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HEATING BY LANDAU WAVE DAMPING OF
SLOW MODES IN TOKAMAK PLASMAS

BY

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CYCLOTRON EMISSION DURING ELECTRON HEATING BY
LANDAU WAVE DAMPING OF SLOW MODES IN TOKAMAK PLASMAS

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ABSTRACT

The emission of cyclotron radiation from PLT plasma in the presence of a mildly superthermal tail in the electron velocity distribution is investigated. The emitted spectrum has asymmetries which can be used to investigate the tail formation during slow modes electron Landau damping.

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I. Introduction

As is well known, the direct effect of wave absorption via electron Landau damping is the distortion of the tail of the electron velocity distribution in the direction parallel to the plasma confining magnetic field $\vec{B}_t$. The magnitude of the deformation depends on the wave amplitude, the $k_{||}$ spectrum, and on the effective collision rate. For instance, for large values of D_{rf}/D_c , where D_{rf} and D_c are the quasilinear and collisional diffusion coefficients, respectively, it is found that the electron distribution is constant for parallel velocities near the wave-packet phase velocities. A corresponding deformation in the perpendicular direction is generally associated with the parallel flattening which results from parallel to perpendicular energy and momentum transfer. In the two cases of most interest, namely the slow mode near the lower hybrid frequency and the slow mode resulting from mode conversion in the ion cyclotron range of frequencies, the accurate knowledge of the wave amplitude, the $k_{||}$ spectrum and the origin of the electron effective collision frequency is still an open problem.

In this report, we discuss the properties of the cyclotron radiation emitted by the electron system with a superthermal tail generated during rf heating. Preliminary results obtained in the TFR tokamak¹ suggest that the study of the cyclotron radiation yields useful information about the superthermal electron population which may help for a better understanding of the tail formation. To anticipate the effect of the superthermal electrons on the cyclotron radiation, we first present a qualitative discussion of the emitted radiation by using the resonance condition for a tokamak magnetic field. We first consider the emission

of radiation from a tokamak plasma in the torus equatorial plane in the direction normal to $\vec{B}_t$ (the x-direction). Let R_0 be the torus large radius, thus $B_t(x) = B_t(0) (1 - x/R_0)$ where $x = 0$ is at the plasma center and $|x| < a$, where a is the plasma radius. A wave of frequency $\omega = s \omega_c$ ($s = 1, 2$) resonates with the electrons of momentum p if

$$(p/mc)^2 = (s\omega_c/\omega)^2 - 1 = -2x/R_0, \quad (1)$$

where $\omega_c > 0$ is the cyclotron frequency, $\omega = s\omega_c(0)$, and m is the electron rest mass. Equation (1) states that particles with large values of p interact with the wave at large (negative) values of x (the high magnetic side). Let T_b and T_s be the temperatures of the thermal body and the superthermal components, respectively, and $T_s \gg T_b$. Thus, the emission and absorption of the two components do not overlap, and one component may screen the other depending on the direction of emission. If the body is optically thick, the emission in the low magnetic side is blackbody at the temperature $T_r = T_b$, the radiation of the superthermal electrons being reabsorbed by the body. The emission in the high magnetic side, on the other hand, is mainly due to the superthermal electrons and $T_r > T_b$. In this case, the radiation from the thermal electron is reabsorbed by the superthermal electrons. If the body is optically thin, the emission in the low magnetic side is partially due to the superthermal electrons. If the superthermal component is localized in the region where $\omega > \omega_c$, the emission in the two sides is due to the body only. This reasoning can be repeated for any frequency ω and may serve to localize the spatial distribution of the superthermal electrons. We now consider the emission of radiation at an angle θ with respect to the direction of $\vec{B}_t$. The resonance condition is given by

$$(1 + p^2/m^2 c^2)^{1/2} - (s\omega_c/\omega) - n_{||} p_{||}/mc = 0 \quad (2)$$

where $n_{||} = \cos\theta$. Since $p_{\perp} > 0$, we obtain that $p_{||}/mc$ lies in the range determined by

$$p_{\pm}/mc = \frac{n_{||} (s\omega_c/\omega) \pm [n_{||}^2 - (1 - s^2 \omega_c^2/\omega^2)]^{1/2}}{1 - n_{||}^2}.$$

