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SURFACE MOTION AND CONFINEMENT POTENTIAL FOR A MICROWAVE CONFINED CORONA

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Potential for a Microwave Confined Corona

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Abstract

[Approximate time dependent solutions for surface velocities and potentials are given for a plane polarized microwave field confining a hot, overdense plasma corona.]

Steady state solutions to Poissons' equation can be applied to the time dependent case, provided transit time effects are included.

The product of ion pressure and potential wave (surface) velocity gives an average heating rate $\sim 7/32 NKT_0 V_0$ directly to the ions.

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I. Introduction

In a previous note¹ solutions to Poissons' equation were given for the charge distribution of one sign, given the other. The motivation for this was primarily to apply the results to the present case.

We want to understand the time dependent behavior of the ion accelerating potential as a result of reflecting microwave radiation from the electrons, with $\omega \ll \omega_{pi}$. Because the microwave field is "slowly varying" we expect the field pressure, $\sim \frac{B^2(t)}{8\pi}$, to be in approximate equilibrium with the particle pressures. Because of the small charge to mass ratio of the ions compared to electrons, the field, $\vec{A}(t)$, couples primarily to the electrons. That is, the interaction of the potential ϕ , and transverse electromagnetic field, $\vec{A}(t)$ is

$$H = q\phi + \frac{(\vec{p} - q\vec{A})^2}{2Mq}$$

with charged particle q . Then the ions will be considered to interact only via ϕ , the plasma potential wave, and electrons via both ϕ and $\vec{A}$.

II. Plasma Potential Wave

At any point in the field cycle the momentum flux of the $\vec{A}$ field is coupled to the electrons. The electrons are in turn coupled to the ions via the plasma potential, $\eta \cdot (\eta = e\phi/KT)$. The equilibrium condition in the wave frame is then

$$\frac{B^2(t)}{8\pi} = (\text{electron pressure} + \text{ion pressure}) = P_e(t) + P_i(t)$$

the shape of the wave will be assumed the same as the corresponding static solution¹. The characteristics of interest are that in the wave frame (static solution) the potential eigenvalue (potential where the charge densities cross) is always $\eta(0) = 1$ and measures the average ion pressure.

The other general initial conditions assumed are equal ion and electron densities (N) and temperatures (T_0) deep in the plasma where $\eta \rightarrow 0$ (i.e. well outside the wave region). Average confinement is assumed, and a steady outward Maxwellian flux is assumed coming from the interior. The only inward flux of particles is that reflected from the wave. The wave velocity is V_ω with respect to the plasma center-of-mass.

Since the ion pressure can accommodate a change, in the wave frame, of 3:1 with a subsonic V_ω , the electron pressure component is essentially constant in the wave frame and $P_e(t) \approx NKT_0 = N\eta_0$, where η_0 is the eigenvalue when $V_\omega = 0$. The ion pressure then satisfies, in units of $N\eta_0$,

$$1) \quad \begin{cases} P_i(t) = (4 \sin^2 \omega t - 1) & \omega t > \pi/6 \\ = 0 & < \pi/6 \end{cases}$$

(see Figs. I and II)

The length scale for the entire corona is assumed much greater than the boundary region so corona volume changes accompanying the surface oscillation are negligible and the particle density is constant.

The essence of the calculation is that when the wave moves toward the c.m., $V_\omega < 0$, it sees an increased ion flux and a greater pressure. When $V_\omega > 0$ it sees a decreasing flux and reduced pressure.

