NON-LINEAR CONVERGING RESONANCE CONES

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ABSTRACT

The trajectory modification of a focused resonance cone due to ponderomotive force effects was investigated in the Princeton L-3 device. A single ring pulsed at $f_o \sim 50$ MHz ($f_{lh} \ll f_o \ll f_{pe}, f_{ce}$) launches a converging resonance cone in an argon plasma with $n_e \sim 10^9 - 10^{10}$, $B \sim 400G - 1kG$, $T_e \sim 1 - 3eV$. At high driving power the strong localized electric fields modify the density through the ponderomotive force on the electrons, leading to a modification of the resonance cone trajectory. The cone is seen to bend in towards the axis and the focus moves toward the ring in agreement with the recent theory of Wang and Kuehl.¹ The density perturbation at the focus propagates away and excites ion-acoustic waves. The wave dispersion relation was verified in He^+ , Ne^+ , Ar^+ , and Kr^+ plasmas. Electron heating was observed which is consistent with collisional heating.

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I. INTRODUCTION

Resonance cones were first predicted theoretically^{2,3} and measured experimentally⁴ for a short dipole antenna. The electric field maximum for this kind of excitation occurs at the antenna, thus making nonlinear studies difficult. Stenzel and Gekelman⁵⁻⁷ realized that by using a ring to launch the resonance cone, the electric field maximum would occur at the focus (Fig. 1) away from the exciter, making it easier to study non-linear effects without the antenna interfering.

Previous experiments⁵⁻⁷ concentrated on the existence of a density depression and the particle dynamics. They concluded that the axial component of the ponderomotive force on the electrons pushed them out of the focal region leading to a density depression. This hole was not stable and during long rf bursts a general turbulence resulted. We investigate here the reaction of the density modification back upon the resonance cones themselves. The focus is found to move toward the ring and the trajectory bends toward the axis in agreement with the recent theory of Wang and Kuehl.¹ By modulating the resonance cone amplitude, the density perturbation can couple to electrostatic ion cyclotron waves ($\omega^2 = \Omega_{ci}^2 + k_{\perp}^2 C_s^2$) whose frequency is determined by the modulation frequency. The wavelengths of the propagating density perturbation were measured at various frequencies and the wave dispersion was verified with various ion species plasmas (He^+ , Ne^+ , Ar^+ , Kr^+).

In addition the variation of the wavelength with applied power indicated heating which was consistent with estimates of collisional heating.

II. THEORY

In cylindrical geometry the electric potential satisfies

$$\frac{1}{r} \partial_r (r \kappa_{\perp} \partial_r \phi) + \partial_z (\kappa_{\parallel} \partial_z \phi) = 0 \quad (1)$$

for a cold plasma. In the region of interest

$$\omega_{pi} \ll \omega \ll \omega_{pe} \ll \omega_{ce} \quad \kappa_{\perp} \approx 1 - \frac{\omega_{pi}^2}{\omega^2} \quad \text{and} \quad \kappa_{\parallel} \approx 1 - \frac{\omega_{pe}^2}{\omega^2}.$$

The potential is singular along the characteristics of equation (1)

$$\tan \theta = \left(\frac{\kappa_{\perp}}{\kappa_{\parallel}} \right)^{1/2} \approx \frac{\omega}{\omega_{pe}} \alpha_n^{-1/2}$$

where $\theta = \tan^{-1} \left(\frac{r}{z} \right)$. Excitation by a ring will therefore give a cone-like structure.⁸ The cone angle is seen to be inversely proportional to the square root of the electron density.

In the non-linear regime the ponderomotive force leads to a density change¹

$$\frac{\delta n}{n} \approx \frac{\epsilon_0 (1 - \kappa_{\parallel})}{4 n_0 \kappa (T_e + T_i)} \left| \partial_z \phi \right|^2 \quad (2)$$

