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MECHANISTIC MODELING AND CORRELATIONS FOR
POOL-ENTRAINMENT PHENOMENON

by

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ABSTRACT

Entrainment from a liquid pool with boiling or bubbling is of considerable practical importance in safety evaluation of nuclear reactor under off-normal transients or accidents such as loss-of-coolant and loss-of-flow accidents. Droplets which are suspended from a free surface are partly carried away by streaming gas and partly returned back to free surface by the gravity. A correlation is developed for the pool entrainment amount based on simple mechanistic modeling and a number of data. This analysis reveals that there exist three regions of entrainment in the axial direction from a pool surface. In the first region (near surface region), entrainment is independent of height and gas velocity. In the second region (momentum controlled region), the amount of entrainment decreases with increasing height from the free surface and increases with increasing gas velocity. In the third region (deposition controlled region), the entrainment increases with increasing gas velocity and decreases with increasing height due to deposition of droplets. The present correlation agrees well with a large number of experimental data over a wide range of pressure for air-water and steam-water systems.

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Title

Phenomenological Modeling of Two-phase Flow in Water Reactor
Safety Research

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NOMENCLATURE

A_d	Projected area of droplet
A_H	Heated Area (heated perimeter times heated length)
A_v	Cross sectional area of vessel
C_D	Drag coefficient of droplet
C	Coefficient in Eq. (64)
C_E	Droplet concentration in gas space (Kg/m^3)
C_K	Coefficient in Eq. (2)
C_m	Coefficient in Eq. (92)
C_1	Coefficient in Eq. (105)
$\dot{d}$	Deposition rate of droplets ($\text{Kg/m}^2/\text{s}$)
D	Droplet diameter
D^*	Dimensionless droplet diameter defined by Eq. (37)
D_B	Bubble diameter
D_c	Critical droplet diameter whose terminal velocity is equal to gas velocity
D_c^*	Dimensionless critical droplet diameter defined by Eq. (12)
D_H	Hydraulic diameter of vessel
D_H^*	Dimensionless hydraulic diameter of vessel defined by Eq. (5)
$D_{\max}$	Maximum droplet diameter
$D_{\max}^*$	Dimensionless maximum droplet diameter defined by Eq. (86)
E_{fg}	Entrainment defined by Eq. (1)
$E_{fg}(h, j_g)$	Entrainment at height h and gas velocity j_g
$E_{fg}(h, j_g, D)$	Entrainment at height h and gas velocity j_g which consists of droplets whose diameter is less than D (given by Eq. (27))
$E_{fg\infty}(j_g)$	Limiting value of $E_{fg}(h, j_g)$ for $h \rightarrow \infty$
$E_o(D, j_g)$	Entrainment at interface which consists of droplets whose diameter are less than D (given by Eq. (25))
$f(D, j_g)$	Droplet size distribution function for j_g
f_D	Frequency of entrained droplet (in Eq. (31))
f_i	Interfacial friction factor between gas and liquid ligament
F_D	Drag force acting on droplet
g	Acceleration due to gravity

NOMENCLATURE (Cont'd)

$g(v_i, D, j_g)$	Initial velocity distribution function for diameter D and gas velocity j_g
h	Height above the pool surface
h^*	Dimensionless height above the pool surface defined by Eq. (7)
h_m	Maximum height of rising droplet
h_m^+	Dimensionless maximum height of rising droplet defined by Eq. (75)
j_{gc}	Superficial gas velocity at burnout
j_{gc}^*	Dimensionless superficial gas velocity at burnout defined by Eq. (132)
j_{fe}	Superficial velocity of liquid flowing upward as droplet
j_f	Total liquid volumetric flux
j_{fd}	Droplet volumetric flux (superficial velocity) of falling drops
j_g	Superficial gas velocity
j_g^*	Dimensionless superficial gas velocity defined by Eq. (3)
j_g^+	Dimensionless superficial gas velocity defined by Eq. (76)
k_d	Deposition coefficient of droplet (m/s)
K	Parameter defined by Eq. (11)
ℓ	Length of liquid ligament
$n_1 \sim n_3$	Exponents in Eq. (61)
n_4	Exponent in Eq. (92)
n_5	Exponent in Eq. (105)
N	Site density of entrained droplet (in Eq. (31))
$N_{\mu g}$	Gas viscosity number defined by Eq. (15)
$N_{\mu f}$	Liquid viscosity number defined by Eq. (8)
p	Escape probability of entrained droplet (in Eq. (31))
P	System pressure
P_0	Pressure around the bubble
q_c	Critical heat flux
Re_D	Droplet Reynolds number defined by Eq. (34)
Re_g	Gas Reynolds number defined by Eq. (90)
S	Velocity ratio between gas and liquid defined by Eq. (A.5)
t	Time
t_B	Bubble burst time

NOMENCLATURE (Cont'd)

v	Velocity
v_f	Velocity of element of liquid ligament
v_{fe}	Droplet velocity
v_g	Gas velocity
v_g^*	Dimensionless gas velocity defined by Eq. (52)
$v_h^{(D,j_g,h)}$	Initial velocity of droplet (diameter D) necessary to rise more than height h under gas velocity j_g
v_h^*	Dimensionless form of v_h defined by Eq. (81)
v_i	Initial velocity of droplet at pool surface
v_i^*	Dimensionless initial velocity defined by Eq. (51)
v_r	Relative velocity between droplet and gas defined by Eq. (69)
$v_{r\infty}$	Terminal velocity of droplet given by Eq. (78)
v_{ri}^+	Dimensionless initial relative velocity of droplet defined by Eq. (77)
We_{max}	Weber number based on maximum droplet diameter defined by Eq. (85)
y	Vertical position above pool surface
z	Vertical coordinate attached to the liquid ligament

Greek Symbols

α	Void fraction in liquid pool
α_E	Liquid fraction of droplet in gas space
β	Parameter defined by Eq. (113)
$\delta(x)$	Delta function defined by Eq. (66)
ΔH_{fg}	Latent heat of vaporization
$\Delta \rho$	Density difference between gas and liquid
$\dot{\epsilon}(j_g)$	Entrainment rate at pool surface for j_g ($\text{Kg/m}^2/\text{s}$)
μ_f	Viscosity of liquid
μ_g	Viscosity of gas
ν_f	Kinematic viscosity of liquid
ν_g	Kinematic viscosity of gas
ρ_f	Density of liquid
ρ_g	Density of gas
σ	Surface tension
τ_i	Interfacial shear stress between gas and liquid ligament

EXECUTIVE SUMMARY

Entrainment of liquid droplets from a bubbling or boiling pool can be quite important in a number of engineering fields. These include the analyses of radioactive material transports in nuclear engineering systems, heat and mass transfer processes and chemical reactors. Recently, this pool entrainment phenomenon has been recognized also to be quite important in accident analyses of both light water and fast breeder reactors. Examples are the thermo-hydraulic analyses of the reflooding phase of loss-of-coolant accidents in LWRs, loading and coolability analyses of a reactor during hypothetical core disruptive accidents in LMFBRs, and analyses of steam generators under off-normal or accident conditions. The entrainment from a liquid pool can significantly alter thermo-hydraulic phenomena in the region above the pool. In case of the reflooding in LWR accidents, it can be said that the entrained droplets in the post critical heat flux (CHF) regime act as a heat sink to superheated vapor. Thus entrainment is one of the key parameters in determining the dryout heat flux, post CHF heat transfer, vapor superheat and effectiveness of the emergency core cooling systems.

There are several correlations for the amount of entrainment mainly developed for boiler and chemical reactor applications. However, these correlations are not based on physical modeling of the pool entrainment phenomenon, therefore, their applicability may be limited. A number of shortcomings of existing correlations are pointed out in detail. In view of these, a new correlation based on a simple physical model is developed in this research by considering the droplet size distribution, initial droplet velocity, droplet motion and droplet deposition. From this detailed model, a practical correlation is obtained in collaboration with a large number of data. It is shown that there are three different regions in terms of entrainment above a bubbling pool, i.e., near surface, momentum controlled and deposition controlled regions. Furthermore, depending on the two-phase flow regimes in the pool, the correlation for the momentum controlled region is divided into three different correlations. This is due to the changes in droplet generation mechanisms which can be either bubble burst, momentum exchange droplet ejection or pseudo-jet disintegration. The present correlations are shown to agree well with a large number of experimental data.

I. INTRODUCTION

Entrainment of liquid by gas flow is often encountered in various areas of engineering applications associated with heat and mass transfer. For example, in safety evaluations of nuclear reactors, entrainment can play an important role on analyzing heat and mass transfer processes under both off-normal operational and accident conditions. There are several different entrainment mechanisms depending on two-phase flow regimes [1,2]. For annular dispersed flow, an onset of entrainment criterion [1], a correlation for the amount of entrained droplets [3], a correlation for droplet size and its distribution [4], and a correlation for entrainment rate [5] have been recently developed based on the shearing-off mechanism of roll-wave crests by a highly turbulent gas flow [1,6]. Coupled with a theoretical formulation of two-fluid model [7], these correlations provide accurate predictions of thermohydraulics of annular dispersed flow. For example, the dryout heat flux, post dryout heat transfer coefficient [8-13], vapor superheat, and the effectiveness of the emergency core cooling in light water reactors [14-17] can be evaluated using these correlations in the formulation.

Besides the above mentioned phenomena, there is another important mechanism of entrainment. That is, the entrainment from a liquid pool by gas flow in boiling or bubbling. Earlier, in the field of nuclear engineering, this pool entrainment was studied in relation to radioactive carryover in a boiling water reactor [18], decontamination factors in evaporation of radioactive liquid waste in a natural circulation evaporator [19-22], and steam generator performance. In the above two cases, entrainment has detrimental effects on reduction of radioactivity.

Recently the importance of the pool entrainment has been recognized associated with heat and mass transfer processes during loss-of-coolant accidents (LOCA), in particular, during the recovery phase of these accidents through reflooding of a core. In this case, pool entrainment may improve heat transfer, since droplets act as a heat sink through droplet evaporation. This will lead to lower vapor superheat and improved cooling of fuel pins which is quite important in terms of safety. Therefore, some researches have been carried out on pool entrainment during LOCA [23,24] and reflooding [25,26]. However, there have been no

satisfactory correlations which predict entrainment amount at given heat flux (or vapor velocity) and given distance from the free liquid surface.