III. Calculation of $V_\omega(t)$

a) Take the wave direction along the x-axis. The ion pressure in the wave frame can be written, after integrating over v_y , v_z , and $a_i = \frac{m_i}{2KT}$

$$2) \quad \left\{ \begin{array}{l} P_i = 2N M_i \left(\frac{a_i}{\pi} \right)^{1/2} \int_0^{v_m} (v_x + V_\omega)^2 e^{-a_i v_x^2} dv_x \quad V_\omega < 0 \\ \\ = 2N M_i \left(\frac{a_i}{\pi} \right)^{1/2} \int_{V_\omega}^{v_m} (v_x - V_\omega)^2 e^{-a_i v_x^2} dv_x \quad V_\omega > 0 \end{array} \right.$$

These equations come about because the flux of ions incident on the wave is $f(v_x) (v_x \pm V_\omega)$ and the change in momentum on reflection is $2 M_i (v_x \pm V_\omega)$. Note that the integral for $V_\omega < 0$ is not extended to $-V_\omega$ because ions are only incident from one side ($v_x > 0$) and the only ions present with inward velocity are those previously reflected from the wave. When $V_\omega > 0$, however, the wave "runs away" from the c.m. and so the integral must extend from V_ω to account for the portion of outgoing flux that never reaches the wave.

Note also that in the limit $\omega \rightarrow 0$ the ions all have time to explore the entire x-axis of the wave and the upper limits $v_m \rightarrow \infty$. This would

then result in the well known case² of a Maxwellian gas translating along the x-axis with a uniform hydrodynamic velocity, v_w . This can be seen, for instance, by transforming to the c.m. frame the second equation above (i.e. let $z \equiv (v_x - v_w)$ and

$$\int_{v_w}^{\infty} (v_x - v_w)^2 e^{-av_x^2} dv_x \rightarrow \int_0^{\infty} z^2 e^{-a(z-v_w)^2} dz \approx \int_0^{\infty} f(z) z^2 dz \quad \text{c.m.}$$

in other words, the pressure on an elastic wall (the wave) depends only on the relative velocity between the wall and particles it confines.

b) Ion transit time.

In the wave frame a particle incident with energy E will require penetration of the wave potential to a distance $\mathcal{L}$ in order to be reflected. Take the average velocity during reflection as $v_i/2$, where v_i is the incident ion velocity ($v_i = v_x \pm v_w$). The transit time, τ , is then defined as ($X = \mathcal{L}\lambda_D$)

$$\tau = \frac{2\mathcal{L}\lambda_D}{v_i/2} = \frac{4\mathcal{L}\lambda_D}{v_i}$$

but $v_i = \sqrt{\frac{2E}{m_i}}$, so the transit time becomes $\tau = \frac{4\mathcal{L}\lambda_D}{\sqrt{2E/m_i}}$

we require that the ion transit time be small compared to the half-fold time of the potential wave amplitude, in order that those ions contribute to the charge density producing the assumed wave amplitude. The previously obtained solutions for the potential correspond to the wave frame and are:

$$n = 2 \ln \left[\frac{\sqrt{2}}{2} \mathcal{L} + \sqrt{e} \right] = E/KT'$$

for $E/KT' > 1$

KT' is the average energy of the incident ion flux. This only coincides with KTo when $V_\omega = 0$. We have made the correspondence that the average energy of the incident particle flux of the dynamic problem is equivalent to a "temperature" T' of the static solution, which gives the same pressure.

An implicit equation for the penetration depth, $\mathcal{L}$, required for a transit time τ is then,

$$\mathcal{L} = \sqrt{2} \left[e^{\frac{m_i}{\pi N e^2 \tau^2} \mathcal{L}^2} - \sqrt{e} \right] \quad \text{--- 3)}$$

where $\lambda_D^2 = KT'/4\pi N e^2$ was used. $\omega t > \pi/4$ range: ($V_\omega < 0$)

For this portion of the cycle, we will need an integral.³

$$\int_0^\infty (v_x + v_\omega)^2 e^{-av_x^2} dv_x = \left[\frac{\sqrt{\pi}}{4a^{3/2}} + \frac{v_\omega}{a} + \frac{v_\omega^2 \sqrt{\pi}}{2a^{1/2}} \right]$$