Levine et al. used this expression to calculate the density in all space.⁸ This problem should however be solved self-consistently. The density changes in Eq. 2 should be used to modify the dielectric components in Eq. 1, giving a new equation for the potential. Wang and Kuehl¹ have derived the correct equation for a strongly magnetized warm plasma with $(\omega_{pi} \ll \omega \ll \omega_{pe} \ll \omega_{ce})$. This equation is

$$\frac{1}{r} \partial_r (r \partial_r \phi) + \kappa_{n0} \partial_z^2 \phi + \frac{3\omega_{pe}^2 v_{th}^2}{2\omega^4} \partial_z^4 \phi + (\kappa_{n0} - 1) \partial_z \times \left(\frac{\delta n}{n} \partial_z \phi \right) = 0 \quad (3)$$

where the third term is the leading thermal correction and the last term brings in the density modification. This equation was solved numerically in a self-consistent fashion using the linear solution as the boundary condition near the ring. This last assumption is justified since the maximum non-linearity occurs near the focus, not the ring. The results of this calculation agree with a crude picture consisting of the following: the electric field gives a density depression which through $\tan \theta \propto \frac{1}{n_e}$ leads to a steeper cone angle and hence a bending of the trajectory toward the axis and a shift of the focus toward the exciter.

III. EXPERIMENTAL ARRANGEMENT

The experiment was performed in the Princeton L-3 device. The plasma source is a multi-filament discharge⁹ which converges into a 4 meter long, 10 cm diameter plasma column. Typical plasma parameters were electron density $n_e \sim 10^9 - 10^{10}$, magnetic field $B \sim 400G - 1KG$, electron temperature $T_e \sim 1-3ev$, ion temperature $T_i < 0.1ev$, filling pressure $\sim 2 \times 10^{-4}$ torr Argon.

Figure 2 shows a schematic of the experimental setup. The resonance cone was launched from one ring of a 36-ring antenna located at one end of the plasma. By changing which ring was used the focus could be moved axially in discrete steps. The rings behaved identically if the forward powers were matched as long as the end rings were not used.

The rings were driven with a burst of rf, duration $\sim .5-10$ μ secs and repetition rate 1KHZ. The experiment was performed in this pulsed mode to avoid ionization of the weakly ionized ($< 1\%$) plasma. Axially and radially driven double tip probes are used to detect the electric field. The received signal is amplified, then applied to a crystal detector whose output is put into a boxcar averager triggered by the rf burst to give spatial and temporal resolution.

In order to verify that the observed trajectory modification was due to a density modification, several methods were employed. A single tip of the radial or axial probe was used as a Langmuir probe. A buffer amplifier that floated at the bias voltage was placed as near the probe as possible and its output was differentially amplified and applied to the boxcar. The bias voltage could be fixed (useful for ion saturation current), pulsed (for electron saturation current), or swept in a repeated ramp (for complete probe characteristics).

It was found that the probe measurements were not reliable during the rf burst so an alternate scheme was devised. Using the probes as antenna and receiver, a resonance cone at a higher frequency (~ 90 Mhz) was propagated through the region of interest. The angle between the two probes would give the average density between them, and the separation between the main peak and the thermal peak^{10,11} the average electron temperature. $(\Delta r_{sep} \sim (\frac{V_{th}}{r\omega_p})^{2/3} (\tan\theta_c)^{1/3}$ where r is the separation between the probes and θ_c is the main peak cone angle.) This method yields only spatially averaged values, but works during the rf burst. In addition, a 10% variation can be easily resolved without ambiguity.

IV. EXPERIMENTAL RESULTS

A. Resonance Cone Trajectories:

Typical radial and axial scans of the electric field amplitude are shown in Fig. 3. A measure of the non-linearity is the quantity $s = \frac{\epsilon_0 E_{rf}^2}{n k T_e}$, the field intensity normalized to the kinetic particle energy density evaluated at the focus. The linear results are for $s \sim .02$ and the non-linear for $s \sim .4$. By following the time evolution, it is found that after $\sim 3.5 \mu\text{sec}$ a steady state is reached. The non-linear results are shown for a time 4 μsecs after the beginning of the rf burst. In Fig. 4a we plot the location of the electric field maximum for both linear and non-linear powers. The theoretical curve is taken from Wang and Kuehl¹ for $s = .4$.

One can see that the non-linear trajectory has moved inward and the new focal point has shifted toward the exciter. The non-linearity parameter s was determined in two ways. First the electric field at the focus is measured by calibrating the probe signal with a known electric field in a parallel plate capacitor in air. This method usually leads to an under-estimation due to sheath effects and probe perturbations on the plasma. A second method is to estimate the power flow through the focus by measuring the power transmitted by the ring, the focal diameter and the group velocity.⁶ Then $s = \frac{4P_t}{n k T_e (\pi d^2)}$,

where P_t is the power transmitted, v_g is the group velocity and d is the diameter of the focus. In general the power flow method leads to slightly larger figures, and these are quoted throughout this paper.