In the field of chemical engineering, pool entrainment has been studied in relation to the efficiency of the gas liquid contacting equipments, e.g., plate columns, etc., [27-29] and fluidized beds [30,31]. Some correlations have been recommended [32] in this field. However, geometry and operational conditions of these chemical engineering equipments are quite different from those of boilers and nuclear reactor systems. Therefore, these correlations may not be directly applied to later cases.

As for the pool entrainment in boilers, some experimental works have been carried out and several empirical correlations have been proposed [33-49]. However, these correlations are not based on physical modeling, therefore their applicability may be limited.

In view of these, a correlation for the pool entrainment is developed here from a simple physical model by considering droplet size distribution, initial velocity of entrained droplets, droplet motion and droplet deposition. It is shown to correlate a number of data over a wide range of experimental conditions.

II. PREVIOUS WORKS

In boiling or bubbling pool systems, droplets are entrained by the mechanisms of bubble bursting, splashing and foaming near the top of a pool. Some part of these entrained droplets fall back to the surface of a pool and the other part is carried away by streaming gas. The entrainment E_{fg} which is the ratio of the droplet upward mass flux $\rho_f j_{fe}$ to the gas mass flux $\rho_g j_g$, has been measured experimentally by some researchers [34-49]. Here j_{fe} is the superficial velocity of liquid flowing as droplets, and j_g is the superficial velocity of gas. Thus the entrainment E_{fg} is defined by

$$E_{fg} \equiv \frac{\rho_f j_{fe}}{\rho_g j_g} \quad (1)$$

The experimental data show that the entrainment E_{fg} is a strong function of the gas superficial velocity j_g and the distance from the surface of a pool h . When the entrainment is plotted against the gas superficial velocity at a fixed distance h , at least three entrainment regimes can be observed [33,38,49]. In a low gas flux regime, the entrainment is small and entrained liquid consists of very fine droplets. In this regime, E_{fg} is approximately proportional to the gas flux. In an intermediate gas flux regime, larger drops are ejected from a pool and E_{fg} increases with j_g^{3-4} . At a higher gas flux, large gas slugs are formed and a pool is highly agitated. Then a considerable amount of liquid can be entrained by splashing. In this high gas flux regime, E_{fg} increases very rapidly with the gas flux, i.e., $E_{fg} \propto j_g^{7-20}$.

As for the effect of the distance from the surface of a pool, there are at least two distinct regions. In the first region (momentum controlled region), entrainment consists of larger droplets which rise due to their initial momentum at the surface and smaller droplets which are carried away by streaming gas. In this region, E_{fg} decreases rapidly with increasing distance, i.e., $E_{fg} \propto h^{-3}$. In the second region (deposition controlled region), entrainment consists only of small droplets whose terminal velocity is smaller than the gas velocity. Entrainment in this region decreases gradually due to the droplet depositions. This trend can be expressed as an exponential decay function of the height.

Although there have been no theoretical methods of predicting entrainment, several semi-empirical correlations [37,40,46] based on various data [35,38,39,41,44,49] and dimensional analyses have been proposed.

Kruzhilin [37] proposed a dimensionless correlation for the intermediate gas flux regime and momentum controlled region based on a dimensional analysis. With an assumption that the initial velocity of droplets at the interface is determined by the kinetic energy of vapor $\rho_g j_g^2/2$, he obtained

$$E_{fg} = C_K j_g^{*4} \sqrt{\frac{\Delta \rho}{\rho_g}} \sqrt{\frac{\Delta \rho}{\rho_f}} \quad (2)$$

Here j_g^* is the dimensionless gas flux defined by

$$j_g^* \equiv j_g / \left(\frac{\sigma g \Delta \rho}{\rho_g^2} \right)^{1/4}, \quad (3)$$

where σ , g and $\Delta \rho$ are the surface tension, acceleration due to the gravity, and difference between the gas and liquid densities, respectively. In Eq. (2), C_K must be determined from experimental data. However, it has been found that the value of C_K depends on the distance from the interface. Although Eq. (2) shows the effects of velocity and pressure on E_{fg} , it does not give any information on the effect of height, which is important in the momentum controlled region.

Sterman [40] correlated the experimental data of entrainment for steam water systems at pressures from 0.1 MPa to 18 MPa, and proposed a dimensionless correlation. His correlation depends on the average void fraction in the pool, therefore, he has also proposed several void fraction correlations applicable to the problem. For a reasonably large vessel satisfying the condition given by

$$D_H^* \left(\frac{\Delta \rho}{\rho_g} \right)^{0.2} > 260, \quad (4)$$

where the dimensionless diameter of vessel D_H^* is given by

$$D_H^* \equiv D_H / \sqrt{\frac{\sigma}{g \Delta \rho}}, \quad (5)$$

the Sterman correlation can be simplified to

$$E_{fg} = 6.09 \times 10^9 j_g^{*2.76} h^{*-2.3} \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.26} N_{\mu f}^{2.2} \left(\frac{\Delta \rho}{\rho_f} \right)^{1.1}. \quad (6)$$

Here the dimensionless height h^* is given by

$$h^* \equiv h / \sqrt{\frac{\sigma}{g \Delta \rho}}, \quad (7)$$

and the liquid viscosity number $N_{\mu f}$ based on liquid viscosity μ_f is given by

$$N_{\mu f} \equiv \frac{\mu_f}{(\rho_f \sigma \sqrt{\sigma/g\Delta\rho})^{1/2}} \quad (8)$$

Equation (6) is applicable to the intermediate gas flux regime and momentum controlled region. The boundary between the intermediate and high gas flux regimes occurs at

$$j_g^* = 1.08 \times 10^{-5} h^{*0.83} \left(\frac{\Delta\rho}{\rho_g} \right)^{-0.2} N_{\mu f}^{-0.92} \left(\frac{\rho_f}{\Delta\rho} \right)^{0.46} \quad (9)$$

The Stermann correlation shows a strong dependence of E_{fg} on the liquid viscosity through the liquid viscosity number $N_{\mu f}$. According to this correlation, entrainment increases with the liquid viscosity as $E \propto \mu_f^{2.2}$. However, experimental data on the effect of the liquid viscosity on entrainment [42] does not show such a strong viscosity dependence. Furthermore, Eq. (6) fails to predict experimental data of air-water systems [41].

Rozen et al. [46] proposed a correlation for the deposition controlled region, which is given by

$$E_{fg} = 3.8 \times 10^{-5} [K^{0.5} + 530 K^{2.1}] \sqrt{\frac{\Delta\rho}{\rho_g}} e^{-0.23h/D_H} \quad (10)$$

where

$$K \equiv D_c^* j_g^* \quad (11)$$

with the dimensionless critical droplet diameter defined by

$$D_c^* \equiv D_c / \sqrt{\frac{\sigma}{g\Delta\rho}} \quad (12)$$

Here D_c is the critical droplet diameter whose terminal velocity is equal to gas velocity. Therefore, for this correlation the size of droplet should be specified. By matching the settling velocity of wake regime droplets [5,50] to the gas flux, the following correlation for a drop size can be obtained

$$D_c = \frac{4j_g}{\left[\frac{(g\Delta\rho)^2}{\mu_g \rho_g} \right]^{1/3}} \quad (13)$$

Then the K parameter can be specified as

$$K \equiv 4 j_g^{*2} N_{\mu g}^{1/3} \quad (14)$$

where the gas viscosity number based on the gas viscosity μ_g is defined by

$$N_{\mu g} \equiv \frac{\mu_g}{(\rho_g \sigma \sqrt{\sigma/g\Delta\rho})^{1/2}} \quad (15)$$

In view of Eq. (14), the Rozen correlation can be modified to

$$E_{fg} = 7.6 \times 10^{-5} \{ j_g^* N_{\mu g}^{1/6} + 4870 j_g^{*4.2} N_{\mu g}^{0.7} \} \sqrt{\frac{\Delta\rho}{\rho_g}} e^{-0.23h/D_H} \quad (16)$$

The correlation given by Eq. (16) shows basically two regimes depending on the value of j_g^* . For small j_g^* , Eq. (16) can be approximated as

$$E_{fg} = 7.6 \times 10^{-5} j_g^* N_{\mu g}^{1/6} \sqrt{\frac{\Delta\rho}{\rho_g}} e^{-0.23h/D_H} \quad (17)$$

On the other hand, for large j_g^* Eq. (16) can be simplified to

$$E_{fg} = 0.37 j_g^{*4.2} N_{\mu g}^{0.7} \sqrt{\frac{\Delta\rho}{\rho_g}} e^{-0.23h/D_H} \quad (18)$$

The transition occurs at

$$j_g^* = 0.071 N_{\mu g}^{1/6} \quad (19)$$

As shown above, these semi-empirical correlations based on dimensional analyses are applicable to limited ranges of operational parameters. Furthermore, some parametric dependencies predicted by these correlations may not be correct if they are applied beyond the ranges of the data base. In view of the importance of entrainment from a liquid pool in various engineering problems as well as safety analyses of nuclear reactors, an accurate correlation applicable over wide ranges of operational parameters is highly desirable. For this general purpose, the correlation should be based on realistic modeling of droplet behaviors in the vapor space and at the liquid vapor interface.

Based on these observations, a correlation for pool entrainment is developed from mechanistic modeling in this study. It takes into account the droplet diameter distribution, initial velocity of droplets and droplet motion. Thus the present model reflects more realistic mechanisms of pool entrainment than those proposed previously.

III. BASIC EQUATION

When boiling or bubbling occurs in a liquid pool, droplets are ejected from a pool surface by bursting of bubbles, splashing or foaming. These droplets have varying diameters and ejection velocities. Each droplet goes through its own trajectory depending on its mass, initial velocity, and drag force exerted by streaming gas. The entrained droplet flux is determined by the collective behavior of each droplet, which can be analyzed by solving an equation of motion of each droplet using initial condition at the interface. However, there are an enormous number of droplets ejected from the surface of a liquid pool, thus it is impractical to treat the movement of each droplet separately. Therefore, a statistical treatment has been adopted here.