First, when $V_\omega = 0$, $P_i = 1$, $\omega t = \pi/4$. Later, as $P_i \rightarrow 2$, $\omega t = \pi/3$ and we must solve for V_ω from

$$P_i = 2N M_i \left(\frac{a}{\pi}\right)^{1/2} \int_0^{v_m} (v_x + v_\omega)^2 e^{-av_x^2} dv_x = 2NKT$$

which means we need v_m , the maximum outward c.m. ion velocity which can transit the wave in time to contribute to the charge density of the wave. For this we use Eq. 3. The available half-fold time is set by the field cycle time and between $\omega t = \pi/6$ for P_i to change by 1 gives a half-fold time of $T_{1/2} = \pi/24\omega$. If we use this as the maximum allowable transit time, τ , in Eq. 3 we obtain a stopping distance $\omega' \approx 7$ which yields $\eta = 3.773$ and $E = KT'\eta \approx (2KT'_0) 3.773$ and therefor an incident velocity $v_i = 2.747v_\phi$ and a $v_m = 2.747v_\phi - v_\omega \gg 1/a^{1/2}$. Then for this range of the cycle we can extend the integral $v_m \rightarrow \infty$ with little error.* This means the wave velocity must satisfy

$$\left[\frac{v_\omega^2 \sqrt{\pi}}{2a^{1/2}} + \frac{v_\omega}{a} + \frac{\sqrt{\pi}}{4a^{3/2}} \right] 2NM_i \left(\frac{a}{\pi}\right)^{1/2} = 2NKT_0$$

or

$$v_\omega \approx \frac{1}{a^{1/2}} \left[\sqrt{\frac{1}{\pi} + \frac{1}{2}} - \frac{1}{\sqrt{\pi}} \right] \quad \text{when } \omega t \approx \pi/3$$

(numerically, $v_\omega = - .340v_\phi$)

Still later, similar considerations yield

$$\text{for } \omega t \approx \pi/2 ; \quad v_i \approx 3.3643v_\phi, \quad v_m \rightarrow 1/a^{1/2}$$

*See Appendix A

and with even less error,

$$v_{\omega} \approx \frac{1}{a^{1/2}} \left[\sqrt{\frac{1}{\pi} + 1} - \sqrt{\frac{1}{\pi}} \right] \quad \text{when } \omega t \approx \pi/2$$

(numerically, $v_{\omega} = - .584 v_c$)

and this is approximately the maximum inward wave speed.

$v_{\omega} > 0$, $\omega t < \pi/4$ range:

For this portion of the cycle, we need another integral:³

$$\int_{v_{\omega}}^{\infty} (v_x - v_{\omega})^2 e^{-av_x^2} dv_x = \left[(x^2 + 1/2) (1 - \phi(x)) - \frac{xe^{-x^2}}{\sqrt{\pi}} \right] \frac{\sqrt{\pi}}{2a^{3/2}}$$

where $\phi(x)$ = error function and $x = \sqrt{a} v_{\omega}$

Following the same procedure as for the previous section, we soon find that the distribution is cut-off and pressure equilibrium between field and particles is lost within about $\pm \omega_{pi}^{-1}$ of $t = \frac{\pi}{6\omega}$. The reason for this is that the available half-fold time must vanish at $t = \frac{\pi}{6\omega}$, when the wave amplitude vanishes. The ions which can transit the wave in time to contribute to its charge density are those of progressively lower energy as the time in the cycle approaches that for the potential to reverse sign ($t \approx \pi/6\omega$)

There is a portion of the cycle between $\omega t = \pi/6$ and $\pi/4$, however, where a reasonable value $v_{\omega} > 0$ can be calculated using known integrals. From $t = \frac{.2098\pi}{\omega}$ ($P_i = 1/2$) to $t = \frac{\pi}{4\omega}$ gives $T_{1/2} \approx .0677/\omega$ and $\mathcal{L} \approx 3.1$

hence $\eta = 2.6908$, $E \approx (\frac{KT_0}{2}) 2.6908 = 1.3454 KT_0$

so $v_m = 1.16 v_0 + v_w > 1/a^{1/2}$; and again if we extend the integral we obtain the equation satisfied approximately by v_w ,

$$2NKT_0 \left[(1/2 + x^2) (1 - \phi(x)) - \frac{xe^{-x^2}}{\sqrt{\pi}} \right] \approx \frac{MKT_0}{2}$$

which results in $v_w \approx .290v_0$. This gives then $v_m \approx 1.16v_0 + .290v_0 = 1.45v_0$ and justifies the approximation.*