In the drifting frame and for $n_{||}^2 \ll 1$, $s\omega_c/\omega = 1$, we obtain

$$p_{\pm}/mc = n_{||} - u_d \pm [n_{||}^2 - (1 - s^2 \omega_c^2/\omega^2)]^{1/2}, \quad (3)$$

where $u_d = v_d/c$ and v_d is the drift velocity. We see from Eq. (3) that $n_{||} - u_d$ plays the role of an effective parallel index, determined by the relative values of $n_{||}$ and u_d . This suggests that the emission and absorption of radiation may depend on the sign of $n_{||}$. For instance, the intensity at θ differs from that at $\pi - \theta$ since $n_{||} = \cos\theta = -\cos(\pi - \theta)$. Note that for a Maxwellian distribution or more generally for a plasma with a symmetric electron distribution $F(p_{\perp}, p_{||}) = F(p_{\perp}, -p_{||})$, we have $I_{\omega}(\theta) = I_{\omega}(\pi - \theta)$, where I_{ω} is the specific intensity. For a plasma, located in a magnetic field of moderate amplitude (PLT), the case of the ordinary mode at $\omega = \omega_c$ is of special interest. For emission towards the low magnetic side at an angle $n_{||} = u_d$ the contribution of the drifting superthermal electron is similar to that of perpendicular propagation, whereas, the contribution of the body is that of oblique propagation. For $n_{||} = -u_d$, the contribution of the superthermal electron is also that of oblique

propagation. Since for the ordinary mode wave absorption is maximum for perpendicular propagation, we expect an asymmetric pattern for the emitted radiation. A similar behavior is expected for the extraordinary mode at $2\omega_c$. This asymmetry may be used to investigate the possible variation of the drift velocity and may be useful for studies of rf current generation.^{2,3}

A quantitative analysis of the properties of the emitted radiation is made by using a trial distribution function with a number of adjustable parameters to be determined from the best fit with the experimental results. In this report, we represent the momentum distribution as the sum of two Maxwellian representing the body of the electrons and a small fraction η of superthermal drifting electrons, respectively.

$$F = \left[(\mu_b/2\pi)^{3/2} / (mc)^3 \right] \exp \left[-\mu_b (p/mc)^2 / 2 \right] \\ + \eta \left[(\mu_s/2\pi)^{3/2} / (mc)^3 \right] \exp \left\{ -\mu_s (p_{\perp}/mc)^2 / 2 - \mu_s (p_{\parallel}/mc - u_d)^2 / 2 \right\},$$

(4)

where $\mu_b = mc^2/T_b$, $\mu_s = mc^2/T_s$ and $\eta \ll 1$. The theory of the cyclotron radiation for a mildly relativistic non-Maxwellian distribution function is investigated in Refs. 4 and 5. For simplicity, we use the same notation as in Ref. 5.

It is worth mentioning that the presence of a superthermal tail may also occur during electron cyclotron heating for oblique propagation and for any angle in the high magnetic side in a large ($R_0 > 10^2$ cm) hot plasma ($T_e > 1$ keV) tokamak.⁵

This report is organized as follows. In Sec. II, we briefly discuss the theory of the cyclotron emission at $\omega = \omega_c, 2\omega_c$. In Sec. III, we present the numerical results obtained for PLT. In Sec. IV, we give the conclusion.

II. Theory of the Electron Cyclotron Emission

We consider perpendicular or quasi-perpendicular emission $|n_{||}| \ll 1$ and use the slab geometry, thus $\vec{B}_t(x) = B_t(x) \hat{e}_z$, $\nabla n_0 \perp \nabla T \perp \hat{e}_z$, where n_0 is the electron density. We define

$$P_\omega(\pm a) = I_\omega(\pm a) / [\omega^2 T_b(x_c) / (2\pi)^3 c^2], \quad (5)$$

where $\pm a$ are the plasma boundaries, and x_c is defined by the relation $\omega = s\omega_c(x_c)$. For an optical thick plasma and $\eta = 0$, $P_\omega(a) = P_\omega(-a) = 1$. Of course, for the extraordinary mode at $\omega = \omega_c$ we only have $P_\omega(-a)$. For $\eta \neq 0$, the electron system is not Maxwellian and the evaluation of I_ω is obtained from the theory of the nonthermal emission.^{4,5} We obtain

$$P_\omega(a) - P_{0\omega} = \{J(-a, a; n_{||}) + J(a, -a; n_{||}) \exp[-\tau(n_{||})]\} \\ \times \{1 - \gamma^2 \exp[-2\tau(n_{||})]\}^{-1}, \quad (6)$$