In order to treat the entrainment problem statistically, one needs to introduce important physical parameters and distribution functions at the interface such as the entrainment rate at the interface, droplet size distribution function, droplet initial velocity distribution, and necessary initial velocity of a droplet to rise more than height, h . The entrainment rate at the interface, $\dot{e}(j_g)$, is the mass flux of droplets at the interface and considered to be a function of the gas velocity.

Droplets ejected from the interface have various diameters represented by droplet size distribution function $f(D, j_g)$, which is the fraction of droplets whose diameter lies between D and $D + dD$. This function is considered to depend on the gas velocity. Furthermore, $f(D, j_g)$ satisfies the following relation.

$$\int_0^{\infty} f(D, j_g) dD = 1 \quad . \quad (20)$$

The initial velocity v_i which is the velocity of the droplet just ejected from the interface has its own distribution function, $g(v_i, D, j_g)$. This function represents the fraction of droplets whose velocity lies between v_i and $v_i + dv_i$ at the interface. It also satisfies

$$\int_0^{\infty} g(v_i, D, j_g) dv_i = 1 \quad . \quad (21)$$

In view of the mechanisms of the droplet ejection, this function is considered to depend on the droplet diameter and gas velocity.

The velocity of a droplet necessary to rise more than h , $v_h(D, j_g, h)$ can be obtained by solving the equation of motion of a single droplet with an appropriate drag coefficient. Thus it should be a function of the droplet diameter, gas velocity and height from the interface.

Using the above mentioned statistical parameters, the entrainment at distance h from the interface can be given by the following integral

$$E_{fg}(h, j_g) = \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_0^{\infty} \int_{v_h(D, j_g, h)}^{\infty} g(v_i, D, j_g) f(D, j_g) dv_i dD \quad . \quad (22)$$

The entrainment given by Eq. (22) consists of two groups of droplets. The first group of droplets are the ones whose diameters are larger than the critical droplet diameter D_c . Here D_c is the diameter of a droplet whose

terminal velocity is equal to the gas velocity. Therefore, this group of droplets arrive at the height by the initial momentum gained at the ejection. The second group of droplets have their diameters less than the critical droplet diameter D_c . This group of droplets can be carried away by the streaming gas to any height unless they are deposited to the wall.

For large h , entrainment should consist only of those droplets having a diameter less than D_c . Then, for this case, one gets the limiting value of E_{fg} from Eq. (22) which is given by

$$\lim_{h \rightarrow \infty} E_{fg}(h, j_g) \equiv E_{fg\infty}(j_g) = \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_0^{D_c} f(D, j_g) dD \quad . \quad (23)$$

Here $E_{fg\infty}$ gives the entrainment amount far from the pool surface with the negligible deposition effect. This value may be physically reached in a large diameter vessel. For a small diameter vessel some modification must be incorporated into the expression of $E_{fg\infty}$. This will be discussed later.

Using Eq. (23), one can rewrite Eq. (22) as

$$E_{fg}(h, j_g) = \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_{D_c}^{\infty} \int_{v_h}^{\infty} g(D, j_g, v_i) f(D, j_g) dv_i dD + E_{fg\infty}(j_g) \quad . (24)$$

The integral term of the right hand side of Eq. (24) represents the entrainment due to the first group of droplets with a diameter larger than D_c . The second term represents the droplets with a diameter smaller than D_c . Again, this equation is applicable to the system where the deposition is negligible. This formulation gives a more clear image of pool entrainment and is convenient in developing an entrainment correlation which is presented in the following sections.

IV. DROPLET DIAMETER DISTRIBUTION AND ENTRAINMENT RATE AT INTERFACE

The droplet diameter distribution function $f(D, j_g)$ and entrainment rate at interface $\dot{\epsilon}(j_g)$ are key factors to determine entrainment at any

height from the pool surface. However, these two parameters are difficult quantities to analyze or measure directly. Thus one needs to consider through a more directly observable parameter. For this purpose the interface entrainment $E_0(D, j_g)$ is introduced. $E_0(D, j_g)$ represents the entrainment consisting of droplets whose diameters are less than the stated value D . Then it can be expressed as

$$E_0(D, j_g) = \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_0^D f(D, j_g) dD \quad . \quad (25)$$

$E_0(D, j_g)$ is an observable parameter. For example, comparing Eq. (23) and Eq. (25) one immediately obtains that

$$E_0(D_c, j_g) = E_{fg\infty}(j_g) \quad . \quad (26)$$

Furthermore, the entrainment due to droplets with a diameter less than D denoted by $E_{fg}(h, j_g, D)$ can be related to this parameter $E_0(D, j_g)$ as shown below

$$E_{fg}(h, j_g, D) = \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_0^D \int_{v_h(D, j_g, h)}^{\infty} g(v_i, D, j_g) f(D, j_g) dv_i dD \quad . \quad (27)$$

By considering the droplets with a diameter less than D_c and using the definition of $v_h(D, j_g, h)$, one gets

$$v_h(D, j_g, h) = 0 \quad (D < D_c) \quad . \quad (28)$$

From Eqs. (21), (27) and (28), $E_{fg}(h, j_g, D)$ can be simplified to the following expression for $D < D_c$;

$$E_{fg}(h, j_g, D) = \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_0^D f(D, j_g) dD \quad . \quad (29)$$

This implies that for $D < D_c$

$$E_o(D, j_g) = E_{fg}(h, j_g, D) \quad (30)$$

It is noted that Garner et al. [35] have measured $E_{fg}(h, j_g, D)$.

Now, instead of using a completely empirical correlation, a simple model for $E_o(D, j_g)$ is developed by considering the mechanisms of entrainment. By introducing the droplet site density N , mean frequency f_D , escape probability p and drag force F_D , the parameter $E_o(D, j_g)$ may be related to these by

$$E_o(D, j_g) \sim N f_D p F_D \quad (31)$$

The drag force F_D [50] is given by

$$F_D = -\frac{1}{2} C_D \rho_g v_r |v_r| A_d \quad (32)$$

where C_D , v_r , and A_d are the drag coefficient, relative velocity, and projected area of a droplet, respectively. For a wake regime ($Re_D = 5 \sim 1000$), the drag coefficient C_D [5,50] is approximately given by

$$C_D = \frac{10.67}{Re_D^{0.5}} \quad (33)$$

Here Re_D is the droplet Reynolds number for a dilute suspension defined by

$$Re_D \equiv \frac{\rho_g v_r D}{\mu_g} \quad (34)$$

For a simple model, the following approximations may be used.

$$\begin{aligned} v_r |v_r| &\sim j_g^2 \\ A_d &\sim D^2 \end{aligned} \quad (35)$$

Substituting Eqs. (32) through (35) into Eq. (31) one obtains

$$E_o(D, j_g) \sim (j_g D)^{1.5} \quad (36)$$

In view of Eqs. (30) and (36), the experimental data of Garner et al. [35] for $E_{fg}(h, j_g, D)$ are plotted against $D^* j_g^*$ in Fig. 1. Here j_g^* is the nondimensional gas flux defined by Eq. (3) and D^* is the nondimensional drop diameter given by

$$D^* = D / \sqrt{\frac{\sigma}{g \Delta \rho}} \quad (37)$$

As predicted by Eq. (36), experimental data for $D < D_c$ can be well correlated by

$$E_{fg}(h, j_g, D) = 0.3975 (D^* j_g^*)^{1.5} \quad (38)$$

From Eqs. (30), (25) and (38) one obtains for $D < D_c$ the following expression;

$$\frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_0^D f(D, j_g) dD = 0.3975 (D^* j_g^*)^{1.5} \quad (39)$$

By differentiating Eq. (39) with respect to D ,

$$\sqrt{\frac{\sigma}{g \Delta \rho}} \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} f(D, j_g) = 0.5963 j_g^{*1.5} D^{*0.5} \quad (40)$$

This is an important correlation relating the entrainment rate and droplet diameter distribution function to the gas flux and droplet diameter. However, it should be noted that the correlation given by Eq. (38) is based on a data set taken at one pressure. As Eq. (2) indicates, there should be an additional pressure effect through the density ratio $\Delta \rho / \rho_g$ in Eqs. (38) and (40). This becomes clear in the latter analyses and some modifications on Eqs. (38) and (40) are made subsequently (see Section VIII).

Fig. 1. Entrainment below Stated Value $E_{fg}(h, j_g, D)$ vs. Dimensionless Diameter $D^* j_g^*$ for the Data of Garner et al. [35]

V. DROPLET VELOCITY AT INTERFACE

When the gas flux j_g is small, the flow regime in a pool is a bubbly flow. In this regime, discrete bubbles rise up to the surface of the pool and collapse there. This mechanism of the bubble burst and subsequent entrainment have been studied previously and an expression for the velocity of entrained droplets has been developed empirically or theoretically [51-57]. According to Newitt et al. [54], the initial velocity of entrained droplet due to bubble burst is given by

$$v_i = \frac{3}{2} \frac{t_B}{D_B \rho_f} \left(\frac{4\sigma}{D_B} + P_0 \right), \quad (41)$$

where t_B , D_B , and P_0 are bubble burst time, bubble diameter, and pressure around the bubble.

However, for the pool entrainment, the bubbly flow regime is limited to a very small gas velocity. For example, in an air-water system at the atmospheric pressure, the flow regime transition from bubbly to churn turbulent flow occurs at j_g in the order of 10 cm/sec [58]. Applying the drift flux model [59] to a bubbling system and using the transition criterion from bubble to churn turbulent flow regime [60] given by $\alpha \approx 0.3$, the transition gas flux becomes

$$j_g^* = 0.325 \left(\frac{\rho_g}{\rho_f} \right)^{1/2}. \quad (42)$$

These indicate that the churn turbulent flow may be the most dominant flow regime in a bubbling pool. In case of the churn turbulent flow, the initial velocity of entrained droplets is not determined by the bubble burst mechanism, but by a momentum exchange mechanism suggested by Nielsen et al. [61].

