phi(R, Z). \quad (\text{A.1})$$

Fig. A-1. Coordinate system and reference surface for definition of the flux function.

It is convenient to work in the Coulomb gauge, $\nabla \cdot \vec{A} = 0$. If the problem under consideration has azimuthal symmetry with all the magnetic fields in the poloidal direction, it can be shown that only the $\hat{\phi}$ component of the vector potential need be retained; that is, under these conditions one can always make a limited gauge transformation (without changing $\nabla \cdot \vec{A} = 0$) to eliminate the other two components. Even with this restriction one can still add a constant ψ_0 to the flux function if $-\psi_0/2\pi R$ is added to A_ϕ . Since

$$\nabla \times \frac{\hat{\phi}}{R} = - \frac{\delta(R) \hat{z}}{R}, \quad (\text{A.2})$$

adding a constant to ψ is equivalent to adding an unphysical, delta function, magnetic field at $R = 0$. It is permissible if $R = 0$ is outside the region of interest. This applies in our case since we only want to match ψ_{ext} inside the plasma as in Sec. 2.3, but is not usually the case when inductances are calculated directly from the flux function, since the integrals involved are over all space (or derived from integrals over all space). Unless explicitly stated otherwise, the flux functions in this paper are defined as in Eq. (A.1) and go to zero at $R = 0$. Since the current distributions are localized, they in addition go to zero at infinity at least as fast as $1/R$.⁹

The vector potential in the Coulomb gauge for an azimuthally symmetric current distribution is given by⁹

$$A_\phi(R, Z) = \frac{\mu}{4\pi} \int \frac{J_\phi(R', Z') (\cos \phi') R' dR' dZ' d\phi'}{|\vec{r} - \vec{r}'|}. \quad (\text{A.3})$$

This equation can be integrated over ϕ' and combined with Eq. (A.1) to give

$$\psi = - \mu \int \frac{J_\phi(R', Z') [AK(q) - BE(q)] dR' dZ'}{\sqrt{B}}, \quad (\text{A.4})$$

$$q^2 = \frac{4RR'}{(R + R')^2 + (Z - Z')^2}, \quad (\text{A.5})$$

$$A = R^2 + R'^2 + (Z - Z')^2, \quad (\text{A.6})$$

$$B = (R + R')^2 + (Z - Z')^2. \quad (\text{A.7})$$

The functions $K(q)$ and $E(q)$ are complete elliptic integrals of the first and second kinds.

For the case of a coil which is sufficiently small that R and Z do not vary appreciably over the coil, Eq. (A.4) reduces to

$$\psi(R, Z) = \frac{\mu I [AK(q) - BE(q)]}{\sqrt{B}}, \quad (\text{A.8})$$

where I is the current in the coil.

A.2 Calculation of Inductances

Given two current distributions labeled by α and β and having magnetic fields $\vec{B}_\alpha$ and $\vec{B}_\beta$ the inductances are defined so that

$$\frac{1}{2} L_{\alpha\beta} I_\alpha I_\beta = \int d^3V \frac{\vec{B}_\alpha \cdot \vec{B}_\beta}{2\mu}, \quad (\text{A.9})$$

where the integral is over all space. The total stored energy is given by

$$U = \frac{1}{2} \sum_{\alpha, \beta} L_{\alpha\beta} I_\alpha I_\beta. \quad (\text{A.10})$$

Equation (A.9) may be written

$$\begin{aligned} \frac{1}{2} L_{\alpha\beta} I_\alpha I_\beta &= \frac{1}{2\mu} \int d^3V (\nabla \times \vec{A}_\alpha) \cdot \vec{B}_\beta \\ &= \frac{1}{2\mu} \int d^3V \{ \nabla \cdot [\vec{A}_\alpha \times \vec{B}_\beta] + \vec{A}_\alpha \cdot (\nabla \times \vec{B}_\beta) \}. \end{aligned} \quad (\text{A.11})$$

The first term can be transformed into a surface integral and vanishes

provided the integrand goes to zero sufficiently fast at infinity. This should be the case for localized current distributions (provided the "arbitrary" constant in ψ is zero). The inductances are then given by

$$L_{\alpha\beta} I_{\alpha} I_{\beta} = \int d^3V \vec{J}_{\beta} \cdot \vec{A}_{\alpha} = \int d^3V \vec{J}_{\alpha} \cdot \vec{A}_{\beta}, \quad (\text{A.12})$$

and the integral need only be taken over the part of space where the current is non-zero.