When this procedure is tried for $P_i \approx 1/4$, we obtain $\mathcal{L} \approx 1.4$ and $v_w \approx .55v_0$ and hence the integral above is not even approximately correct. An ion which can only travel $\sim \lambda_D$ and has $v_i \approx v_0$ spends only ω_{pi}^{-1} in the wave and thus violates the assumed equilibrium condition, and the wave shape is not even approximately that of the static solutions, for $E/KT' > 1$.

The solutions for $E/KT' < 1$, the "leading edge" of the wave, have an entirely different spatial dependence.¹ Unlike the slow logarithmic dependence, a very rapidly rising, nearly exponential shape occurs in this energy range. An approximate value for v_w near $\omega t = \pi/6$, can be obtained by setting the wave pressure equal to that ω_{pi}^{-1} after $\omega t = \pi/6$ and only allow those ions which can traverse $\sim \lambda_D$ to be counted in the wave charge density. For this purpose we need another integral,*

$$\int_{v_w}^{v_m} (v_x - v_w)^2 e^{-av_x^2} dv_x = \frac{\sqrt{\pi}}{2a^{3/2}} \left\{ (x^2 + 1/2) [\phi(1+x) - \phi(x)] + e \frac{-(1+x)^2}{(x-1)} - \frac{xe^{-x^2}}{\sqrt{\pi}} \right\}$$

*See Appendix A

where $x = \sqrt{a} v_w$ and $v_m = 1/a^{1/2}$. The ion pressure needed to balance that from the field and electrons at $\omega t = \pi/6 + \omega/\omega_{pi}$ can be estimated by making an expansion of $4 \sin^2 \omega t$ about $\pi/6$, with the result

$$P_i(\pi/6 + \omega/\omega_{pi}) \approx 4 \frac{\omega}{\omega_{pi}} \cdot .866 = .0693 \text{ for } \omega/\omega_{pi} = 2 \times 10^{-2}$$

using the above integral we then find $x \approx .578$ and $V_w \approx .578 v_0$.

Between $\omega t = 0$ and $\sim \pi/6$, the relatively simple situation of ion inertial confinement of the electrons holds (see Fig. II). The potential again appears in the wave (now ion free expansion) frame, as essentially the static solution with $\eta < 0$ everywhere. As the field pressure varies between $\omega t = 0$ and $\pi/6$, it is in equilibrium with the potential and electron pressure. The field momentum at $\omega t = \pi/6$ is balanced by the thermal electron momentum. As the field momentum diminishes, the electrons hand their momentum to the ions, and finally at $\omega t = 0$ all the electron momentum is reflected by the ions. The ion expansion speed remains essentially constant because all the electrons can do is hand their momentum to the ions.

Finally, an additional clue as to the approximate free expansion speed of the ions, between $\omega t = 0$ and $\pi/6$, can be ascertained by the observation that

$$\int_{\text{cycle}} V_w dt' = 0$$

for average confinement and we know the value of this integral over enough of the cycle that, together with the condition that $V_w \approx \text{constant}$ during

this portion, $v_w \approx v_0/2$.

IV. Summary

The behavior of the ion accelerating potential wave, resulting from reflecting a plane polarized microwave field from the electrons, can be understood by using steady state charge distributions to estimate transit time effects in the wave. This allows approximate time dependent solutions to be generated. Accurate wave velocities can be calculated for the latter half of the cycle, and estimates made of the first half.

The wave reaches a peak inward velocity of $\sim .584v_0$ and a free expansion velocity of $\sim v_0/2$, which also happens to coincide with the root mean square speed of a maxwellian ion distribution. A graph of $v_p(t)$ over the entire field cycle is shown below in Fig. III.