This momentum exchange mechanism is shown schematically in Fig. 2. The equation of motion for an element of the liquid ligament is given by,

Fig. 2. Schematic Diagram of Droplet Entrainment

$$\rho_f D \frac{dv_f}{dt} = \tau_i - \Delta \rho g D \quad , \quad (43)$$

where v_f is the velocity of an element of a liquid ligament at z . And the interfacial shear stress τ_i is given in terms of interfacial friction factor f_i as

$$\tau_i = f_i \frac{1}{2} \rho_g v_g^2 \quad . \quad (44)$$

When the gravity term is negligible compared with the interfacial term, Eq. (43) can be rewritten as

$$\frac{dv_f}{\frac{1}{2} \frac{\rho_g}{\rho_f} \frac{1}{D} f_i v_g^2} = dt = \frac{dz}{v_f} \quad . \quad (45)$$

At $z = \ell$, this velocity is equal to the initial velocity of the droplet, that is

$$v_f(z=\ell) = v_i \quad . \quad (46)$$

Integrating Eq. (45) from $z = 0$ to $z = \ell$, one can obtain

$$\frac{1}{2} \rho_f v_i^2 D = \frac{1}{2} f_i \rho_g v_g^2 \ell \quad . \quad (47)$$

Equation (47) implies that the kinetic energy of the droplet entrained is equal to the work exerted on the element of a liquid ligament by gas flow.

The ligament of liquid at the pool interface, as shown in Fig. 2, can be regarded as a sequence of several droplets which are about to be entrained. Therefore, the interfacial shear stress may be related to the drag coefficient for a droplet in the wake regime, i.e., Eq. (33), thus

$$f_i \sim C_D \sim Re_D^{-0.5} \quad . \quad (48)$$

The length of the liquid ligament is assumed to be proportional to the width of the ligament which is on the order of the droplet diameter in analogy with the Rayleigh instability of a liquid jet. Then,

$$\lambda \sim D \quad . \quad (49)$$

Substituting Eqs. (48) and (49) into Eq. (47), one obtains

$$v_i^* \sim v_g^{*3/4} N_{\mu g}^{1/4} D^{*-1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} \quad , \quad (50)$$

where

$$v_i^* \equiv v_i \left/ \left(\frac{\sigma g \Delta \rho}{2 \rho_g} \right)^{1/4} \right. \quad (51)$$

and

$$v_g^* \equiv v_g \left/ \left(\frac{\sigma g \Delta \rho}{2 \rho_g} \right)^{1/4} \right. \quad . \quad (52)$$

The gas velocity v_g is related to the superficial gas velocity j_g in terms of void fraction α in the liquid pool as

$$v_g = \frac{j_g}{\alpha} \quad . \quad (53)$$

Then Eq. (50) can be rewritten as

$$v_i^* \sim j_g^{*3/4} \alpha^{-3/4} N_{\mu g}^{1/4} D^{*-1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} \quad . \quad (54)$$

Equation (54) is derived from the consideration of the momentum exchange mechanism at the interface. However, there are some experimental data for v_i , which support the dependence of v_i on j_g and D as given by Eq. (54).

Akselrod et al. [62] and Cheng et al. [28] measured indirectly (calculated from the maximum height of a droplet) the droplet initial velocity in an air-water system at the atmospheric pressure. In their experiment, the

liquid level is very low, i.e., 5-6 mm for Akselrod et al. [62] and 38 mm for Cheng et al. [28]. Under these conditions, a steady gas jet should form at the pool interface as observed by Muller et al. [63]. For this case α is almost independent of j_g . Hence

$$v_i^* \sim j_g^{*3/4} N_{\mu g}^{1/4} D^{*-1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} \quad (55)$$

In Fig. 3, the experimental data of Akselrod et al. [62] and Cheng et al. [28] are plotted in $v_i^* / \left\{ j_g^{*3/4} N_{\mu g}^{1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} \right\}$ vs. D^* plane. It can be seen that the experimental data for the initial velocity of droplets are well correlated by

$$v_i^* = 205 j_g^{*3/4} N_{\mu g}^{1/4} D^{*-1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} \quad (56)$$

For a system with a much higher liquid level, which is of considerable practical importance, the void fraction is a function of the gas superficial velocity j_g . For this case a correlation between α and j_g is necessary. The void fraction in a liquid pool generally shows lower values than those predicted by the one-dimensional drift-flux model. This is mainly due to the recirculation of liquid in the pool. Some empirical correlations for the void fraction are available [64-67] for this kind of flow.

Margulova [64] proposed the following dimensional correlation for a steam water system;

$$\alpha = (0.576 + 0.00414 P) j_g^{0.75} \quad (57)$$

where P is pressure in atm and j_g in m/s. Kurbatov [65] presented a dimensionless correlation given by

Fig. 3. Initial Velocity of Droplet in $v_i^* / \left\{ j_g^{*3/4} N_{\mu g}^{*1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} \right\}$ vs. D^* Plane for the Data of Akselrod et al. [62] and Cheng et al. [28]

$$\alpha = 0.67 j_g^{*2/3} \left(\frac{\rho_g}{\rho_f} \right)^{-1/3} \left(\frac{v_g}{v_f} \right)^{-2/9} D_H^{*-1/6} , \quad (58)$$

where D_H^* is the dimensionless hydraulic diameter of vessel defined by $D_H^* = D_H / \sqrt{g \Delta \rho}$. The correlation of Sterman [66] is given by

$$\alpha = 1.07 j_g^{*0.8} D_H^{*-0.25} \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.23} . \quad (59)$$

Wilson, et al. [67] proposed a dimensionless correlation similar to Sterman's;

$$\alpha = 0.68 \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.14} D_H^{*-0.1} j_g^{*0.62} . \quad (60)$$

These correlations indicate that the void fraction depends on the dimensionless velocity, hydraulic diameter and density ratio as

$$\alpha \sim j_g^{*n_1} D_H^{*-n_2} \left(\frac{\rho_g}{\Delta \rho} \right)^{-n_3} , \quad (61)$$

where n_1 , n_2 , and n_3 have the ranges of

$$n_1 = 0.62 \sim 0.8$$

$$n_2 = 0.1 \sim 0.25 . \quad (62)$$

$$n_3 = 0.14 \sim 0.23$$

Yeh and Zuber [33] recommend the value of $n_1 = 2/3$ which gives satisfactory agreements with a number of experimental data used in the above correlations. Then Eq. (61) becomes

$$\alpha \sim j_g^{*2/3} D_H^{*-n_2} \left(\frac{\rho_g}{\Delta \rho} \right)^{-n_3} . \quad (63)$$

Substituting Eq. (63) into Eq. (54), one obtains

$$v_i^* = C j_g^{*1/4} N_{\mu g}^{1/4} D^{*-1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} D_H^{*3n_2/4} \left(\frac{\rho_g}{\Delta \rho} \right)^{3n_3/4} \quad (64)$$

Here C is a proportionality constant which should be determined in collaboration with experimental data.

It is noted here that actually the initial velocity of droplet at the pool interface should have its distribution as discussed in Section III. However, apparently no data are available due to experimental difficulties. Therefore, as a first approximation, only the mean value of the initial velocities expressed by the above correlation is used in this analysis. This may also be justified for the simplicity of the model which can be checked by experimental data. Then droplet velocity distribution function, $g(v_i, D, j_g)$ is given by the delta function in the following form

$$g(v_i, D, j_g) dv_i = \delta \left(v_i^* - C j_g^{*1/4} N_{\mu g}^{1/4} D^{*-1/4} \times \left[\frac{\rho_g}{\rho_f} \right]^{1/2} D_H^{*3n_2/4} \left[\frac{\rho_g}{\Delta \rho} \right]^{3n_3/4} \right) dv_i^* \quad (65)$$

where

$$\left. \begin{aligned} \int_{-\infty}^{\infty} \delta(x) dx &= 1 \\ \delta(x) &= 0 \text{ for } x \neq 0 \end{aligned} \right\} \quad (66)$$

In others words, the initial velocity distribution over the same size droplets is neglected.

VI. MAXIMUM HEIGHT OF RISING DROPLET

The maximum height which can be attained by a rising droplet in gas flowing vertically upward can be calculated by solving the equation of motion of the droplet by specifying the drag coefficient [68,69]. In this study, an analytical solution for a practical range of the droplet Reynolds number ($Re_D = 5 \sim 1000$) has been obtained. From the maximum height of a rising droplet, the droplet velocity $v_h(D, j_g, h)$ necessary to rise more than height h can be calculated as an inverse function.

For the situation illustrated in Fig. 4, the equation of motion of a droplet in gas stream is formulated by

$$\frac{dv}{dt} = - \frac{\Delta \rho}{\rho_f} g - \frac{3}{4} C_D \frac{1}{D} \frac{\rho_g}{\rho_f} (v - v_g) |v - v_g| \quad (67)$$

and

$$\frac{dy}{dt} = v \quad (68)$$

Here v , t , and y are the droplet velocity, time, and height from the pool surface. In the above equations, it may be assumed that the droplet concentration is relatively small such that $v_g \approx j_g$. This implies that the relative velocity can be given by

$$v_r = v - v_g \approx v - j_g \quad (69)$$

Initial conditions are given by

$$\left. \begin{array}{l} v = v_i \\ y = 0 \end{array} \right\} \text{ at } t = 0 \quad (70)$$

Equations (67) and (68) can be rewritten in terms of the relative velocity and drag coefficient for wake regime given by Eq. (33) as

$$\frac{dv_r}{dt} = - \frac{\Delta \rho}{\rho_f} g - \frac{3}{4} \cdot \frac{10.67}{D} \left(\frac{\mu_g}{\rho_g D} \right)^{0.5} \frac{\rho_g}{\rho_f} v_r |v_r|^{0.5} \quad (71)$$