In particular, the mutual inductance of two thin coils is given by

$$\begin{aligned} M_{ab} I_a I_b &= \int d^3V J_a A_{b,\phi} \\ &= \int 2\pi R' dR' dZ' J_a(R', Z') \left[-\frac{\psi_b(R', Z')}{2\pi R'} \right] \\ &= -I_a \psi_b(a), \end{aligned} \quad (\text{A.13})$$

provided ψ_b does not vary appreciably over the coil a. Using Eq. (A.8) for ψ_b , the mutual inductance becomes

$$M_{ab} = \frac{\mu[AK(q) - BE(q)]}{\sqrt{B}}, \quad (\text{A.14})$$

with R , R' , Z , and Z' in Eqs. (A.5) and (A.6) replaced by R_a , R_b , Z_a , and Z_b , respectively. This expression differs only by some algebra from Eq. (2.12).

It is also apparent that the mutual inductance of a thin coil (labeled by a) and a general current distribution (labeled by α) is the generalization of Eq. (A.13)

$$M_{a\alpha} I_{\alpha} = -\psi_a(a), \quad (\text{A.15})$$

where I_{α} is the equivalent current of the distribution. The equivalent current is somewhat arbitrary. Changing the definition of the equivalent current changes the inductances, but the two should always appear in combination so that physical results are invariant. Equation (A.15) is useful in calculating the mutual inductances of an EF coil with the plasma or with

combination so that physical results are invariant. Equation (A.15) is useful in calculating the mutual inductances of an EF coil with the plasma or with the entire OH system.

The self-inductances are harder to calculate since the flux function does vary over the current distribution. Two cases of interest are given: The self-inductance of a large aspect ratio coil with a constant current distribution is calculated in many textbooks to be

$$L_{aa} \equiv L_a = \mu R_a \left[\log \frac{8R_a}{a} - \frac{7}{4} \right]. \quad (A.16)$$

This is a good approximation to the OH and EF coils in a tokamak provided the aspect ratio of the coils is larger than about five.

The self-inductance of the plasma can be calculated from the MHD equilibrium using the total flux function as a metric with

$$d^3V = \frac{d\psi \, dl \, R \, d\phi}{2\pi R B_p}, \quad (A.17)$$

where dl is a distance increment around a flux line, and B_p is the magnitude of the total poloidal field, given by

$$\vec{B}_p = \frac{\hat{\phi} \times \nabla \psi}{2\pi R}. \quad (A.18)$$

In that case Eq. (A.12) gives

$$L_p = \frac{1}{2} \int_{\psi_m}^{\psi_x} d\psi \oint \frac{dl}{B_p} - \frac{\psi_{int} J}{2\pi R}, \quad (A.19)$$

where ψ_{int} is the flux function (normalized to zero at infinity) of the internal field (see Sec. 2.2) and ψ_m and ψ_x are the values of the total flux

at the magnetic axis and limiter, respectively. Most MHD codes are set up to evaluate flux line averages such as the one which appears in Eq. (A.19).

Of course, if the total energy of a current distribution is known, the self-inductances can be calculated directly from

$$\frac{1}{2} L_{\alpha} I_{\alpha}^2 = U_{\alpha} \equiv \frac{1}{2\mu} \int B_{\alpha}^2 d^3V. \quad (\text{A.20})$$

A.3 Equivalent Inductances

It is often convenient to treat systems of coils as a single equivalent system rather than to treat the coils individually. For example, the EF coils in this report have been treated as coil pairs consisting of a coil and its mirror image on the other side of the midplane. In addition, it is sometimes convenient to treat the EF system as a single equivalent system. Doing this is largely a matter of bookkeeping, and it is convenient to have a systematic manner of converting between the various views of the system.

First consider the relationship between the self-inductances of individual thin coils (labeled by a or a') and the self-inductance of the system (labeled by α). From Eq. (A.12) with $L_{\alpha} \equiv L_{\alpha\alpha}$ and $L_a \equiv L_{aa}$

$$\begin{aligned} L_{\alpha} I_{\alpha}^2 &= \int_{\text{all coils}} d^3V \mathbf{J}_{\alpha} \cdot \mathbf{A}_{\alpha} \\ &= \sum_{a'} \int_{\text{all coils}} d^3V \mathbf{J}_a \cdot \mathbf{A}_{a'}, \\ &= \sum_a \sum_{a'} \int_{\text{coil } a} d^3V \mathbf{J}_a \cdot \mathbf{A}_{a'}, \\ &= \sum_a L_a I_a^2 + \sum_a \sum_{a \neq a'} M_{aa'} I_a I_{a'}. \end{aligned} \quad (\text{A.21})$$

It is assumed the superposition in the second step is possible. The equivalent current I_{α} is again arbitrary; however, the inductances are different for different definitions of the equivalent currents.