The axial location of the focus versus s is shown in Fig. 4b. The solid curve is the theoretical result from Wang and Kuehl and the dots are experimental points. Agreement is seen to be quite good.

The density modifications were measured to verify the explanation of the trajectory modification. Ion-saturation current measurements similar to Reference 5 were made at first, but it was found that the magnitude of the measured density hole was dependent on bias voltage as shown in Fig. 5a. This was interpreted as a reduction in current due to fast electrons accelerated by the rf penetrating the sheath. It could also be due to change of plasma potential.

Electron saturation current measurements were next attempted. The time evolution of the electron saturation current can be seen in Fig. 5b. The current signal was actually greater during the rf, but dropped abruptly upon turnoff to indicate a density hole. The behavior during the rf is probably due to sheath rectification. By examining the full probe characteristic during the rf (Fig. 6), it can be seen that the whole characteristic is grossly distorted by the rf and neither the density nor the floating potential can be reliably inferred. After the rf turnoff the characteristic quickly returns to the

usual shape and eventually to identically that of before the rf burst. The density depression measured one μsec after turn-off vs. power is plotted in Fig. 7. The measured value at full power is consistent with our estimates of $\frac{E^2}{nkT}$. This and the linearity of the curve leads us to believe that the density modification is due to the ponderomotive force. One motivation for the present experiment was to investigate the competition between ponderomotive force effects and parametric decay processes.¹² However, the density hole exists only for a few (~ 7) μsecs and breaks up before parametric instabilities can occur.

As a second check and as a way to make measurements during the rf, the test wave method described above was also used. Examples of this measurement are shown in Fig. 8. The change in the location of the main peak in Fig. 8b gives the average density change through $\tan\theta_c \sim \frac{\omega}{\omega_{pe}}$ and therefore $\frac{n_{e2}}{n_{e1}} = \left(\frac{\theta_{c2}}{\theta_{c1}}\right)^2$. The change in the separation of the thermal peak from the main peak in Fig. 8a gives the temperature from

$$\Delta\theta \approx \left(\frac{v_{th}}{v_{0p}}\right)^{2/3} \tan^{1/3} \theta_c, \text{ leading to } \frac{T_2}{T_1} = \left(\frac{\text{separation 2}}{\text{separation 1}}\right)^3 \left(\frac{\theta_{c2}}{\theta_{c1}}\right)^3.$$

The results of these measurements are summarized in Fig. 9.

Fig. 9a shows that the density hole forms rather quickly during the burst. The apparent filling in of the hole during the burst is due to the shift of the hole out of the test wave path due to the trajectory modification. The depth of the hole is in agreement with the estimated s and the electron saturation current

measurements made after the burst's conclusion. These results are in agreement with previous work.^{6,7} In Fig. 9b however, we see the evolution of the electron temperature as determined by the test waves. The observed heating is in contrast to previous conclusions of no electron heating; however, plasma conditions in the two experiments may not be identical. The rise in T_e is further confirmed by the acoustic wave data shown later in this paper (see Fig. 15).

Since the density modification is present and is consistent with ponderomotive effects, we conclude that the trajectory modification is due to ponderomotive effects.

One way to exploit these effects (and a further way to verify their existence) is to drive the density modification in such a way as to couple to the normal modes of the plasma. One can then infer plasma parameters from the characteristics of these modes. By increasing the repetition rate of our rf bursts until a 50% duty cycle was reached, we created an oscillating density hole at the focus. The repetition rate was varied from 63KHZ - 1MHZ. Too fast or slow a chopping rate or too strong an rf burst led to ionization which masked the desired effect. Only small density variations were needed, so the rf power could be reduced from that used in the first part of this experiment. By scanning radially at the axial location of the focus, we detected a fluctuating ion density

at the chopping frequency. The density perturbation starts from the focus and propagates outside the resonance cone. The perpendicular ($\perp \bar{B}$) velocity is very close to the acoustic speed C_s .

These measurements were repeated in several gases (He^+ , Ar^+ , Ne^+ , Kr^+) yielding appropriate phase velocities as can be seen in Fig. 13. The 10-20% increase above C_s measured in the absence of rf is taken to be due to heating during the rf (see Fig. 16).