Fig. 4. Schematic Diagram of Droplet Motion in Gas Stream

and

$$y = \int_0^t (v_r + j_g) dt \quad . \quad (72)$$

Equation (71) can be analytically solved and the maximum height of a rising droplet h_m can be obtained by integrating Eq. (72). Thus for $v_i > j_g$

$$\begin{aligned} h_m^+ = & 2\sqrt{v_{ri}^+} + 2\sqrt{j_g^+} - \frac{1}{3}(1+j_g^+) \ln \frac{(1+\sqrt{v_{ri}^+})^2}{(1-\sqrt{v_{ri}^+} + v_{ri}^+)} \\ & + \frac{1}{3}(1-j_g^+) \ln \frac{(1-\sqrt{j_g^+})^2}{(1+\sqrt{j_g^+} + j_g^+)} - \frac{2}{\sqrt{3}}(1-j_g^+) \tan^{-1} \left(\frac{2\sqrt{v_{ri}^+}-1}{\sqrt{3}} \right) \\ & - \frac{2}{\sqrt{3}}(1+j_g^+) \tan^{-1} \left(\frac{2\sqrt{j_g^+}+1}{\sqrt{3}} \right) + \frac{4}{\sqrt{3}} j_g^+ \tan^{-1} \frac{1}{\sqrt{3}} \quad . \end{aligned} \quad (73)$$

For $v_i < j_g$

$$\begin{aligned} h_m^+ = & 2(\sqrt{j_g^+} - \sqrt{-v_{ri}^+}) + \frac{2}{3}(1-j_g^+) \ln \frac{(1-\sqrt{j_g^+})}{(1-\sqrt{-v_{ri}^+})} \\ & - \frac{1}{3}(1-j_g^+) \ln \frac{(1+\sqrt{j_g^+} + j_g^+)}{(1+\sqrt{-v_{ri}^+} - v_{ri}^+)} \\ & - \frac{2}{\sqrt{3}}(1+j_g^+) \left\{ \tan^{-1} \left(\frac{2\sqrt{j_g^+}+1}{\sqrt{3}} \right) - \tan^{-1} \left(\frac{2\sqrt{-v_{ri}^+}+1}{\sqrt{3}} \right) \right\} \quad . \end{aligned} \quad (74)$$

where

$$h_m^+ = \frac{h_m g}{v_{r\infty}^2} \frac{\Delta \rho}{\rho_f} \quad (75)$$

$$j_g^+ = \frac{j_g}{v_{r\infty}} \quad (76)$$

$$v_{ri}^+ = \frac{v_i - j_g}{v_{r\infty}} \quad (77)$$

Here $v_{r\infty}$ is the terminal velocity of a single droplet. For the wake regime it is given by [5,50]

$$v_{r\infty} = \frac{1}{4} D \left[\frac{(g \Delta \rho)^2}{\mu_g \rho_g} \right]^{1/3} \quad (78)$$

In Table I, the maximum height of a droplet for the Stokes regime and Newton's regime is also tabulated.

In Fig. 5, the maximum height calculated from Eq. (73) for $v_i/j_g = 1$, $v_i/j_g = 2$, and $v_i/j_g = 10$ is plotted against the diameter ratio D/D_c where D_c is the critical diameter having the terminal velocity equal to j_g .

As mentioned above, the initial velocity of a droplet necessary to rise more than height h is obtained as an inverse function of Eqs. (73) or (74). It is a complicated function of D , j_g , and h , therefore, the analytical solution is not presented here. However, calculations based on Eqs. (73) and (74) indicate that the effect of j_g is not so strong for the practical range of j_g . From this observation, therefore, $v_h(D, j_g, h)$ may be approximated by the following simple expression for the range of j_g corresponding to bubbly or churn turbulent flow.

$$\begin{aligned} v_h &= 0 & (D < D_c) \\ v_h &= \sqrt{2gh \frac{\Delta \rho}{\rho_f}} & (D > D_c) \end{aligned} \quad (79)$$

Fig. 5. Maximum Height of Rising Droplet Calculated by Eq. (73) for Wake Regime

Equation (79) can be rewritten in a dimensionless form as

$$\begin{aligned} v_h^* &= 0 & (D^* < D_C^*) \\ v_h^* &= \sqrt{2h^*} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} & (D^* > D_C^*) \end{aligned} \quad (80)$$

where

$$v_h^* \equiv v_h / \left(\frac{\sigma g \Delta \rho}{2 \rho_g} \right)^{1/4} \quad (81)$$

For the wake regime D_C^* is given by

$$D_C^* = 4 j_g^* N_{\mu g}^{1/3} \quad (82)$$

VII. CORRELATION FOR ENTRAINMENT AMOUNT

In previous sections, the entrainment rate and droplet size distribution at the pool interface, initial droplet velocity, and velocity necessary to rise more than height h , are obtained. Now the amount of entrainment can be calculated using the basic expression for entrainment and these results. Hence by substituting Eqs. (40), (65), and (80) into Eq. (22) one obtains

$$\begin{aligned} E_{fg}(h, j_g) &= \int_0^\infty \int_{v_h^*}^\infty \delta \left(v_i^* - C j_g^{1/4} N_{\mu g}^{1/4} D^{*-1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} D_H^{*3n_2/4} \left(\frac{\rho_g}{\Delta \rho} \right)^{3n_3/4} \right) \\ &\quad \times 0.5963 j_g^{*1.5} D^{*0.5} dv_i^* dD^* \quad (83) \end{aligned}$$

Using Eq. (24), the above equation can be rewritten as

$$\begin{aligned}
E_{fg}(h, j_g) = & 0.5963 \int_0^{D_c^*} j_g^{*1.5} D^{*0.5} dD^* \\
& + 0.5963 \int_{D_c^*}^{\infty} \int_{v_h^*}^{\infty} \delta \left(v_i^* - C j_g^{*1/4} N_{\mu g}^{1/4} D^{*-1/4} \left(\frac{\rho_g}{\rho_f} \right)^{1/2} D_H^{*3n_2/4} \left(\frac{\rho_g}{\Delta \rho} \right)^{3n_3/4} \right) \\
& \times j_g^{*1.5} D^{*0.5} dv_i^* dD^* .
\end{aligned} \tag{84}$$

Droplets have finite diameters, therefore, the range of the integration over D^* will be limited by the maximum diameter of droplets $D_{\max}$. $D_{\max}$ is often correlated in terms of the maximum droplet Weber number [70,4] defined by

$$We_{\max} \equiv \frac{D_{\max} \rho_g j_g^2}{\sigma} = D_{\max}^* j_g^{*2} , \tag{85}$$

where

$$D_{\max}^* \equiv \frac{D_{\max}}{\sqrt{\frac{\sigma}{g \Delta \rho}}} . \tag{86}$$

For example, for falling droplets [70] the maximum droplet diameter is given by

$$We_{\max} = 22 . \tag{87}$$

This implies that

$$D_{\max}^* = 22 j_g^{*-2} . \tag{88}$$

On the other hand, in annular-dispersed flow, the maximum droplet diameter is given by [4]

$$We_{\max} = 0.031 Re_g^{2/3} \left(\frac{\rho_g}{\rho_f} \right)^{-1/3} \left(\frac{\mu_g}{\mu_f} \right)^{2/3}, \quad (89)$$

where Re_g is the gas Reynolds number given by

$$Re_g \equiv \frac{\rho_g j_g D_H}{\mu_g}. \quad (90)$$

Equation (89) shows that $D_{\max}^*$ can be scaled by

$$D_{\max}^* \sim j_g^{*-4/3}. \quad (91)$$

In view of the entrainment mechanism discussed in the previous sections, the maximum droplet diameter in pool entrainment is assumed to be given by a form similar to Eqs. (88) and (91). Thus

$$D_{\max}^* = C_m j_g^{*-n_4}, \quad (92)$$

where C_m and n_4 should be determined in collaboration with experimental data in the absence of a detailed hydrodynamic model of the droplet generation.

Integrating Eq. (84) with Eq. (92), one obtains the following results:

$$\text{For } h^* < \frac{C^2}{2} \frac{1}{\sqrt{C_m}} j_g^{*\frac{n_4+1}{2}} N_{\mu g}^{1/2} D_H^{*3n_2/2} \left(\frac{\rho_g}{\Delta p} \right)^{3n_3/2}$$

$$E_{fg}(h, j_g) = 0.398 C_m^{1.5} j_g^{*1.5-3n_4/2} \quad (93)$$

$$\text{For } h^* > \frac{C^2}{4} N_{\mu g}^{1/3} D_H^{*3n_2/2} \left(\frac{\rho_g}{\Delta \rho} \right)^{3n_3/2}$$

$$E_{fg}(h, j_g) = 3.18 j_g^{*3} N_{\mu g}^{1/2} \quad (94)$$

$$\text{For } \frac{C^2}{2} \frac{1}{\sqrt{C_m}} j_g^{*\frac{n_4+1}{2}} N_{\mu g}^{1/2} D_H^{*3n_2/2} \left(\frac{\rho_g}{\Delta \rho} \right)^{3n_3/2} < h^* < \frac{C^2}{4} N_{\mu g}^{1/3} D_H^{*3n_2/2} \left(\frac{\rho_g}{\Delta \rho} \right)^{3n_3/2}$$

$$E_{fg}(h, j_g) = 0.0497 C^6 j_g^{*3} h^{*-3} N_{\mu g}^{1.5} D_H^{*4.5n_2} \left(\frac{\rho_g}{\Delta \rho} \right)^{4.5n_3} \quad (95)$$

Equations (93) through (95) show that there are three regions in terms of the height from the pool interface. The first region (near surface region) is limited to small h . The entrainment and limit on height are given by Eq. (93). In this region, entrainment consists of all droplets which are entrained at the pool interface. The second region (momentum controlled region) is limited to intermediate h . In this region, entrainment consists of droplets which can attain height h due to the initial momentum of droplets. As the correlation given by Eq. (95) indicates, the entrainment amount increases with increasing gas velocity and with decreasing height in the second region. The third region (deposition controlled region) applies to large h . In this region, entrainment consists of droplets whose terminal velocity is less than gas velocity. Equation (94) indicates that E_{fg} is independent of h . In actual system, droplets can deposit on the vessel wall and thus E_{fg} should decrease gradually with increasing height.

In Fig. 6, the general trend of the entrainment E_{fg} is shown schematically. Indeed, the same trend is also observed in various experiments [43,44]. Equations (93) through (95) indicate significant dependence of the entrainment amount on the height above the pool surface and gas velocity. For practical applications the proportionality constants and effects of physical properties of liquid and gas reflected in exponents n_2 , n_3 and n_4 should be correlated in collaboration with experimental data.