$$J(-a, a; n_{||}) = \int_{-a}^a dx [T_x(x)/T_b(x_c)] (2k'') \exp(-2 \int_x^a dx' k''),$$

$$\tau(n_{||}) = 2(\omega/c) \int_{-a}^a dx n''(x),$$

where $P_{0\omega}$ is the contribution of a possible source of radiation due to mode conversion, γ is the reflection coefficient of the material boundaries, $k'' = (\omega/c) n''$, and τ is the optical thickness. Equation (6) yields the emission at $x = a$; by changing a into $-a$ and k'' into $-k''$, we obtain the emission at $-a$. Note that $J(a, -a; n_{||})$ is obtained from $J(-a, a; n_{||})$ using the same sign convention.

Using Eq. (4), the radiation temperature and n'' are evaluated and for $\omega \approx \omega_c$, $2\omega_c$ and $n_{||} = 0$, the results are given below.

Ordinary mode.

$$T_r = \frac{T_b \psi_b + \eta T_s \psi_s}{\psi_b + \eta \psi_s},$$

$$2n'' = (1 - \omega_p^2 / \omega^2)^{1/2} (\psi_b + \eta \psi_s), \quad (7)$$

where

$$\begin{aligned} \psi_b = & \left[(2\pi\mu_b)^{1/2} / 30 \right] (\omega_p^2 / \omega_c \omega) \mu_b^2 \left(\frac{\omega_c}{\omega} - 1 \right)^{5/2} \exp \left[-\mu_b (\omega_c^2 / \omega^2 - 1) / 2 \right] \\ & + \left(1 - \frac{\omega_p^2}{\omega^2} \right) (\pi/8)^{1/2} (\omega_p^2 \omega / \omega_e^3) (\mu_b^{5/2} / 105) \left(4 \frac{\omega_c^2}{\omega} - 1 \right)^{7/2} \exp \left[-\mu_b \left(\frac{4\omega_c^2}{\omega^2} - 1 \right) / 2 \right], \end{aligned}$$

$$\psi_s = (\pi\mu_s / 2u_d^{3/2}) (\omega_p^2 / \omega^2) \left(\frac{\omega_c}{\omega} - 1 \right)^{7/4} [I_{7/2}(\xi_1) + I_{5/2}(\xi_1) / \xi_1]$$

$$\begin{aligned}
& \times \exp \left[-\mu_s \left(\frac{\omega_c^2}{\omega^2} - 1 + u_d^2 \right) / 2 \right] \\
& + \left(1 - \frac{\omega_b^2}{\omega^2} \right) \frac{(2\pi\mu_s)^{1/2}}{4\mu_s^5 \mu_d^7} (\omega_p^2 \omega / \omega_c^3) \left(\frac{\pi \epsilon_2}{2} \right)^{1/2} \left[-9 \epsilon_2 (10 + \epsilon_2^2) I_{3/2}(\epsilon_2) \right. \\
& \left. + \epsilon_2^2 (30 + \epsilon_2^2) I_{1/2}(\epsilon_2) \right] \exp \left[-\mu_s \left(\frac{4\omega_c^2}{\omega^2} - 1 + u_d^2 \right) / 2 \right],
\end{aligned}$$

where $\epsilon_n = \mu_s u_d (n^2 \omega_c / \omega^2 - 1)^{1/2}$, $n = 1, 2$. For the ordinary mode $P_{O\omega} = 0$.

Extraordinary mode.

For $\omega = \omega_c$

$$T_r = \frac{T_b \psi_b + \eta T_s \psi_s}{\psi_b + \eta \psi_s},$$

(8)

$$2 n'' = n_x^{-1} \left| 1 - i \frac{\epsilon_{xy}}{\epsilon_{xx}} \right|^2 (\psi_b + \eta \psi_s),$$

where

$$n_x^2 = 1 + \frac{(\omega_p^2 / \omega^2) (\omega_p^2 / \omega^2 - 1)}{1 - \omega_p^2 / \omega^2 - \omega_c^2 / \omega^2},$$

$$\epsilon_{xx} = \epsilon'_{xx} + 1 (\psi_b + \eta \psi_s), \quad \epsilon_{xy} = \epsilon'_{xy} + (\psi_b + \eta \psi_s),$$

$$\psi_b = \left[(2\pi)^{1/2}/6 \right] (\omega_p^2 \omega_c/\omega^3) \mu_b^{5/2} \left(\frac{\omega_c^2}{\omega^2} - 1 \right)^{3/2} \exp[-\mu_b (\omega_c^2/\omega^2 - 1)/2],$$