Next consider the mutual inductances of a general current distribution, α , with a second current distribution, β , consisting of thin coils labeled by b . From Eq. (A.12) and (A.15)

$$\begin{aligned}
 M_{\alpha\beta} I_{\alpha} I_{\beta} &= \int_{\text{coils } b} d^3V \vec{J}_{\beta} \cdot \vec{A}_{\alpha} \\
 &= \int_{\text{coils } b} dR' dZ' J_{\beta} \psi_{\alpha} \\
 &= - \sum_b I_b \psi_{\alpha}(b) \\
 &= \sum_b M_{ab} I_{\alpha} I_b, \tag{A.22}
 \end{aligned}$$

or

$$M_{\alpha\beta} I_{\beta} = \sum_b M_{ab} I_b. \tag{A.23}$$

Finally consider the mutual inductances of a current distribution α consisting of thin coils labeled by a and a current distribution β consisting of thin coils labeled by b . As above

$$\begin{aligned}
 M_{\alpha\beta} I_{\alpha} I_{\beta} &= \int_{\text{all coils } b} d^3V \vec{J}_{\beta} \cdot \vec{A}_{\alpha} \\
 &= \sum_a \int_{\text{all coils } b} d^3V \vec{J}_{\beta} \cdot \vec{A}_a \\
 &= \sum_a \int_{\text{all coils } b} dR' dZ' J_{\beta} \psi_a
 \end{aligned}$$

$$= - \sum_a \sum_b I_b \psi_a(b)$$

$$= \sum_a \sum_b M_{ab} I_a I_b. \quad (A.24)$$

Again, the superposition in the second step is assumed possible.

It can be seen that the relations in Eqs. (A.23) and (A.24) are valid whether coils or coil pairs are considered, since by symmetry the flux functions which appear in the derivations are the same at the upper and lower coils of the pair. If, however, Eqs. (A.21) and (A.24) are applied to the individual coils of a coil pair, the relations in Eq. (2.24) are obtained provided the equivalent current is taken the same as the current in either of the individual coils.

The mutual inductance of the OH and EF systems, for example, can be obtained from Eq. (A.24)

$$M_{OH,EF} I_{OH} I_{EF} = \sum_i^{EF} \sum_j^{OH} M_{ij} I_i I_j. \quad (A.25)$$

The mutual inductance of an individual EF coil and the OH system is given in Eq. (A.23)

$$M_{OH,i} I_{OH} = \sum_j^{OH} M_{ij} I_i, \quad (A.26)$$

so that

$$M_{OH,EF} I_{EF} = \sum_i^{EF} M_{OH,i} I_i. \quad (A.27)$$

The EF and OH systems are decoupled ($M_{OH,EF} = 0$) if the right-hand side of Eq. (A.27) is made to vanish. (This criterion combined with Eq. (A.15) is

equivalent to the criterion given in Eq. (A.7). Either could be used.) If it were desired to decouple each of the EF coils from the OH system, it would be necessary to use the more restrictive criterion that the right-hand side of Eq. (A.26) be zero.

Appendix B INPUT FILE

```

Procedure Study: Final Case using SAMPLMHD
* NAMELIST follows
idcple=0 icoils=0 imodel=0 imode=2 itrpnt=0 irpnt=1 iclim=2
delphi=10. rr(1)=4.0 0.0 8.0
deltfc=.94 delbsi=1.2 delbso=0.0 deld=3.41 delv=.2 ntfc=12
dist=1.0 nmcoils=20
$ end
      1.7156      0.4159
      2.8114      6.8352
      5.0181      8.2552
      13.5854      4.0233
      14.3254      1.7587
3      7.0000      0.0000      0.1500      14.8000      14.1000      0.1500

```

Appendix C

OUTPUT FILE

EFC: Poloidal Coil Design Code

sample Time=10:51:42 Date=11/14/89 Machine= Suffix=c

Input file echo

```

Procedure Study: Final Case using SAMPLMHD
* NAMELIST follows
idcple=0 icolls=0 imode=0 imode=2 itrpnt=0 lrpnt=1 iclim=2
delphi=10. rr(1)=4.0 0.0 0.0
deltfc=.94 delbsi=1.2 delbso=0.0 deld=3.41 delv=.2 ntfc=12
dist=1.0 nmcoils=20
$ end
1.7150 0.4159
2.8114 0.8352
6.0181 8.2552
13.5654 4.0233
14.3254 1.7587
3 7.0000 0.0000 0.1500 14.0000 14.1000 0.1500