VIII. ENTRAINMENT AMOUNT IN MOMENTUM CONTROLLED REGIME

Several experiments have been carried out to study entrainment amount in a bubbling or boiling pool. Most of the experimental data fall in the momentum controlled region which is most important in view of practical applications. It is also noted that in this region the entrainment is a very strong function of the height, whereas in the other two regimes the effects of the height is not very important. In view of these, the entrainment in this region will be discussed first.

Among available data, those of Kolokoltsev [49] (steam-water, 0.129 MPa), Garner et al. [35] (steam-water 0.101 MPa), Sterman et al. [39,40] (steam-water 1.72 ~ 18.7 MPa), Golub [44] (air-water 0.101 MPa) and Styrikovich et al. [41] (air-water 0.11 ~ 5.0 MPa) are used for the present correlation purpose because in these experiments the gas velocity and measurement location above the pool surface have been varied systematically. The key parameters of these experiments are summarized in Table II.

In Figs. 7 through 17, experimental data for the entrainment are plotted against a parameter j_g^*/h^* . However, Fig. 18 shows the data of Golub [44] in E_{fg}/j_g^{*3} vs. h^* plane. This is because in his experiments h^* has been varied extensively, and in this plot the dependence of E_{fg} on h^* can be easily examined. It is noted that his data also include the entrainment amount in the deposition controlled region. Figure 18 verifies the dependence of the entrainment on the height above the pool surface as predicted by Eqs. (93) through (95) and indicated schematically in Fig. 6.

Fig. 6. General Trend of Effects of Height, h , on Entrainment E_{fg}

As shown in Figs. 7 through 17, the entrainment amount increases with the third power of j_g^*/h^* , which is also predicted by the present model by Eq. (95). In collaboration with these experimental data, the proportionality constant C , and exponents n_2 and n_3 in Eq. (95) are determined. Then the final form of the correlation for the entrainment in the momentum controlled region is given by

$$E_{fg}(h, j_g) = 5.417 \times 10^6 j_g^{*3} h^{*-3} N_{\mu g}^{1.5} D_H^{*1.25} \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.31} \quad (96)$$

Figure 19 shows the comparison of various data to the above correlation in $E_{fg} / \left\{ N_{\mu g}^{1.5} D_H^{*1.25} \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.31} \right\}$ vs. j_g^*/h^* plane. As can be seen from this comparison, Eq. (96) satisfactorily correlates the wide range of experimental data for entrainment in the momentum controlled region. Further comparisons of data to the correlation are shown in Figs. 7 through 18. This correlation applies to the intermediate gas flux regime.

Figure 19 shows that at low values of j_g^*/h^* , the dependence of the entrainment on j_g^*/h^* changes. This change is caused by the flow regime transition in a liquid pool. As mentioned in Section V, when the gas velocity is small, the flow regime becomes bubbly flow. The discrete bubbles rise up to the pool interface and collapse there. This is different from the mechanisms under which Eq. (96) is derived. In this bubbly flow regime, the droplet size distribution and initial velocity distribution may be quite different from those predicted by Eqs. (40) and (65). This regime corresponds to the low gas flux regime previously mentioned. Indeed, the change from the low to intermediate gas flux regime occurs approximately at j_g^* predicted by Eq. (42) which is derived from the criteria of flow regime transition from bubbly to churn turbulent flow in a pool.

Although data are scarce and considerable scattering is observed, the entrainment in the low gas flux regime may be correlated by

Fig. 7. Comparison of Experimental Data of Kolokoltsev [49] with Predicted Entrainment at 0.129 MPa

Fig. 8. Comparison of Experimental Data of Garner et al. [35] with Predicted Entrainment at 0.101 MPa

Fig. 9. Comparison of Experimental Data of Sterman et al. [39,40] with Predicted Entrainment at 1.72 MPa

Fig. 10. Comparison of Experimental Data of Sterman et al. [39,40] with Predicted Entrainment at 3.75 MPa

Fig. 11. Comparison of Experimental Data of Sterman et al. [39,40] with Predicted Entrainment at 9.22 MPa

Fig. 12. Comparison of Experimental Data of Sterman et al. [39,40] with Predicted Entrainment at 11.1 MPa

Fig. 13. Comparison of Experimental Data of Sterman et al. [39,40] with Predicted Entrainment at 18.7 MPa

Fig. 14. Comparison of Experimental Data of Styrikovich et al. [41] with Predicted Entrainment at 0.11 MPa

Fig. 15. Comparison of Experimental Data of Styrikovich et al. [41] with Predicted Entrainment at 0.3 MPa

Fig. 16. Comparison of Experimental Data of Styrikovich et al. [41] with Predicted Entrainment at 2.5 MPa

Fig. 17. Comparison of Experimental Data of Styrikovich et al. [41] with Predicted Entrainment at 5.0 MPa

Fig. 18. Comparison of Experimental Data of Golub [44] with Predicted Entrainment at 0.101 MPa

Fig. 19. Comparison of Experimental Data of Various Researchers [35,39,40, 41,49] with Predicted Entrainment in $E_{fg} / \left\{ N_{\mu g}^{1.5} D_H^{*1.25} \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.31} \right\}$ vs. j_g^* / h^* Plane

$$E_{fg}(h, j_g) = 2.213 N_{\mu g}^{1.5} D_H^{*1.25} \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.31} j_g^* h^{*-1} \quad (97)$$

In Fig. 19, overall comparisons between the various experimental data and the above correlations are shown.

From Eqs. (96) and (97) the transition criterion between the low and intermediate gas flux regimes can be obtained, thus

$$\frac{j_g^*}{h} = 6.39 \times 10^{-4} \quad (98)$$

This criterion gives a higher value of j_g^* than that predicted by Eq. (42) over the range of h^* appeared in the experimental data. This is because in a boiling or bubbling pool system, the void fraction is generally lower than those predicted by the standard drift flux model due to the significant internal circulation of liquid in the pool. Here h^* appears in the criterion due to the fact that the information at the interface to reach h^* requires a certain gas flux.

Figure 20 shows the various experimental data for entrainment in E_{fg} vs. $(j_g^*/h^*) N_{\mu g}^{0.5} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{-0.10}$ plane. The solid line in this figure represents the correlation given by Eq. (96). This figure indicates that the correlation generally agrees well with the data, however, there is a systematic deviation at high values of j_g^*/h^* . This actually signifies the appearance of the high gas flux regime. This transition occurs approximately at

$$\left(\frac{j_g^*}{h} \right) = 5.7 \times 10^{-4} N_{\mu g}^{-0.5} D_H^{*-0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{0.10} \quad (99)$$

or at

$$E_{fg} \approx 1.0 \times 10^{-3} \quad (100)$$

Fig. 20. Comparison of Experimental Data of Various Researchers [35,39,40, 41,49] with Predicted Entrainment in E_{fg} vs. $(j_g^*/h^*) N_{\mu g}^{0.5} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho}\right)^{-0.10}$ Plane

In the regime where (j_g^*/h^*) exceeds the value given by Eq. (99), the entrainment increases with (j_g^*/h^*) very rapidly as given by

$$E_{fg} \propto (j_g^*/h^*)^{7 \sim 20} \quad (101)$$

The transition from the intermediate to high gas flux regime may be attributed to a flow regime transition in the liquid pool. The transition from churn-turbulent to annular, pseudo jet or fountain flow [2,73,74] is indicated.

Now, based on the above analysis and correlation development, some of the underlining assumptions are reexamined. First, from Eqs. (95) and (96), the various constants in the correlation are now identified as

$$C = 21.86 \quad (102)$$

$$n_2 = 0.279 \quad (103)$$

and

$$n_3 = -0.0693 \quad (104)$$

When these values are compared to the results from the void fraction correlations given by Eq. (62), interesting observations can be made. The value of n_2 from Eq. (103) almost coincides with that given in Eq. (62), whereas the value of n_3 is quite different from that given in Eq. (62). This discrepancy should come from neglecting the effect of the density ratio $(\rho_g/\Delta\rho)$ in some of the correlations used to develop Eq. (95).

A careful reexamination of Eqs. (40), (65), and (80) leading to Eq. (95), indicates that Eq. (40) should have an additional $(\rho_g/\Delta\rho)$ term which reflects the effects of the pressure. Note that Eq. (40) is obtained from a simple model and the experimental data of steam-water system at 1 atm. There are no experimental data available at elevated pressures for this correlation. Thus Eq. (40) should be modified to account for the pressure effects as

$$\sqrt{\frac{\sigma}{g\Delta\rho}} \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} f(D, j_g) = C_1 \left(\frac{\rho_g}{\Delta\rho} \right)^{n_5} j_g^{*1.5} D^{*0.5} \quad (105)$$

From Eqs. (62), (95), and (96) C_1 and n_5 can be determined. Therefore, Eq. (105) becomes

$$\sqrt{\frac{\sigma}{g\Delta\rho}} \frac{\dot{\epsilon}(j_g)}{\rho_g j_g} f(D, j_g) = 3.72 \times 10^{-4} \left(\frac{\rho_g}{\Delta\rho} \right)^{-1.0} j_g^{*1.5} D^{*0.5} \quad (106)$$

This correlation is used instead of Eq. (40) in analysis of the entrainment for the deposition controlled region and near surface region which is discussed in the following section. With these modifications, there are no inconsistencies between the present entrainment correlation and previously developed void fraction correlations for a bubbling pool.

IX. ENTRAINMENT AMOUNT IN DEPOSITION CONTROLLED REGION

As shown in Section VII, beyond a certain height from the pool surface the entrainment consists only of the droplets having the terminal velocity less than the gas velocity. This phenomenon is also observed in some experiments [43,44]. By neglecting the effect of the droplet deposition, the upper limit of the entrainment in this region can be expressed by the following equation

$$E_{fg}(h, j_g) = 1.99 \times 10^{-3} j_g^{*3} N_{\mu g}^{0.5} \left(\frac{\rho_g}{\Delta\rho} \right)^{-1.0} \quad (107)$$

This is the modified form of Eq. (94) in view of Eq. (106).