$$\psi_s = (\pi \mu_s/2 u_d^{3/2}) (\omega_p^2 \omega_c/\omega^3) \left(\frac{\omega_c^2}{\omega^2} - 1 \right)^{3/4} I_{3/2}(\xi_1)$$

$$\times \exp \left[-u_s \left(\frac{\omega_c^2}{\omega^2} - 1 + u_d^2 \right) / 2 \right],$$

and ϵ'_{xx} , ϵ'_{xy} are given in Ref. 5. In this case, $\gamma = 0$ and $P_{0\omega} = \exp(-\tau)$. This contribution was found experimentally⁶ and is attributed to wave conversion of Bernstein modes into transverse electromagnetic radiation. For $\omega = 2\omega_c$

$$T_r = \frac{T_b \psi_b + \eta T_s \psi_s}{\psi_b + \eta \psi_s},$$

(9)

$$2n'' = n_x \left(\frac{6 - \omega_p^2/\omega_c^2}{6 - 2\omega_p^2/\omega_c^2} \right)^2 (\psi_b + \eta \psi_s),$$

where

$$\psi_b = [(2\pi)^{1/2}/15] (\omega_p^2/\omega\omega_c) \mu_b^{5/2} \left(\frac{4\omega_c^2}{\omega^2} - 1\right)^{5/2} \exp \left[\mu_b \left(\frac{4\omega_c^2}{\omega^2} - 1\right)/2\right],$$

$$\psi_s = (\pi/u_d^{5/2}) (\omega_p^2/\omega\omega_c) \left(\frac{4\omega_c^2}{\omega^2} - 1\right)^{5/2} I_{5/2}(\xi_1) \exp \left[-\mu_s \left(\frac{4\omega_c^2}{\omega^2} - 1 + u_d^2\right)\right].$$

The validity of Eqs. (9) is restricted by the condition that mode conversion between the extraordinary mode and the longitudinal mode⁷ is forbidden. As noted in Ref. 8, this put an upper limit in the value of ω_p^2/ω_c^2 . For perpendicular propagation and $\eta = 0$, $\omega_p^2/4\omega_c^2 \approx 0.28$ which is less than the cutoff value 0.5. The case of $n_{||} \neq 0$ is much more involved and is discussed in Ref. 5.

III. Numerical results for PLT

To investigate the cyclotron emission in PLT, we choose the following basic parameters ($\gamma = 0.95$)

$$n_b(x) = n_b(0) \left[1 - \left(\frac{x}{40}\right)^2\right], \quad n_b(0) = 2 \times 10^{13} \text{ cm}^{-3},$$

$$T_b(x) = T_b(0) \left[1 - \left(\frac{x}{40}\right)^2\right]^2, \quad T_b(0) = 1 \text{ keV}, \quad (10)$$

$$B_t(x) = B_t(0)/(1 + x/130), \quad B_t(0) = 28 \text{ kG}.$$

For the superthermal beam we consider two different situations

$$n_s(x) = n n_b(o) \left(1 - \frac{x^2}{b^2}\right), \quad n = 5 \times 10^{-2}, b = 10$$

$$T_s(x) = T_s(o) \left(1 - \frac{x^2}{b^2}\right)^2, \quad T_s(o) = 4 \text{ keV} \quad (11)$$

$$u_d(x) = u_d(o) \left(1 - \frac{x^2}{b^2}\right), \quad u_d(o) = 0.18.$$

Equations (11) describe a beam of superthermal electrons located in the vicinity of $x = 0$ and may occur even in the absence of rf in a low density plasma. The presence of the rf may shift the region where the superthermal electrons are located. To describe this situation we consider the following profiles

$$n_s(x) = n n_b(o) \left\{ \left[1 - \left(\frac{x - x_o}{b}\right)^2\right] + \left[1 - \left(\frac{x + x_o}{b}\right)^2\right] \right\}, \quad b = x_o$$

$$T_s(x) = T_s(o) \left\{ \left[1 - \left(\frac{x - x_o}{b}\right)^2\right]^2 + \left[1 - \left(\frac{x + x_o}{b}\right)^2\right]^2 \right\}, \quad (12)$$

$$u_d(x) = u_d(o) \left\{ \left[1 - \left(\frac{x - x_o}{b}\right)^2\right] + \left[1 - \left(\frac{x + x_o}{b}\right)^2\right] \right\}.$$