Since chopping the rf does not lead to a sinusoidal density modulation, higher harmonics of the chopping frequency should be present. By using a notch filter set at the second harmonic, we were able to make interferograms for this component as well (Fig. 14).

In Fig. 15 we plot the amplitude of the oscillation normalized to s vs. power. At these power levels in fluctuation seems to be a constant fraction of s .

One application of this technique can be seen in Fig. 16. By measuring the wavelength of the excited oscillation at various rf powers, we can measure the electron temperature as a function of power. The results are in agreement with the test wave measurements quoted above, but in principle can give better spatial resolution if the excited wavelengths are short enough, and in addition eliminates the need for a launching probe.

To estimate the electron heating in a short burst we calculate the power input due to collisional damping of the z-component of the electric field. The power input can be written¹³ $P = \frac{-i\omega}{16\pi} (\kappa_{zz} - 1) |E_z|^2 + \text{c.c.}$ Taking

$$\kappa_{zz} = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)} \quad \text{then } P = \frac{\omega_p^2 \nu}{8\pi} \frac{E_{oz}^2}{\omega^2} \quad (\nu \ll \omega)$$

where ν is the electron-neutral collision frequency. For Argon¹⁴ $\nu \approx \nu_0 T^{3/2}$ in our temperature range. Then from

$$\frac{n}{2} \frac{\partial T}{\partial t} = P \quad \text{we get}$$

$$T(t) = \frac{T(0)}{\left(1 - \frac{Ct T(0)}{2}\right)^{1/2}} \quad \text{where } C = \frac{e^2 E_{oz}^2}{m\omega^2} \nu_0.$$

For the conditions of Fig. 9 $\omega = 2\pi \times 50\text{MHz}$, $E_{oz} \approx 10 \text{ V/cm}$
 $\nu \approx 3 \times 10^5 \text{ sec}^{-1}$, $T(0) \approx 1.5\text{eV}$, $t \sim 1\mu\text{sec}$ then $T(t) \approx 1.5 T(0)$
 in fair agreement with Fig. 9.

We conclude from this that the electron heating we observe is consistent with collisional heating.

V. CONCLUSION

In this paper we investigated the trajectory modification of a converging resonance cone due to ponderomotive force effects. The measured change was shown to be in agreement with the recent theory of Wang and Kuehl.¹ The density modification was measured by several techniques and used to excite electrostatic ion-cyclotron waves whose dispersion properties gave further confirmation of the te t wave measurements.

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(a) Point Exciter

(b) Ring Exciter

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Fig. 1. Schematic of resonance cones from (a) point source and (b) ring source. The magnetic field is along the symmetry axis and θ is the resonance cone angle.

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Fig. 2. A schematic of the experimental set-up.

(a) RADIAL SCAN

(b) AXIAL SCAN

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Fig. 3. Linear and non-linear electric field amplitude scans (a) radial scans (b) axial scans.

Fig. 4. (a) Location of peak electric field in the r - Z plane for linear ($s \sim .02$) and nonlinear ($s \sim .4$) powers (b) axial location of peak electric field vs. s .

Fig. 5. (a) $\delta n/n$ as determined by the ion-saturation current
 (b) electron saturation current during rf burst (c) electron saturation current
 during rf burst (d) electron saturation current during rf burst

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Fig. 6. Langmuir probe characteristics at various times during rf burst with the probe located in the cone.

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Fig. 7. $\frac{\delta n}{n}$ from electron saturation current measured 1 μ s after the end of the rf burst vs. power.

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Fig. 8. Examples of test wave data for (a) Temperature determination and (b) Density determination.

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Fig. 2. Results of test wave measurements: (a) n_e/n_{e0} vs. time (b) T_e/T_{e0} initial vs. time.

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Fig. 10. Schematic of fluctuating ion-density interferometry set-up.

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Fig. 11. Radial interferograms of the fluctuating ion density at various delay times in Argon. $f_{\text{ch}} = 138 \text{ KHZ}$.

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Fig. 12. Dispersion relation in Helium determined from fluctuating ion-density inteferograms.

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 Fig. 13. Phase velocity normalized to sound speed (without rf) vs. mass of gas used.

Fig. 14. First and second harmonic radial interferogram in Argon at $f_{ch} = 142$ KHZ.

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Fig. 15. Peak amplitude of fluctuating ion density normalized to S_n vs. input power.

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Fig. 16. Electron temperature from interferograms vs. input power for $f_{ch} = 124$ KHZ in Ar.