In an actual system, the entrainment amount decreases gradually with height h due to the deposition. The mass balance equation in the deposition controlled region is given by Eq. (108) assuming that there are no phase changes between liquid and gas, and no additional entrainment from the liquid film.

$$\frac{dE_{fg}}{dh} = - \frac{4}{D_H} \frac{\dot{d}}{\rho_g j_g} \quad . \quad (108)$$

Here $\dot{d}$ is the deposition rate of droplets ($K_g/m^2/s$) and related to the mass concentration of droplets in the gas $C_E (K_g/m^3)$ by

$$\dot{d} = k_d C_E \quad , \quad (109)$$

where k_d is droplet deposition coefficient (m/s) [2,5]. When the velocity of droplets is approximately equal to the gas velocity (Appendix A), the droplet concentration C_E is given in terms of E_{fg} as

$$C_E = \rho_g E_{fg} \quad . \quad (110)$$

Substituting Eqs. (109) and (110) into Eq. (108) one obtains

$$\frac{dE_{fg}}{d(h/D_H)} = -4 \left(\frac{k_d}{j_g} \right) E_{fg} \quad . \quad (111)$$

Integration of Eq. (111) leads to an exponential decay characteristic given by

$$E_{fg} \propto e^{-\beta(h/D_H)} \quad , \quad (112)$$

with

$$\beta \equiv 4 k_d / j_g \quad . \quad (113)$$

In view of Eqs. (107) and (112) experimental data of entrainment for the deposition controlled region [35,44] are plotted in

$E_{fg} / \left\{ j_g^{*3} N_{\mu g}^{0.5} \left(\frac{\rho_g}{\Delta p} \right)^{-1.0} \right\}$ vs. h/D_H plane in Fig. 21. Although the data scatter considerably due to experimental uncertainties, they can be correlated by

Fig. 21. Experimental Data of Entrainment in Deposition Controlled Region [35,44]

$$E_{fg}(h, j_g) = 7.13 \times 10^{-4} j_g^{*3} N_{\mu g}^{0.5} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} \exp(-0.205(h/D_H)) \quad (114)$$

and $k_d = 0.051 j_g$.

The deposition coefficient calculated from Eqs. (113) and (114) gives higher values of k_d than those observed in normal annular dispersed flow [75]. It is considered that droplets entrained from a bubbling or boiling pool have higher random velocity in the lateral direction when they are entrained. The deposition coefficient should increase due to this initial random momentum in the lateral direction. This situation is similar to the case where droplets are injected from a nozzle [76] into a pipe. In the latter case, the very high deposition rate near the injection nozzle is well known [2,5,76].

Comparing Eqs. (96) and (114) the height above the pool surface at which the momentum controlled region changes to deposition controlled regime is given by

$$h^* \exp(-0.068(h^*/D_H^*)) = 1.97 \times 10^3 N_{\mu g}^{0.33} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{0.23} \quad (115)$$

When the droplet deposition is small, Eq. (115) can be approximated by

$$h^* \approx 1.97 \times 10^3 N_{\mu g}^{0.33} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{0.23} \quad (116)$$

X. ENTRAINMENT AMOUNT NEAR SURFACE REGION

In the near surface region, entrainment consists of all the entrained droplets from the pool surface. It is given by Eq. (93), which should be rewritten as follows in view of Eq. (106).

$$E_{fg}(h, j_g) = 2.48 \times 10^{-4} C_m^{1.5} j_g^{*1.5-3n_4/2} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} \quad (117)$$

Measuring the entrainment amount in this region is difficult because the pool surface is highly agitated. Therefore, the discrimination of entrained droplets from the agitated pool is quite difficult. The only data available in this near surface region are those of Rozen et al. [45,46] who have obtained the data by extrapolating the measured entrainment in the momentum controlled region. The data are for an air-water system at the atmospheric pressure. Their results show no dependence of E_{fg} on j_g and are given by

$$E_{fg} \approx 4 \quad . \quad (118)$$

In view of Eqs. (117) and (118), one obtains the correlation for the near surface region as

$$E_{fg}(h, j_g) = 4.84 \times 10^{-3} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} \quad . \quad (119)$$

Equation (119) indicates that C_m and n_4 in Eq. (93) are given by

$$C_m = 7.24 \quad (120)$$

$$n_4 = 1 \quad . \quad (121)$$

From the above results, the maximum drop size correlation given by Eq. (92) can be rewritten as

$$D_{\max}^* = 7.24 j_g^{*-1} \quad . \quad (122)$$

Equation (122) implies that the maximum droplet size is fairly large, however, such large droplets only exist in the near surface region and their life time is considered to be very short.

From Eqs. (96) and (119), the transition point between the near surface and momentum controlled regions is given by

$$h^* = 1.038 \times 10^3 j_g^* N_{\mu g}^{0.5} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{0.23} \quad (123)$$

Since Eq. (119) is obtained from a limited number of experimental data, further experimental works may be needed to verify the validity of the correlation given by Eq. (119) at high pressures.

XI. SUMMARY AND CONCLUSIONS

Correlations for the droplet entrainment and carryover from a bubbling or boiling pool by streaming gas have been developed based on a simple mechanistic model of entrainment in collaboration with experimental data.

The analysis reveals that there are three regions of entrainment depending on the height above the pool surface. For each region the correlation for the entrainment amount has been developed in terms of the dimensionless gas velocity j_g^* , height above surface h^* , gas viscosity number $N_{\mu g}$, vessel diameter D_H^* , and density ratio $\left(\frac{\rho_g}{\Delta \rho} \right)$. These are defined by

$$j_g^* \equiv j_g / \left(\frac{\sigma g \Delta \rho}{\rho_g^2} \right)^{1/4}$$

$$h^* \equiv h / \sqrt{\frac{\sigma}{g \Delta \rho}}$$

$$N_{\mu g} \equiv \mu_g / (\rho_g \sigma \sqrt{\sigma / g \Delta \rho})^{1/2}$$

and

$$D_H^* \equiv D_H / \sqrt{\frac{\sigma}{g \Delta \rho}} \quad .$$

The results for the entrainment amount are summarized below:

(1) Near Surface Region

This region is limited to the vicinity of the pool surface given by

$$0 \leq h^* \leq 1.038 \times 10^3 j_g^* N_{\mu g}^{0.5} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{0.23} .$$

In this region, entrainment consists of all droplets entrained at the pool surface and given by Eq. (119) as

$$E_{fg}(h, j_g) = 4.84 \times 10^{-3} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} .$$

(2) Momentum Controlled Region

This region is limited to the intermediate height range given by

$$1.038 \times 10^3 j_g^* N_{\mu g}^{0.5} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{0.23} \leq h^* \\ \leq 1.97 \times 10^3 N_{\mu g}^{0.33} D_H^{*0.42} \left(\frac{\rho_g}{\Delta \rho} \right)^{0.23} .$$

In this regime, entrainment consists partly of the droplets which attain height h due to the initial momentum and partly of the droplets whose terminal velocity is less than the gas velocity. This region is subdivided into three regimes, depending on the gas velocity:

(2-1) For the low gas flux regime limited to

$$j_g^*/h^* < 6.39 \times 10^{-4}$$

the entrainment is given by Eq. (97) as

$$E_{fg}(h, j_g) = 2.213 N_{\mu g}^{1.5} D_H^{*1.25} \left(\frac{\rho_g}{\Delta p} \right)^{-0.31} j_g^* h^{*-1} .$$

(2-2) For the intermediate gas flux regime bounded by

$$6.39 \times 10^{-4} < j_g^*/h^* \lesssim 5.7 \times 10^{-4} N_{\mu g}^{-0.5} D_H^{*-0.42} \left(\frac{\rho_g}{\Delta p} \right)^{0.10}$$

the entrainment is given by Eq. (96) as

$$E_{fg}(h, j_g) = 5.417 \times 10^6 j_g^{*3} h^{*-3} N_{\mu g}^{1.5} D_H^{*1.25} \left(\frac{\rho_g}{\Delta p} \right)^{-0.31} .$$

(2-3) For the high gas flux regime limited to

$$j_g^*/h^* \gtrsim 5.7 \times 10^{-4} N_{\mu g}^{-0.5} D_H^{*-0.42} \left(\frac{\rho_g}{\Delta p} \right)^{0.10}$$

the entrainment amount increases very rapidly as

$$E_{fg} \propto (j_g^*/h^*)^{7 \sim 20} .$$

(3) Deposition Controlled Region

Above the height given by

$$h^* \gtrsim 1.97 \times 10^3 N_{\mu g}^{0.33} D_H^{*0.42} \left(\frac{\rho_g}{\Delta p} \right)^{0.23}$$

the deposition becomes the main factor determining the amount of entrainment. In this regime, the entrainment consists of droplets whose

terminal velocity is less than the gas velocity. E_{fg} decreases gradually with h due to deposition and is given by Eq. (114) as

$$E_{fg}(h, j_g) = 7.13 \times 10^{-4} j_g^{*3} N_{\mu g}^{0.5} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} \exp(-0.205(h/D_H)) \quad .$$

(4) Entrainment rate and droplet size distribution

From the above development for entrainment, now it is possible to obtain the entrainment rate and droplet size distribution at the pool surface. By integrating Eq. (106) over D with the limit given by Eq. (122), one gets

$$\frac{\dot{\epsilon}(j_g)}{\rho_g j_g} \int_0^\infty f(D, j_g) dD = 4.84 \times 10^{-3} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} \quad . \quad (124)$$

In view of Eq. (20), the entrainment rate becomes

$$\dot{\epsilon}(j_g) = 4.84 \times 10^{-3} \rho_g j_g \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} \quad . \quad (125)$$

Thus the droplet size distribution becomes

$$f(D, j_g) = 0.077 j_g^{*1.5} D^{*0.5} \sqrt{\frac{g \Delta \rho}{\sigma}} \quad (126)$$

which applies to $D^* < D_{\max}^*$ where $D_{\max}^* = 7.24 j_g^{*-1}$.

XII. SUPPLEMENTARY REMARKS ON APPLICATION TO PRACTICAL CASES

When the correlations developed here are applied to a practical system, additional considerations may be necessary for each case. Some of the important cases are discussed below.

(1) Total Droplet Flux

In this work, the entrainment is correlated in terms of the upward droplet flux j_{fe} . This parameter itself is very important in studying the carryover droplet mass. However, it is noted that the total liquid volumetric flux j_f may be different from the upward droplet flux j_{fe} due to the droplets and liquid film which are falling back to the pool. For quasi-steady state conditions, it is straight forward to extend the present analysis to obtain the total liquid flux as discussed below.