References

1. ~~UCIR-1185~~ "Equilibrium Plasma Corona Surfaces". *UCID-18228*
2. K. Huang, "Statistical Mechanics", John Wiley & Sons, 1963 p.72.
3. I. S. Gradshteyn, "Table of Integrals Series and Products", Academic Press, N.Y., 1965 pp 317, 338.

Appendix A

$\omega t > \pi/4$

Some integrations were extended from v_m to ∞ and we want to estimate the error involved. v_m is the maximum c.m. ion velocity allowed to contribute to the wave charge density from the outgoing Maxwellian distribution. (See Ref. 3 for integrals.)

$$\begin{aligned} \int_0^{v_m} (v_x + v_\omega)^2 e^{-av_x^2} dv_x &= \int_0^\infty (v_x + v_\omega)^2 e^{-av_x^2} dv_x - \int_{v_m}^\infty (v_x + v_\omega)^2 e^{-av_x^2} dv_x \\ &= \frac{\sqrt{\pi}}{4a^{3/2}} + \frac{v_\omega}{a} + \frac{v_\omega^2 \sqrt{\pi}}{2a^{1/2}} - \frac{\sqrt{\pi}}{4a^{3/2}} - \frac{v_m e^{-av_m^2}}{2a} - \frac{v_\omega e^{-av_m^2}}{a} - \frac{v_\omega^2 \sqrt{\pi}}{2\sqrt{a}} (1 - \sqrt{a} v_m) \end{aligned}$$

Using the $v_m \rightarrow \infty$ value of v_ω in the correction where $v_m = v_i + v_\omega$, v_i is incident speed on wave from transit time, the corrections are:
 $v_\omega \approx .346v_0$ rather than $.340v_0$ for $\omega t = \pi/3$ a 1.7% correction. Values for v_ω in this portion of the cycle are quite accurate.

$\omega t < \pi/4$

We want $\int_{v_\omega}^{v_m} (v_x - v_\omega)^2 e^{-av_x^2} dv_x$, rather than extending the upper

limit to ∞ . (Refer to Ref. 3 for the integrals used.) set $z = (v_x - v_\omega)$

$$\int_{v_{\omega}}^{v_m} (v_x - v_{\omega})^2 e^{-av_x^2} dv_x = \int_0^{(v_m - v_{\omega})} z^2 e^{-a(z + v_{\omega})^2} dz = \int_0^{\infty} z^2 e^{-a(z + v_{\omega})^2} dz - \int_{(v_m - v_{\omega})}^{\infty} z^2 e^{-a(z + v_{\omega})^2} dz$$

which results in, with $\alpha = (v_i + v_{\omega})$

$$\int_0^{v_m - v_{\omega}} z^2 e^{-a(z + v_{\omega})^2} dz = \frac{\sqrt{\pi}}{2a^{3/2}} \left\{ \left(\frac{1}{2} + x^2 \right) [\Phi \sqrt{a} \alpha - \phi(x)] - \frac{x e^{-x^2}}{\sqrt{\pi}} + \frac{x e^{-ax^2}}{\sqrt{\pi}} \left(2 - \frac{\alpha}{v_{\omega}} \right) \right\}$$

using the extended integral, at $\omega t = .2098\pi$, $P_i = 1/2$ gave $v_{\omega} \approx .29v_{\theta}$.

Using this to compute the correction, with $v_i = 1.16v_{\theta}$ and $v_{\omega} \approx .29v_{\theta}$

$\alpha = 1.45v_{\theta}$, and the iterated value on $v_{\omega} \approx .21v_{\theta}$, a 20% correction.

The above equation can be used directly for v_{ω} without making any correction, as was done for $v_i = 1/a^{1/2}$ and resulted in $v_{\omega} \approx .578v_{\theta}$ for $\omega t = \pi/6 + \omega/\omega_{p1}$.

FIG. 1 $\omega/\omega_p > 1/6$

Fig. III, V_w