```

Switches and scaling

```

iclim = 2
imode = 2
idcple = 0
iscale = 0
iloop = 1
iron = 0
iapnt = 0
ibpnt = 0
irpnt = 1
ibound = 0
iplot = 1
itrpnt = 0
indpnt = 0
letot = 0
lohsys = 0
icolls = 0
imodel = 0
itfc = 1
rr = 4. 0. 0.
r00 = 0.
f00 = 0.
delphi = 10.
curden = 15.
err0 = 3.e-04
acmax = 0.5
deltfc = 0.94
delbsi = 1.2
delbso = 0.
deld = 3.41
delv = 0.2
ntfc = 12
dist = 1.
nmcoils = 20
$

```

Equilibrium and geometry parameters

```

sipa= 1.400 beta= 1.551 alphasj= 0.476 cappa= 1.600 deity= 0.200 asp= 3.600
r0= 7.000 fwai= 40.950 betat= 0.637 q(a)= 2.502 iplas= 1.480e+07

```

Magnet parameters

```

deltfc= 0.940 delbsi= 1.200 delbso= 0.000 deld= 3.410 delv= 0.200 ntfc= 12
rm= 3.656 r1tc= 3.186 r2tc=13.024 height= 7.530 rplas= 1.944 bmax=11.202

```

TF coil coordinates

r	z	r	z	r	z	r	z	r	z
1.26e+01	0.	1.25e+01	4.58e-01	1.25e+01	9.13e-01	1.24e+01	1.36e+00	1.23e+01	1.80e+00
1.22e+01	2.24e+00	1.21e+01	2.65e+00	1.19e+01	3.00e+00	1.17e+01	3.45e+00	1.15e+01	3.82e+00
1.12e+01	4.17e+00	1.10e+01	4.49e+00	1.07e+01	4.80e+00	1.04e+01	5.08e+00	1.01e+01	5.33e+00
9.83e+00	5.57e+00	9.52e+00	5.77e+00	9.21e+00	5.90e+00	8.90e+00	6.11e+00	8.59e+00	6.25e+00
8.28e+00	6.36e+00	7.98e+00	6.45e+00	7.68e+00	6.51e+00	7.39e+00	6.56e+00	7.10e+00	6.59e+00
6.83e+00	6.60e+00	6.50e+00	6.59e+00	6.31e+00	6.50e+00	6.00e+00	6.52e+00	5.83e+00	6.47e+00
5.61e+00	6.41e+00	5.41e+00	6.33e+00	5.21e+00	6.25e+00	5.03e+00	6.16e+00	4.85e+00	6.00e+00
4.71e+00	5.95e+00	4.56e+00	5.84e+00	4.43e+00	5.72e+00	4.31e+00	5.60e+00	4.20e+00	5.48e+00
4.10e+00	5.35e+00	4.02e+00	5.22e+00	3.94e+00	5.09e+00	3.87e+00	4.90e+00	3.81e+00	4.82e+00
3.70e+00	4.69e+00	3.72e+00	4.55e+00	3.69e+00	4.42e+00	3.67e+00	4.28e+00	3.65e+00	4.14e+00
3.65e+00	4.01e+00	3.60e+00	0.						
1.35e+01	0.	1.35e+01	5.17e-01	1.34e+01	1.03e+00	1.33e+01	1.54e+00	1.32e+01	2.04e+00
1.31e+01	2.53e+00	1.29e+01	3.00e+00	1.27e+01	3.40e+00	1.25e+01	3.90e+00	1.23e+01	4.32e+00
1.20e+01	4.72e+00	1.17e+01	5.09e+00	1.14e+01	5.44e+00	1.11e+01	5.76e+00	1.07e+01	6.00e+00
1.04e+01	6.33e+00	1.00e+01	6.67e+00	9.67e+00	6.78e+00	9.30e+00	6.97e+00	8.94e+00	7.12e+00