The superthermal electrons carry a small fraction of the total current ($\approx 60kA$). In Figs. 1 and 2, we show $k'' = (\omega/c) n''$ vs x for the O and X mode for the situation described by Eqs. (10) and (11) and $n_{||} = 0$, $\omega = \omega_c(o)$. For $\eta = 0$ we obtain the damping of the thermal electrons (a). For $\eta = 5 \times 10^{-2}$ (curve b), the contribution of the superthermal

tail appears for large (negative) x . It is clear that the radiation observed at $x = a$ is mostly due to the body of the electrons since the radiation generated by a radiating element situated at $x < -2$ cm has little chance to emerge at $x = a$. Vice versa, the radiation emerging at $x = -a$ is mainly due to the superthermal electrons. However, if the body of the electron distribution is not optically thick, the radiation at $x = a$ is partially due to the superthermal tail. The emitted spectrum is presented in Fig. 3. Figures 3 (a,c) refer to the O mode at $x = a$ and $x = -a$, respectively. The emission of the X mode at $x = -a$ is presented in Fig. 3b. We first note that the maximum of emission is shifted toward the positive x . This is expected since the superthermal beam is centered at $x = 0$. The second feature is that the intensity at $x = -a$ is larger than at $x = a$. This asymmetry is the result of the presence of the superthermal component. Note also that the effect of the tail is manifest at $x = a$ since the body of the electrons is not completely optically thick. We now consider the emission for the situation described by Eqs. (10) and (12) with $x_0 = 10$ cm. In Fig. 4 (a,c) we present the O mode at $x = \pm a$ and in Fig. 4b we present the X mode at $x = -a$. It is evident that Fig. 4 is similar to Fig. 3 except that the beam contributes approximately twice. The emission at an angle with respect to the direction of $\vec{B}_t$ for the case of Eqs. (10) and (11) is presented in Figs. 5, 6, and 7. We consider three angles $\theta = 75^\circ, 90^\circ, 105^\circ$, i. e., $n_{||} = 0, \pm 0.26$. In Fig. 5, we present the emission for the O mode at $x = a$. We see that the value and the position of the maximum are a sensitive function of $n_{||}$. The emission at $x = -a$ (Fig. 6) shows a similar dependence on $n_{||}$. In Fig. 7, we show the emission for the X mode at $x = -a$. The value $P_{x\omega}(-a) = 1$ for $\theta = 75^\circ$ is due to the

presence of the source term P_{0w} which we assume is the same as for $\theta = 0$. Without this thermal source P_{0w} ($n_{||} = 0$) $\ll 1$ as expected for a plasma confined by a relatively small magnetic field.

IV. Conclusions

We have investigated the emission of radiation at the electron gyrofrequencies in the PLT tokamak plasma with a small fraction of mildly superthermal electrons. The emitted spectrum displays the peculiar asymmetries due to the relativistic mass variation and the spatial variation of the confining magnetic field. Using a trial distribution function for the electron system with a number of adjustable parameters to be determined by the best fit with the measured spectra, it is possible in principle to study the modification of the electron tail during rf heating and current generation via electron Landau damping. The tail formation can also be investigated during ion cyclotron heating in the presence of a mode conversion layer. In this case the superthermal component should be localized near the conversion layer, and the emitted radiation has the pattern shown in Fig. 4.

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Fig. 1 $k'' = (\omega/c)n''$ vs x for the O mode for $\omega = \omega_c$ (a) and $n_{||} = 0$ and the plasma parameters given by Eqs. (10) and (11).

81 X 0292

Fig. 2 The same as Fig. 1 for the X mode.

Fig. 3 Emitted spectrum vs ω (or x_c) for the O mode towards the low magnetic side (a) and high magnetic side (c) and for the X mode (b) for the parameters given by Eqs. (10) and (11).

81 X 0290

Fig. 4 Emitted spectrum vs ω for the O mode (a,c) and X mode (b) for the parameters given by Eqs. (10) and (12).

81 X 0288

Fig. 5 Emitted spectrum vs ω for the O mode for $\theta = 75^\circ, 90^\circ, 105^\circ$ at $x = a$ for the parameters given by Eqs. (10) and (11).

81X0287

Fig. 6 The same as in Fig. 5 for $x = -a$.

81X 0289

Fig. 7 Emitted spectrum vs ω for the X mode.