For simplicity, the effect of the deposition is neglected. In this case, it can easily be shown from the continuity relation that the total liquid flux can be given by the droplet flux at the end of the momentum controlled region, thus from Eq. (107)

$$\frac{\rho_f j_f}{\rho_g j_g} = 2.0 \times 10^{-3} j_g^{*3} N_{\mu g}^{0.5} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} \quad (127)$$

This is because all the droplets arriving at the end of momentum controlled region are fully suspended and cannot fall back without the deposition. Then the droplet volumetric flux of the falling drops are given by

$$j_{fd} = j_f - j_{fe} \quad (128)$$

which is generally negative in the near surface or momentum controlled regions. this indicates that there are a definite number of droplets falling downward in these regions.

(2) Burnout in Steady State

In the case where vapor is produced by pool boiling, the critical heat flux (CHF) gives an additional limit for j_g^* . The pool boiling CHF criterion developed by Zuber [77] or Kutateladze [78] is given by

$$\frac{q_c}{\rho_g \Delta H_{fg}} = 0.16 \left[\frac{\sigma g \Delta \rho}{\rho_g^2} \right]^{1/4} \quad (129)$$

Here q_c is the critical heat flux and ΔH_{fg} is the latent heat of vaporization. Then the superficial gas velocity at CHF j_{gc} is given by

$$j_{gc} = \frac{q_c}{\rho_g \Delta H_{fg}} \frac{A_H}{A_V} , \quad (130)$$

where A_H is the heated area and A_V is the cross sectional area of a vessel. From Eqs. (129) and (130), one obtains

$$j_{gc}^* = 0.16 \left(\frac{A_H}{A_V} \right) , \quad (131)$$

where j_{gc}^* is defined by

$$j_{gc}^* \equiv j_{gc} / \left(\frac{\sigma g \Delta \rho}{2 \rho_g} \right)^{1/4} . \quad (132)$$

(3) Flooding

When the gas flux increases up to a certain value, the flooding can occur and all the liquid is expelled from the vessel. This forms another upper limit for j_g^* . From the standard flooding correlations [58,79,80] for pool boiling or bubbling one obtains for $D_H^* > 30$

$$j_g^* \sim 4 , \quad (133)$$

and for $3 < D_H^* < 20$

$$j_g^* \sim (0.5 \sim 1) \sqrt{D_H^*} . \quad (134)$$

It is noted that usually the value of j_g^* given by Eqs. (133) or (134) is much larger than the upper limits of j_g^* such as the boundary between the low and

intermediate gas flux regimes given by Eq. (98) or the limit due to CHF given by Eq. (131).

(4) Entrainment amount in high gas flux regime

At a high gas flux in momentum controlled region, the entrainment increases rapidly with j_g , as given by $E_{fg} \propto j_g^{7 \sim 20}$. It may be possible to make a dimensionless correlation based on experimental data for this case. However, such correlation is not very useful because it may give an unreasonably high value of entrainment at large j_g^* due to its high power dependence on j_g^* . Although no data are available, there should be an upper limit for entrainment in this regime. Here, as a first approximation, it is assumed that all the droplets produced at the pool surface are carried away. This limit is also predicted by the basic formulation given by Eq. (83) at $j_g^* \rightarrow \infty$ with the condition given by Eq. (122). Thus, the limit is obtained as

$$\begin{aligned}
 E_{fg}(h, j_g) &\sim \lim_{j_g^* \rightarrow \infty} \int_0^{8.45 j_g^{*-1}} \int_{v_h^*}^{\infty} \delta \left(v_i^* - 22 j_g^{*1/4} N_{\mu g}^{1/4} D^{*-1/4} \right. \\
 &\quad \left. \left[\frac{\rho_g}{\rho_f} \right]^{1/2} D_H^{*0.21} \left[\frac{\rho_g}{\Delta \rho} \right]^{0.17} \right) \\
 &\quad \times 3.72 \times 10^{-4} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} j_g^{*1.5} D^{*0.5} d v_i^* d D^* \\
 &= 4.84 \times 10^{-3} \left(\frac{\rho_g}{\Delta \rho} \right)^{-1.0} .
 \end{aligned} \tag{135}$$

This value is identical to that in the near surface regime which is given by Eq. (119). Therefore, for practical purposes, the entrainment amount in the high gas flux might well be estimated by the correlation for the near surface region.

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APPENDIX A

Derivation of Eq. (110)

Equation (110) can be obtained by considering mass balance of liquid and gas phase. The mass concentration of droplet C_E is given by

$$C_E = \rho_f \alpha_E \quad . \quad (A.1)$$

Here α_E is the droplet volumetric fraction in the gas space above the pool. On the other hand, in the deposition controlled region

$$j_{fe} = v_{fe} \alpha_E \quad (A.2)$$

and

$$j_g = v_g (1 - \alpha_E) \quad . \quad (A.3)$$

Here v_{fe} and v_g are the velocity of droplets and gas in gas space.

From Eqs. (A.2) and (A.3) one obtains

$$\alpha_E = \frac{j_{fe}}{j_{fe} + S j_g} \quad . \quad (A.4)$$

Here S is the velocity ratio between droplet and gas

$$S = \frac{v_{fe}}{v_g} \quad . \quad (A.5)$$

When $j_g \gg j_{fe}$ and $S = 1$, which may be applicable in the present analysis, Eq. (A.4) can be rewritten as

$$\alpha_E = \frac{j_{fe}}{j_g} \quad . \quad (A.6)$$

Substituting Eq. (A.6) into Eq. (A.1) one finally obtains

$$C_E \doteq \rho_f \frac{j_{fe}}{j_g} = \rho_g E_{fg} \quad . \quad (A.7)$$

Table I. Maximum Height of Rising Droplet for Various Regimes

Regime	Range of Reynolds Number	C_D	$V_{r\infty}$	h_m^+
Stokes	$Re_D < 5$	$\frac{24}{Re_D}$	$\frac{1}{18} \frac{D^2 \Delta \rho g}{\mu_g}$	$v_{ri}^+ + j_g^+ - (1-j_g^+) \ln \frac{(1+v_{ri}^+)}{(1-j_g^+)}$
Wake	$5 < Re_D < 1000$	$\frac{10.67}{Re_D^{0.5}}$	$\frac{D}{4} \left[\frac{(g \Delta \rho)^2}{\mu_g \rho_g} \right]^{1/3}$	For $v_i > j_g$
				$2 \sqrt{v_{ri}^+} + 2 \sqrt{j_g^+} - \frac{1}{3} (1+j_g^+) \ln \frac{(1+\sqrt{v_{ri}^+})^2}{(1-\sqrt{v_{ri}^+} + v_{ri}^+)}$
				$+ \frac{1}{3} (1-j_g^+) \ln \frac{(1-\sqrt{j_g^+})^2}{(1+\sqrt{j_g^+} + j_g^+)} - \frac{2}{\sqrt{3}} (1-j_g^+) \tan^{-1} \left(\frac{2 \sqrt{v_{ri}^+} - 1}{\sqrt{3}} \right)$
Newton's	$1000 < Re_D$	0.45	$1.72 \sqrt{gD \frac{\Delta \rho}{\rho_g}}$	$- \frac{2}{\sqrt{3}} (1+j_g^+) \tan^{-1} \left(\frac{2 \sqrt{j_g^+} + 1}{\sqrt{3}} \right) + \frac{4}{\sqrt{3}} j_g^+ \tan^{-1} \frac{1}{\sqrt{3}}$
				For $v_i < j_g$
				$2 \left(\sqrt{j_g^+} - \sqrt{-v_{ri}^+} \right) + \frac{2}{3} (1-j_g^+) \ln \frac{(1-\sqrt{j_g^+})}{(1-\sqrt{-v_{ri}^+})}$
Newton's	$1000 < Re_D$	0.45	$1.72 \sqrt{gD \frac{\Delta \rho}{\rho_g}}$	$- \frac{1}{3} (1-j_g^+) \ln \frac{(1+\sqrt{j_g^+} + j_g^+)}{(1+\sqrt{-v_{ri}^+} - v_{ri}^+)}$
				$- \frac{2}{\sqrt{3}} (1+j_g^+) \left\{ \tan^{-1} \left(\frac{2 \sqrt{j_g^+} + 1}{\sqrt{3}} \right) - \tan^{-1} \left(\frac{2 \sqrt{-v_{ri}^+} + 1}{\sqrt{3}} \right) \right\}$
				For $v_i > j_g$
Newton's	$1000 < Re_D$	0.45	$1.72 \sqrt{gD \frac{\Delta \rho}{\rho_g}}$	$\frac{1}{2} \ln (1+v_{ri}^{+2}) (1-j_g^{+2}) + j_g^+ (\tanh^{-1} v_{ri}^+ + \tanh^{-1} j_g^+)$
				For $v_i < j_g$
				$- \frac{1}{2} \ln \frac{(1-v_{ri}^{+2})}{(1-j_g^{+2})} + j_g^+ (\tanh^{-1} v_{ri}^+ + \tanh^{-1} j_g^+)$

Table II. Summary of Various Experiments on Entrainment Amount from Liquid Pool

Reference	Fluid	Vessel Diameter D_H (m)	Height above Pool Surface, h (m)	Pressure (MPa)	j_g (m/s)
Kolokoltsev [49]	Steam-Water	0.30	0.5 ~ 0.6	0.129	1.0 ~ 1.7
Garner et al. [35]	Steam-Water	0.30	0.5 ~ 1.0	0.101	0.3 ~ 1.3
Sterman et al. [39,40]	Steam-Water	0.24	0.5 ~ 0.9	1.72 ~ 18.7	0.01 ~ 1.3
Styrikovich et al. [41]	Air-Water	0.10	0.26 ~ 0.72	0.11 ~ 5.0	0.1 ~ 1.7
Golub [44]	Air-Water	0.20	0.1 ~ 2.2	0.101	0.5 ~ 2.0
Rozen [47]	Air-Water	0.20	0 ~ 0.35	0.101	0.6 ~ 3.0

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