8.57e+00	7.25e+00	8.21e+00	7.35e+00	7.66e+00	7.44e+00	7.51e+00	7.49e+00	7.16e+00	7.53e+00
8.83e+00	7.54e+00	8.50e+00	7.53e+00	8.19e+00	7.50e+00	5.89e+00	7.45e+00	5.85e+00	7.38e+00
5.32e+00	7.30e+00	5.85e+00	7.21e+00	4.81e+00	7.10e+00	4.58e+00	6.98e+00	4.35e+00	6.85e+00
4.16e+00	6.71e+00	3.97e+00	6.56e+00	3.79e+00	6.41e+00	3.63e+00	6.25e+00	3.48e+00	6.09e+00
3.34e+00	5.90e+00	3.22e+00	5.73e+00	3.11e+00	5.54e+00	3.02e+00	5.35e+00	2.94e+00	5.17e+00
2.87e+00	4.98e+00	2.81e+00	4.79e+00	2.77e+00	4.59e+00	2.74e+00	4.46e+00	2.72e+00	4.26e+00
2.72e+00	4.61e+00	2.72e+00	0.						

Coil configuration

n	type	fixed?	rcoil	zcoil	acoil	acoilm	ascoil	icoil	icoil
1	ef		1.716	0.416	0.150	0.560	11.437	0.	0.
2	ef		2.811	0.835	0.150	0.560	18.743	0.	0.
3	ef		5.018	0.255	0.150	0.560	33.454	0.	0.
4	ef		13.555	4.023	0.150	0.560	90.436	0.	0.
5	ef		14.325	1.759	0.150	0.560	95.503	0.	0.
6	plasma	yes	7.068	0.000	1.944	1.944	3.686	1.41e-05	1.48e+07

Calculations

rrr= 1.00e-04 -log10(rrr)= 4.00e+00 gamma= 6.19e-03 curden= 1.50e+07

EF system

r	z	a	r/a	i	i norm
1.72	0.42	0.403	4.25	-7.669e+06	-0.153
2.81	0.84	0.333	8.43	5.246e+06	0.105
5.02	0.26	0.290	17.31	3.950e+06	0.079
13.57	4.02	0.271	50.03	-3.464e+06	-0.069
14.33	1.76	0.317	45.20	-4.734e+06	-0.094

lambda= 0.

err= 4.58e-03 errmax= 2.76e-02 at nf = 201 rms err= 2.00e-04 psimax= 7.37e+01

Sum of EF currents= -6.07e+06 Sum of positive EF currents= 9.20e+06 Total EF Amp-turns= 5.81e+07

EF stored energy= 5.62e+09 EF coil volume= 1.36e+02

Energy breakdown (joules)

EF self energy= 5.62e+09

Plasma self energy= 1.54e+09

EF-Plasma or Plasma-EF mutual energy= -6.88e+08

Total energy= 5.79e+09

Plasma Volt-seconds = 2.69e+02

EF Volt-seconds to plasma= -9.29e+01

Error messages for rrr= 1.00e-04, -log10(rrr)= 4.00e+00:

There is at least one coil with aspect ratio < 5

rcur= 4.00e+00 rms err= 2.00e-04 rmin= 0. rmax= 8.00e+00

rrr= 1.00e-03 -log10(rrr)= 3.00e+00 gamma= 6.02e-03 curden= 1.50e+07

EF system

r	z	a	r/a	i	i norm
1.72	0.42	0.216	7.89	-2.230e+06	-0.077
2.81	0.84	0.245	11.48	2.028e+06	0.098
5.02	0.26	0.206	24.41	1.992e+06	0.066
13.57	4.02	0.256	53.00	-3.087e+06	-0.107
14.33	1.76	0.303	47.24	-4.334e+06	-0.150

lambda= 0.

err= 2.59e-02 errmax= 1.12e-01 at nf = 201 rms err= 1.44e-03 psimax= 7.37e+01

Sum of EF currents= -4.83e+06 Sum of positive EF currents= 4.02e+06 Total EF Amp-turns= 2.89e+07

EF stored energy= 4.16e+09 EF coil volume= 1.05e+02

Energy breakdown (joules)

EF self energy= 4.16e+09

Plasma self energy= 1.54e+09

EF-Plasma or Plasma-EF mutual energy= -6.27e+08

Total energy= 4.45e+09

Plasma Volt-seconds = 2.69e+02

EF Volt-seconds to plasma= -8.48e+01

Error messages for rrr= 1.00e-03, -log10(rrr)= 3.00e+00:

rcur= 3.00e+00 rms err= 1.44e-03 rmin= 0. rmax= 8.00e+00

Iterations

rrr= 1.07e-04 -log10(rrr)= 3.97e+00 gamma= 0.19e-03 curden= 1.50e+07

EF system

r	z	a	r/a	i	i norm
1.72	0.42	0.401	4.28	-7.570e+00	-0.152
2.81	0.84	0.332	8.47	5.189e+00	0.104
5.02	0.26	0.289	17.35	3.943e+00	0.079
13.57	4.02	0.270	50.22	-3.438e+00	-0.069
14.33	1.76	0.317	45.13	-4.748e+00	-0.095

lambda= 0.

err= 4.77e-03 errmax= 2.87e-02 at nf = 201 rms err= 2.98e-04 psimax= 7.37e+01

Sum of EF currents= -6.63e+00 Sum of positive EF currents= 9.13e+00 Total EF Amp-turns= 4.98e+07

EF stored energy= 5.60e+09 EF coil volume= 1.36e+02

Energy breakdown (Joules)

EF self energy= 5.60e+09

Plasma self energy= 1.54e+09

EF-Plasma or Plasma-EF mutual energy= -6.87e+08

Total energy= 5.77e+09

Plasma Volt-seconds = 2.09e+02

EF Volt-seconds to plasma= -9.28e+01

Error messages for rrr= 1.07e-04, -log10(rrr)= 3.97e+00:

There is at least one coil with aspect ratio < 5

1	rlow= 3.00e+00	rcur= 3.97e+00	rhigh= 4.00e+00
	erlow= 1.44e-03	ecur= 2.98e-04	erhigh= 2.00e-04

rrr= 1.08e-04 -log10(rrr)= 3.97e+00 gamma= 0.19e-03 curden= 1.50e+07

EF system

r	z	a	r/a	i	i norm
1.72	0.42	0.401	4.28	-7.564e+00	-0.152
2.81	0.84	0.332	8.48	5.183e+00	0.104
5.02	0.26	0.289	17.35	3.941e+00	0.079
13.57	4.02	0.270	50.25	-3.435e+00	-0.069
14.33	1.76	0.317	45.12	-4.749e+00	-0.095

lambda= 0.

err= 4.79e-03 errmax= 2.88e-02 at nf = 201 rms err= 3.00e-04 psimax= 7.37e+01

Sum of EF currents= -6.62e+00 Sum of positive EF currents= 9.12e+00 Total EF Amp-turns= 4.97e+07

EF stored energy= 5.59e+09 EF coil volume= 1.36e+02

Energy breakdown (Joules)

EF self energy= 5.59e+09

Plasma self energy= 1.54e+09

EF-Plasma or Plasma-EF mutual energy= -6.87e+08

Total energy= 5.76e+09

Plasma Volt-seconds = 2.09e+02

EF Volt-seconds to plasma= -9.28e+01

Error messages for rrr= 1.08e-04, -log10(rrr)= 3.97e+00:

There is at least one coil with aspect ratio < 5

2	rlow= 3.97e+00	rcur= 3.97e+00	rhigh= 4.00e+00
	erlow= 2.98e-04	ecur= 3.00e-04	erhigh= 2.00e-04

EFC: Poloidal Coil Design Code

sample Time=10:51:42 Date=11/14/89 Machine= Suffix=c

Procedure Study: Final Case using SAMPLMHD

rrr= 1.00e-04 -log10(rrr)= 3.97e+00 gamma= 0.10e-03 curden= 1.50e+07

EF system

r	z	a	r/a	j	i norm
1.72	0.42	0.401	4.20	-7.564e+00	-0.152
2.01	0.04	0.332	0.49	5.183e+00	0.184
5.02	0.20	0.209	17.35	3.041e+00	0.070
13.57	4.02	0.270	50.25	-3.435e+00	-0.060
14.33	1.70	0.317	45.12	-4.749e+00	-0.095

lambda= 0.

err= 4.79e-03 errmax= 2.00e-02 at nf = 201 rms err= 3.00e-04 psimax= 7.37e+01

Sum of EF currents= -0.02e+00 Sum of positive EF currents= 9.12e+00 Total EF Amp-turns= 4.97e+07

EF stored energy= 5.59e+09 EF coil volume= 1.36e+02

Energy breakdown (joules)

EF self energy= 5.59e+09

Plasma self energy= 1.54e+09

EF-Plasma or Plasma-EF mutual energy= -0.07e+00

Total energy= 5.76e+09

Plasma Volt-seconds = 2.09e+02

EF Volt-seconds to plasma= -9.20e+01

Error messages for rrr= 1.00e-04, -log10(rrr)= 3.97e+00:

There is at least one coil with aspect ratio < 5

Appendix D
PLOT OUTPUT